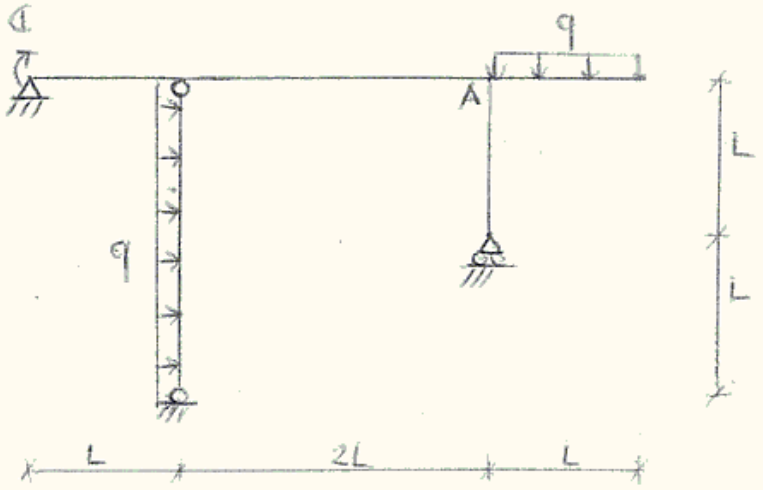
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA IN ITINERE DI STATICA
FERRARA, 16/12/2011

A



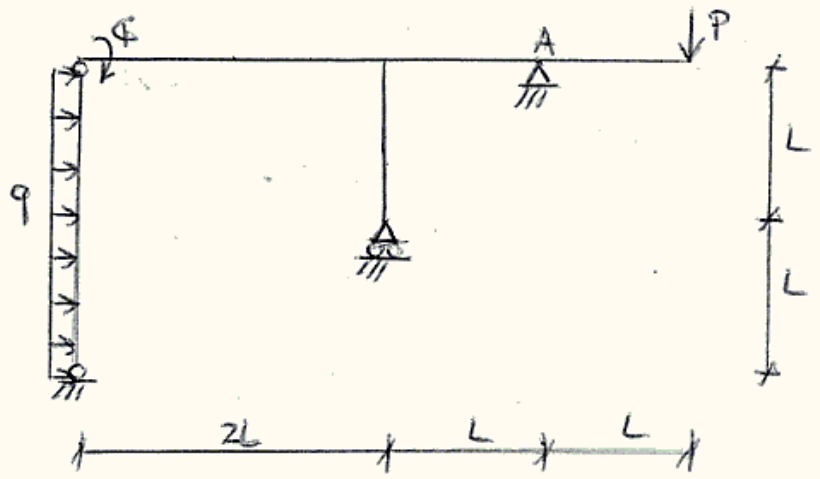
L = 2 m, q = 5 kN/m, C = 20 kNm,
E = 210 GPa, $\alpha = 10^{-5}$ 1/°C

La travatura iperstatica di figura è realizzata con profilati IPE 160.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei carichi q, e C e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo A.
3. Risolvere nuovamente la travatura considerando anche un riscaldamento della biella di 20°C e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q, C che del carico termico.

CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA IN ITINERE DI STATICA
FERRARA, 16/12/2011

B



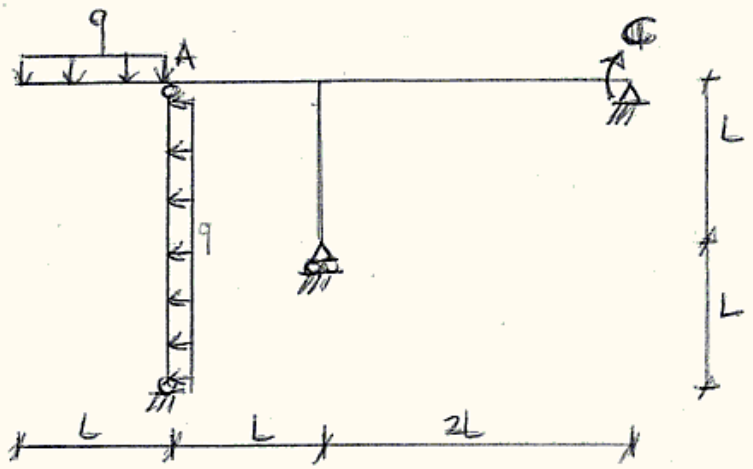
L = 1 m, q = 20 kN/m, C = 20 kNm, P = 20 kN
E = 210 GPa, $\alpha = 10^{-5}\ °C^{-1}$

La travatura iperstatica di figura è realizzata con profilati IPE 180.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei carichi q, C e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo A.
3. Risolvere nuovamente la travatura considerando anche un riscaldamento della biella di 20°C e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q, C e P che del carico termico.

CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA IN ITINERE DI STATICA
FERRARA, 16/12/2011

C



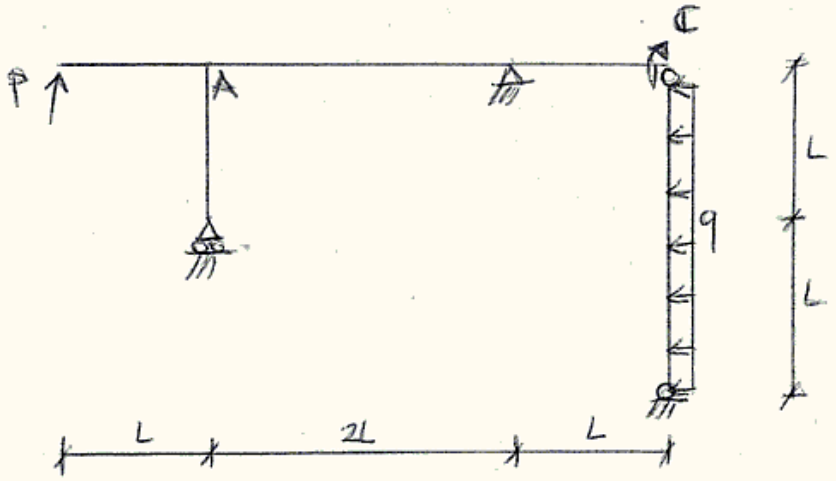
L = 2 m, q = 15 kN/m, C = 60 kNm
E = 210 GPa, $\alpha = 10^{-5}\,°C^{-1}$

La travatura iperstatica di figura è realizzata con profilati IPE 240.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei carichi q, C e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo A.
3. Risolvere nuovamente la travatura considerando anche un riscaldamento della biella di 20°C e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q, C che del carico termico.

CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
SECONDA PROVA SCRITTA IN ITINERE DI STATICA
FERRARA, 16/12/2011

D

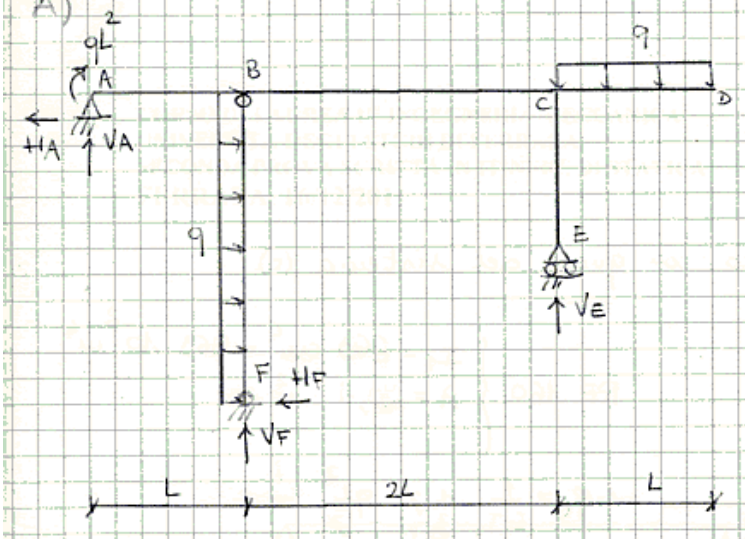


L = 1 m, q = 20 kN/m, P = 20 kN, C = 20 kNm
E = 210 GPa, $\alpha = 10^{-5}\,°C^{-1}$

La travatura iperstatica di figura è realizzata con profilati IPE 200.

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei carichi q, C e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Calcolare la rotazione del nodo A.
3. Risolvere nuovamente la travatura considerando anche un riscaldamento della biella di 20°C e disegnare i nuovi diagrammi delle caratteristiche della sollecitazione (N, T, M) comprensivi sia di q, C e P che del carico termico.

A)



$(B\curvearrowright)_{BF}$ $\quad HF\,2L = \frac{5}{2}qL^2$

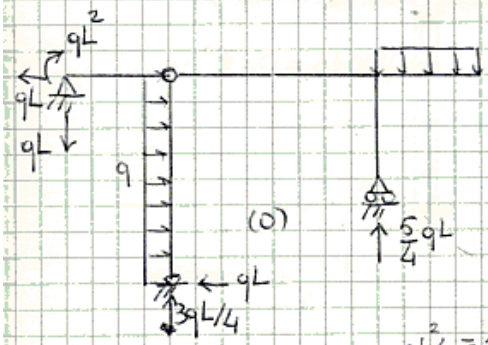
$(\leftarrow)$ $\quad HA = 2qL - HF = qL$

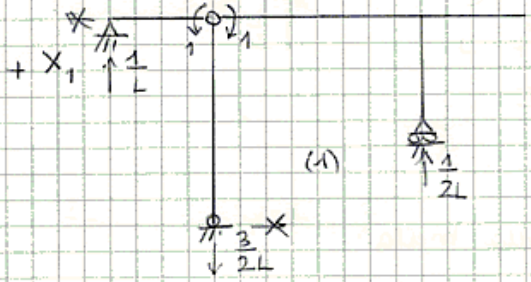
$(\uparrow)$ $\quad VA + VF + VE = qL$

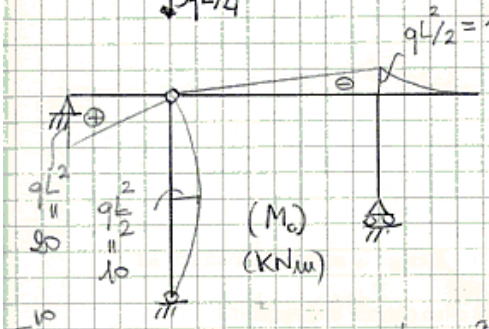
$(B\curvearrowright)$ $\quad VA\,L + qL^2 = VE\,2L - qL\,\frac{5}{2}L$

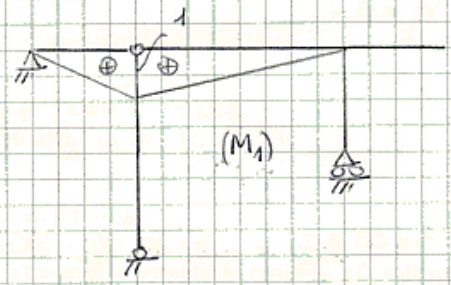
Trovatura 1 volta iperstatica.

