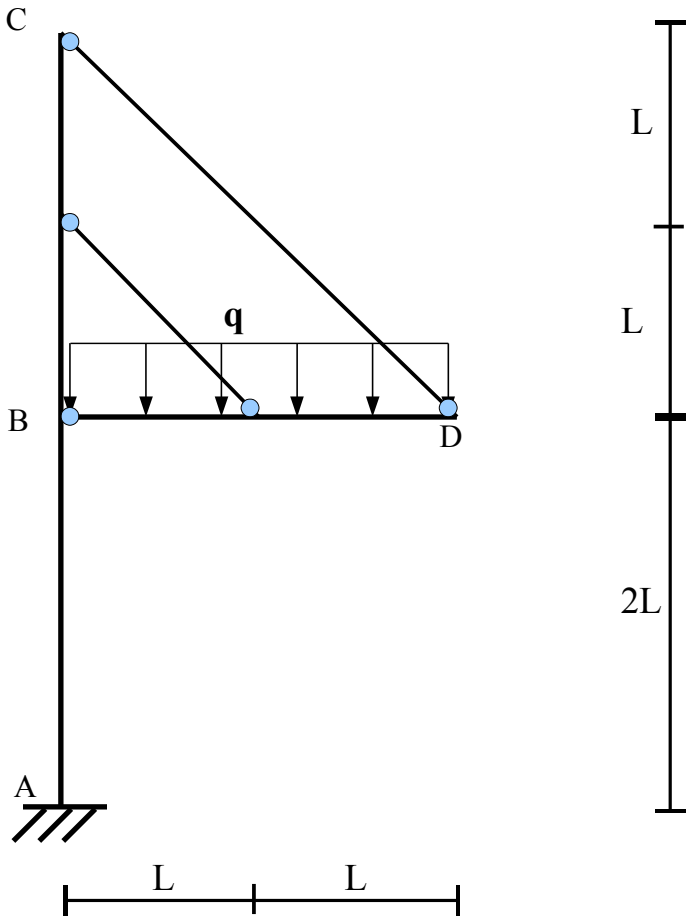
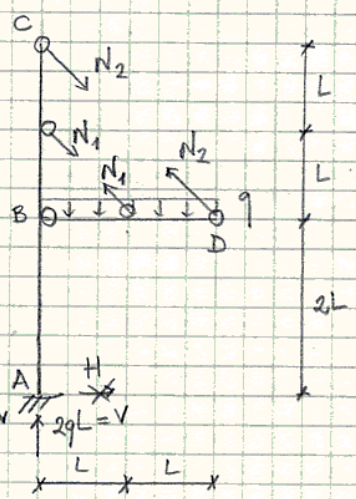
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 09/07/2014



L = 2 m, q = 20 kN/m,
E = 210 GPa, σ_{AMM}= 240 MPa
ΔT = 10°C

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del carico q e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Dimensionare i tratti ABC, e BD con profilati IPE e calcolare la sezione circolare per le bielle che soddisfa il criterio di resistenza.
3. Calcolare lo spostamento verticale in D.
4. Risolvere nuovamente la travatura considerando anche un riscaldamento di entrambe le bielle pari a 10°C.

Equazioni cardinali della Statica:



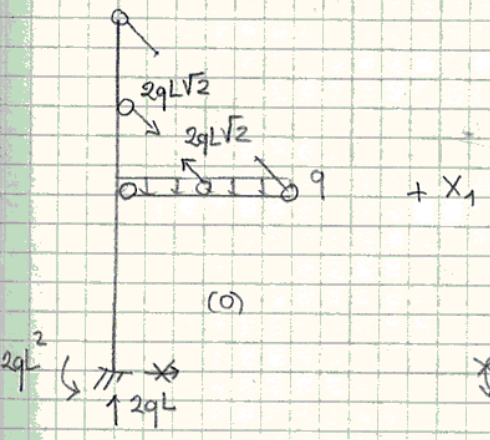
$$\begin{cases} H = 0 \\ V = 2qL \\ \mathcal{C} = 2qL^2 \end{cases}$$

Internamente la struttura è una volta iperstatica.

