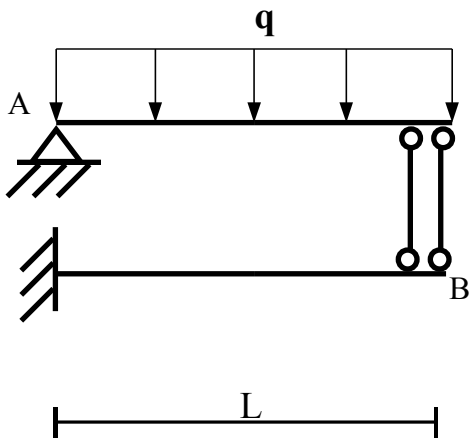
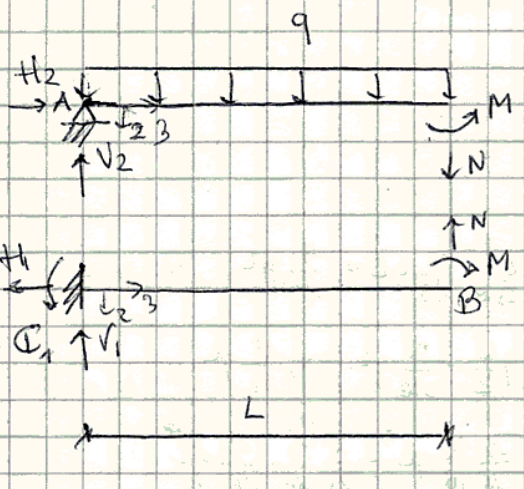
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 17/09/2014



L = 3 m, q = 30 kN/m
E = 210 GPa, σ_{AMM}= 240 MPa
δ = 1cm

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del carico q e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Progettare la travatura con profilati IPE.
3. Calcolare la rotazione della sezione in B.
4. Risolvere nuovamente la travatura considerando anche un abbassamento verticale pari a δ del vincolo in A e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia del carico q che del cedimento vincolare.



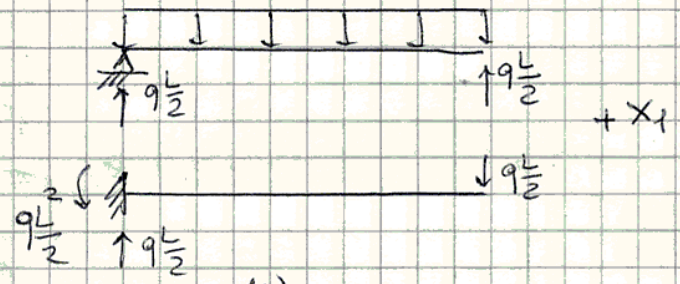
Equi di equilibrio:

$$\begin{cases} V_2 = qL + N \\ H_2 = 0 \\ -V_2 L + q\frac{L^2}{2} + M = 0 \end{cases}$$

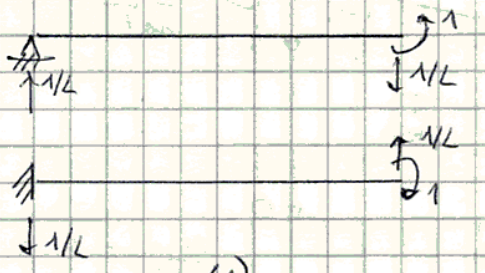
$$\begin{cases} V_1 + N = 0 \\ H_1 = 0 \\ V_1 L + M - C_1 = 0 \end{cases}$$

Incognite: $V_1, V_2, C_1, H_1, H_2, M, N$. Equi 6.

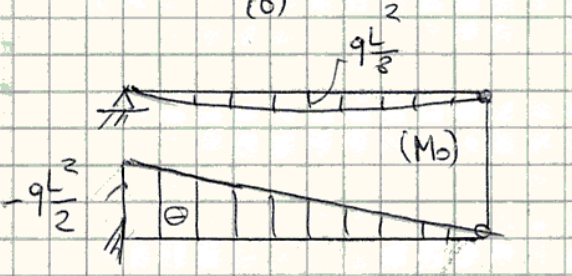
La trave è una volta iperstatica. Incognita iperstatica $X_1 = M$.