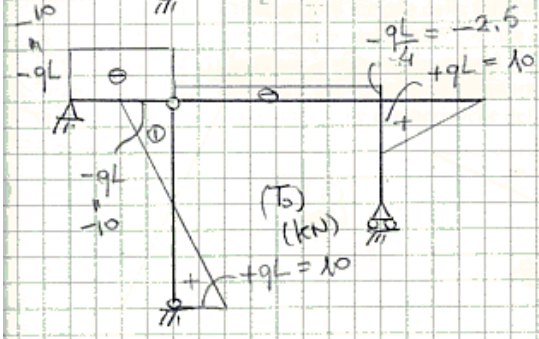
Incognita iperstatica: $X_1 = M_B$

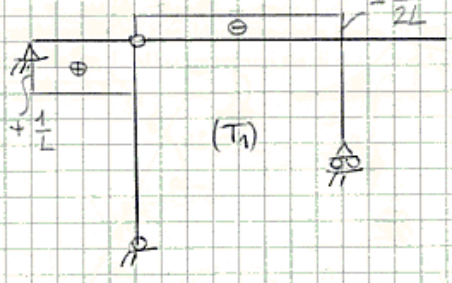


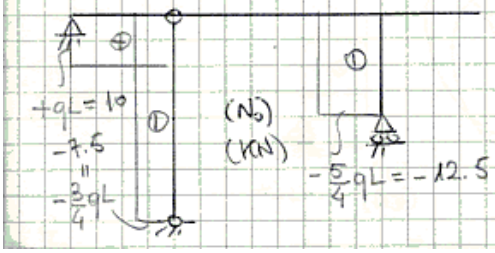


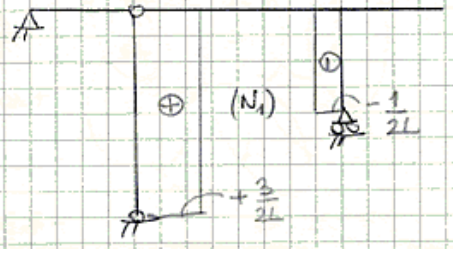












$$EI_1 \eta_{10} = \frac{1}{6} q L^2 \cdot L - \frac{1}{6} q \frac{L^2}{2} 2L = 0$$

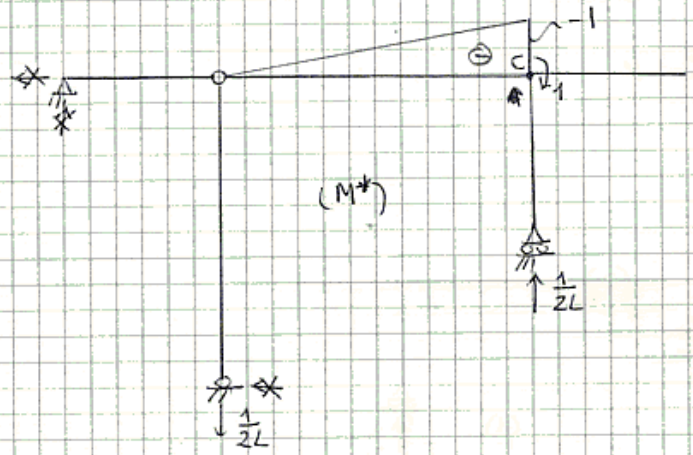
$$EI_1 \eta_{11} = \frac{1}{3} L + \frac{1}{3} 2L = L$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = 0$$

Disegno dello s.c.s. coincidono con quello del sistema (0)

Rotazione in C:

IPE 160 $\begin{cases} I_1 = 869\ cm^4 = 869 \cdot 10^{-8}\ m^4 \\ A = 20{,}1\ cm^2 \end{cases}$



$$1 \cdot \varphi_C = \frac{1}{EI_1} \frac{1}{3} 2L \, q \frac{L^2}{2}$$

$$\varphi_C = \frac{qL^3}{3EI_1} = \frac{5 \cdot 10^3 \cdot 8^3}{210 \cdot 10^9 \cdot 3 \cdot 869 \cdot 10^{-8}} = 0{,}0073 = 0{,}42°$$

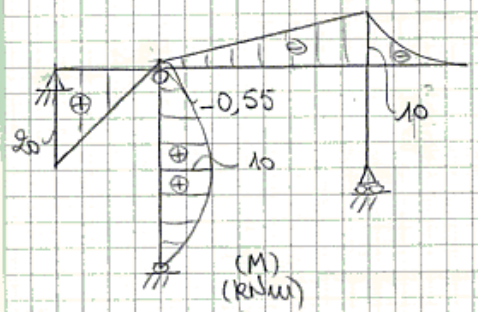
Riscaldamento della biella:

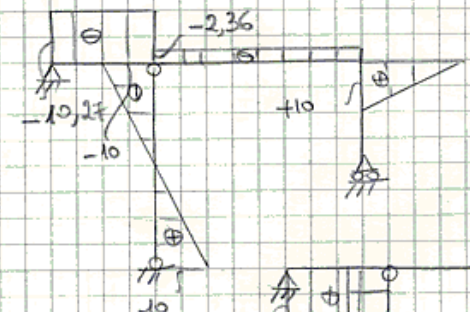
$$\eta_{1t} + \eta_{10}^{\,0} + \eta_{11} X_1 = 0$$

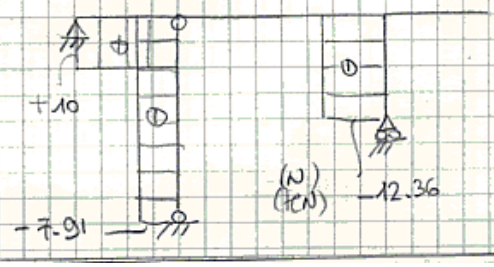
$$\eta_{1t} = \frac{3}{2L} \alpha \Delta T \cdot 2L$$

$$X_1 = -\frac{\eta_{1t}}{\eta_{11}} = -3\alpha\Delta T \frac{EI_1}{L} = -3 \cdot 10^{-5} \cdot \frac{20}{2} \cdot 210 \cdot 10^8 \cdot 869 \cdot 10^{-8}$$

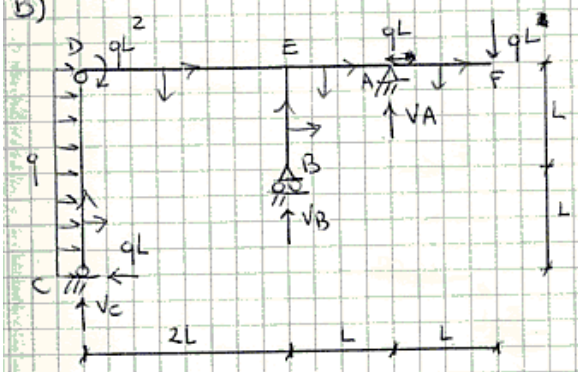
$$= -547\ Nm = -0{,}547\ kNm \simeq -0{,}55\ kNm$$







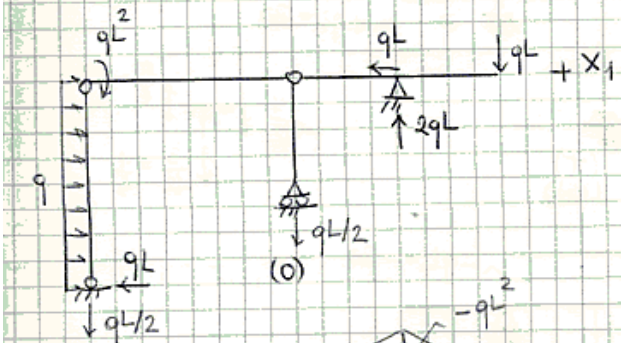
B)

