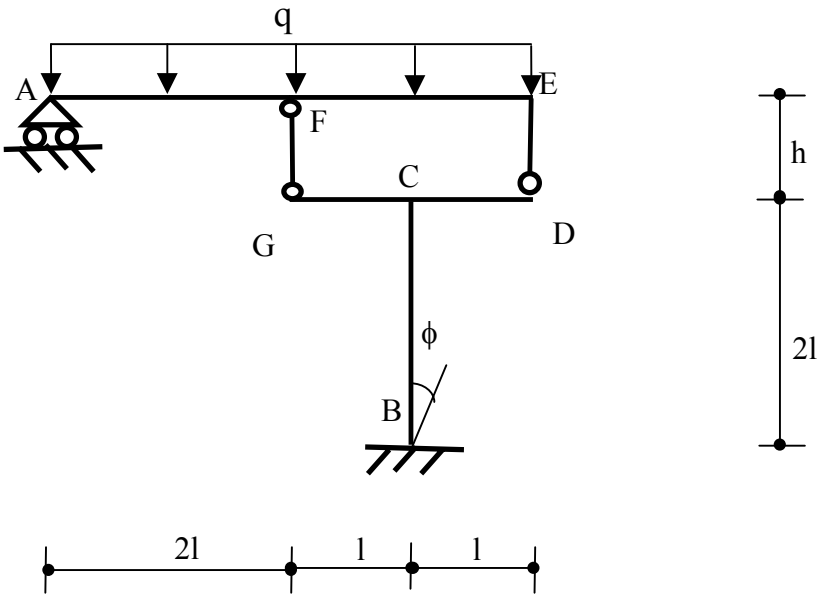
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 14/04/2010

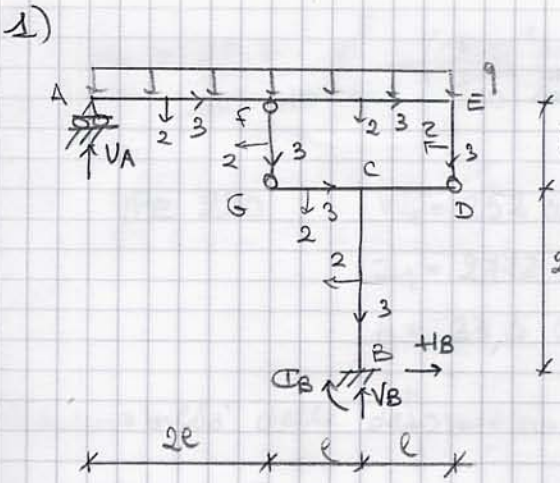


l = 1 m, h= 70 cm, q = 25 kN/m,
E = 200 GPa, σ_{amm} = 160 MPa
ϕ= 1/300

La travatura iperstatica di figura è realizzata con profilati IPE.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del solo carico verticale distribuito q e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). In questa fase è possibile trascurare le deformazioni assiali.
2. Dimensionare la struttura e verificare l'ipotesi di trascurabilità delle deformazioni assiali.
3. Calcolare lo spostamento verticale in E.
4. Risolvere nuovamente la travatura considerando anche un cedimento angolare dell'incastro in B pari a ϕ = 1/300 e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia del carico distribuito q che del cedimento ϕ.

1)



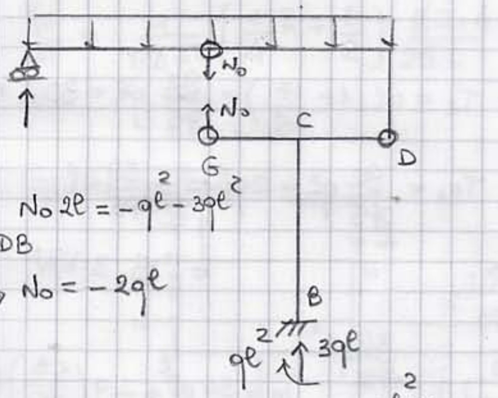
Equazioni cardinali:

