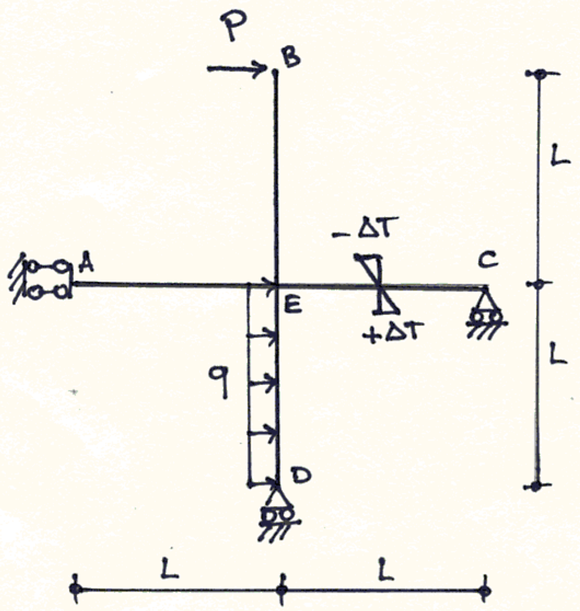
A

CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 21/12/12



$L = 1\,m$ $\quad$ $P = 60\,kN$ $\quad$ $\sigma_{AMM} = 240\,MPa$

$q = 20\,kN/m$ $\quad$ $E = 210\,GPa$ $\quad$ $\Delta T = 20°C$

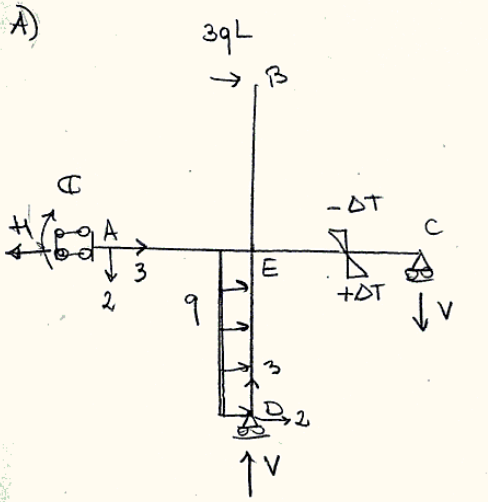
$\alpha = 10^{-5}\,°C^{-1}$

La travatura iperstatica di figura è realizzata con profilati IPE .

1. Utilizzando il metodo delle forze risolvere la travatura in assenza del carico termico e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). In questa fase è consentito trascurare le deformazioni assiali.
2. Dimensionare la struttura.
3. Calcolare la rotazione del nodo E.
4. Risolvere nuovamente la travatura considerando anche il carico termico e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi dei carichi considerati al punto 1 e del carico termico.

NB. Carico termico solo sul tratto EC.

A)



$H = 4qL$

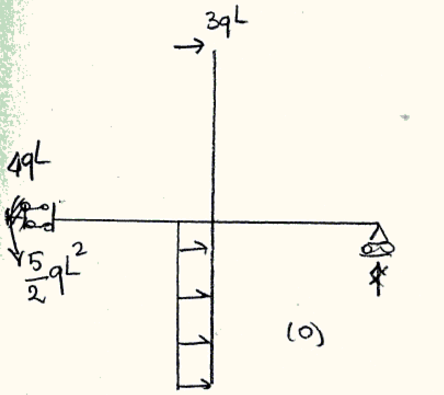
$V - V = 0$

$(E\circlearrowright)\; -VL + q\frac{L^2}{2} - \mathcal{C} - 3qL^2 = 0$

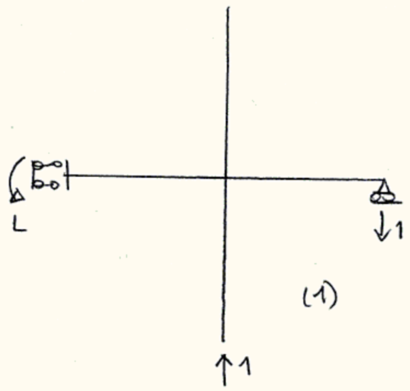
Travatura una volta iperstatica.

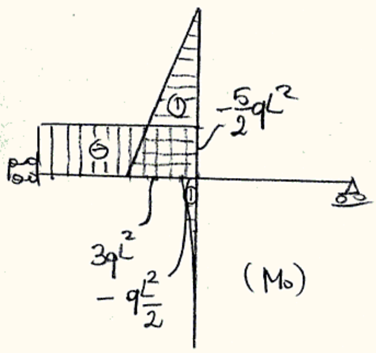
Incognita iperstatica $X_1 = V$

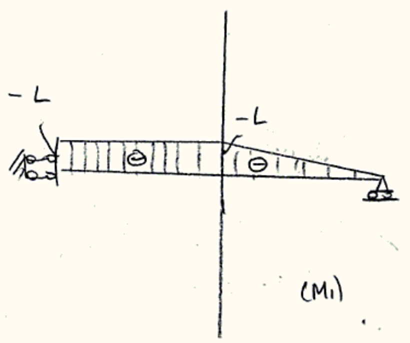
Solo q e 3qL:

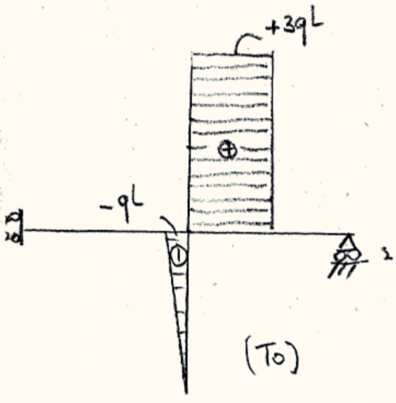


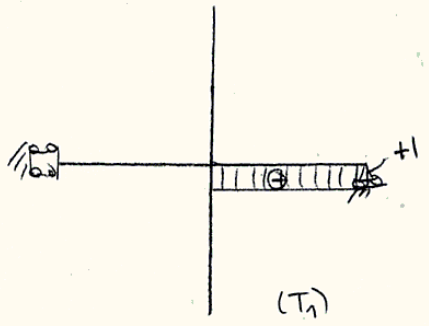
+ X_1

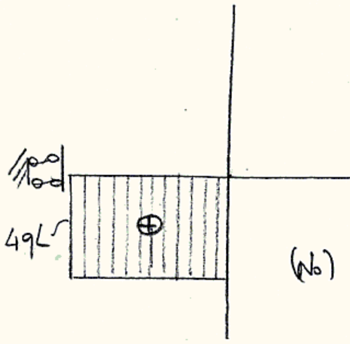


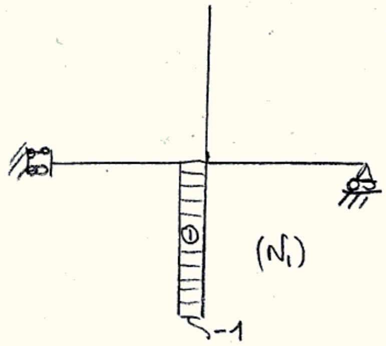












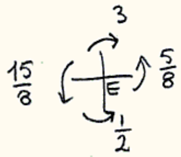
$0 = \eta_{10} + \eta_{11} X_1$

$\eta_{10} = \frac{1}{EI_1} L^2 \frac{5}{2} q L^2 = \frac{5}{2} \frac{qL^4}{EI_1}$

