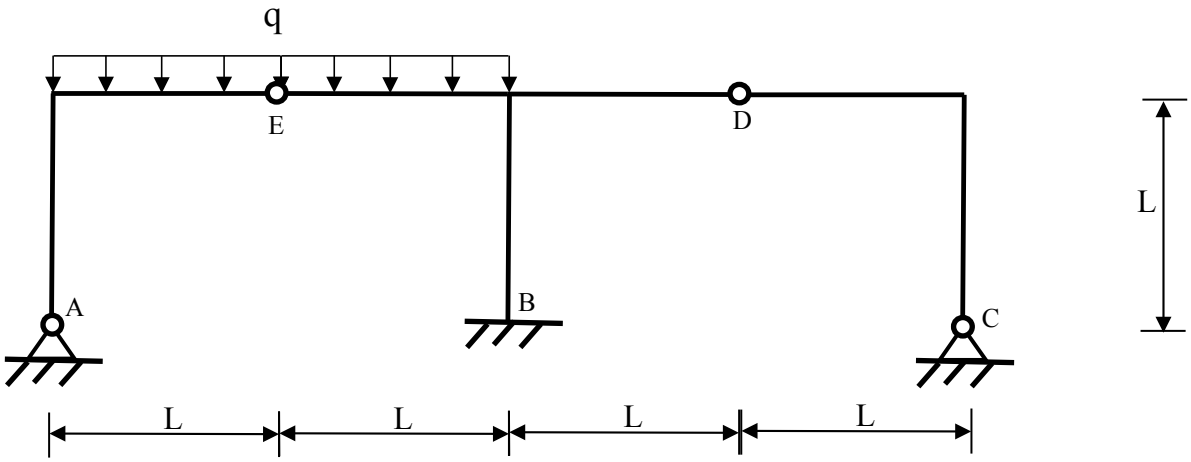
LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA

PROVA SCRITTA DI STATICA

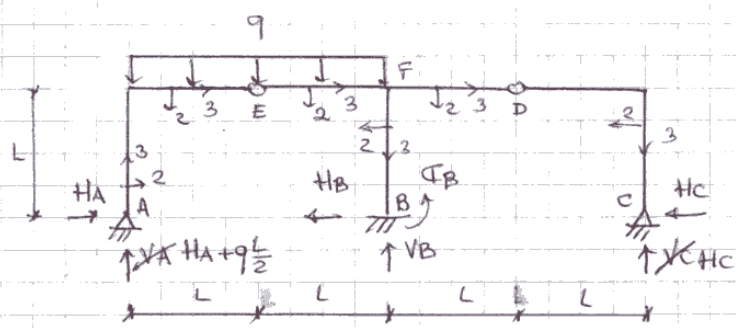
FERRARA, 04/07/2013



L= 3 m, q = 20 kN/m

σ_{AMM} = 240 MPa , E = 210 GPa

1. Utilizzando il metodo delle forze risolvere la travatura iperstatica di figura e disegnarne i diagrammi delle caratteristiche di sollecitazione (N, T, M).
2. Dimensionare la travatura con profilati IPE.
3. Calcolare lo spostamento verticale in E.



$q = 20\ kN/m$
$L = 3\ m$

Eq.ni ausiliarie (sconnessione in D, E):

$V_C L - H_C L = 0 \rightarrow V_C = H_C$

$V_A 4 - H_A 4 - qL\frac{4}{2} = 0 \rightarrow V_A = H_A + q\frac{L}{2}$

Eq.ni cardinali della Statica:

$(\rightarrow)\ H_B + H_C = H_A$

$(\uparrow)\ H_C + V_B + H_A + q\frac{L}{2} - 2qL = 0$

$(B\uparrow)\ M_B + H_C 2L - 2L\left(H_A + q\frac{L}{2}\right) + 2qL^2 = 0$

$$\begin{cases} H_B = H_A - H_C \\ V_B = -H_C - H_A + \frac{3}{2}qL \\ M_B = 2L(H_A - H_C) - qL^2 \end{cases}$$