$H_B = 0$

$V_B + V_A = 4ql$

$(B\circlearrowleft)\ C_B + V_A 3l - 2ql\,3l = 0$

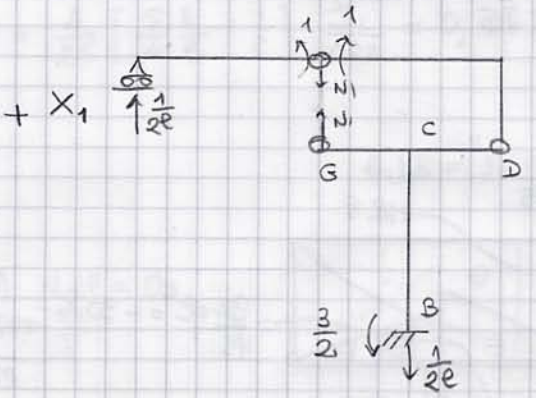
La travatura è una volta iperstatica per vincoli esterni. Internamente è isostatica (una maglia chiusa con 3 cerniere non allineate). Complessivamente la travatura è 1 volta iperstatica. Incognita iperstatica: $X_1 = M_F$



$(D\circlearrowleft)_{GCDB}\ N_0\, 2l = -ql^2 - 3ql^2$

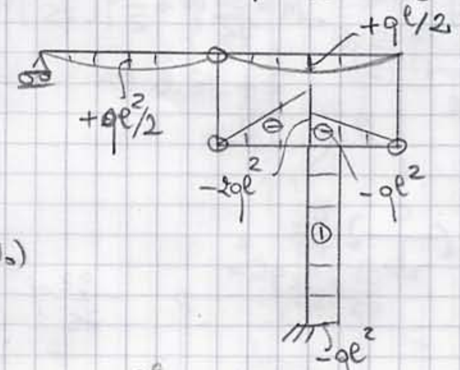
$\hookrightarrow N_0 = -2ql$

$+ X_1$

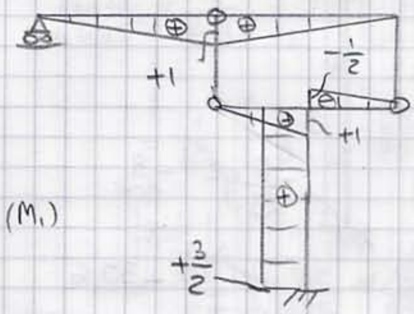


$(D\circlearrowleft)_{GCDB}\ N_1\, 2l = \frac{3}{2} + \frac{1}{2}$

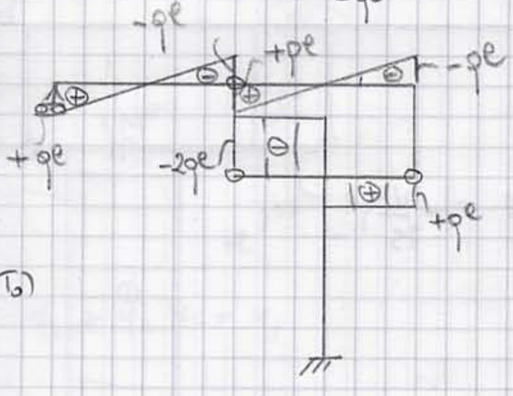
$\hookrightarrow N_1 = \frac{1}{l}$



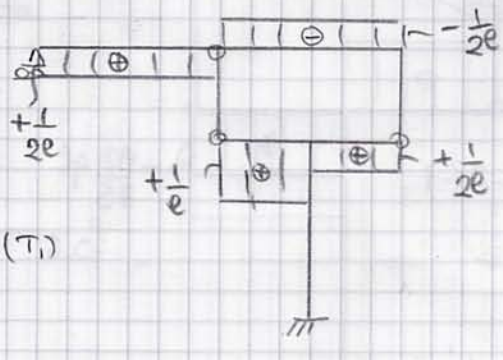
(M_0)



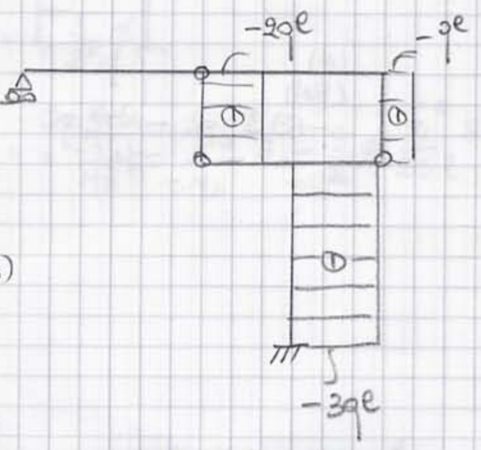
(M_1)