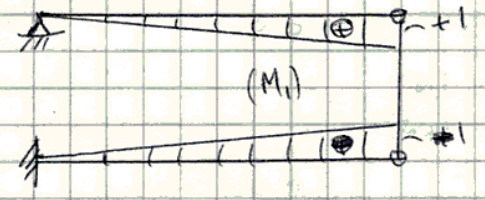


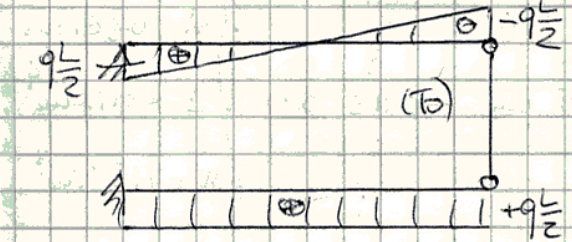
(0)

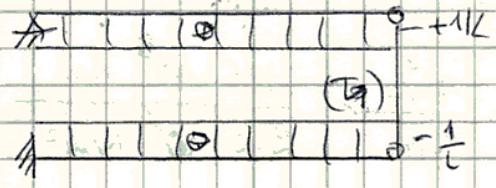


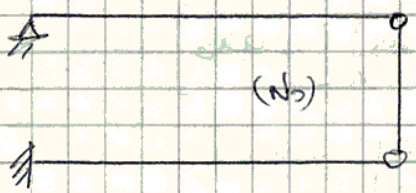
(1)

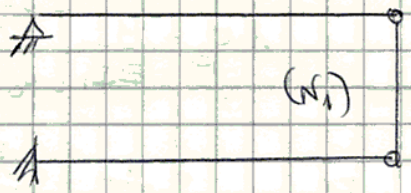












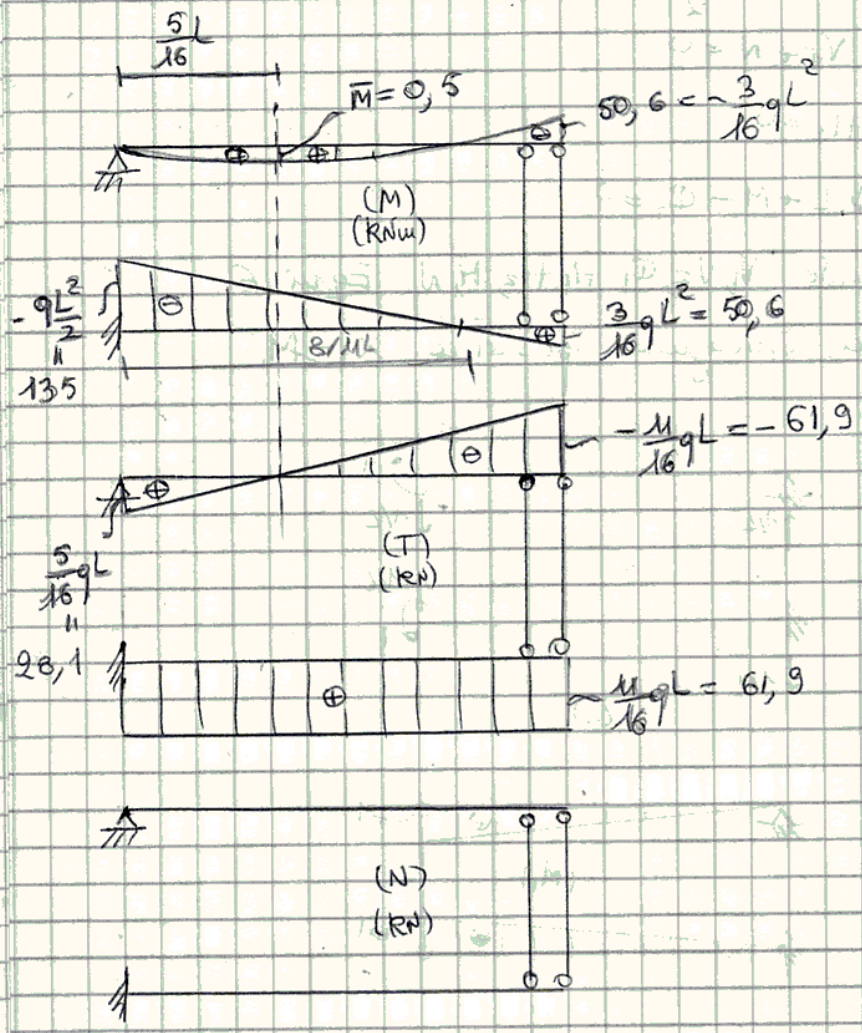
$$EI_1 \delta_{10} = \frac{qL^3}{24} + \frac{1}{6} L \, q\frac{L^2}{2} = \left(\frac{1}{24} + \frac{1}{12}\right) qL^3 = \frac{3}{24} qL^3 = \frac{qL^3}{8}$$