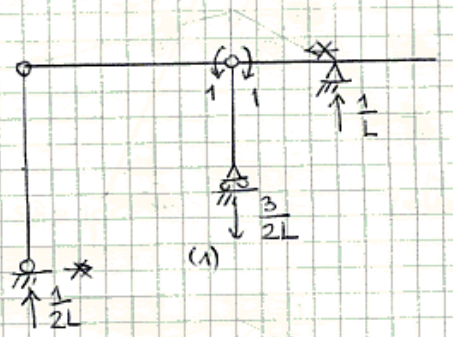


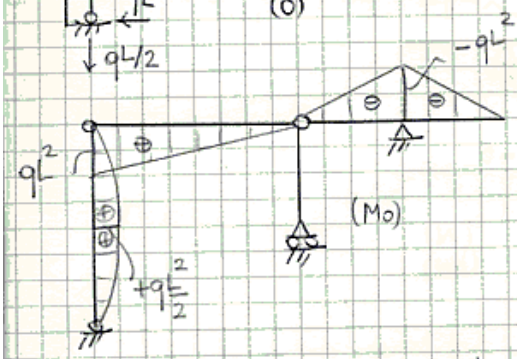
$$\begin{cases} V_A\,3L + V_B\,2L = qL^2 + 4qL^2 \\ V_A + V_B + V_C = qL \end{cases}$$

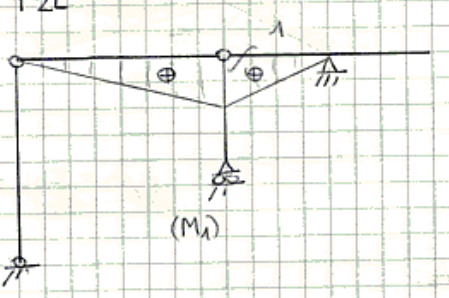
Trovata una volta iperstatica

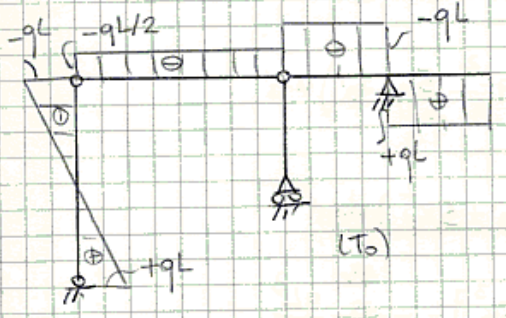
Incognita iperstatica: $X_1 = M_E$.

