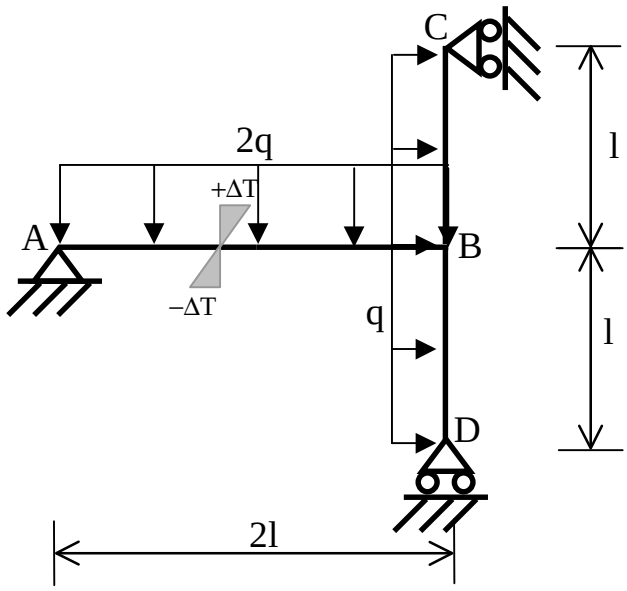
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 21/09/2010

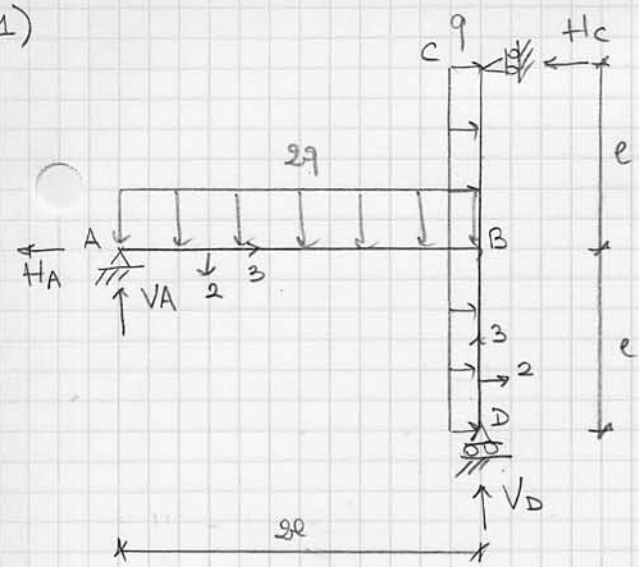


l = 2 m, q = 10 kN/m
E = 210 GPa, σ_{AMM} = 160 MPa
ΔT = 10°C, $\alpha = 10^{-5}\,°C^{-1}$

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e 2q e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Progettare la travatura.
3. Calcolare la rotazione del nodo triplo B.
4. Risolvere nuovamente la travatura considerando anche il carico termico sul tratto AB.

1)



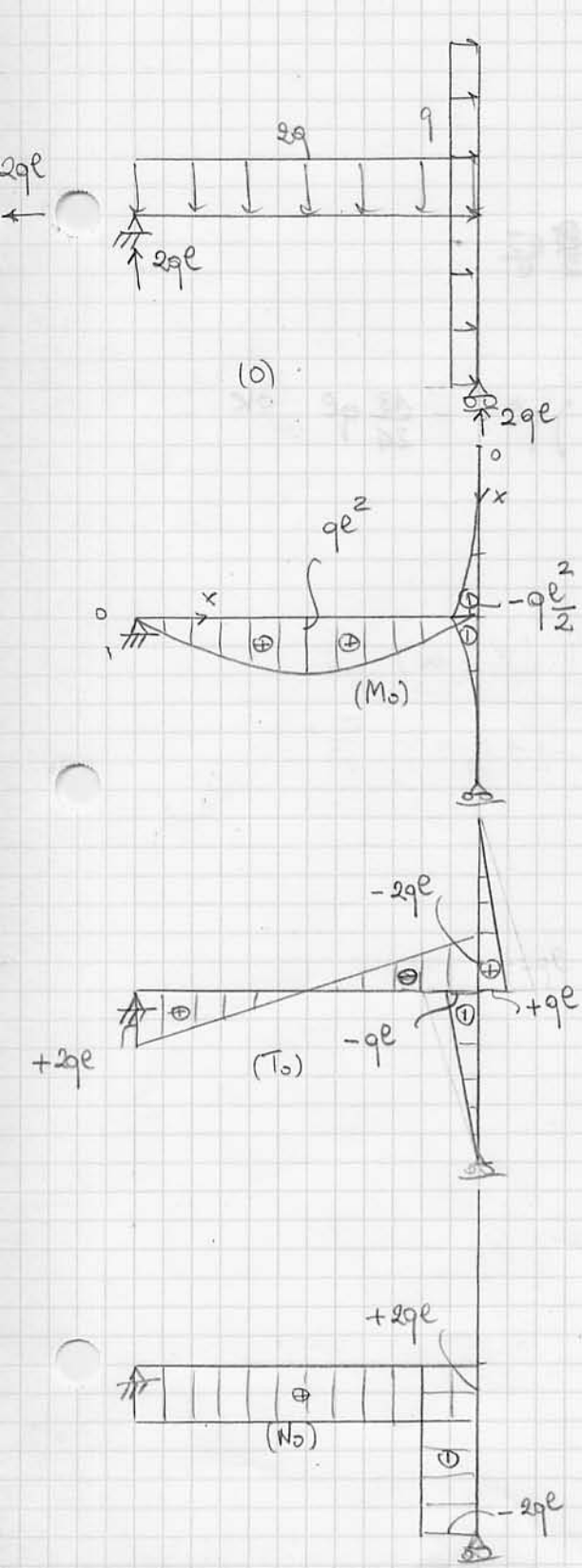
Equazioni cardinali della Statica:

$(\rightarrow)$ $H_A + H_C = 2qe$

$(\uparrow)$ $V_A + V_D = 4qe$

(A) $V_D 2e + H_C e = 4qe^2$

Trasformo una volta iperstatica. $X_1 = H_C$.



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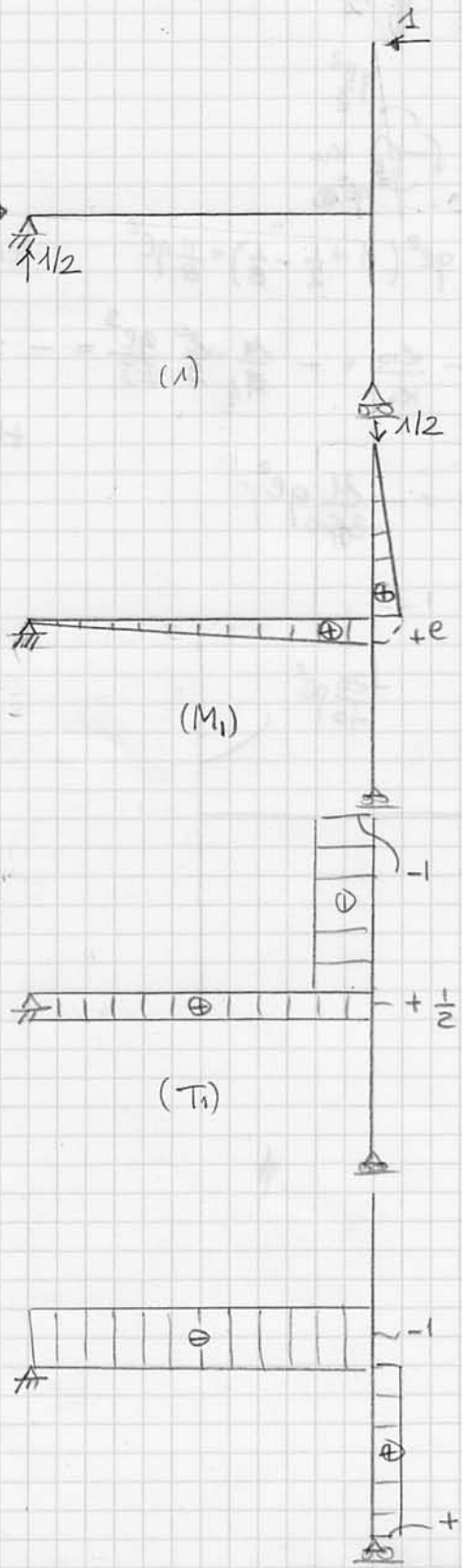


$$EI_1 \delta_{10} = \int_0^{2\ell} \left(\frac{x}{2}\right)\left(2q\ell x - 2q\frac{x^2}{2}\right) dx + \int_0^{\ell} \left(-q\frac{x^2}{2}\right) x\, dx$$

$$= \frac{1}{2}\int_0^{2\ell} \left(2q\ell x^2 - qx^3\right) dx - \frac{q}{2}\int_0^{\ell} x^3 dx$$

$$= \frac{1}{2}\left[2q\ell\frac{x^3}{3} - q\frac{x^4}{4}\right]_0^{2\ell} - \frac{q}{2}\left[\frac{x^4}{4}\right]_0^{\ell} = \frac{1}{2}\left[\frac{16}{3} - 4\right] q\ell^4 - \frac{q}{8}\ell^4 = \left(\frac{1}{2}\frac{4}{3} - \frac{1}{8}\right) q\ell^4 = \frac{16-3}{24} q\ell^4$$

$$= \frac{13}{24} q\ell^4$$

$$EI_1 \delta_{11} = \frac{1}{3} 2\ell\, \ell^2 + \frac{1}{3}\ell^3 = \ell^3$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{13}{24} q\ell = -10{,}83 \text{ kN}$$