Equazione della cerniera in B:

$$(B\curvearrowright)_{BD} \quad N_1 \frac{\sqrt{2}}{2} L + N_2 \frac{\sqrt{2}}{2} 2L = 2qL^2$$

Incognita iperstatica $X_1 = N_2$.



$$EI_1 M_{10} = +\frac{1}{6} L \left(-2qL^2\right)\left(-\frac{\sqrt{2}}{2} L\right) + 2 \int_0^L \left(-q\frac{x^2}{2}\right)\left(\frac{\sqrt{2}}{2} x\right) dx$$

$$= \frac{\sqrt{2}}{6} qL^4 - q\frac{\sqrt{2}}{2}\frac{L^4}{4} = \frac{\sqrt{2}}{24} qL^4$$

$$EI_1 M_{11} = 4 \frac{1}{3} L \left(\frac{\sqrt{2}}{2} L\right)^2 = \frac{2}{3} L$$

$$X_1 = -\frac{M_{10}}{M_{11}} = -\frac{\sqrt{2}}{24}\frac{3}{2} qL = -\frac{\sqrt{2}}{16} qL = -3{,}53 \text{ kN}$$

$$\rightarrow N_1 + 2N_2 = 2\sqrt{2}qL$$

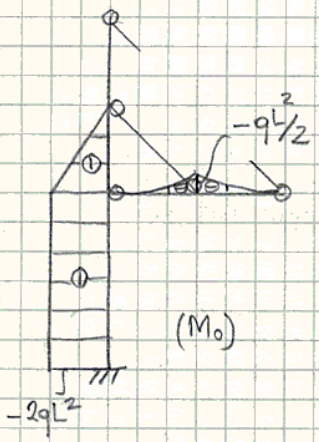
$$N_1 = 2\sqrt{2}qL - 2N_2 = 2\sqrt{2}qL + 2\sqrt{2}\frac{qL}{16} = \frac{34}{16}\sqrt{2}qL = \frac{17}{8}\sqrt{2}qL = 120 \text{ kN}$$

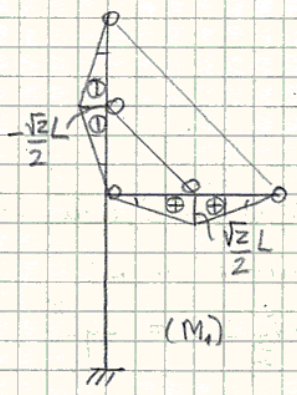
DIMENSIONAMENTO travi:

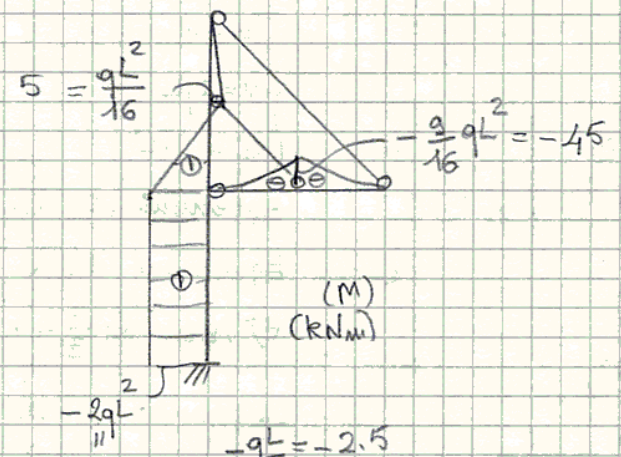
$$W_1 \geq \frac{M_{MAX}}{\sigma_{AMM}} = \frac{2\cancel{160}\cdot 10^3}{\frac{2}{3}\cancel{240}\cdot\cancel{10^6}}\cancel{10^6}\ cm^3 = \frac{2000}{3}\ cm^3 = 667\ cm^3$$

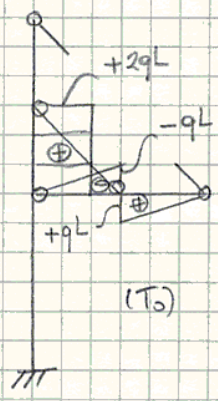
IPE 330 $\begin{cases} W_1 = 713\ cm^3 \\ A = 63\ cm^2 \\ I_1 = 11770\ cm^4 \end{cases}$

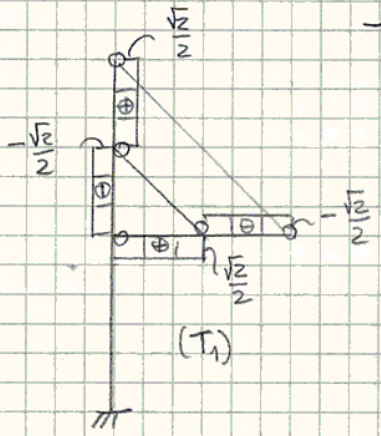
Bielle: $A \geq \frac{N_{MAX}}{\sigma_{AMM}} = \frac{\cancel{120}\cdot 10^3}{\frac{2}{3}\cancel{240}\cdot\cancel{10^6}}\cancel{10^4}\ cm^2 = 5\ cm^2$