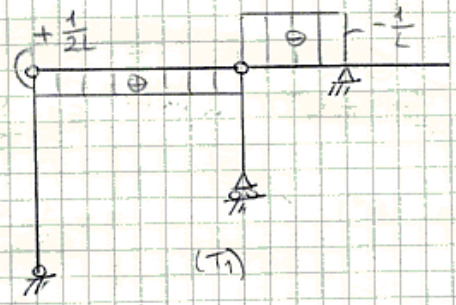


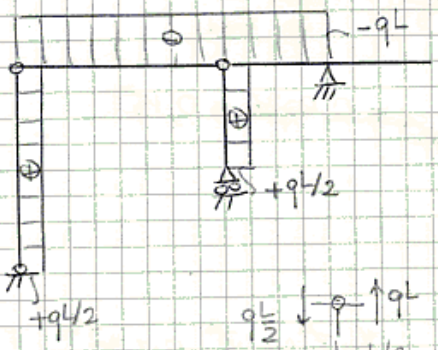


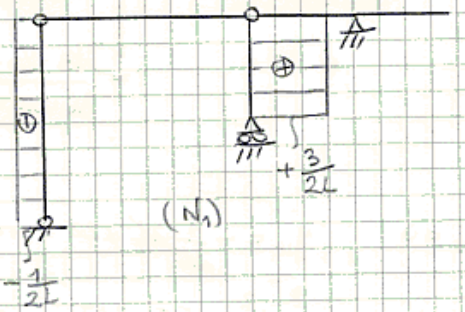










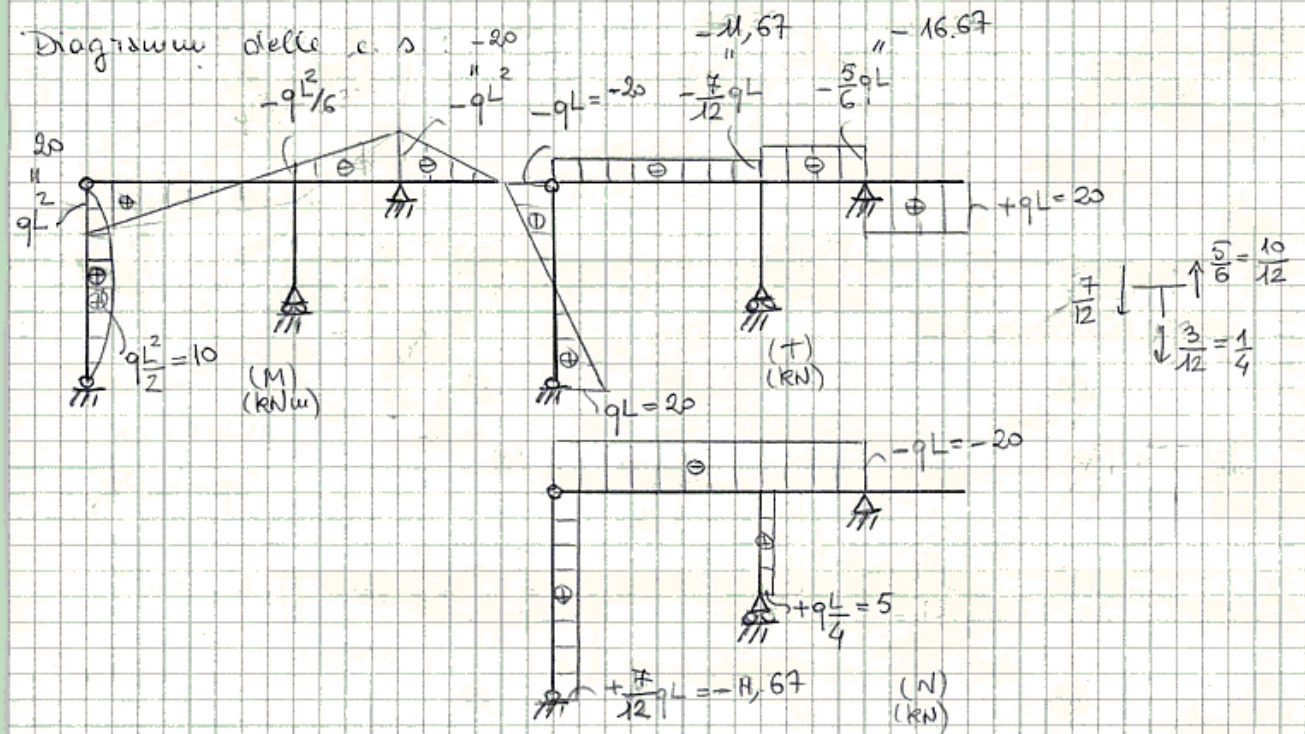


$$EI_1 \gamma_{10} = +\frac{1}{6} 2L\, qL^2 - \frac{1}{6} qL^3 = +\frac{qL^3}{6}$$

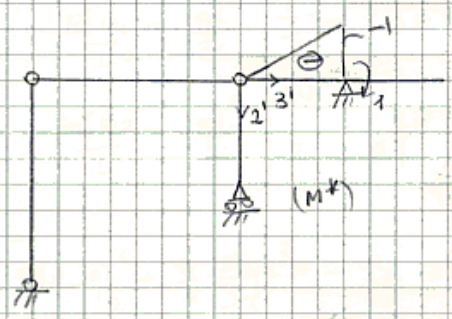
$$EI_1 \gamma_{11} = \frac{1}{3} 2L + \frac{1}{3} L = L$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} = -\frac{qL^2}{6} = -3.33 \text{ kNm}$$

Diagramma delle c.s.



Rotazione in A:



$$1 \cdot \varphi_A = \frac{1}{EI_1} \int_0^L \left(-\frac{x_3'}{L}\right)\left(-\frac{qL^2}{6} - \frac{5}{6} qL x_3'\right) dx_3'$$

$$= +\frac{1}{6EI_1} \frac{q}{L} \int_0^L (Lx_3' + 5x_3'^2)\, dx_3'$$

$$= +\frac{q}{6EI_1} \left[\frac{L^3}{2} + \frac{5L^3}{3}\right] = +\frac{13}{36} \frac{qL^3}{EI_1}$$

$$= \frac{13 \cdot 20 \cdot 10^{3\,2}}{36 \cdot 210 \cdot 10^{8} \cdot 1317 \cdot 10^{-8}} = 0,0026 = 0,15^\circ$$

Carico termico:

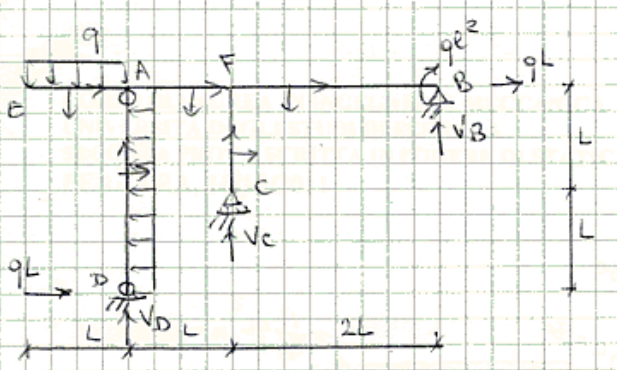
$$\gamma_{1t} + \gamma_{10} + \gamma_{11} X_1 = 0 \quad , \quad \gamma_{1t} = -\frac{1}{2L} 2L\, \alpha \Delta T = -\alpha \Delta T$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} - \frac{\gamma_{1t}}{\gamma_{11}} = -\frac{qL^2}{6} + \frac{EI_1 \alpha \Delta T}{L} = -3.33 + 210 \cdot 10^{6} \cdot 12 \cdot 10^{-6} \cdot 20 \cdot 15 \cdot 1317 \cdot 10^{-8}$$

$$\simeq (-3.33 + 0,55) \text{ kNm} = -2.77 \text{ kNm}$$

Occorre rifare i diagrammi.

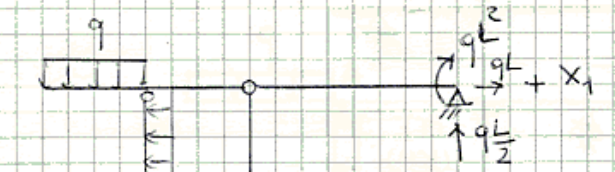
C)

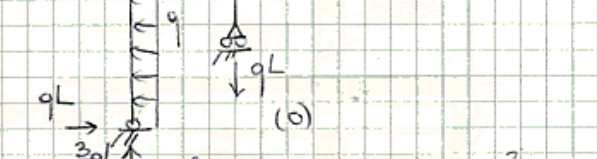


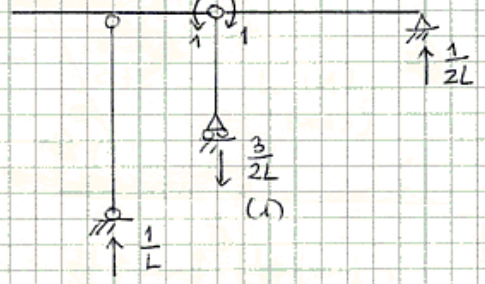
$$\begin{cases} V_B 2L - qL^2 + 2qL^2 - V_D L - 2qL^2 + 2qL^2 = 0 \\ V_B + V_C + V_D = qL \end{cases}$$