$\eta_{11} = \frac{1}{EI_1}\left(L^3 + \frac{L^3}{3}\right) = \frac{4}{3} \frac{L^3}{EI_1}$

$X_1 = -\frac{\eta_{10}}{\eta_{11}} = -\frac{5}{2} qL \frac{3}{4} = -\frac{15}{8} qL$

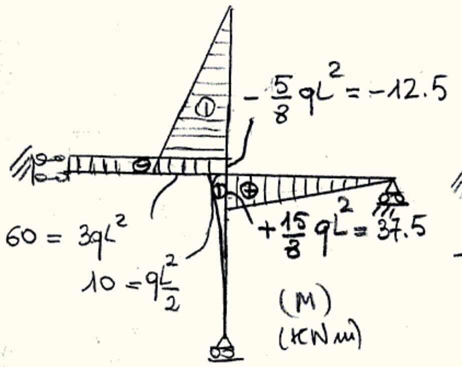
$M_{EA} = \frac{4 \cdot 5}{4 \cdot 2} qL^2 - \frac{5}{8} qL^2 = \frac{15}{8} qL^2$

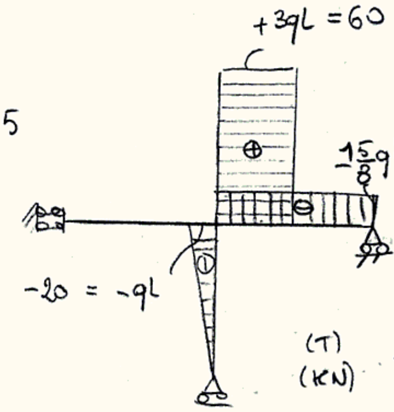


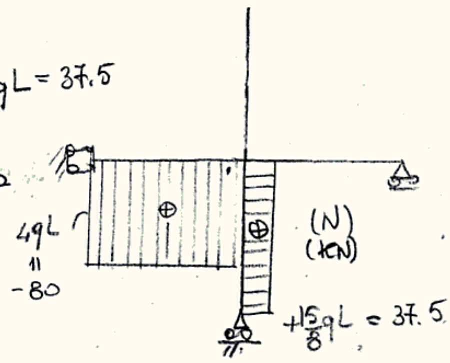
$\frac{20}{8} + \frac{4}{2 \cdot 4} = \frac{24}{8} = 3$ ok

$qL^2 = 20$ kNm

$qL = 20$ kN





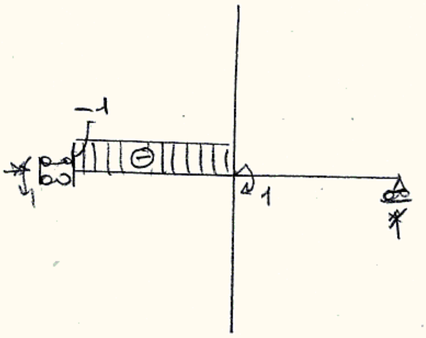


Dimensionamento: $W_1 \geq \frac{3qL^2}{\sigma_{AMM}} = \frac{3 \cdot 20 \cdot 10^3}{240 \cdot 10^6} \cdot 10^6 \text{ cm}^3 = 250 \text{ cm}^3$

IPE 240
- $W_1 = 324{,}3 \text{ cm}^3$
- $A = 39{,}12 \text{ cm}^2$
- $I_1 = 3892 \text{ cm}^4$
- $H = 240$ mm

Rotazione:

$$1 \cdot \varphi_E = \frac{1}{EI_1} \frac{5}{8} q L^2 \cdot L = \frac{1}{EI_1} \frac{5}{8} q L^3$$



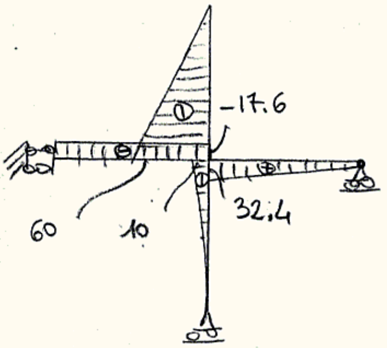
Carico termico : $0 = \eta_{1t} + \eta_{10} + \eta_{11} X_1$

$$\eta_{1t} = - \frac{2\alpha \Delta T}{H} \frac{L^2}{2}$$

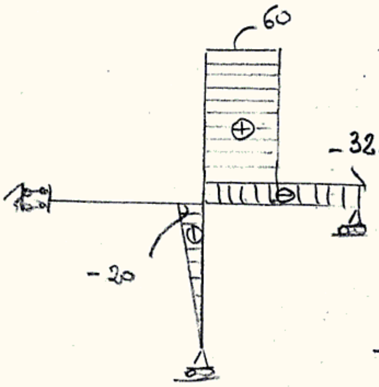
$$\hookrightarrow X_1 = -\frac{5}{8} q L + \frac{\alpha \Delta T L^2}{H} \frac{3 EI_1}{4 L^3}$$

$$= -\frac{15}{8} q L + \frac{3 \alpha \Delta T E I_1}{4 L H} = \left(-37.5 + \frac{3 \cdot 10^{-4} \cdot 20 \cdot 210 \cdot 10^{3} \cdot 3892 \cdot 10^{-8}}{4 \cdot 0,24} \right) kN$$

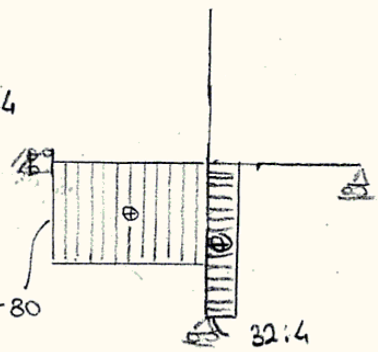
$$= (-37.5 + 5,1)\ kN = -32.4\ kN$$



(M)
(kNm)



(T)
(kN)



(N)
(kN)

B

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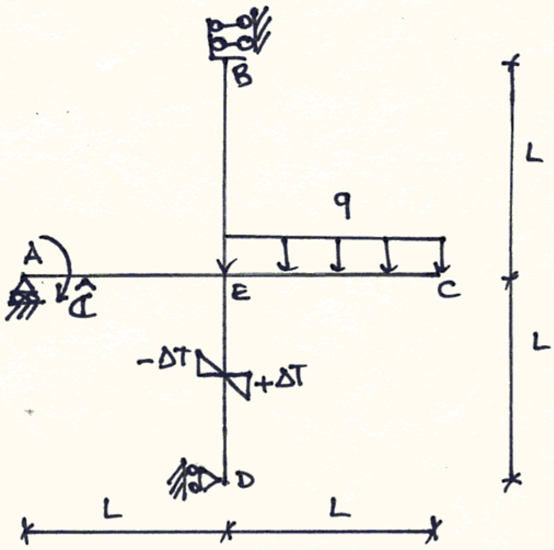

$L = 2\,m$ $\quad E = 210\,GPa$ $\quad \Delta T = 20\,°C$

$q = 10\,kN/m$ $\quad \sigma_{AMM} = 240\,MPa$ $\quad \alpha = 10^{-5}\,°C^{-1}$

$\hat{C} = 20\,kNm$

La travatura iperstatica di figura è realizzata con profilati IPE .