$$EI_1 \delta_{11} = \frac{2}{3} L$$

$$\delta_1 = 0$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} = -\frac{3}{16}qL^2 = -51\ kNm$$

Diagrammi quotati:



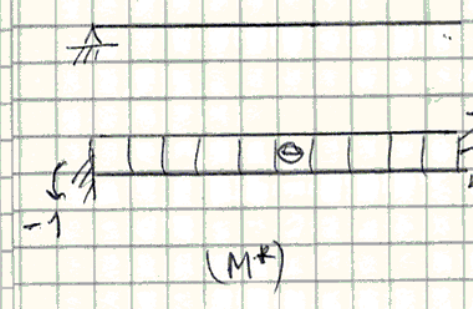
- $q\frac{L}{2}\cdot\frac{L}{8} - \frac{3}{16}qL = \frac{5}{16}qL$
- $-q\frac{L}{2} - \frac{3}{16}qL = -\frac{11}{16}qL$
- $M = \frac{5}{16}\frac{5}{16}qL^2 - \frac{1}{2}\frac{5}{16}\cdot\frac{5}{16}qL^2$
  $= \frac{25}{512}qL^2 = 0,5\ kNm$

Dimensionamento:

$$W_1 \geq \frac{M_{max}}{\sigma_{AMM}} = \frac{qL^2/2}{\sigma_{AMM}} = \frac{135\cdot 10^3\ Nm}{240\cdot 10^6\ N/m^2} = 562,5\ cm^3 \rightarrow IPE\ 330$$

$$\begin{cases} A = 62,61\ cm^2 \\ I_1 = 11770\ cm^4 \\ W_1 = 713,1\ cm^3 \end{cases}$$

Rotazione in B:



$$1\cdot\varphi_B = \frac{1}{EI_1}\left[\int_0^L \left(-q\frac{L^2}{2} + \frac{11}{16}qLx_3\right)\cdot(-1)\,dx_3\right]$$

$$= \frac{1}{EI_1}\left[q\frac{L^2}{2}L - \frac{11}{16}qL\frac{L^2}{2}\right]$$

$$= \frac{5}{32}\frac{qL^3}{EI_1} = \frac{5\cdot 30\cdot 10^3\cdot 27}{32\cdot 210\cdot 10^6\cdot 11770\cdot 10^{-8}}$$

$$= 0,00512 = 0,3°$$

Abbassamento verticale:

$$\eta_1 = -\frac{\delta}{L}$$

$$M_0 + M_{11} X_1 = M_1$$

$$X_1 = + \frac{M_1}{M_{11}} - \frac{M_{10}}{M_{11}} = -\frac{3}{16} qL^2 - \frac{\delta}{L} \frac{3EJ_1}{2L}$$

$$= \left(-50,6 - \frac{0,01}{3} \frac{3}{2} \cdot 10^{\cancel{8}} \cdot 1770 \cdot \cancel{10^{-8}} \cdot 10^{-3} \right) \text{kNm}$$

$$= -50,6 - 41,2 = -92 \text{ kNm}$$