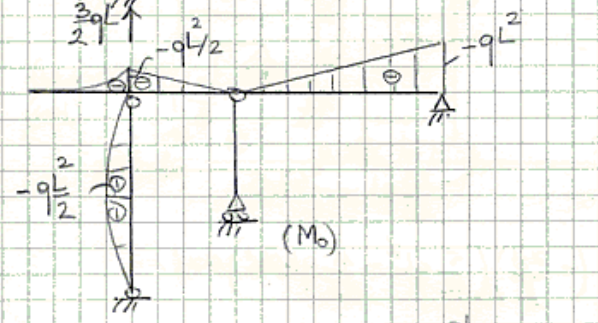
Struttura una volta iperstatica

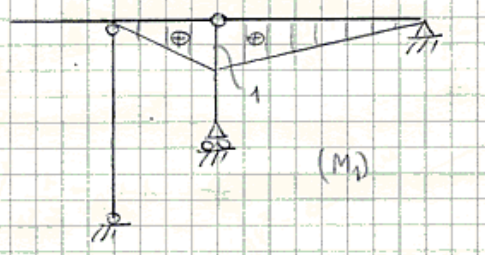
Incognita iperstatica: $X_1 = M_F$

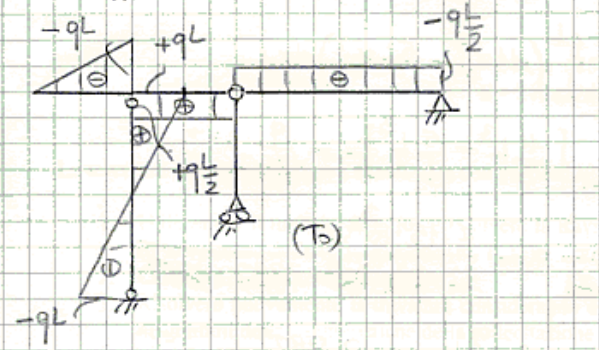


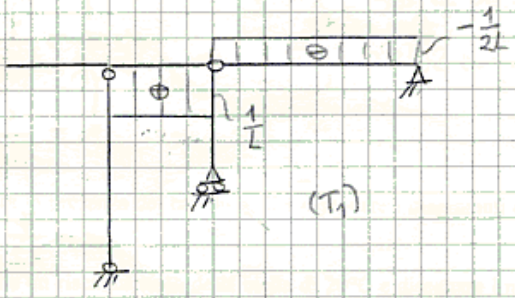


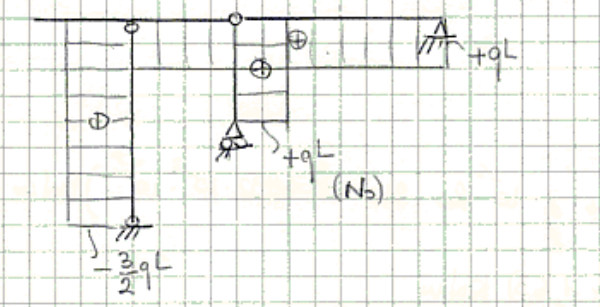


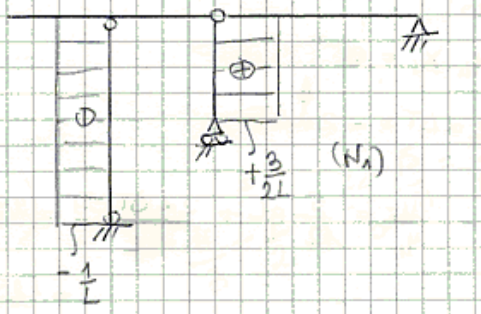


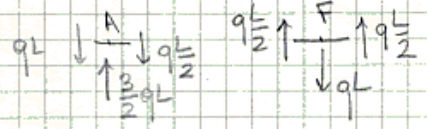










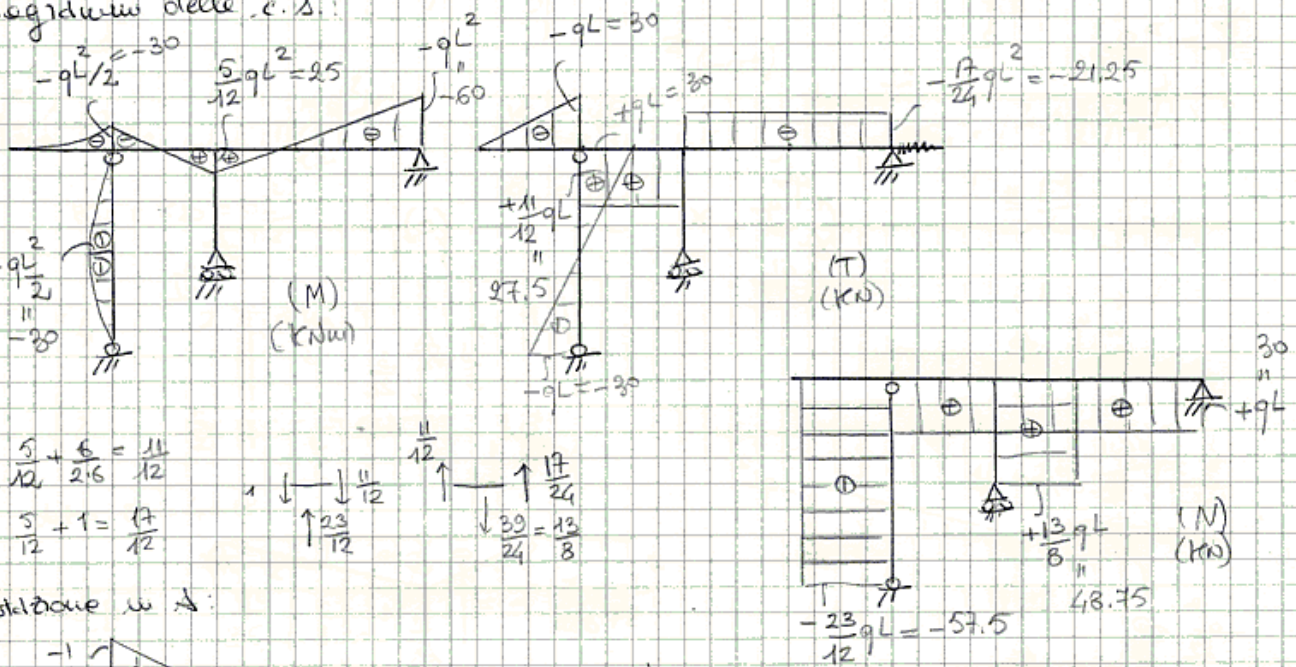


$$EI_1 \gamma_{10} = -\frac{1}{6} L \, qL^2/2 - \frac{1}{6} 2L \, qL^2 = -\frac{5}{12} qL^3$$

$$EI_1 \gamma_{11} = \frac{L}{3} + \frac{2}{3} L = L$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} = \frac{5}{12} qL^2 = 25 \text{ kNm}$$

