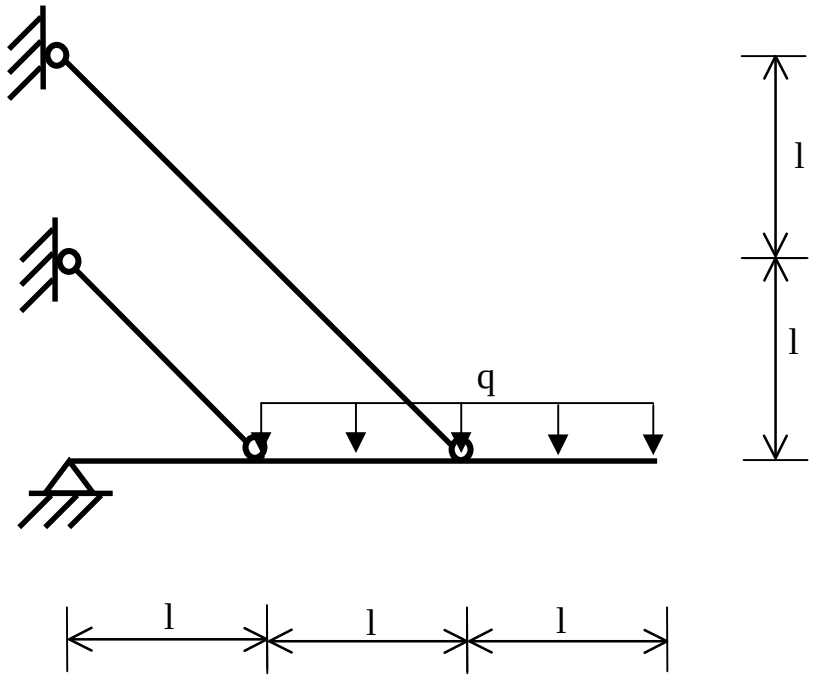
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 16/06/2010

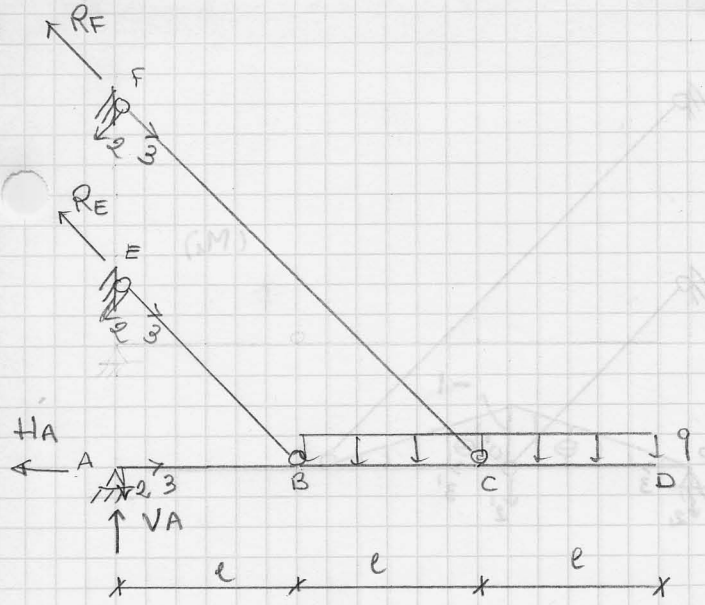


l = 1 m, q = 25 kN/m
E = 210 GPa, σ_{AMM}= 160 MPa, $\alpha = 10^{-5}$ °C^{-1}

Nella travatura iperstatica di figura la trave orizzontale è realizzata con profilati IPE, mentre le bielle sono realizzate con tubi a sezione circolare di area pari a 5.23 cm^2.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del carico q e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). In questa fase è possibile trascurare le deformazioni assiali.
2. Progettare la trave orizzontale.
3. Calcolare lo spostamento verticale dell'estremo libero.
4. Risolvere nuovamente la travatura considerando anche un riscaldamento uniforme della biella più lunga pari a $\Delta T = 20°C$.

