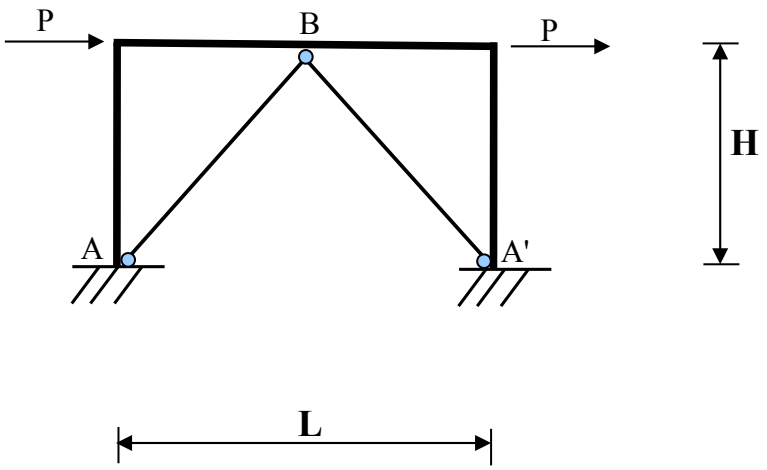
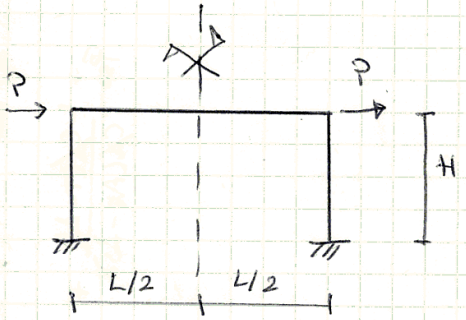
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
12/06/2012



L= 6 m, H = 3 m, P =30 kN
E = 210 GPa, σ_{amm} = 240 MPa, $A_{AB}= A_{BA'}$

La travatura in figura deve essere realizzata con profilati IPE.

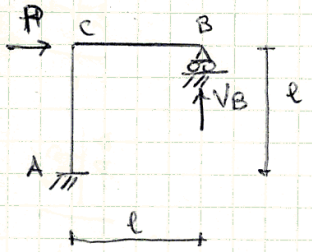
- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in assenza dei controventi AB e BA'.
- Dimensionare la travatura.
- Calcolare lo spostamento orizzontale del telaio.
- Impostare la risoluzione della travatura considerando anche la presenza dei due controventi.



$H = L/2$.

Struttura simmetrica caricata in modo antisimmetrico.

È possibile ricondursi allo studio di metà struttura

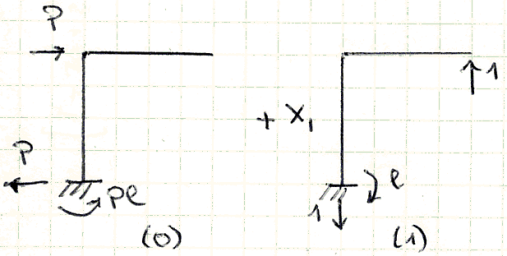


con $\ell = H = L/2$

Struttura una volta iperstatica

Incognita iperstatica: $X_1 = V_B$

(Si trascurano le def. assiali.)



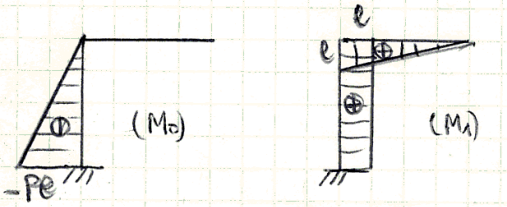
Eq.ne di congruenza:

$$\eta_{10} + \eta_{11} X_1 = \eta_1$$

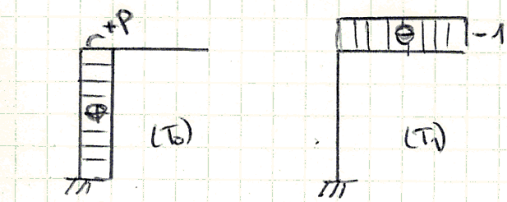
$\eta_1 = 0$

$$\eta_{10} = \frac{1}{EI_1}\,\ell\left(-Pe\,\frac{\ell}{2}\right) = -\frac{Pe^3}{2EI_1}$$

$$\eta_{11} = \frac{1}{EI_1}\left(\ell^3 + \frac{\ell^3}{3}\right) = \frac{4\ell^3}{3EI_1}$$