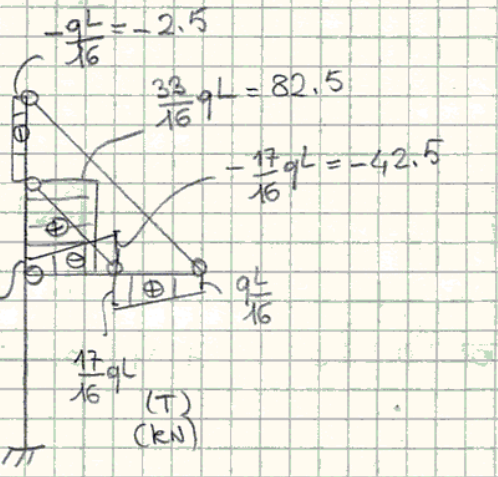

$-qL^2/2$
(M_0)
$-2qL^2$

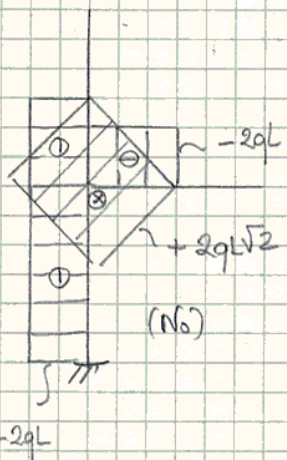

$-\frac{\sqrt{2}}{2}L$
$\frac{\sqrt{2}}{2}L$
(M_1)

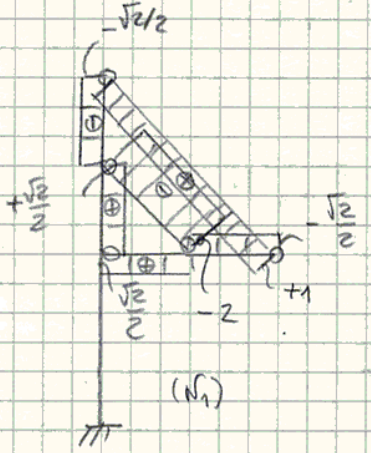

$5 = \frac{qL^2}{16}$
$-\frac{9}{16}qL^2 = -45$
(M)
(kNm)
$-2qL^2$
$= -160$

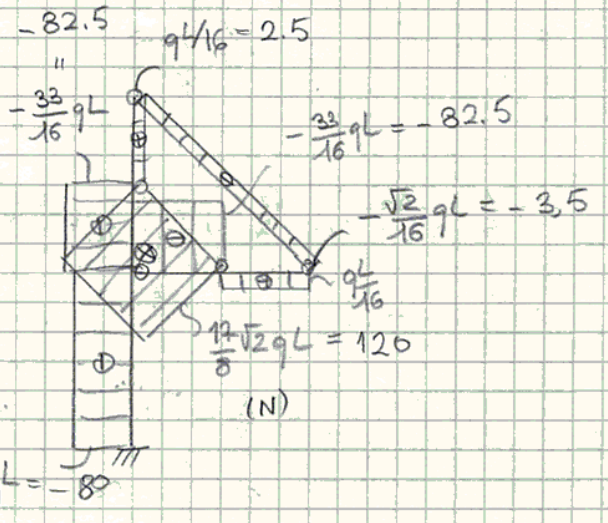

$+2qL$
$-qL$
$+qL$
(T_0)

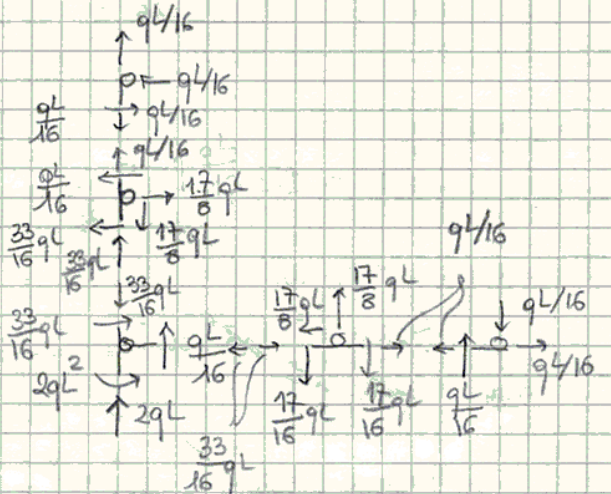

$\frac{\sqrt{2}}{2}$
$-\frac{\sqrt{2}}{2}$
$-\frac{\sqrt{2}}{2}$
$\frac{\sqrt{2}}{2}$
(T_1)