1)



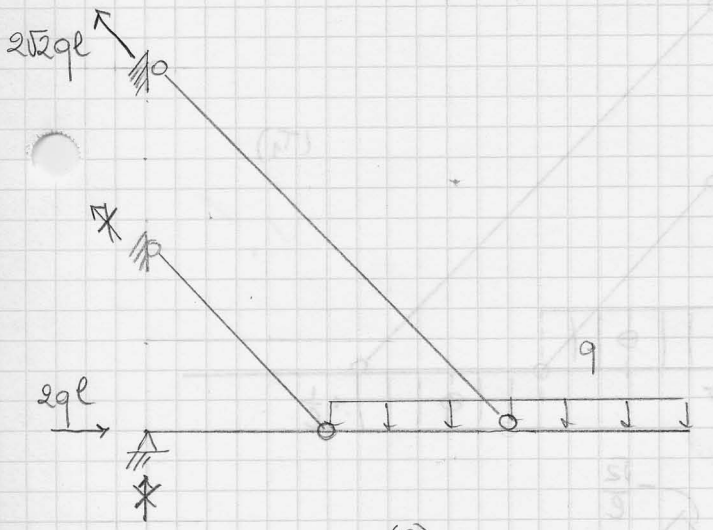
Equazioni cardinali della Statica:

$$(\rightarrow) \quad H_A + R_E \frac{\sqrt{2}}{2} + R_F \frac{\sqrt{2}}{2} = 0 \qquad (*)$$

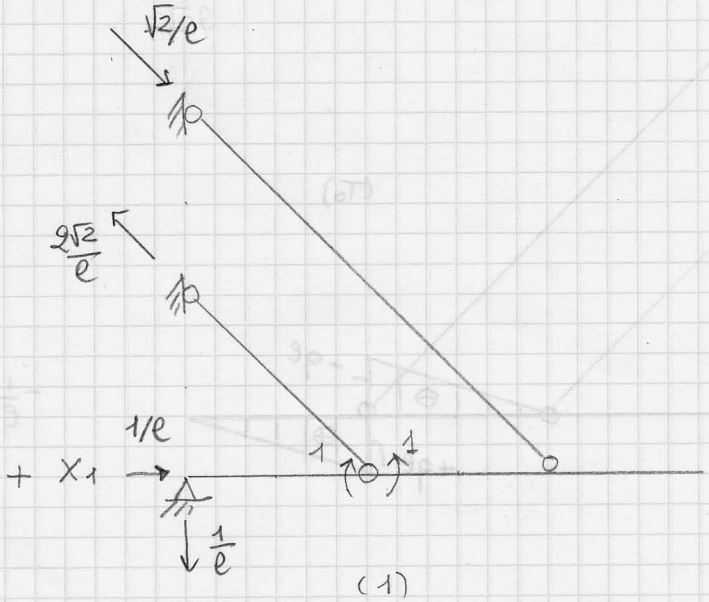
$$(\uparrow) \quad V_A + R_E \frac{\sqrt{2}}{2} + R_F \frac{\sqrt{2}}{2} = 2q\ell \qquad (**)$$

$$(A\uparrow) \quad R_E \frac{\sqrt{2}}{2} \ell + R_F \frac{\sqrt{2}}{2} 2\ell = 4q\ell^2 \qquad (***)$$

Una volta iperstatica. $X_1 = M_B$.



(0)



(1)

Sistema (0):

$$0 = M_B = V_A \cdot \ell \iff V_A = 0$$

$$\begin{cases} R_E \frac{\sqrt{2}}{2} + R_F \frac{\sqrt{2}}{2} = 2q\ell & (**) \\ R_E \frac{\sqrt{2}}{2} \ell + R_F \frac{\sqrt{2}}{2} 2\ell = 4q\ell^2 & (***) \end{cases}$$

$$R_F \frac{\sqrt{2}}{2} = 2q\ell \iff R_F = 2\sqrt{2}q\ell$$

$$R_E \frac{\sqrt{2}}{2} = 2q\ell - 2q\ell = 0$$

$$H_A = - R_F \frac{\sqrt{2}}{2} = -2q\ell$$

Sistema (1):