1. Utilizzando il metodo delle forze risolvere la travatura in assenza del carico termico e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). In questa fase è consentito trascurare le deformazioni assiali.
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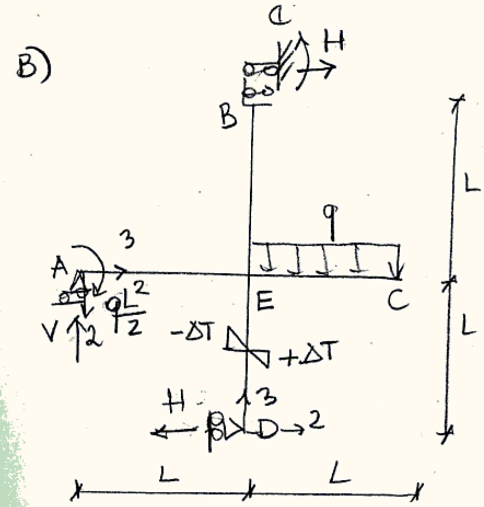
N.B. Carico termico solo su ED.

B)



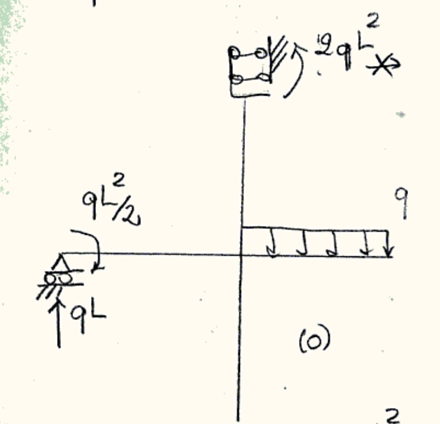
$$\begin{cases} H - H = 0 \\ V = qL \\ (A\circlearrowright)\ -2HL + C - q\frac{L^2}{2} - \frac{3}{2} L\, qL = 0 \end{cases}$$

$$\begin{cases} V = qL \\ C = 2HL + 2qL^2 \end{cases}$$

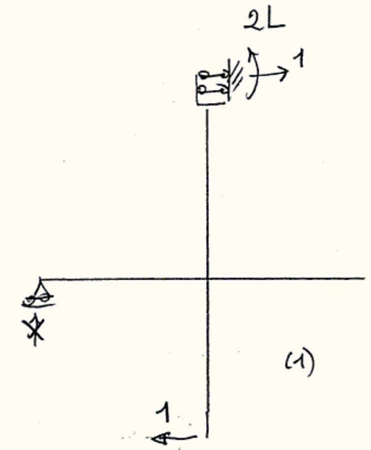
Trovatura una volta iperstatica

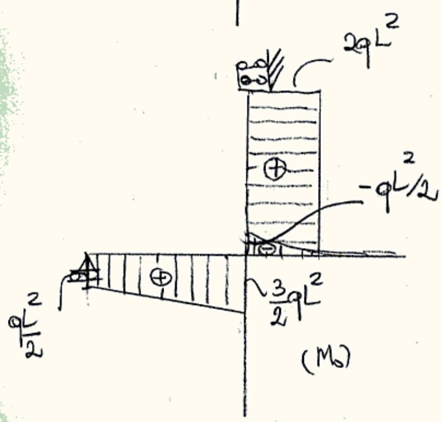
Incognita iperstatica : $X_1 = H$

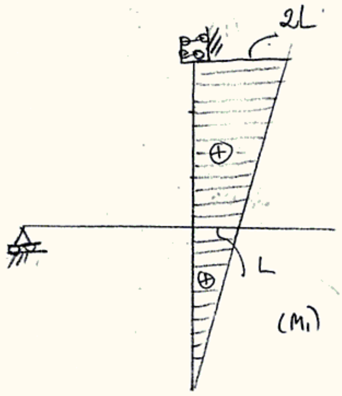
Solo q e coppia in A:

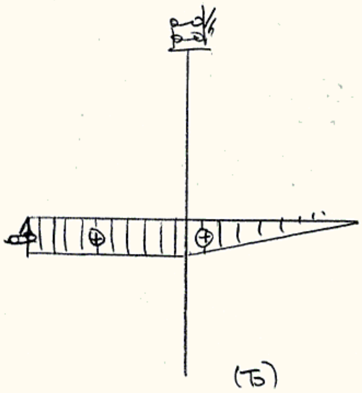


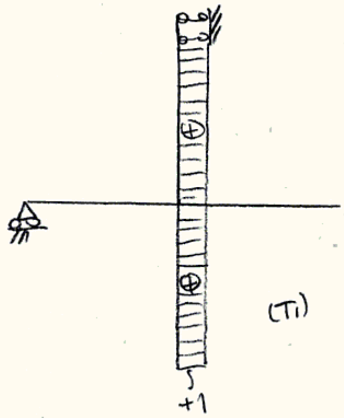
$+ X_1$

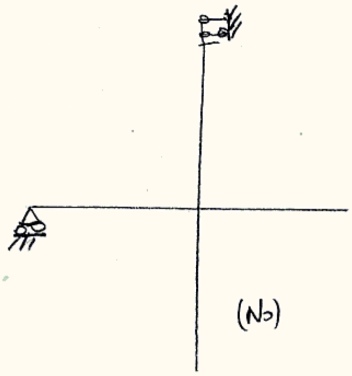


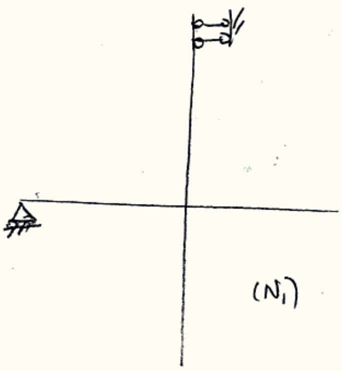












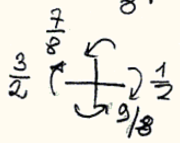
$$\eta_{10} = \cancel{2}qL^2\, L\, \frac{3L}{\cancel{2}EI_1} = \frac{3qL^4}{EI_1}$$

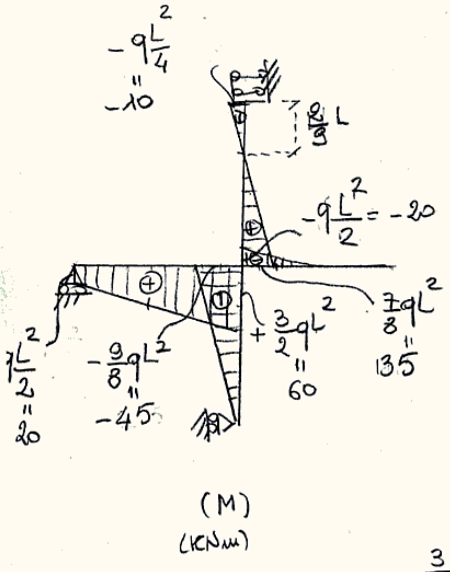
$$\eta_{11} = \frac{1}{3}\, 2L\, (2L)^2 = \frac{8L^3}{3EI_1}$$