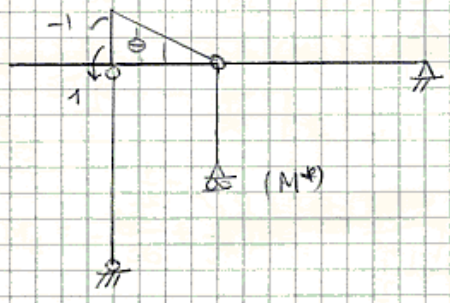
Diagrammi delle c.s.:



$$\frac{5}{12} + \frac{6}{2.16} = \frac{11}{12}$$

$$\frac{5}{12} + 1 = \frac{17}{12}$$

Rotazione in A:



$$1 \cdot \varphi_A = \frac{1}{EI_1} \int_0^L \left(-\frac{x_3'}{L}\right)\left(\frac{5}{12} qL^2 - \frac{11}{12} qL x_3'\right) dx_3'$$

$$= -\frac{q}{12EI_1} \int_0^L \left(11 x_3'^2 - 5L x_3'\right) dx_3'$$

$$= -\frac{q}{12EI_1} \left[\frac{11}{3} L^3 - \frac{5}{2} L^3\right] = -\frac{q}{12EI_1} \cdot \frac{7}{6} L^3$$

$$= -\frac{7}{72} \frac{qL^3}{EI_1} = -\frac{7 \cdot 15 \cdot 10^3 \cdot 8}{72 \cdot 210 \cdot 10^8 \cdot 3892 \cdot 10^{-8}}$$

$$= -0{,}0014 = -0{,}082°$$

Carico termico:

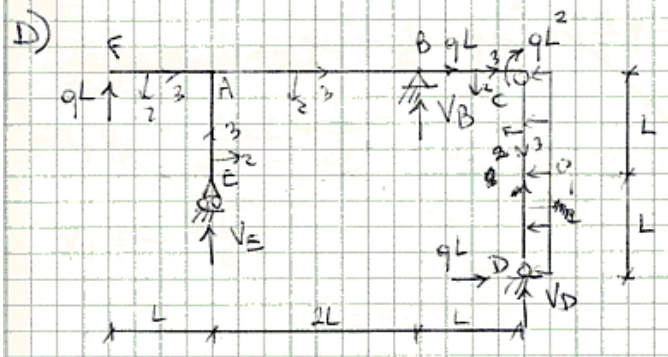
$$\gamma_{1t} + \gamma_{10} + \gamma_{11} X_1 = 0$$

$$\gamma_{1t} = -\frac{1}{L} 2L \alpha \Delta T = -2\alpha \Delta T$$

$$X_1 = -\frac{\gamma_{10}}{\gamma_{11}} - \frac{\gamma_{1t}}{\gamma_{11}} = \frac{5}{12} qL^2 + \frac{2\alpha \Delta T EI_1}{L} = \left(25 + \frac{2}{7} \cdot 10^{-5} \cdot 20 \cdot 210 \cdot 10^9 \cdot 3892 \cdot 10^{-8} \cdot 10^{-3}\right) \text{kNm}$$

$$= (25 + 1{,}63) \text{ kNm}$$

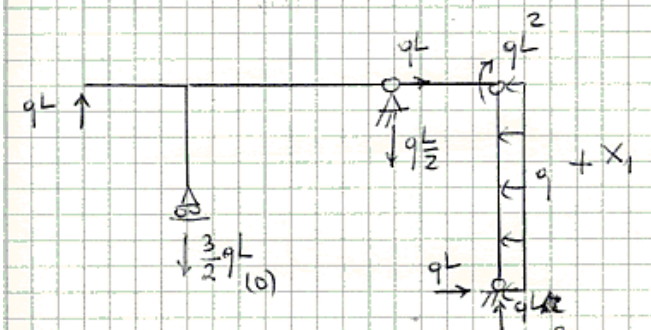
1)

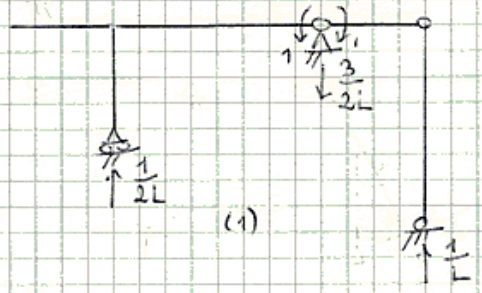


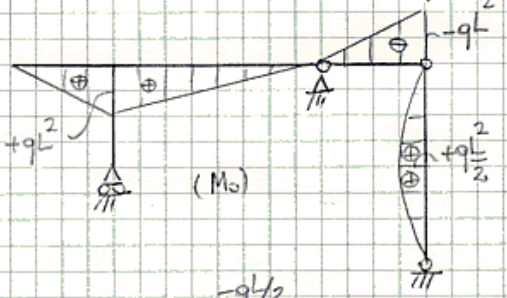
$$\begin{cases} V_B 2L + V_D 3L - qL^2 - qL^2 + 2qL^2 - \frac{3}{2}qL^2 = 0 \\ V_E + V_B + V_D = -qL \end{cases}$$