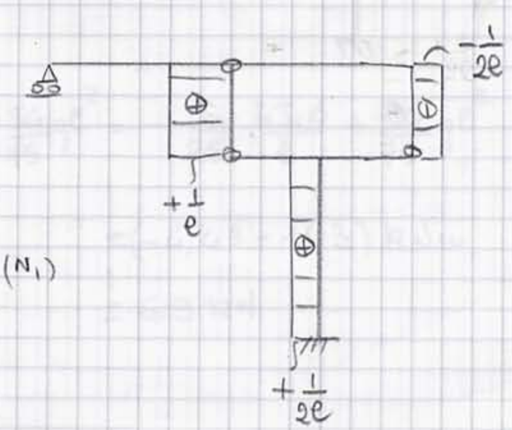
(T_0)



(T_1)



(N_0)



(N_1)

$$EI_1 \delta_{10} = 2\,\frac{1}{24}\,q(2\ell)^3 + \frac{1}{3}\ell(-2q\ell^2)\cdot 1 + \frac{1}{3}\ell(-q\ell^2)\left(-\frac{1}{2}\right) + 2\ell\left(-q\ell^2\right)\left(\frac{3}{2}\right)$$

$$= q\ell^3\left[\frac{2}{3} - \frac{2}{3} + \frac{1}{6} - 3\right] = -\frac{17}{6}q\ell^3$$

$$EI_1 \delta_{11} = 2\,\frac{1}{3}2\ell\cdot 1 + \frac{1}{3}\ell\cdot 1 + \frac{1}{3}\ell\,\frac{1}{4} + 2\ell\,\frac{9}{4}$$

$$= \ell\left[\frac{4}{3} + \frac{1}{3} + \frac{1}{12} + \frac{9}{2}\right] = \frac{25}{4}\ell$$

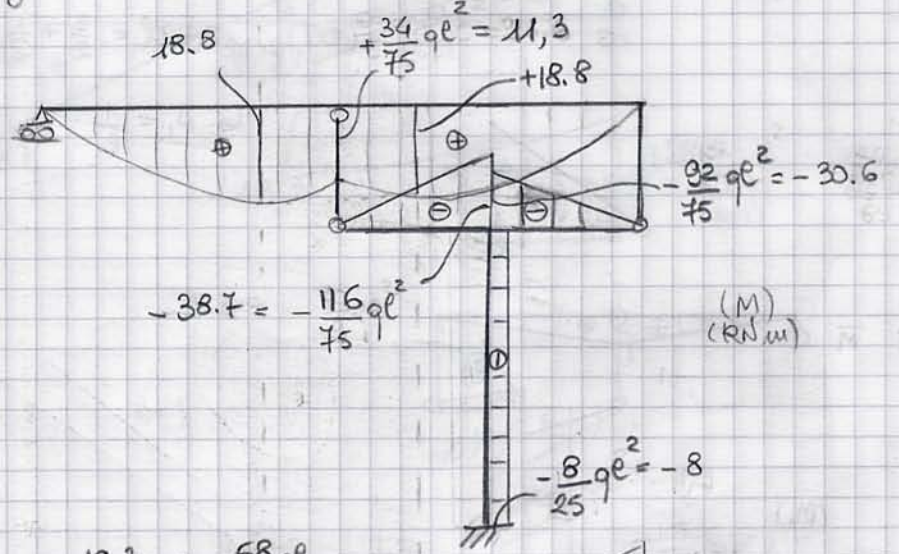
$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{17}{6}q\ell^3\,\frac{4}{25\ell} = \frac{34}{75}q\ell^2 = 11{,}3\ \text{kN}$$

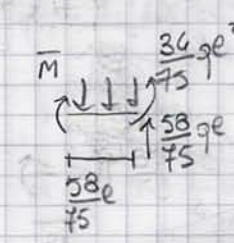
Diagrammi delle c.s.