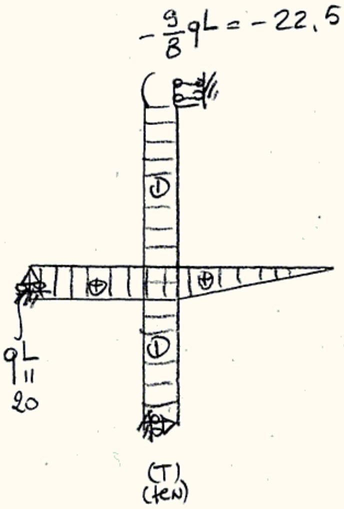
$$0 = \eta_{10} + \eta_{11} X_1 \quad \longrightarrow \quad X_1 = -\frac{\eta_{10}}{\eta_{11}} = -\frac{3qL^4}{EI_1}\,\frac{3EI_1}{8L^3} = -\frac{9}{8}qL$$

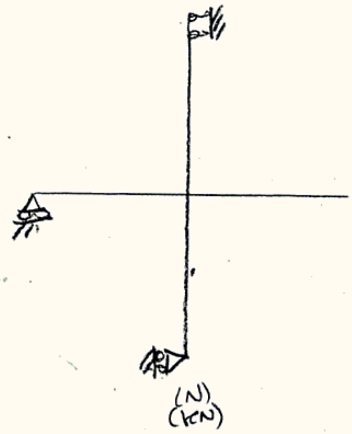
$$M_B = 2qL^2 - \frac{9}{8}\cdot 2qL^2 = -2\cdot\frac{qL^2}{8} = -\frac{qL^2}{4}$$

$$M_{EB} = 2qL^2 - \frac{9}{8}qL^2 = \frac{7}{8}qL^2$$



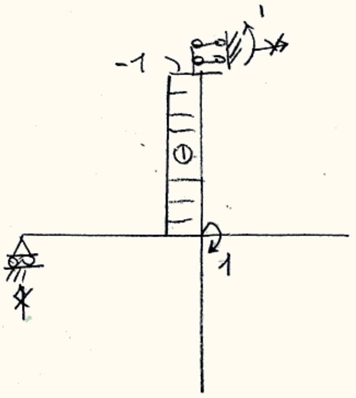






Dimensionamento: $W_1 \geq \frac{\frac{3}{2}qL^2}{\sigma_{AMM}} = \frac{3\cdot 10\cdot 10^3\cdot 4\cdot 10^6}{2\cdot 240\cdot 10^6} = \frac{3000}{12}\,cm^3 = 250\,cm^3$

IPE 240
- $W_1 = 324{,}3\ cm^3$
- $A = 39{,}12\ cm^2$
- $I_1 = 3892\ cm^4$
- $H = 240\ mm$



$$1\cdot\varphi_E = \frac{1}{EI_1}\left[q\frac{L^2}{4}\cdot\frac{1}{2}\cdot\frac{2}{3}L - 7q\frac{L^2}{8\cdot 2}\,\frac{7}{9}L\,\frac{1}{2}\right]$$

$$= \frac{qL^3}{36EI_1}\left[1-\frac{49}{4}\right] = -\frac{45}{144}\frac{qL^3}{EI_1} = -\frac{5}{16}\frac{qL^3}{EI_1}$$

$$= -\frac{5\cdot 10^{3}\cdot 8}{16\cdot 210\cdot 10^{6}\cdot 3892\cdot 10^{-8}} = -0{,}00306 = -0{,}17^\circ$$

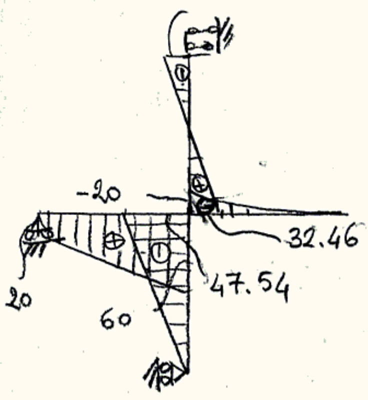
Carico termico su ED: $0 = \eta_{1t} + \eta_{10} + \eta_{11}X_1$

$$\eta_{1t} = \frac{\alpha\Delta T}{H}\,\frac{L^2}{2}$$

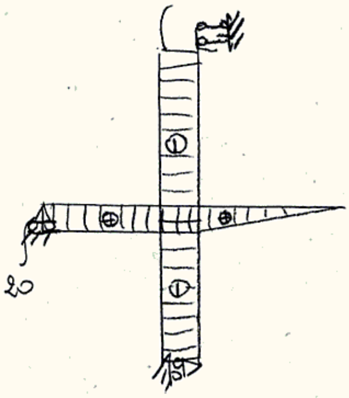
$$X_1 = -\frac{9}{8}qL - \frac{\alpha\Delta T L^2}{H}\,\frac{3EI_1}{8L^3}$$

$$= -\frac{9}{8}qL - \frac{3}{8}\,\frac{\alpha\Delta T EI_1}{LH} = \left(-22{.}5 - \frac{3\cdot 10^{-5}\cdot 20\cdot 210\cdot 10^{6}\cdot 3892\cdot 10^{-8}}{16\cdot 0{,}24}\right)\text{kN}$$

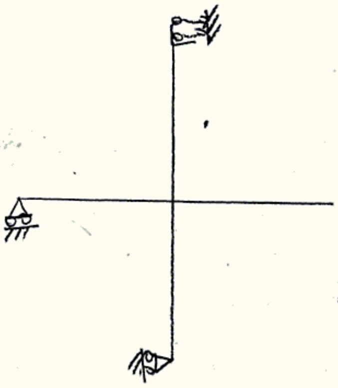
$$= (-22{.}5 - 1{,}27)\ \text{kN} = -23{,}77\ \text{kN}$$



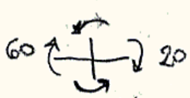
(M)
(kNm)



(T)
(kN)



(N)
(kN)



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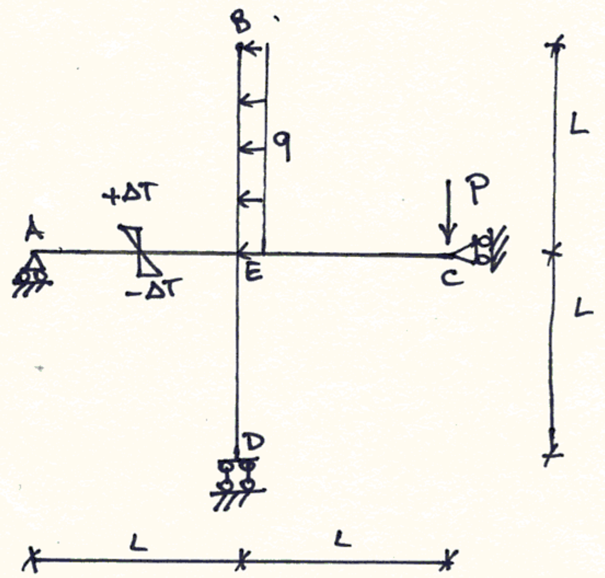

$L = 2\,m$ $\quad E = 210\,GPa$ $\quad \Delta T = 20\,°C$

$q = 15\,kN/m$ $\quad \sigma_{AMM} = 240\,MPa$ $\quad \alpha = 10^{-5}\,°C^{-1}$

$P = 30\,kN$

La travatura iperstatica di figura è realizzata con profilati IPE .