$$0 = M_B = \ell V_A + 1 \iff V_A = -\frac{1}{\ell}$$

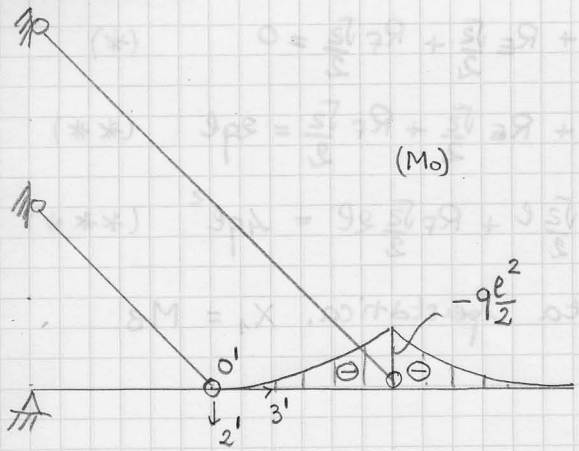
$$\begin{cases} -\frac{1}{\ell} + R_E \frac{\sqrt{2}}{2} + R_F \frac{\sqrt{2}}{2} = 0 & (**) \\ R_E \frac{\sqrt{2}}{2} \ell + R_F \frac{\sqrt{2}}{2} 2\ell = 0 & (***) \end{cases}$$

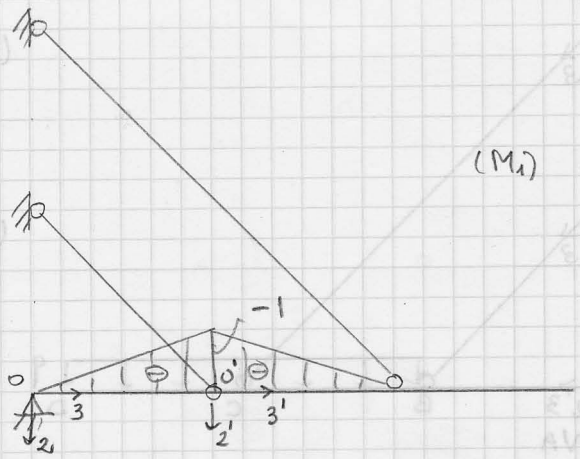
$$\frac{1}{\ell} + R_F \frac{\sqrt{2}}{2} = 0 \iff R_F = -\frac{\sqrt{2}}{\ell}$$

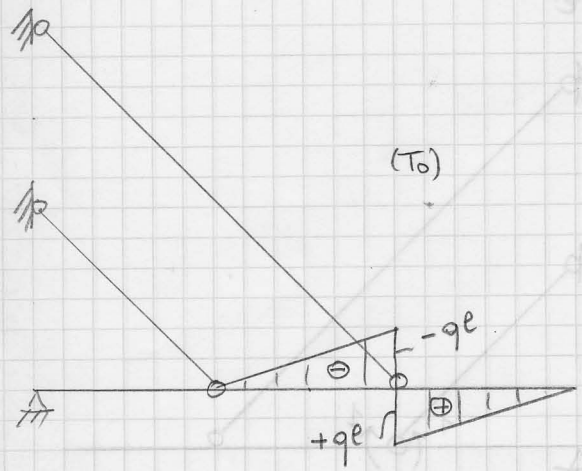
$$R_E \frac{\sqrt{2}}{2} = - R_F \frac{\sqrt{2}}{2} 2 = + \frac{2}{\ell}$$