Diagrammi delle caratteristiche di sollecitazione:

$$M_{BC} = -q\frac{\ell^2}{2} - \frac{13}{24} q\ell^2 = -\frac{25}{24} q\ell^2$$

$$M_{BA} = -\frac{13}{24} q\ell^2$$

$$T_A = 2q\ell - \frac{13}{48} q\ell = \frac{83}{48} q\ell$$

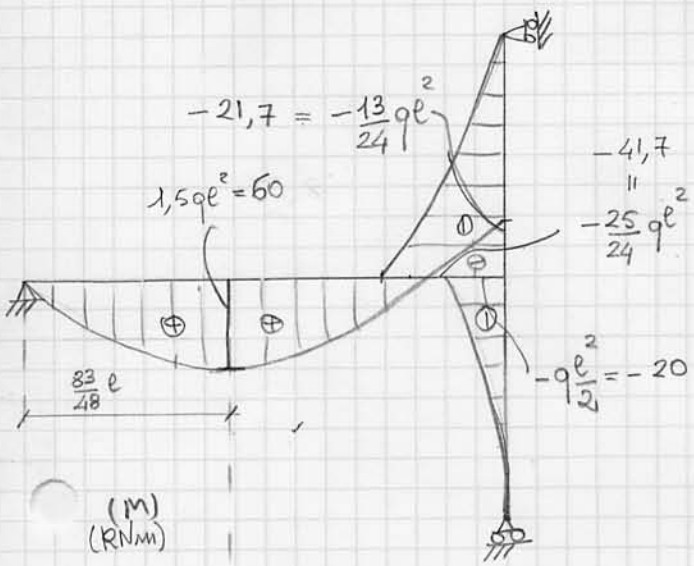
$$T_{BA} = -2q\ell - \frac{13}{48} q\ell = -\frac{109}{48} q\ell$$

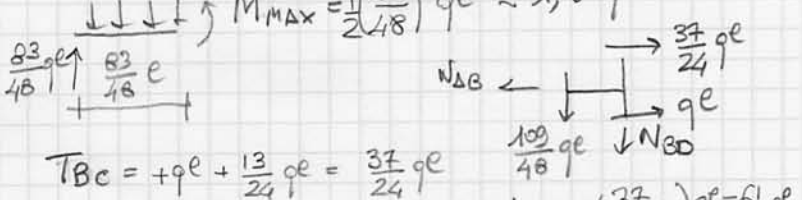
$$M_{MAX} = \frac{1}{2}\left(\frac{83}{48}\right)^2 q\ell^2 \simeq 1{,}5\, q\ell^2$$

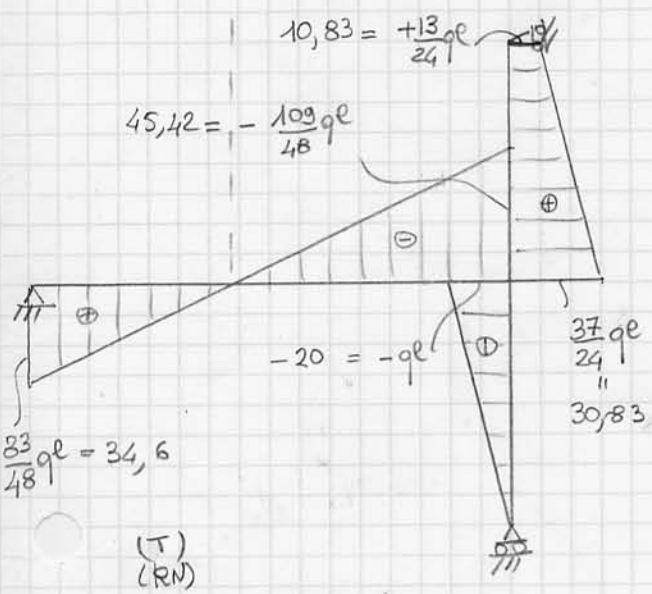
$$T_{BC} = +q\ell + \frac{13}{24} q\ell = \frac{37}{24} q\ell$$