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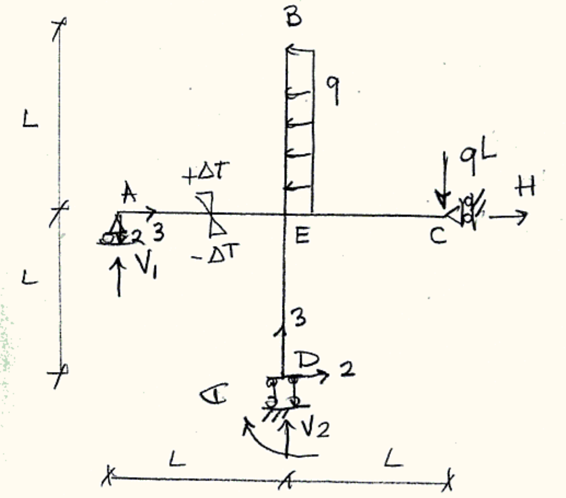
N.B. Carico termico solo su AE.

c)

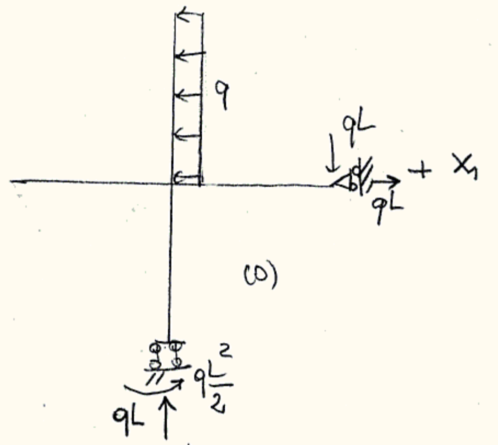


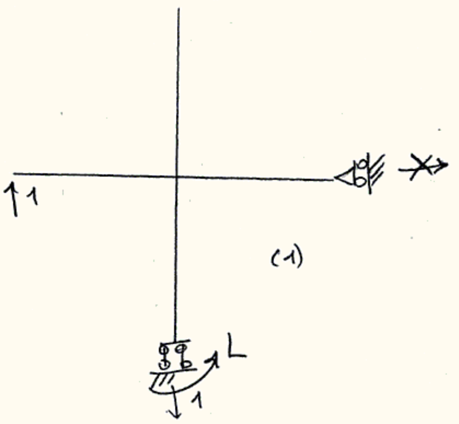
$$\begin{cases} H = qL \\ V_1 + V_2 = qL \\ (E\circlearrowright)\quad q\frac{L^2}{2} - qL^2 - \mathcal{C} - V_1 L = 0 \end{cases}$$

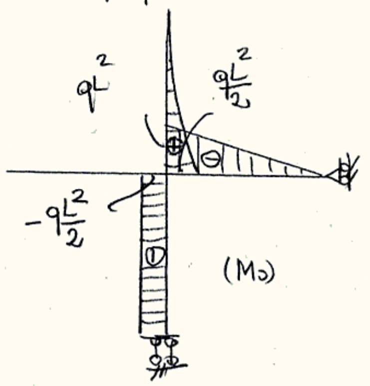
Travatura una volta iperstatica.

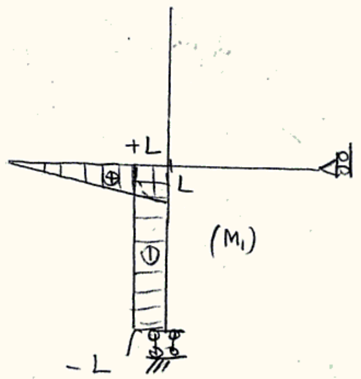
Incognita iperstatica: $X_1 = V_1$

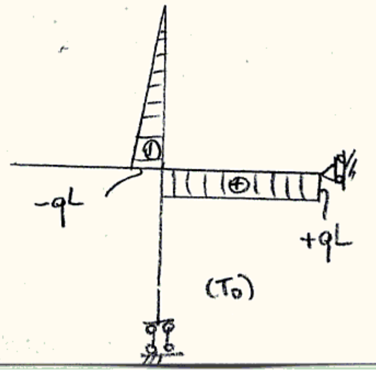
Solo q e qL :

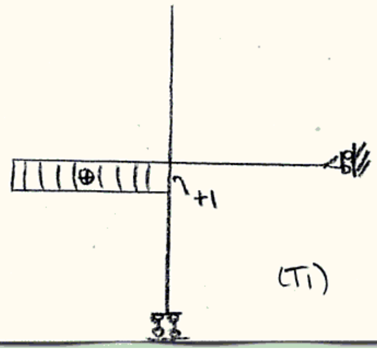


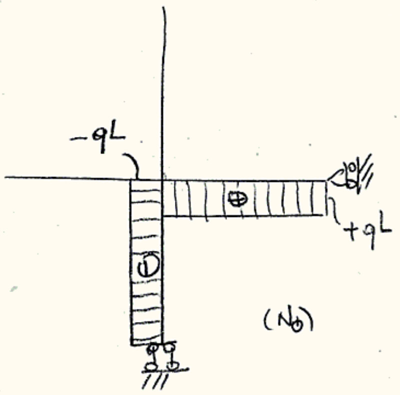


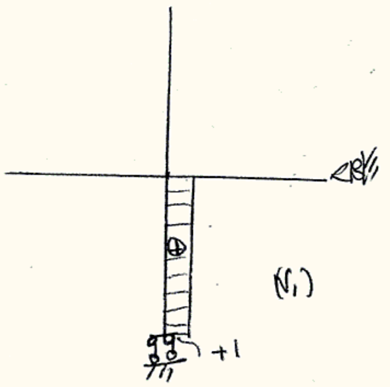












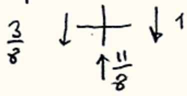
$$0 = \eta_{10} + \eta_{11} X_1$$

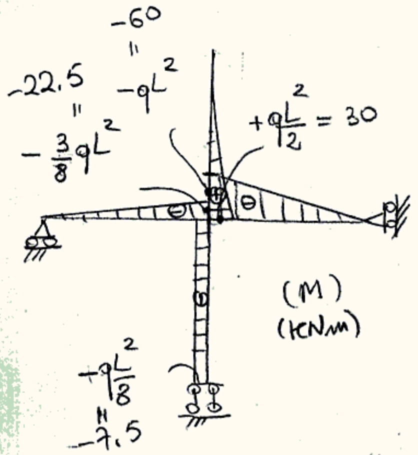
$$\eta_{10} = \frac{1}{EI_1}\, q\frac{L^2}{2} L L = \frac{qL^4}{2EI_1}$$