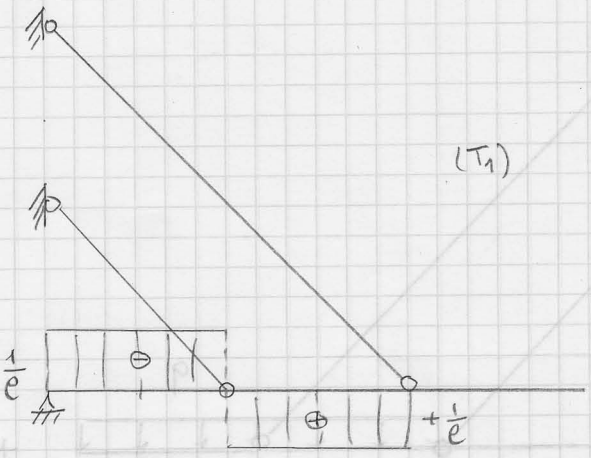
$$\iff R_E = \frac{2\sqrt{2}}{\ell}$$

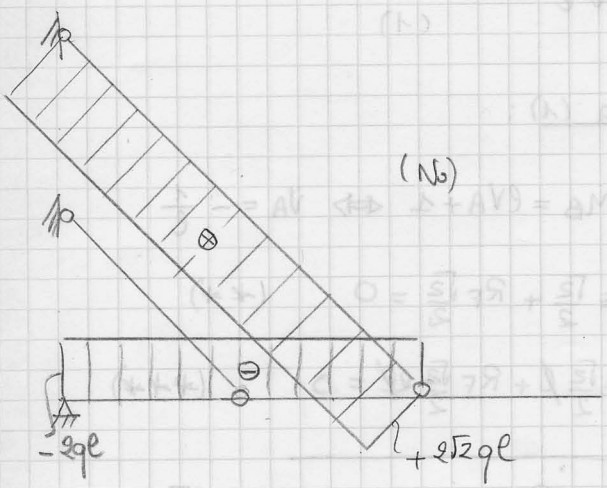
$$H_A = - R_E \frac{\sqrt{2}}{2} - R_F \frac{\sqrt{2}}{2} = -\frac{2}{\ell} + \frac{1}{\ell} = -\frac{1}{\ell}$$

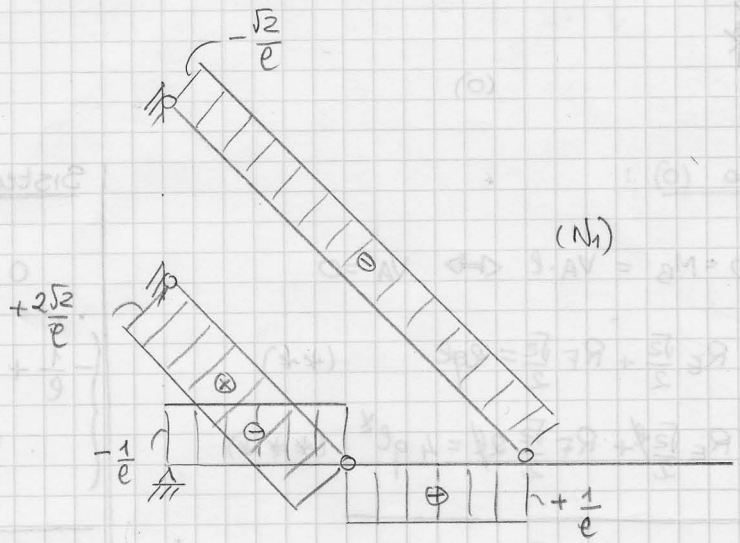












$$EI_1 \delta_{10} = \int_0^{\ell} \left(-\frac{q}{2} x_{3'}^2\right)\left(-1 + \frac{1}{\ell} x_{3'}\right) dx_{3'} = +\frac{q}{2} \int_0^{\ell} \left(x_{3'}^2 - \frac{x_{3'}^3}{\ell}\right) dx_{3'} = \frac{q}{2}\left[\frac{\ell^3}{3} - \frac{\ell^3}{4}\right] = \frac{q\ell^3}{24}$$

$$EI_1 \delta_{11} = 2\int_0^{\ell} \left(\frac{x_3}{\ell}\right)^2 dx_3 = \frac{2\ell^3}{3\ell^2} = \frac{2\ell}{3}$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{q\ell^3}{24}\cdot\frac{3}{2\ell} = -\frac{q\ell^2}{16} = -1{,}56 \text{ kNm}$$

Diagramma delle c. s. :

$$T_{C^+} = -ql - \frac{ql}{16} = -\frac{17}{16}ql = -26,56$$

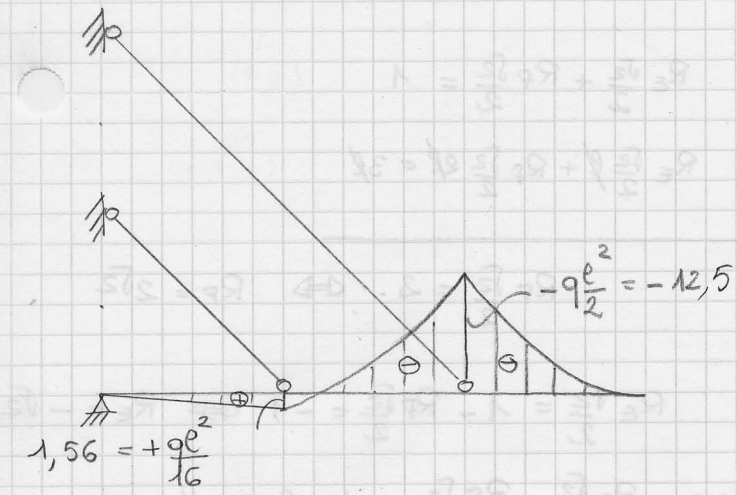
$$N_A = -2ql + \frac{ql}{16} = -\frac{31}{16}ql = -48,44\ kN$$

