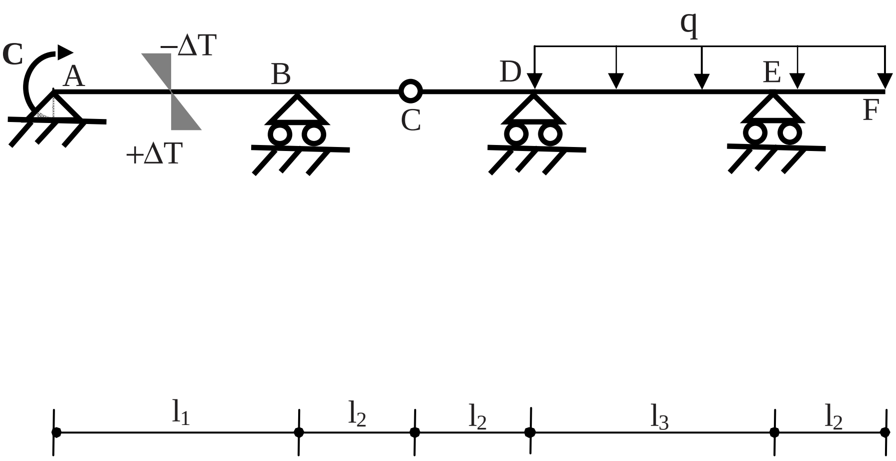
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA PARZIALE DI STATICA
FERRARA, 24/11/2009

D

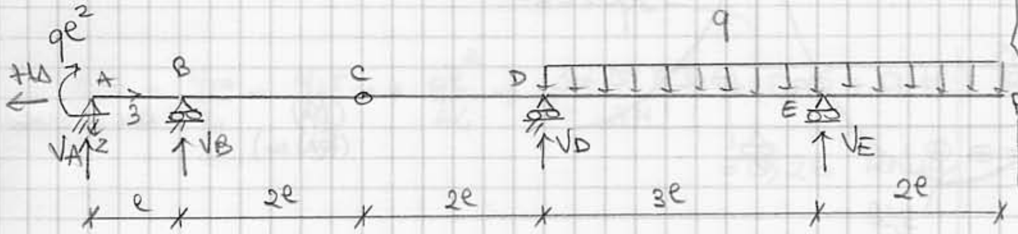


l_1 = 1 m, l_2= 2 m, l_3 = 3 m,
q = 10 kN/m, C = 10 kN m,
E = 2.1 10^3 kN/cm^2, α= 10^{-5} °C^{-1}, ΔT = 20 °C

La travatura iperstatica di figura è realizzata con profilati IPE 180 (H= 180 mm, A = 23.9 cm^2, I_1 = 1317 cm^4).

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e C e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo D.
3. Risolvere nuovamente la travatura considerando anche il carico termico nel solo tratto AB e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q, C che di ΔT.

D1)



$$\begin{cases} H_A = 0 \\ (C\circlearrowleft)_{ABC}\ V_A 3\ell + V_B 2\ell + q\ell^2 = 0 \\ (C\circlearrowleft)_{CDEF}\ V_D 2\ell + V_E 5\ell = 5q\ell \frac{9}{2}\ell \\ (\uparrow)\ V_A + V_B + V_D + V_E = 5q\ell \end{cases}$$