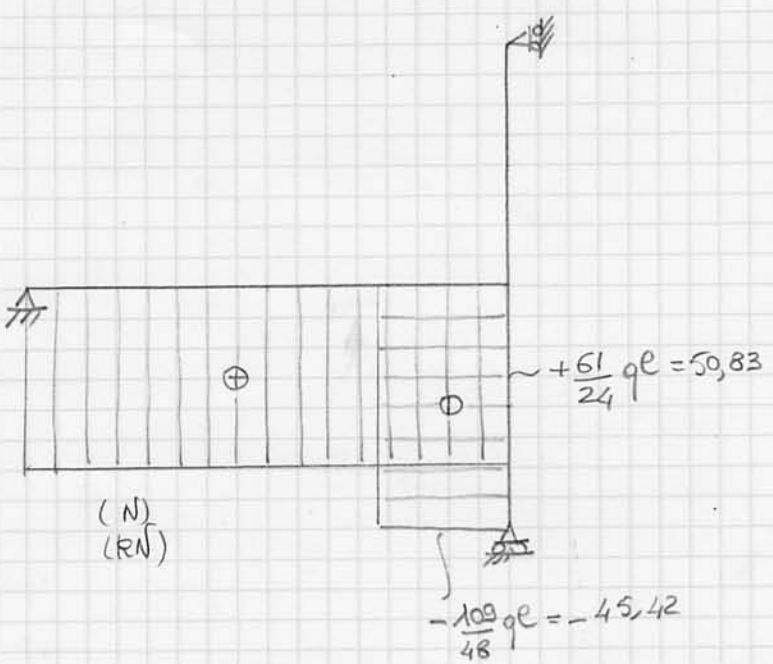
$$T_C = \frac{13}{24} q\ell$$

$$N_{AB} = \left(\frac{37}{24} + 1\right) q\ell = \frac{61}{24} q\ell$$





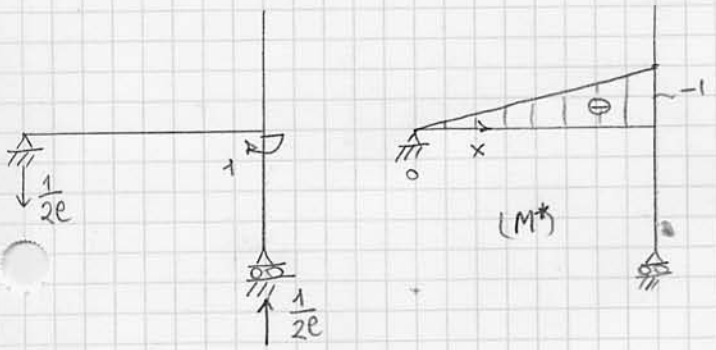




2) Progetto: $W_1 \geq \frac{M_1}{\sigma_{AMM}} = \frac{59{,}8 \cdot 10^3 \ Nm}{160 \cdot 10^6 \ N/m^2} = 3{,}73 \cdot 10^{-4} \ m^3 = 373 \ cm^3$

IPE 270
$W_1 = 429 \ cm^3$
$I_1 = 5790 \ cm^4$
$H = 27 \ cm$

3) Rotazione del nodo B.



$$\varphi_B = \frac{1}{EI_1} \int_0^{2e} \left(-\frac{x}{2e}\right)\left(\frac{83}{48} qex - qx^2\right) dx$$

$$= -\frac{1}{2EI_1 e}\left[\frac{83}{48} qe \frac{x^3}{3} - q\frac{x^4}{4}\right]_0^{2e}$$

$$= -\frac{qe^4}{2EI_1 e}\left[\frac{83}{48_6}\frac{1}{3} 8 - \frac{16^4}{4}\right]$$

$$= -\frac{qe^3}{2EI_1}\left[\frac{83}{18} - 4\right] = -\frac{11}{36}\frac{qe^3}{EI_1}$$

$$= -\frac{11 \cdot 10^2 \cdot (2 \cdot 10^2)^3}{36 \cdot 210 \cdot 10^5 \cdot 5790}$$

$$= -\left(\frac{11 \cdot 8}{36 \cdot 5790 \cdot 210}\right) 10^3$$

$$= 0{,}002 = 0{,}11^\circ$$

4) Carico termico

$$M_{10} + M_{1t} + M_{11} X_1 = 0$$

$$M_{1t} = \int_0^{2e} M_1 \chi_t \, dx = \chi_t \int_0^{2e} M_1 \, dx = \chi_t \left(\frac{1}{2} 2e \, e\right) = -\frac{2\alpha\Delta T \, e^2}{H}$$

$$X_1 = -\frac{M_{10}}{M_{11}} - \frac{M_{1t}}{M_{11}} = -\frac{13}{24} qe + \frac{2\alpha\Delta T e^2 EI_1}{H e^3} = -10{,}83 + 4{,}5 = -6{,}33 \ kN$$

$$\frac{2 \cdot 10^3 \cdot 10 \cdot 210 \cdot 10^6 \cdot 5790}{27 \cdot 200}$$