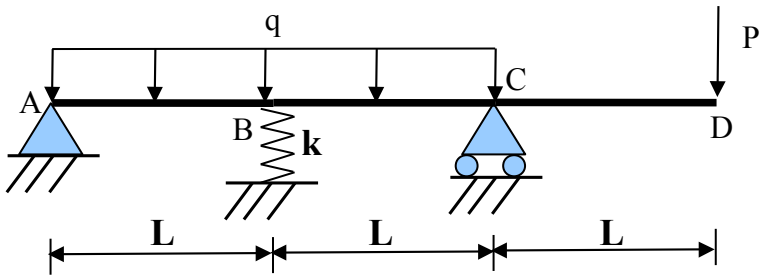
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
17/07/2012

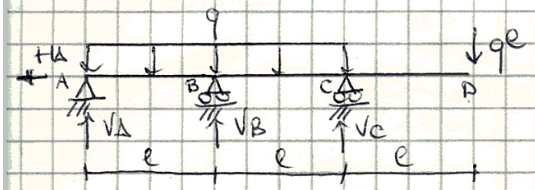


L= 3 m, q = 20 kN/m, P =60 kN
E = 210 GPa, σ_{amm} = 240 MPa, k = 20 MPa m

La travatura in figura presenta un appoggio cedevole elasticamente nel punto B e deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione nel caso k $\rightarrow+\infty$ (appoggio fisso).
- Dimensionare la travatura.
- Calcolare lo spostamento verticale in D nel caso k $\rightarrow+\infty$.
- Risolvere la travatura per il valore indicato di k e ridisegnare i diagrammi delle caratteristiche della sollecitazione.

SCRITTO DEL 17/7/2012

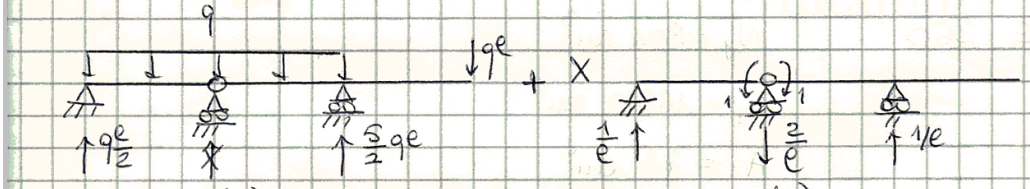


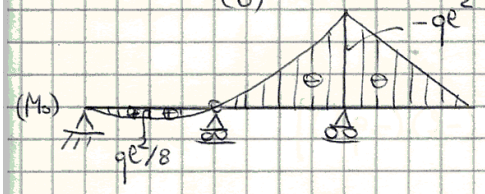
Equazioni cardinali della Statica:

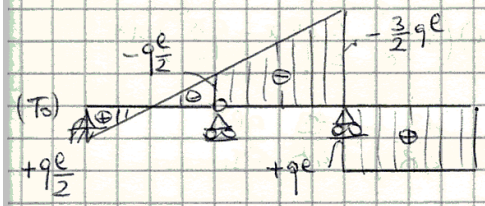
$$\begin{cases} H_A = 0 \\ V_A + V_B + V_C = 3q\ell \\ V_B\ell + V_C 2\ell = 2q\ell^2 + 3q\ell^2 = 5q\ell^2 \end{cases}$$

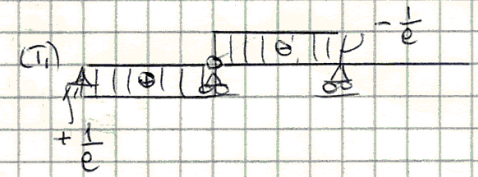
La trave è una volta iperstatica

Incognita iperstatica: $X_1 = M_B$









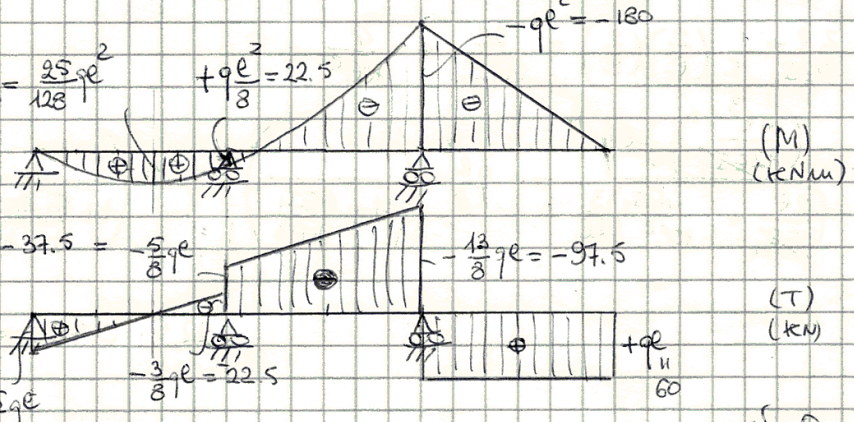
$N_0 = 0$ $\qquad$ $N_1 = 0$

$$EI_1 \gamma_{10} = \frac{1}{24} q\ell^3 + \int_0^\ell \frac{x}{\ell}\left(-q\ell^2 + \frac{3}{2}q\ell x - q\frac{x^2}{2}\right) dx = \frac{q\ell^3}{24} - \frac{q\ell^3}{8} = -\frac{q\ell^3}{12}$$