$-\frac{qL}{16} = -2.5$
$\frac{33}{16}qL = 82.5$
$-\frac{17}{16}qL = -42.5$
$\frac{qL}{16}$
$-\frac{qL}{16}$
$= -2.5$
$\frac{17}{16}qL$
(T)
(kN)

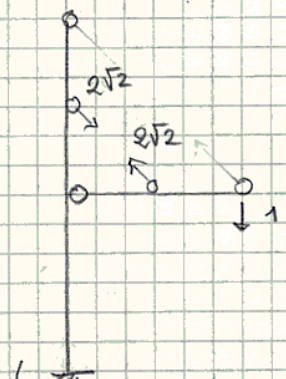

$-2qL$
$+2qL\sqrt{2}$
(N_0)
$-2qL$


$-\sqrt{2}/2$
$+\frac{\sqrt{2}}{2}$
$-\frac{\sqrt{2}}{2}$
$\frac{\sqrt{2}}{2}$
-2
$+1$
(N_1)


-82.5
$= -\frac{33}{16}qL$
$qL/16 = 2.5$
$-\frac{33}{16}qL = -82.5$
$-\frac{\sqrt{2}}{16}qL = -3.5$
$\frac{qL}{16}$
$\frac{17}{8}\sqrt{2}qL = 120$
(N)
$-2qL = -80$


$qL/16$
$\frac{qL}{16}$
$\frac{17}{8}qL$
$\frac{33}{16}qL$
$2qL^2$
$2qL$
$\frac{17}{16}qL$
$\frac{33}{16}qL$

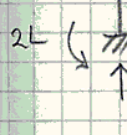
Spostamento verticale in D:



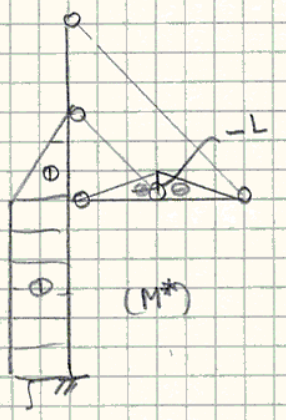
$$1\cdot v_D = \frac{1}{EI_1}\int_S M M^* \, dx$$

$$= \frac{1}{EI_1}\left\{ 2L\left(-2qL^2\right)(-2L) + \int_0^L (-2x)\left(\frac{qL^2}{16}\cdot\frac{3}{8} - \frac{33}{16}qLx\right)dx \right.$$

$$\left. + 2\int_0^L \left(-\frac{qL}{16}x - q\frac{x^2}{2}\right)(-x)\,dx \right\}$$



$$= \frac{1}{EI_1}\left\{ 8qL^4 + \frac{21}{16}qL^4 + \frac{7}{24}qL^4 \right\}$$



$$= \frac{461}{48}\,\frac{qL^4}{EI_1}$$

$$= \frac{461}{48}\,\frac{20\cdot 10^3\cdot 16}{210\cdot 11770\cdot 10^8}\cdot 10^2 \text{ cm} = 12{,}43 \text{ cm}$$

CARICO TERMICO

$$N_{1t} = \int_S N_1 \varepsilon_t \, dx = -2\alpha\Delta T L + 1\alpha\Delta T L = -\alpha\Delta T L$$

$$X_1 = -\frac{N_{10}}{N_{11}} - \frac{N_{1t}}{N_{11}} = -\frac{\sqrt{2}}{16}qL + \frac{2\alpha\Delta T K}{2L}\,3EI_1$$

$$= \left\{ -3{,}53 + \frac{10^{-5}\cdot 10\cdot 3\cdot 210\cdot 10^2\cdot 11770\cdot 10^8\cdot 10^{-3}}{4} \right\} \text{kN}$$

$$= \{-3{,}53 + 1{,}85\}\,\text{kN} = -1{,}67 \text{ kN}$$

Occorre rifare i diagrammi...