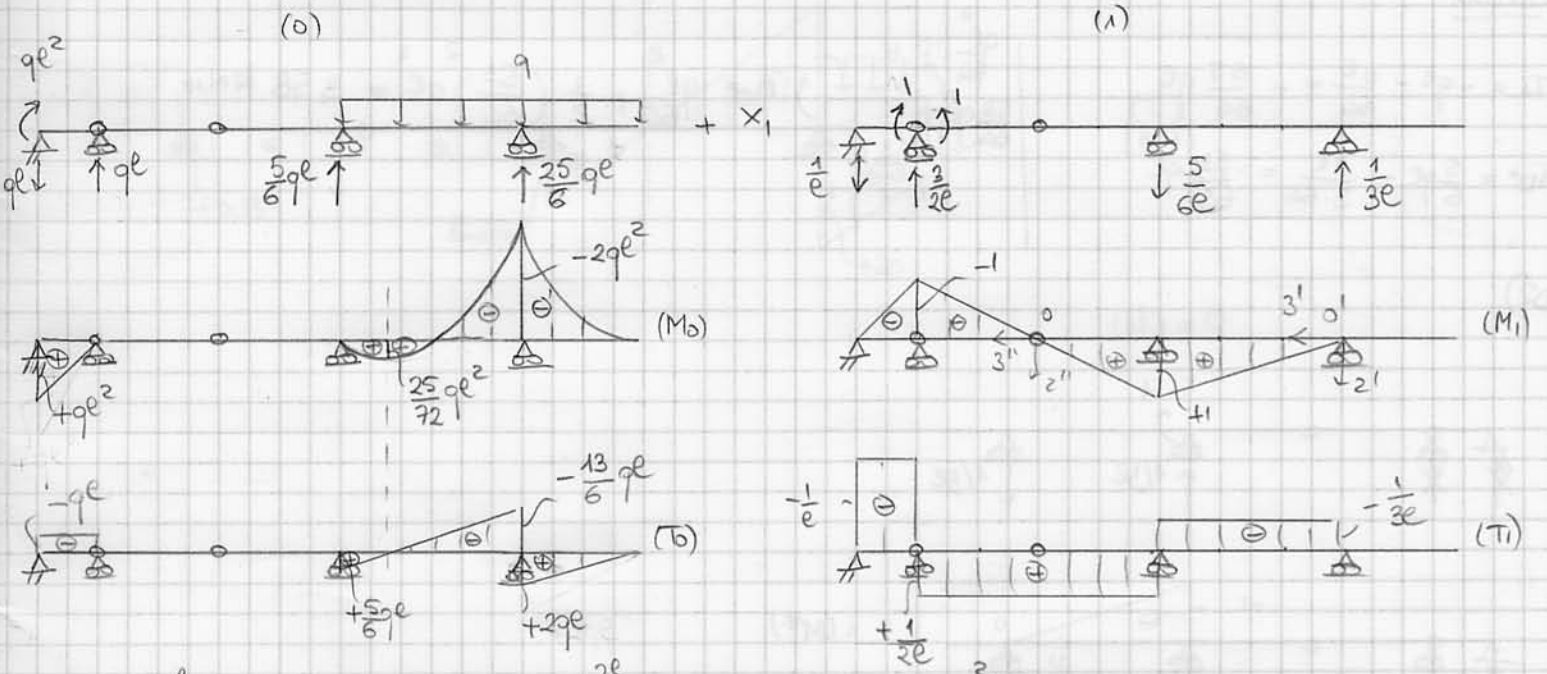
$$\begin{cases} H_A = 0 \\ V_B = -q\frac{\ell}{2} - \frac{3}{2}V_A \\ V_E = \frac{9}{2}q\ell - \frac{2}{5}V_D \\ V_A - q\frac{\ell}{2} - \frac{3}{2}V_A + \frac{9}{2}q\ell - \frac{2}{5}V_D + V_D = 5q\ell \end{cases}$$

$\ell = 1m, \quad q = 10\ kN/m$

Travatura una volta iperstatica

Incognita iperstatica: $X_1 = M_B$.

$$\begin{cases} H_A = 0 \\ V_B = -q\frac{\ell}{2} - \frac{3}{2}V_A = -\frac{9}{5}V_D + \frac{5}{2}q\ell \\ V_E = \frac{9}{2}q\ell - \frac{2}{5}V_D \\ V_A = \frac{6}{5}V_D - 2q\ell \end{cases}$$