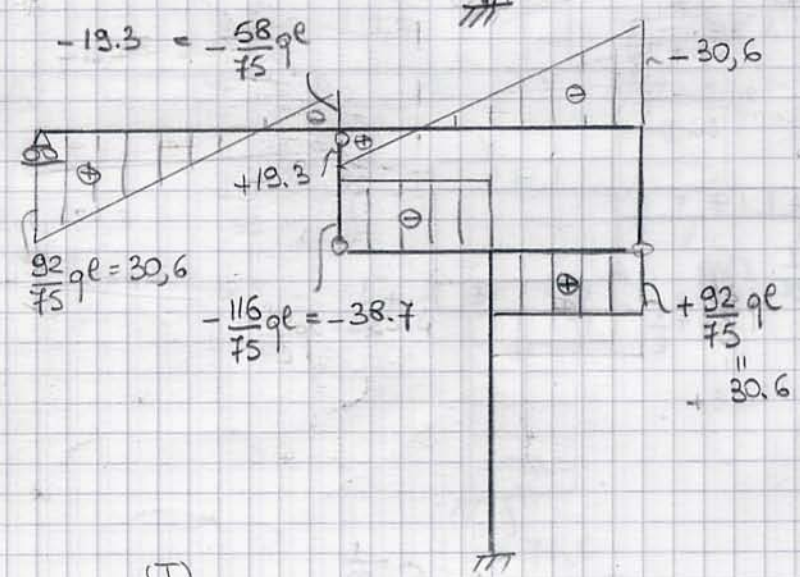
$$T_A = q\ell\left(1 + \frac{17}{75}\right) = \frac{92}{75}q\ell = 30{,}6\ \text{kN}$$

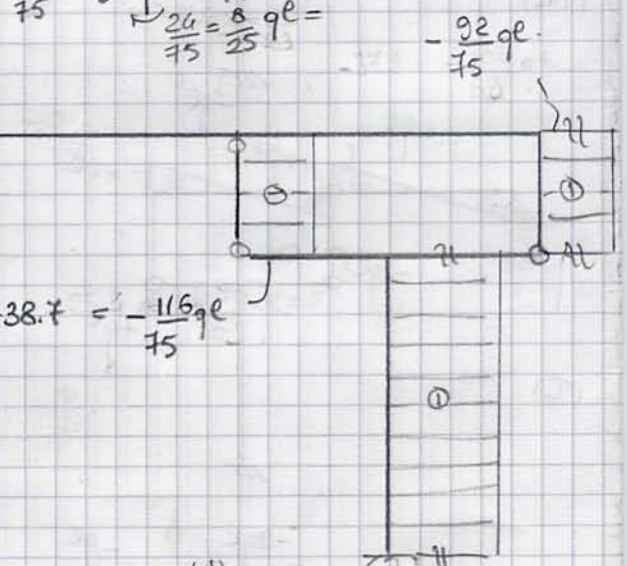
$$T_{FA} = \frac{92}{75}q\ell - 2q\ell = -\frac{58}{75}q\ell = -19{,}3\ \text{kN}$$

$$\bar{M} = \frac{34}{75}q\ell^2 + \frac{1}{2}q\left(\frac{58}{75}\right)\ell^2 = +18.8\ \text{kN}$$









2) $W_1 \geq \frac{38,7 \cdot 10^{-3}}{160 \cdot 10^{8}} m^3 = \frac{38,7}{160 \cdot 10^{5}} 10^{8} cm^3 = 241,9\ cm^3$

IPE 220 $W_1 = 252\ cm^3$

$I_1 = 2772\ cm^4 = 2772\ 10^{-8}\ m^4$

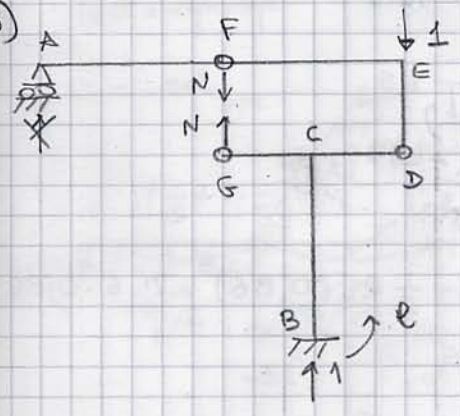
$A = 33,4\ cm^2$

Trascurabilità delle deformazioni assiali

$$\gamma_{11}^N = \frac{1}{EA}\left[h\frac{1}{\ell^2} + h\frac{1}{4\ell^2} + 2\ell\frac{1}{4\ell^2}\right] = \frac{1}{EA\ell^2}\left[\frac{5}{4}h + \frac{\ell}{2}\right]$$