$$N_{B^+} = -2ql + \frac{ql}{16} = -\frac{33}{16}ql = -51,56\ kN$$

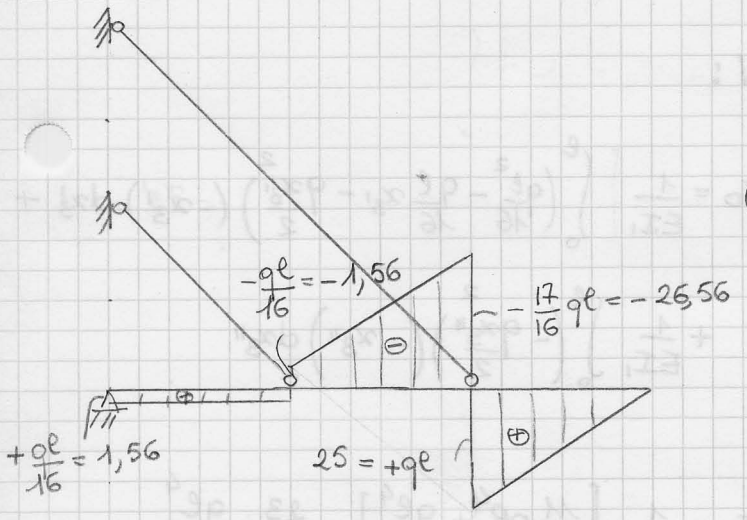
$$N_{BE} = -\frac{2\sqrt{2}}{\ell}\,q\frac{\ell^2}{16} = -\frac{\sqrt{2}}{8}ql = -4,42\ kN$$

$$N_{CF} = 2\sqrt{2}ql + \frac{\sqrt{2}}{16}ql = \frac{33}{16}\sqrt{2}ql = 72,92\ kN$$

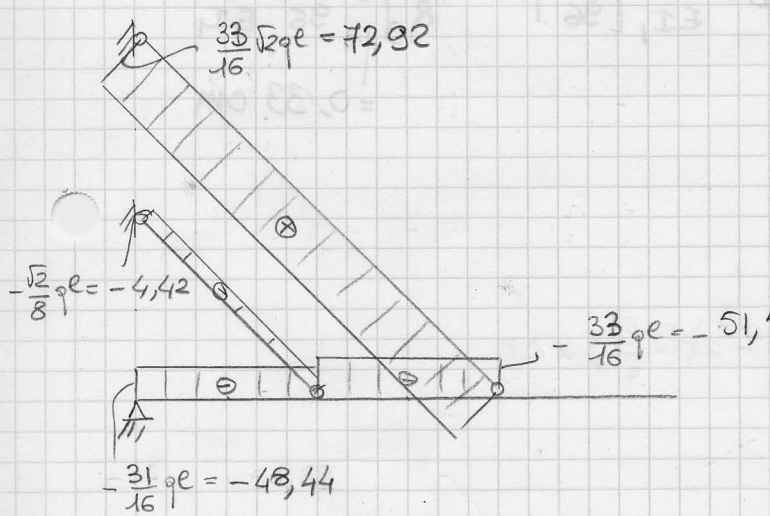
(M) (kN m)



(T) (kN)



(N) (kN)



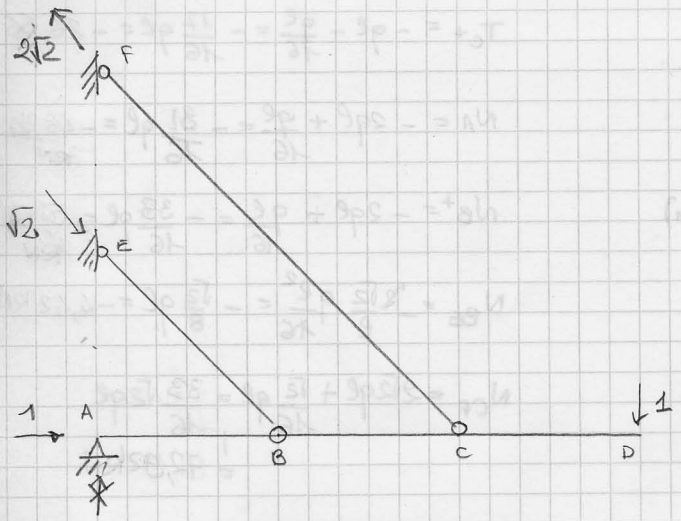
2) Progetto della trave orizzontale :