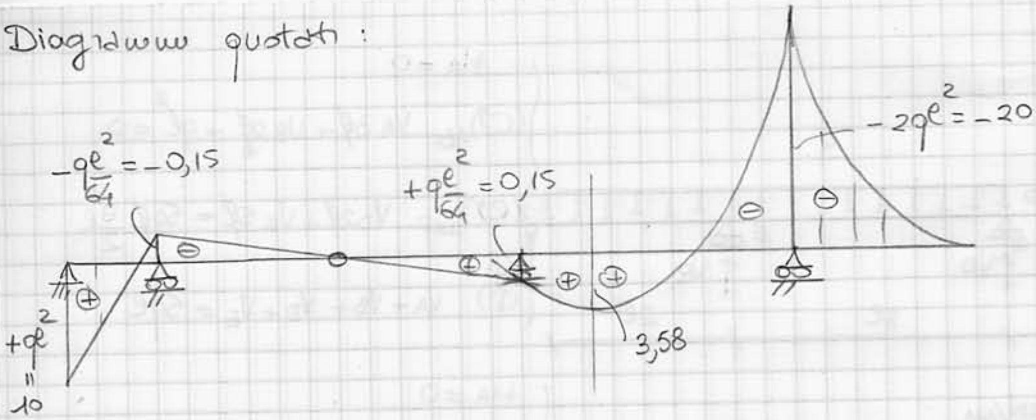
$$EI_1 \eta_{10} = \int_0^\ell \left(-\frac{x_3}{\ell}\right)\left(q\ell^2 - q\ell x_3\right)dx_3 + \int_0^{3\ell}\left(-2q\ell^2 + \frac{13}{6}q\ell x_3' - q\frac{x_3'^2}{2}\right)\left(\frac{x_3'}{3\ell}\right)dx_3'$$

$$= -q\frac{\ell^3}{6} + q\frac{\ell^3}{8} = -\frac{q\ell^3}{24}$$

$$EI_1 \eta_{11} = \int_0^\ell \left(-\frac{x_3}{\ell}\right)^2 dx_3 + 2\int_0^{2\ell}\left(-\frac{x_3''}{2\ell}\right)^2 dx_3'' + \int_0^{3\ell}\left(+\frac{x_3'}{3\ell}\right)^2 dx_3' = \frac{\ell}{3} + 2\cdot\frac{2\ell}{3} + \frac{1}{3}3\ell = \frac{8}{3}\ell$$

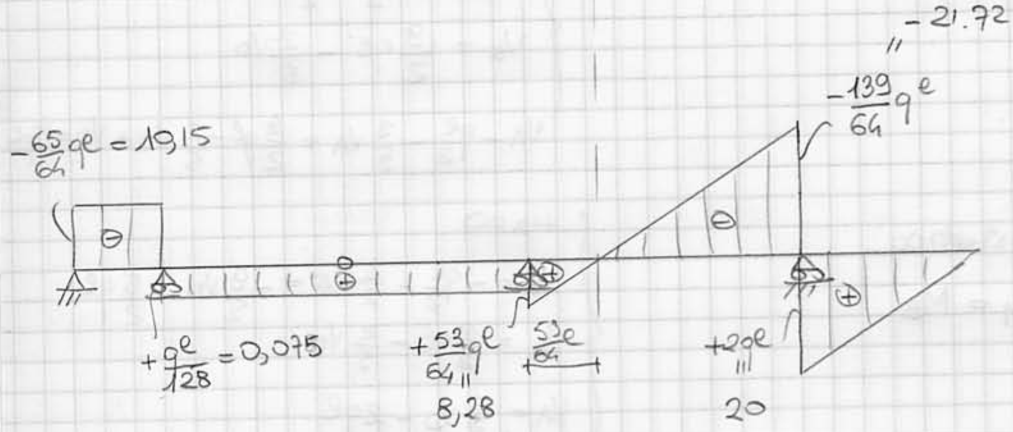
$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{q\ell^3}{24_8}\frac{3}{8\ell} = \frac{q\ell^2}{64} = 0{,}15\ kNm$$

Diagrammi quotati:



(M)
(kN m)

(Non in scala)



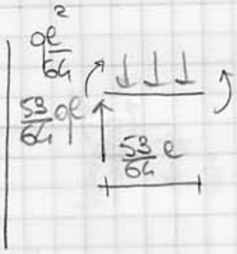
(T)
(kN)

(Non in scala)