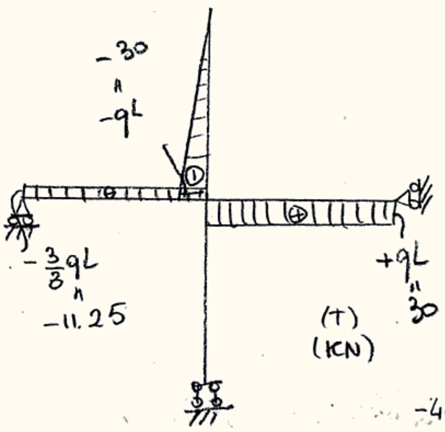
$$\eta_{11} = \frac{L^3}{3} + L^3 = \frac{4}{3}\frac{L^3}{EI_1}$$

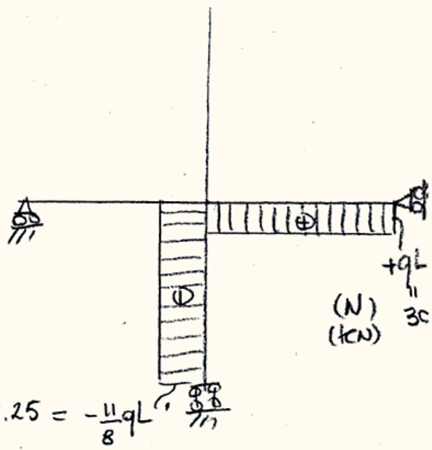
$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = -\frac{qL}{2}\frac{3}{4} = -\frac{3}{8}qL$$

$$M_{ED} = \frac{4}{4}\frac{qL^2}{2} - \frac{3}{8}qL^2 = +q\frac{L^2}{8}$$









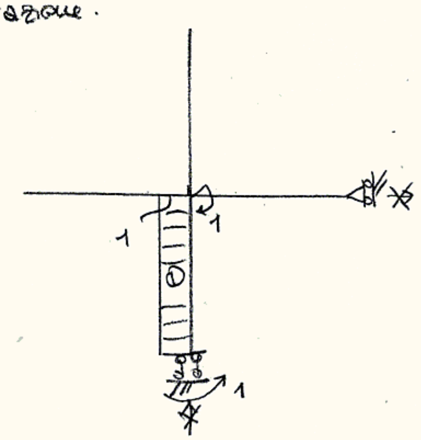
Dimensionamento:

$$W_1 \geq \frac{qL^2}{\sigma_{amm}} = \frac{60 \cdot 10^3 \cdot 10^6}{240 \cdot 10^6} = 250\ cm^3$$

IPE 240

$$\begin{cases} W_1 = 324,3\ cm^3 \\ A = 39,13\ cm^2 \\ I_1 = 3892\ cm^4 \\ H = 240\ mm \end{cases}$$

Rotazione.



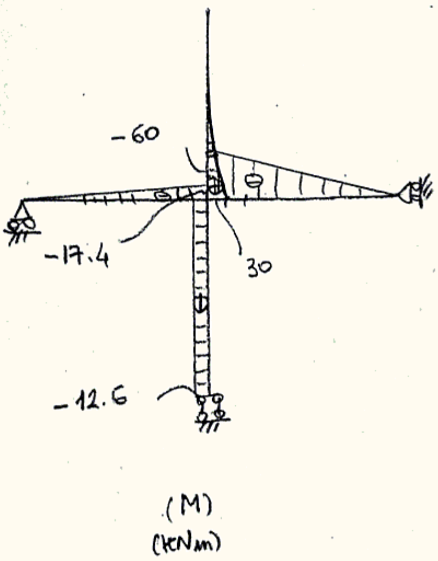
$$1 \cdot \varphi_E = \frac{1}{EI_1}\frac{qL^3}{8} = \frac{15 \cdot 10^{-3} \cdot 8}{8 \cdot 210 \cdot 10^{9} \cdot 3892 \cdot 10^{-8}} = 0.00183 \stackrel{!}{=} 0,105^\circ$$

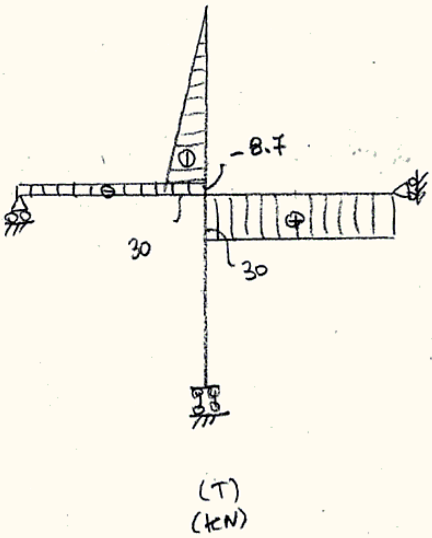
Carico termico in AE: $0 = \eta_{1t} + \eta_{10} + \eta_{11} X_1$

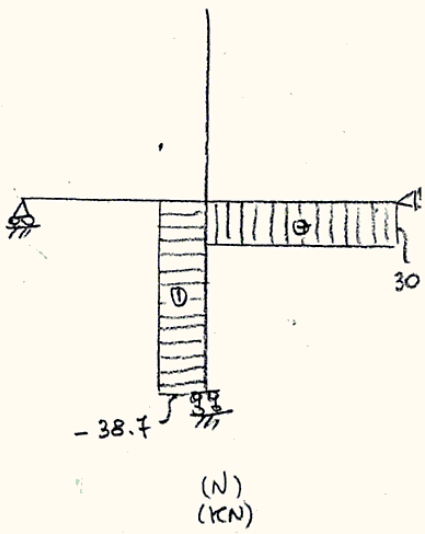
$$\eta_{1t} = -\frac{\alpha \Delta T}{H}\frac{L^2}{2}$$

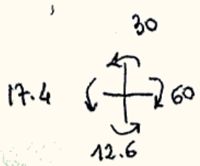
$$X_1 = -\frac{3}{8}qL + \frac{\alpha \Delta T L^2}{H}\frac{3EI_1}{4L^3} \stackrel{!}{=} -\frac{3}{8}qL + \frac{3\alpha\Delta T E I_1}{4HL} = \left(-11.25 + \frac{3 \cdot 10^{-5} \cdot 20 \cdot 210 \cdot 10^{9} \cdot 3892 \cdot 10^{-8}}{4 \cdot 0,24 \cdot 2} \cdot 10^{-3}\right) kN$$

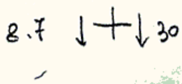
$$\stackrel{!}{=} (-11.25 + 2.55)\, kN = -8.7\ kN$$