Incognite iperstatiche: $X_1 = H_A$

$X_2 = H_C$

Diagrammi portati alla pagina successiva

Calcolo delle reazioni vincolari nel sistema (0):

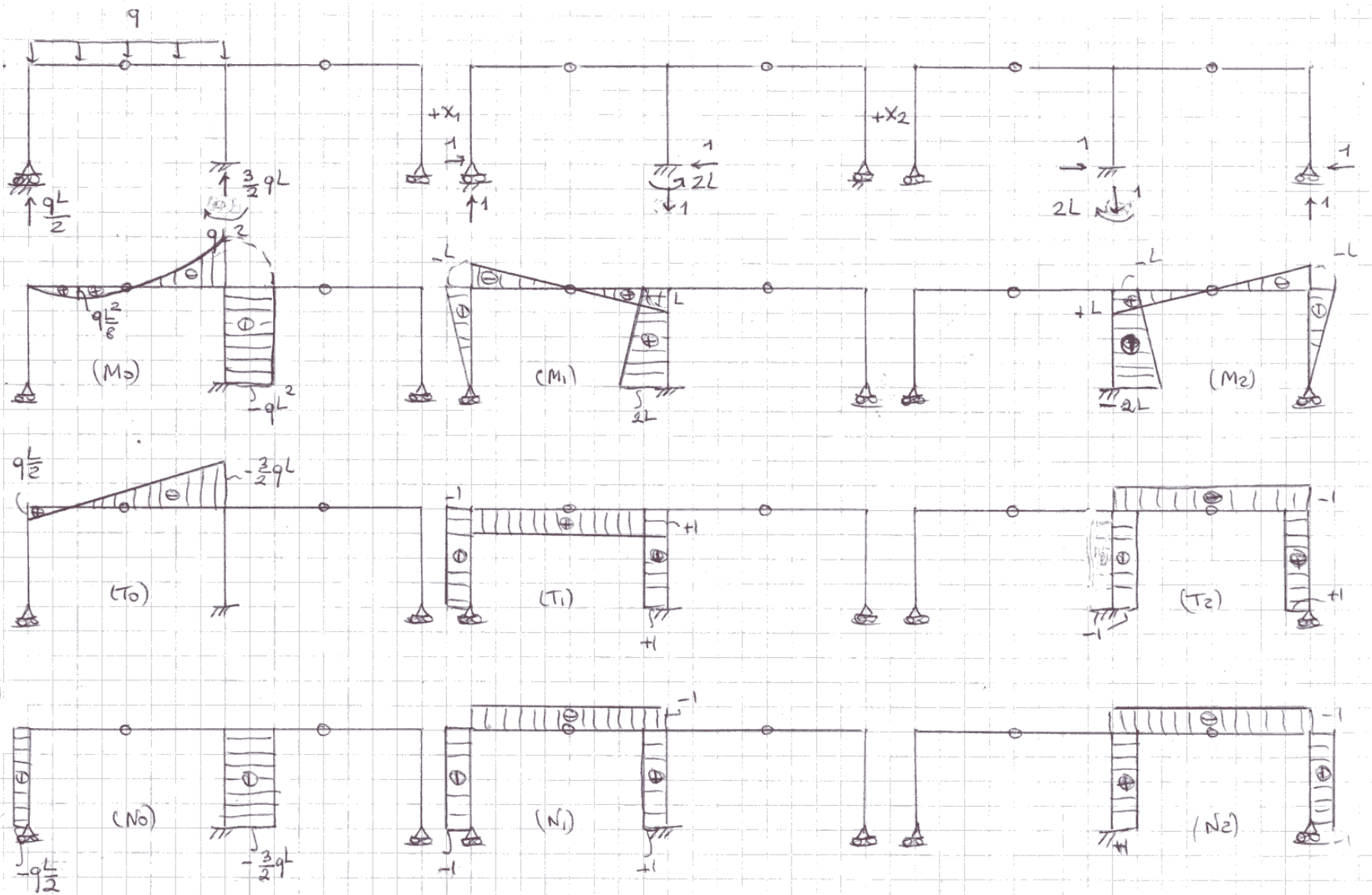
$$\begin{cases} H_A = 0 = H_C \\ H_B = 0 \\ V_B = \frac{3}{2}qL \\ M_B = -qL^2 \end{cases}$$