$$\frac{\gamma_{11}^N}{\gamma_{11}^M} = \frac{1}{\not{4}A\ell^2}\left[\frac{5}{4}h + \frac{\ell}{2}\right]\frac{\not{4}I_1}{25\ell} = \frac{4}{25}\left[\frac{5}{4}\frac{h}{\ell} + \frac{1}{2}\right]\frac{I_1}{A\ell^2} = 0,18\ \% \rightarrow ok$$

3)



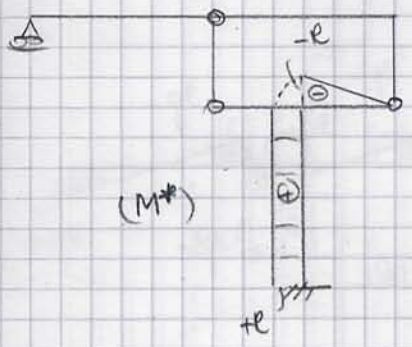
(D) $N 2\ell = 0$

AFED

E.L.V. $1 \cdot v_E = \frac{1}{EI_1}\int_S MM^* dx_3$

$= \frac{1}{EI_1}\left[\frac{1}{3}\ell\left(-\frac{92}{75}q\ell^2\right)(-\ell) + 2\ell\left(-\frac{8}{25}q\ell^2\right)(+\ell)\right]$

$= \frac{q\ell^4}{EI_1}\left[\frac{92}{225} - \frac{16}{25}\right] = -\frac{52}{225}\frac{q\ell^4}{EI_1} = -0.1\ cm$



4) $\gamma_{10} + \gamma_{11} X_1 = \gamma_{11}$

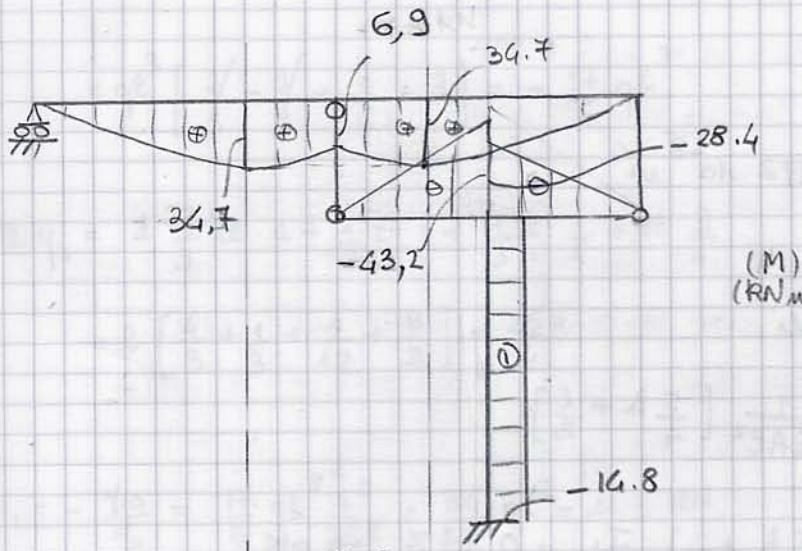
$\gamma_{11} = -\frac{3}{2}\phi$

$$X_1 = +\frac{\gamma_1}{\gamma_{11}} - \frac{\gamma_{10}}{\gamma_{11}} = -\frac{3}{\not{2}}\phi\frac{\not{4}^2}{25\ell}EI_1 + \frac{36}{75}q\ell^2 = -\frac{6}{25}\frac{EI_1\phi}{\ell} + \frac{36}{75}q\ell^2$$

$= (-4,43 + 11,3)\ kNm$

$= 6,9\ kN$

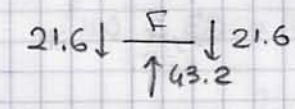
Diagrammi delle c.s. comprensivi di q e di ϕ:

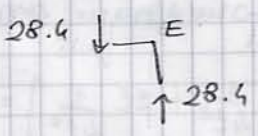


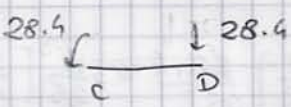
(M)
(KN m)

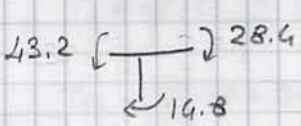
$T_A = 25 + \frac{1}{2} \cdot 6,9 = 28,4$ KN

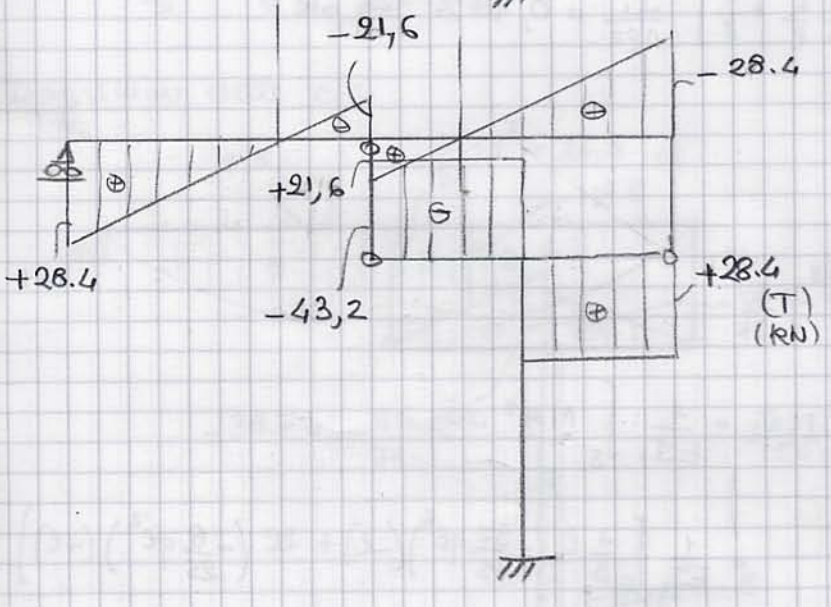
$T_{FA} = 28.4 - 2 \cdot 25 = -21.6$ KN



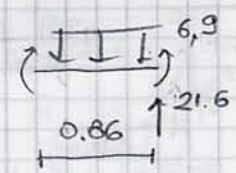






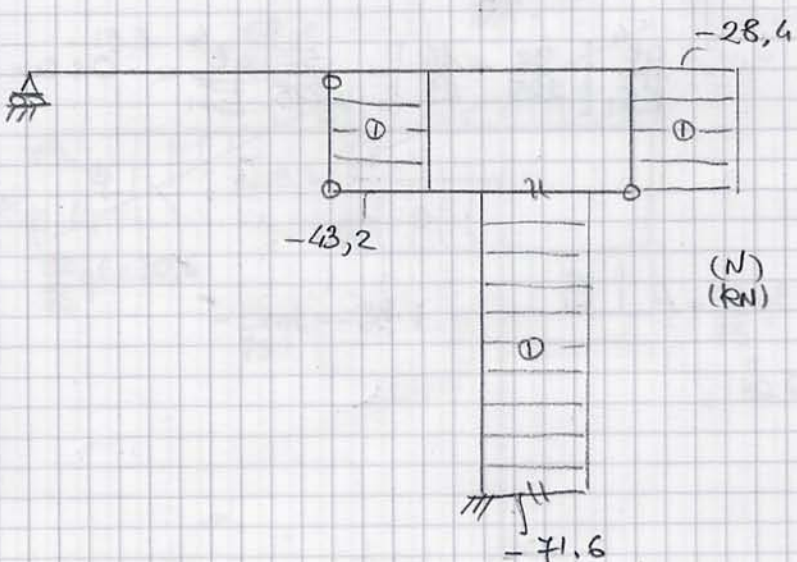


(T)
(KN)



$\overline{M} = 6,9 - \frac{1}{2} 25 (0,86)^2 + 21.6 \cdot 0,86$

$= 34,7$



(N)
(KN)