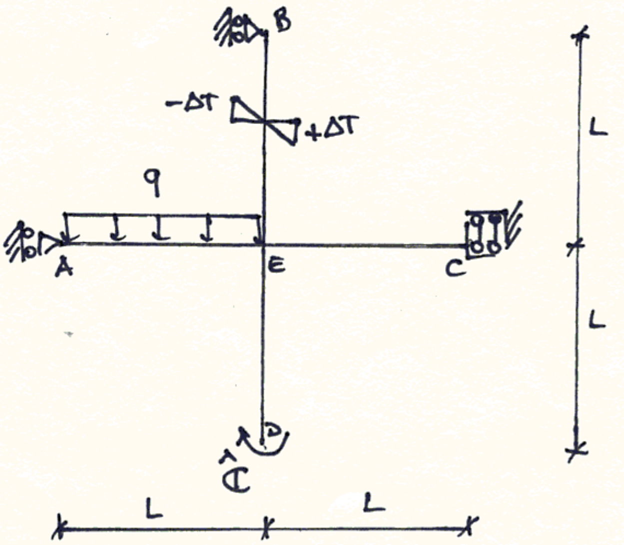






CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 21/12/12

D



$L = 1\ m$ $E = 210\ GPa$ $\Delta T = 20°C$

$q = 25\ kN/m$ $\sigma_{AMM} = 240\ MPa$ $\alpha = 10^{-5}\ °C^{-1}$

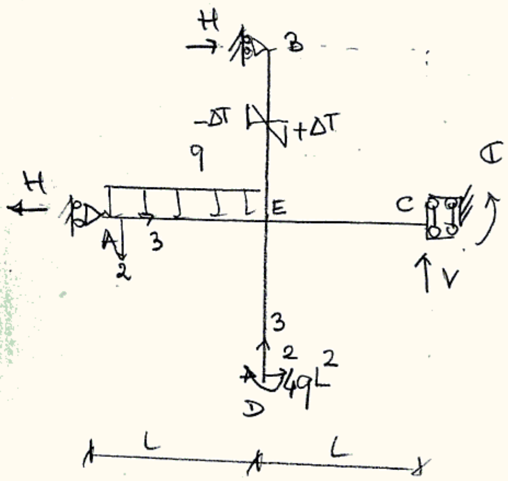
$\hat{C} = 100\ kNm$

La travatura iperstatica di figura è realizzata con profilati IPE .

1. Utilizzando il metodo delle forze risolvere la travatura in assenza del carico termico e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M). In questa fase è consentito trascurare le deformazioni assiali.
2. Dimensionare la struttura.
3. Calcolare la rotazione del nodo E.
4. Risolvere nuovamente la travatura considerando anche il carico termico e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi dei carichi considerati al punto 1 e del carico termico.

N.B. Carico termico solo su BE.

D)

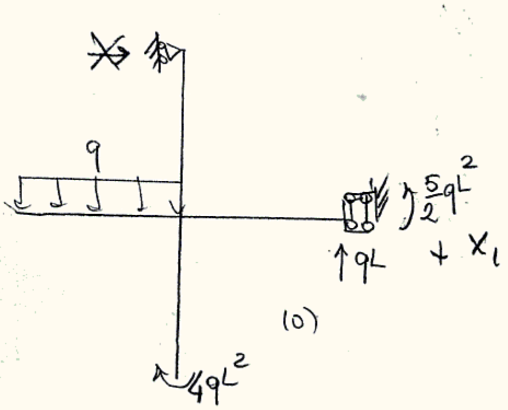


$$\begin{cases} V = qL \\ H - H = 0 \\ (C\circlearrowleft)\ C - 4qL^2 + qL\frac{3}{2}L - HL = 0 \end{cases}$$

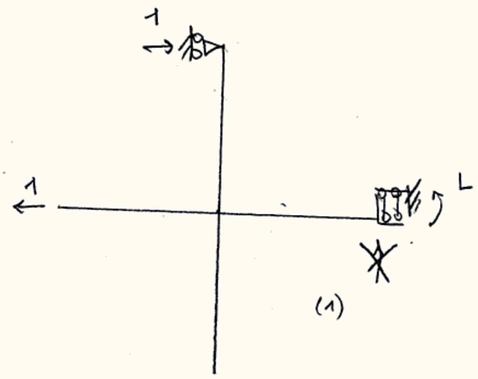
Struttura una volta iperstatica

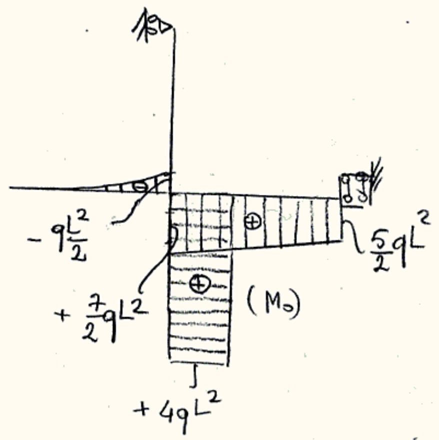
Incognita iperstatica: $X_1 = H$

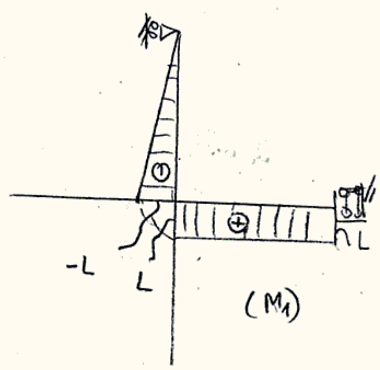
Solo q e $4qL^2$:

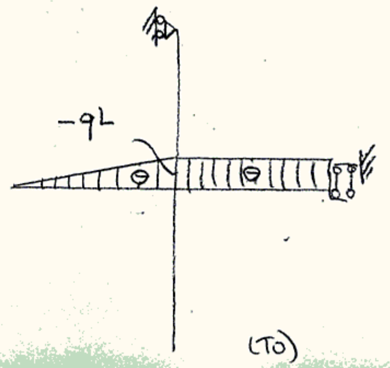


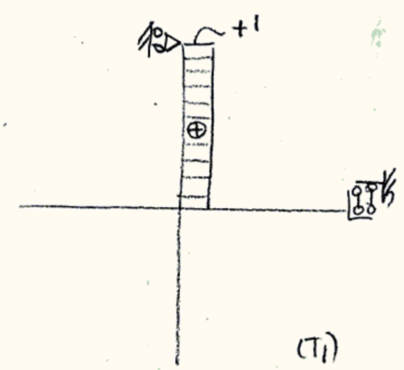
$+ X_1$

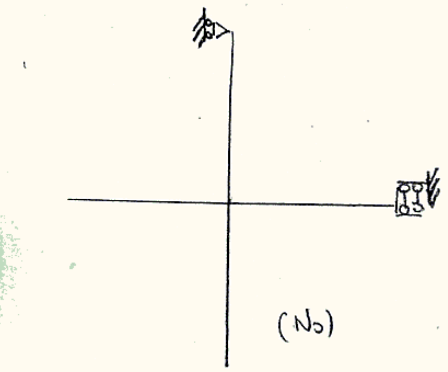


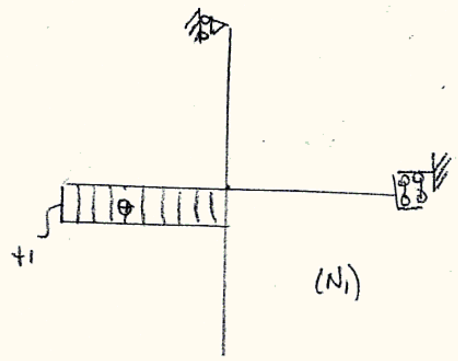












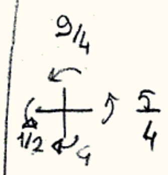
$$\eta_{10} = \frac{1}{EI_1} L \frac{L}{2} \frac{3}{2} qL^2 = 3\frac{qL^4}{EI_1}$$