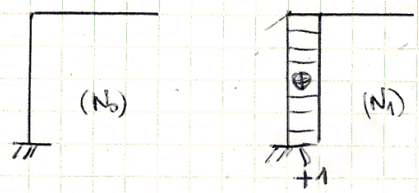
$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{Pe^2}{2EI_1}\,\frac{3EI_1}{4\ell^3} = \frac{3}{8}P$$



Calcolo dei momenti flettenti:

$$M_A = -P\ell + \frac{3}{8}Pe = -\frac{5}{8}Pe$$

$$M_C = \frac{3}{8}Pe$$

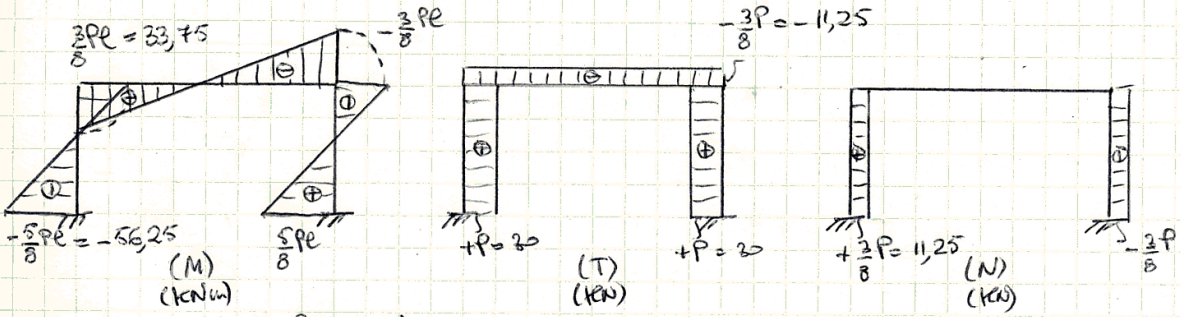


e dei tagli:

$$T_A = +P$$

$$T_C = -\frac{3}{8}P$$

Diagrammi delle c.s.:

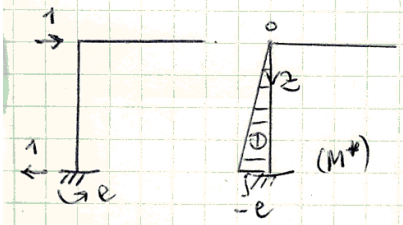


Dimensionamento (solo flessione):

$$W_1 \geq \frac{M_1}{\sigma_{AMM}} = \frac{56,25 \cdot 10^3}{240 \cdot 10^6}\, m^3 = \frac{5625}{24}\, cm^3 = 234,4\, cm^3$$

IPE 220 $\begin{cases} W_1 = 252\, cm^3 \\ I_1 = 2772\, cm^4 \\ A = 33,37\, cm^2 \end{cases}$

Spostamento orizzontale:



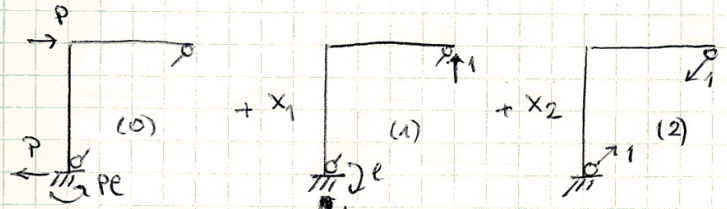
$$1 \cdot v = \frac{1}{EI_1}\int_0^\ell (-z)\left(\frac{3}{8}Pl - Pz\right)dz$$