$$W_1 > \frac{M_{max}}{\sigma_{AMM}} = \frac{q\ell^2/2}{\sigma_{AMM}} = \frac{+12,5\cdot 10^3}{160\cdot 10^8_3} = 78\cdot 10^{-6}\ m^3 = 78\ cm^3$$

IPE 160

$$\begin{cases} W_1 = 109\ cm^3 \\ I_1 = 869\ cm^4 \\ A = 20\ cm^2 \end{cases}$$

3) Freccia in D:



$$0 = M_B = V_A \cdot \ell \iff V_A = 0$$

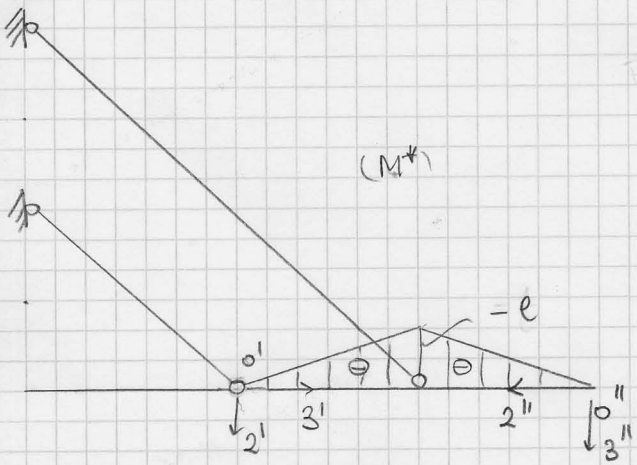
$$(\uparrow) \quad R_E \frac{\sqrt{2}}{2} + R_F \frac{\sqrt{2}}{2} = 1$$

$$(A\curvearrowright) \quad R_E \frac{\sqrt{2}}{2} \ell + R_F \frac{\sqrt{2}}{2} 2\ell = 3\ell$$

$$R_F \frac{\sqrt{2}}{2} = 2 \iff R_F = 2\sqrt{2}$$

$$R_E \frac{\sqrt{2}}{2} = 1 - R_F \frac{\sqrt{2}}{2} = -1 \iff R_E = -\sqrt{2}$$

$$H_A = -R_E \frac{\sqrt{2}}{2} - R_F \frac{\sqrt{2}}{2} = 1 - 2 = -1$$



PLV:

$$1 \cdot v_D = \frac{1}{EI_1} \int_0^\ell \left(q\frac{\ell^2}{16} - q\frac{\ell}{16} x_{3'} - q\frac{x_{3'}^2}{2} \right) (-x_{3'}) \, dx_{3'} +$$

$$+ \frac{1}{EI_1} \int_0^\ell \left(-q\frac{x_{3''}^2}{2} \right) (-x_{3''}) \, dx_{3''}$$

$$1 \cdot v_D = \frac{1}{EI_1} \left[\frac{11}{96} q\ell^4 + q\frac{\ell^4}{8} \right] = \frac{23}{96} \frac{q\ell^4}{EI_1}$$

$$= 0{,}33 \text{ cm}$$

4) Carico termico sulle bielle:

$$\gamma_{1t} + \gamma_{10} + \gamma_{11} X_1 = \gamma_1 \overset{0}{\nearrow}$$

$$\gamma_{1t} = \int_{CF} N_1 \varepsilon_t \, dx_3 = 2\ell\sqrt{2} \left(-\frac{\sqrt{2}}{\ell} \right) \alpha \Delta T$$

$$= -4 \alpha \Delta T$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} - \frac{\gamma_{1t}}{\gamma_{11}} = -\frac{q\ell^2}{16} + \frac{6\alpha\Delta T EI_1}{\ell}$$

$$= (-1{,}56 + 2{,}19) \text{ kNm} = 0{,}63 \text{ kNm}$$