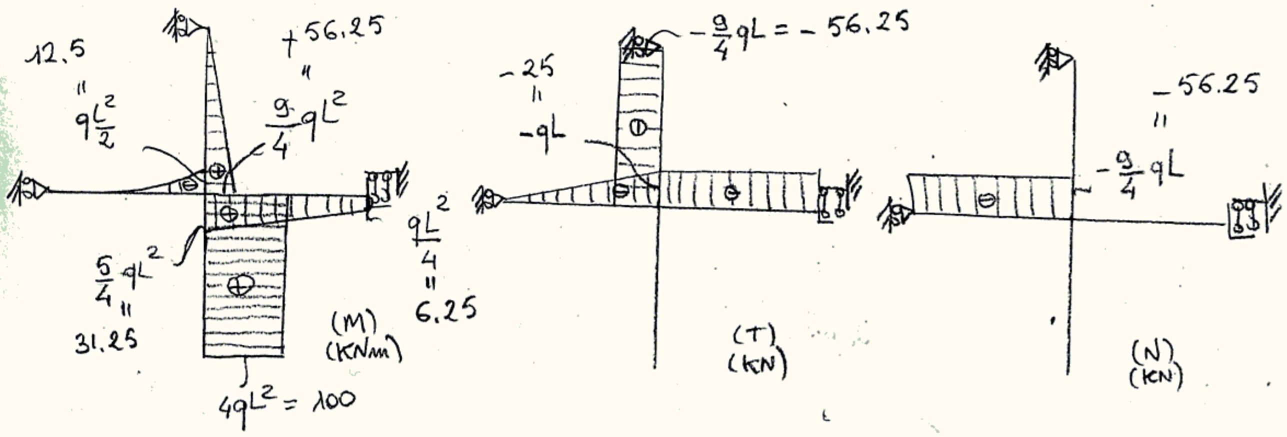
$$\eta_{11} = \frac{1}{EI_1}\left(L^3 + \frac{1}{3}L^3\right) = \frac{4L^3}{3EI_1}$$

$$0 = \eta_{10} + \eta_{11} X_1$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = -3\cdot\frac{3}{4}qL = -\frac{9}{4}qL$$

$$\frac{3\cdot 5}{2\cdot 2} - \frac{9}{4} = \frac{1}{4}$$



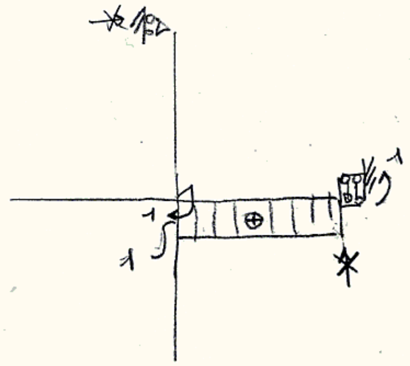


Dimensionamento:

$$W_1 \geq \frac{4qL^2}{\sigma_{amm}} = \frac{4\cdot 25\cdot 10^3\cdot 10^6}{240\cdot 10^6} = 417\ cm^3$$

IPE 270

- $W_1 = 428.9\ cm^3$
- $A = 45.95\ cm^2$
- $I_1 = 5790\ cm^4$
- $H = 270\ mm$



$$1\cdot\varphi_E = \frac{1}{EI_1}\,\frac{L}{2}\,\frac{\alpha^3}{4}\,qL^2 = \frac{3qL^3}{4EI_1} = \frac{3\cdot 25\cdot 10^3\cdot 1}{4\cdot 210\cdot 10^9\cdot 5790\cdot 10^{-8}} = 0.0015 = 0.08^\circ$$

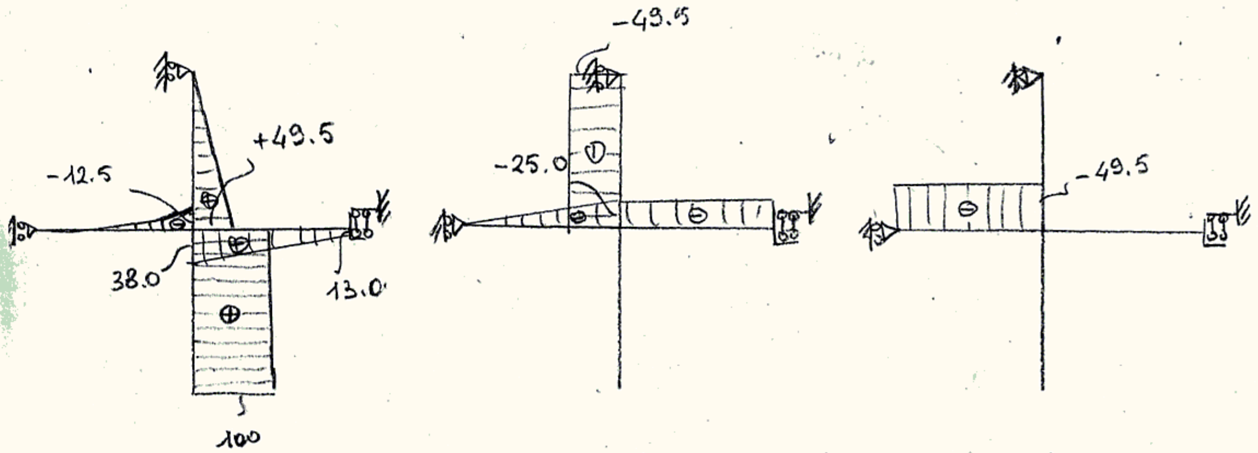
Carico termico: $0 = \eta_{1t} + \eta_{10} + \eta_{11} X_1$

$$\eta_{1t} = -\frac{2\alpha\Delta T}{H}\,\frac{L^2}{2}$$

$$X_1 = -\frac{9}{4}qL + \frac{\alpha\Delta T L^2}{H}\,\frac{3EI_1}{4L^3}$$

$$= \left(-56.25 + \frac{10^{-5}\cdot 20\cdot 3\cdot 210\cdot 10^9\cdot 5790\cdot 10^{-8}}{4\cdot 0.27\cdot 1}\cdot 10^{-3}\right)\text{kN}$$

$$= (-56.25 + 6.75)\,\text{kN} = -49.5\ \text{kN}.$$



(M) (kNm) (T) (kN) (N) (kN)