Calcoli

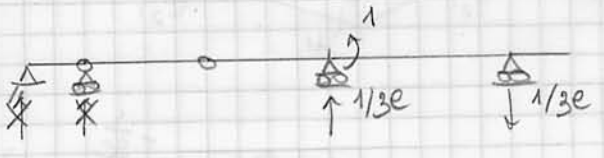
$$T_A = -ql - \frac{ql}{64} = -\frac{65}{64}ql$$

$$T_{D^+} = \frac{5}{6}ql - \frac{ql}{3 \cdot 64} = \frac{53}{64}ql$$



$$\bar{M} = \frac{ql^2}{64} + \frac{1}{2}\left(\frac{53}{64}\right)^2 ql^2 = 3{,}58 \text{ kNm}$$

D2)



(M^*)

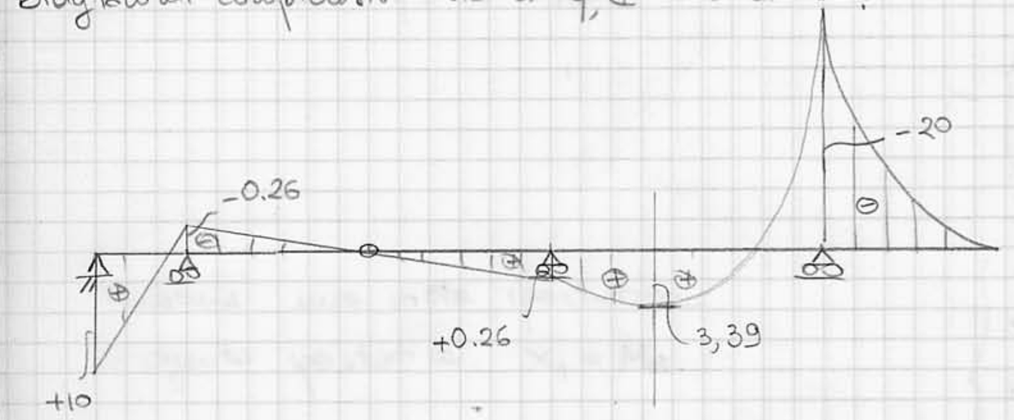
$$1 \cdot \varphi_D = \frac{1}{EI_1}\int_0^{3l}\left(-\frac{x_3'}{3l}\right)\left(-2ql^2 + \frac{139}{64}ql\,x_3' - q\frac{x_3'^2}{2}\right)dx_3' = -\frac{9ql^3}{64EI_1} = -0{,}29°$$

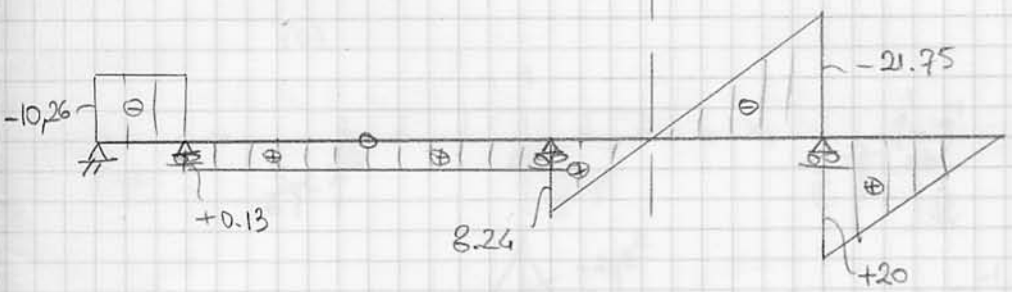
D3) $\eta_{1t} = \int_{AB} M_1 \chi_t = \left(\frac{2\alpha \Delta T}{H}\right) \int_{AB} M_1 = \left(\frac{2\alpha\Delta T}{H}\right)\left(-\frac{l}{8}\right) = -\frac{\alpha \Delta T l}{H}$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} - \frac{\eta_{1T}}{\eta_{11}} = +\frac{ql^2}{64} + \frac{3\alpha \Delta T EI_1}{8H} = (0,15 + 0,11)\ \mathrm{kNm}$$

$$= 0,26\ \mathrm{kNm}$$

Diagrammi complessivi sia di q, I che di ΔT:





(N) = 0