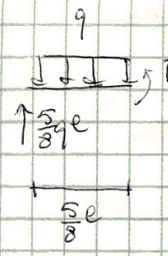
$$EI_1 \gamma_{11} = 2 \cdot \frac{1}{3}\ell$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} = \frac{q\ell^2}{8} = 22.5 \text{ kNm}$$

DIAGRAMMI: (K → +∞)



$N = 0$



$$M = \frac{5}{8}q\ell\,\frac{5}{8}\ell - \frac{1}{2}\,\frac{5}{8}q\ell\,\frac{5}{8}\ell = \frac{25}{128}q\ell^2$$

PROGETTO:

$$W_x \geq \frac{q\ell^2}{\sigma_{AMM}} = \frac{20 \cdot 10^3 \cdot 9}{240 \cdot 10^{*3}}\,m^3 = 750\ cm^3$$

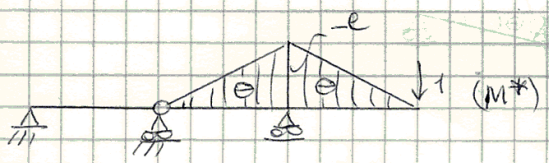
↳ IPE 360

$W_x = 904\ cm^3$

$I_1 = 16270\ cm^4$

$A = 73\ cm^2$

SPOSTAMENTO IN D:



$$1\,\delta_D = \frac{1}{EI_1}\left[\frac{1}{3}\ell\,(-\ell)(-q\ell^2)\right] + \frac{1}{EI_1}\int_0^{\ell}(-x)\left(+q\frac{\ell^2}{8} - \frac{5}{8}q\ell x - q\frac{x^2}{2}\right)dx$$

$$= \frac{1}{EI_1}q\ell^4\left[\frac{1}{3} + \frac{13}{48}\right] = \frac{29}{48}\frac{q\ell^4}{EI_1}$$

$$= \frac{29 \cdot 20 \cdot 10^3 \cdot 81 \cdot 10^{*}}{48 \cdot 210 \cdot 10^{*} \cdot 16270 \cdot 10^{-8}}\ cm$$

$$= 2{,}86\ cm$$

RISOLUZIONE CON VINCOLO ELASTICO:

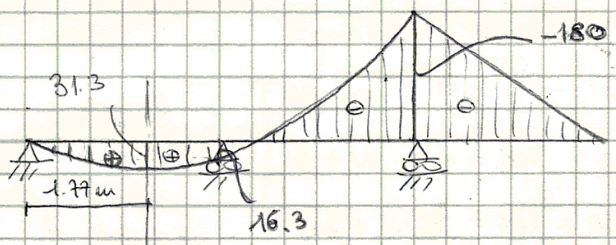
$$M_{10} = -\frac{q\ell^3}{12EI_1} + \frac{1}{K}\left(\frac{2}{\ell}\right)\cdot 0 = -\frac{q\ell^3}{12EI_1}$$

$$M_{11} = \frac{2\ell}{3EI_1} + \frac{1}{K}\left(\frac{2}{\ell}\right)\left(\frac{2}{\ell}\right) = \frac{2\ell}{3EI_1} + \frac{4}{K\ell^2}$$

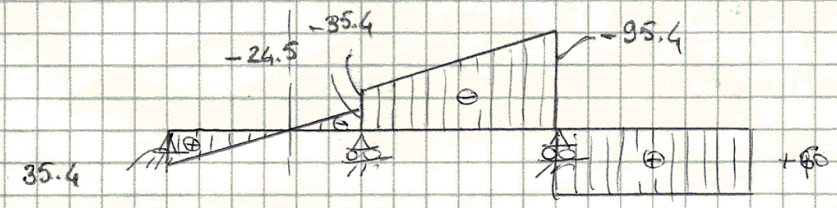
$$X_1 = \frac{\left(\frac{q\ell^3}{12EI_1}\right)\frac{3\ell}{2}}{\left(\frac{2\ell}{3EI_1} + \frac{4EI_1}{K\ell^2}\right)\frac{3}{2\ell}} = \frac{q\ell^2}{8}\left(\frac{1}{1 + \frac{6EI_1}{K\ell^3}}\right) = 22{,}5 \cdot \left(\frac{1}{1 + \frac{6 \cdot 210 \cdot 10^9 \cdot 16270 \cdot 10^{-8}}{20 \cdot 10^6 \cdot 27}}\right)\ kN\,m$$

$$= 22{,}5 \cdot 0{,}724 = 16.3\ kN\,m$$

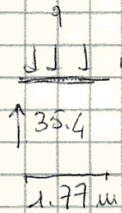
DIAGRAMMI:
(VINCOLO ELASTICO)



(M)
(kNm)



(T)
(kN)



$\curvearrowright \bar{M} = 35.4 \times 1.77 - \frac{1}{2} \cdot 20 \, (1.77)^2$ kNm

$= 31.3$ kNm