Calcolo delle reazioni vincolari in (1):

$$\begin{cases} H_A = 1,\ H_C = 0,\ q = 0 \\ H_B = 1 \\ V_B = -1 \\ M_B = 2L \end{cases}$$

Calcolo delle reazioni vincolari in (2):

$$\begin{cases} H_A = 0,\ H_C = 1,\ q = 0 \\ H_B = -1 \\ V_B = -1 \\ M_B = -2L \end{cases}$$


q
+X1
+X2
(M0)
(M1)
(M2)
(T0)
(T1)
(T2)
(N0)
(N1)
(N2)

$$EI_1 y_{10} = \int_0^{2L} \left(q\frac{L}{2}x_3 - q\frac{x_3^2}{2}\right)(-L+x_3)\,dx_3 + L\left(-qL^2\right)\left(3L\cdot\frac{L}{2}\right)$$

$$= -\frac{qL^4}{3} - \frac{3}{2}qL^4 = -\frac{11}{6}qL^4$$

$$EI_1 y_{20} = L\left(-qL^2\right)\left(-3L\,\frac{L}{2}\right) = \frac{3}{2}qL^4$$

$$EI_1 y_{11} = EI_1 y_{22} = 3\cdot\frac{1}{3}LL^2 + \int_0^L (L+x_3)^2\,dx_3 = L^3 + \frac{7}{3}L^3 = \frac{10}{3}L^3$$

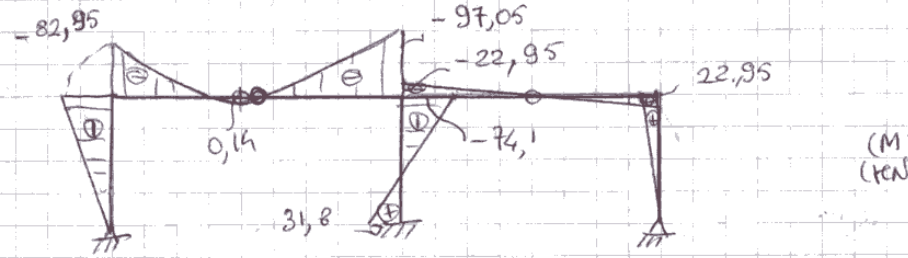
$$EI_1 y_{12} = EI_1 y_{21} = -\int_0^L (L+x_3)^2\,dx_3 = -\frac{7}{3}L^3$$

$$\begin{bmatrix} 10 & -7 \\ -7 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{qL^2}{\theta_2}\begin{bmatrix} 11 \\ -9 \end{bmatrix}$$

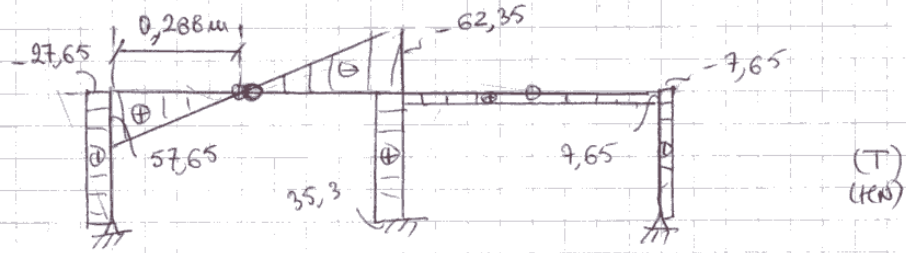
$$\det = 100 - 49 = 51$$

$$x_1 = \frac{qL}{102}\begin{vmatrix} 11 & -7 \\ -9 & 10 \end{vmatrix} = \frac{47}{102}qL = 0{,}46\,qL = 27{,}65 \text{ tenm}$$