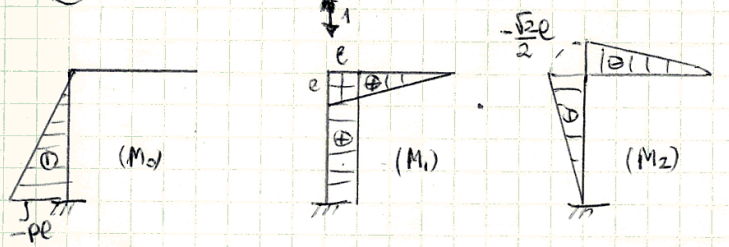
$$= -\frac{P}{EI_1}\int_0^\ell \left(\frac{3}{8}lz - z^2\right)dz = -\frac{Pl^3}{EI_1}\left[\frac{3}{16} - \frac{1}{3}\right] = \frac{7}{48}\frac{Pe^3}{EI_1}$$

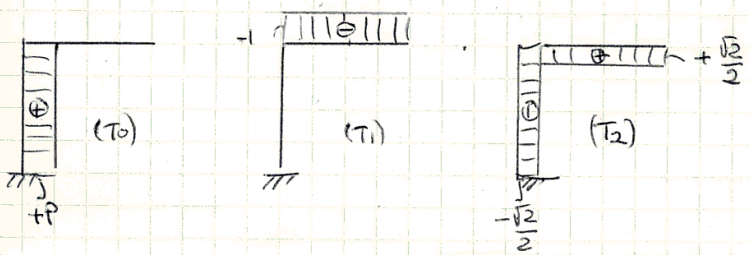
$$v = \frac{7}{48}\frac{Pe^3}{EI_1} = \frac{\cancel{7} \cdot \cancel{20} \cdot 10^{\cancel{3}^{2}} \cdot 27}{48 \cdot \cancel{2}10 \cdot \cancel{10^3} \cdot 2772 \cdot \cancel{10^{-8}}} = \frac{27 \cdot 100}{48 \cdot 2772}\, m$$

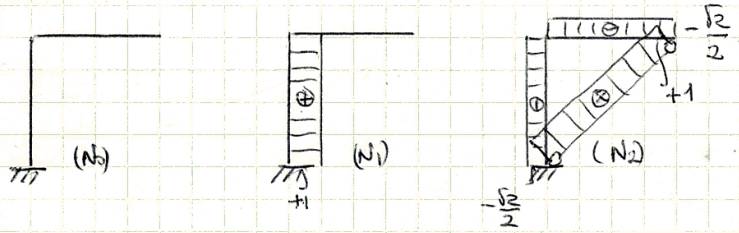
$$= 2,02\, cm$$

Risoluzione con controvento:









$$\begin{cases} \gamma_{10} + \gamma_{11} X_1 + \gamma_{12} X_2 = \gamma_1 \\ \gamma_{20} + \gamma_{21} X_1 + \gamma_{22} X_2 = \gamma_2 \end{cases}$$

$\gamma_1 = 0 = \gamma_2$

$\gamma_{10} = -\frac{Pe^3}{2EI_1}$, $\gamma_{20} = \frac{1}{6}\frac{e}{EI_1}(-Pe)\left(-\frac{\sqrt{2}}{2}e\right) = +\frac{\sqrt{2}}{12}\frac{Pe^3}{EI_1}$

$\gamma_{11} = \frac{4e^3}{3EI_1}$, $\gamma_{12} = \frac{1}{EI_1}\left[e^2\left(-\frac{\sqrt{2}}{2}e\right) + \frac{1}{3}e^2\left(-\frac{\sqrt{2}}{2}\right)\right] = -\frac{4\sqrt{2}\,e^3}{6EI_1} = -\frac{2\sqrt{2}e^3}{3EI_1}$

$\gamma_{22} = \frac{1}{EI_1}\frac{1}{3}e\,e^2 + \frac{1}{EA_{AB}}e\sqrt{2} = \frac{e^3}{3EI_1} + \frac{e\sqrt{2}}{EA_{AB}}$