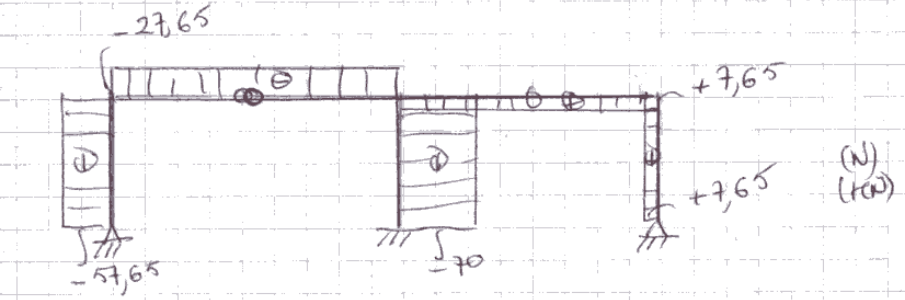
$$x_2 = \frac{qL}{102}\begin{vmatrix} 10 & 11 \\ -7 & -9 \end{vmatrix} = -\frac{13}{102}qL = -0{,}13\,qL = -7{,}65 \text{ tenm}$$



(M)
(tenm)



(T)
(ten)



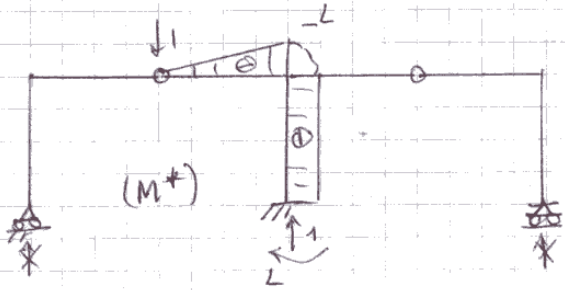
(N)
(ten)

Dimensionamento

$$W_1 \geq \frac{97{,}05 \cdot 10^6}{240 \cdot 10^8}\ m^3 = \frac{97{,}05 \cdot 10^8}{240 \cdot 10^2}\ cm^3 = 97{,}05 \cdot \frac{100}{24}\ cm^3 = 404{,}4\ cm^3$$

IPE 270
- $W_1 = 428{,}9\ cm^3$
- $I_1 = 5790\ cm^4$
- $A = 45{,}95\ cm^2$

Spostamento verticale in E:



$$T_E = -q\frac{L}{2} + \frac{47}{102}qL = -\frac{2}{51}qL$$

$$T_{F_B} = \frac{47}{102}qL + \frac{13}{102}qL = \frac{60}{102}qL = \frac{30}{51}qL$$

$$M_F = -qL^2 + \frac{47}{102}qL^2 + \frac{13}{102}qL^2 \overset{!}{=} \frac{30}{51}qL^2 - qL^2 = -\frac{7}{17}qL^2$$

$$1 \cdot \delta_E = \frac{1}{EI_1}\int_0^L (-x_3)\left(-\frac{2}{51}qLx_3 - q\frac{x_3^2}{2}\right)dx_3 + \frac{1}{EI_1}\int_0^L (-L)\left(-\frac{7}{17}qL^2 + \frac{30}{51}qLx_3\right)dx_3$$

$$= \frac{1}{EI_1}\frac{169}{1224}qL^4 + \frac{1}{EI_1}\frac{2}{17}qL^4 = \frac{313}{1224}\frac{qL^4}{EI_1}$$

$$= 0{,}255 \cdot \frac{20 \cdot 81 \cdot 10^3}{210 \cdot 10^8 \cdot 5790 \cdot 10^{-8}}\ m$$

$$= 0{,}034\ m = 3{,}4\ cm$$