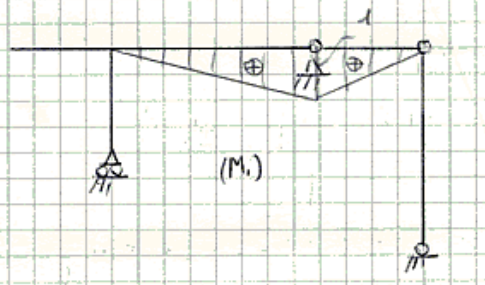
Trave tre volte iperstatica

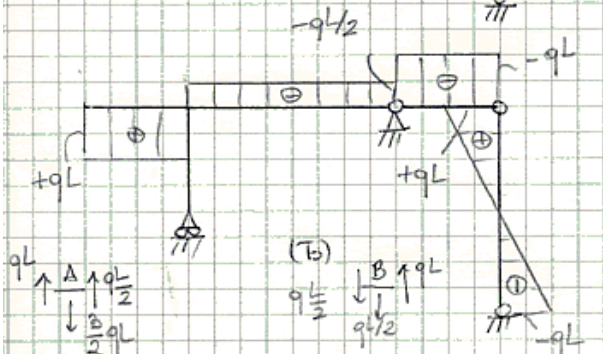
Incognita iperstatica: $X_1 = M_B$

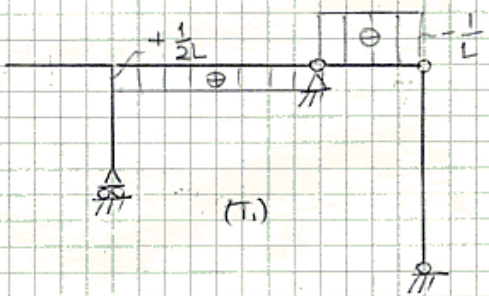


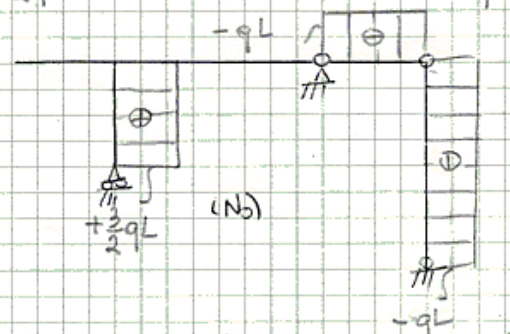


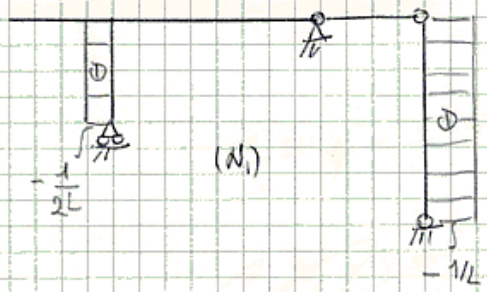










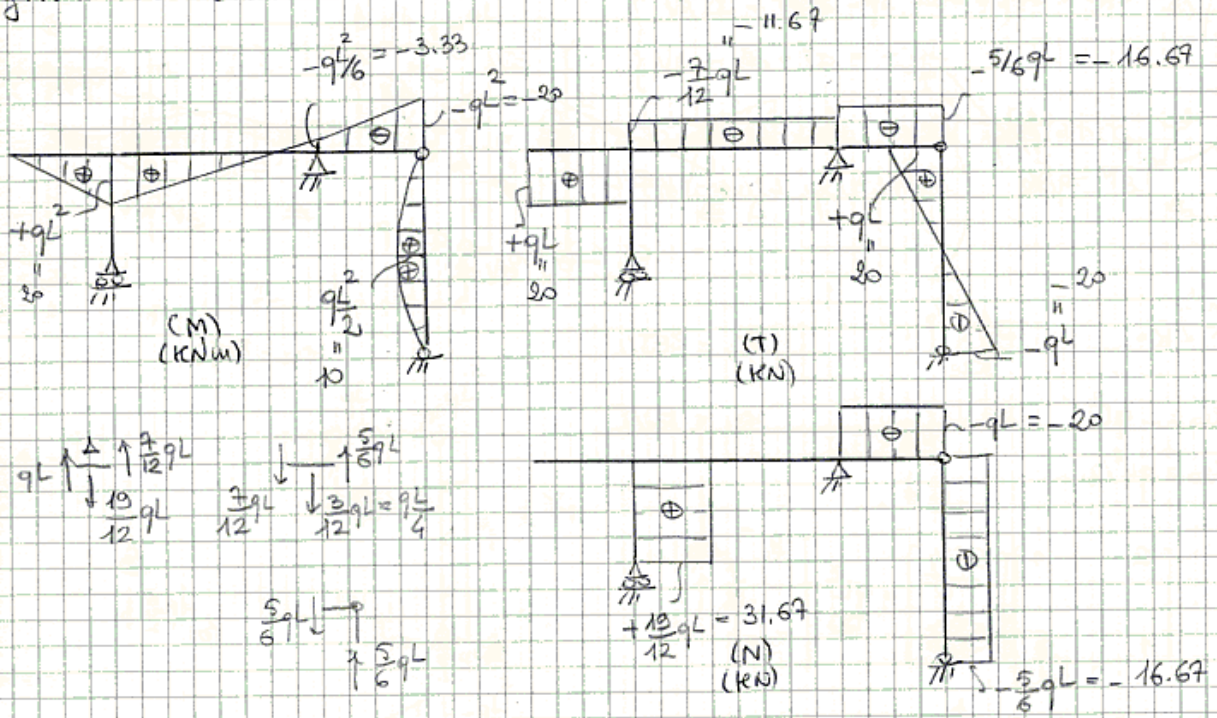


$$EI_1 \gamma_{10} = \frac{1}{6} 2L\, qL^2 - \frac{1}{6} L\, qL^2 = \frac{1}{6} qL^3$$

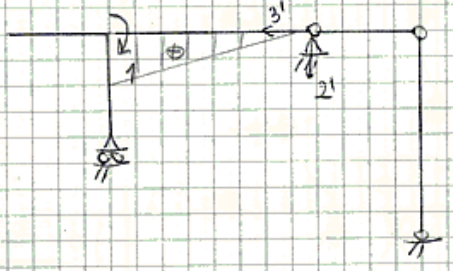
$$EI_1 \gamma_{11} = \frac{1}{3} 2L + \frac{L}{3} = L$$

$$X_1 = -\gamma_{10}/\gamma_{11} = -qL^2/6 = -3.33 \text{ KNw}$$

Diagrammi del c.s.:



Rotazione in A:



$$1 \cdot \varphi_A = \frac{1}{EI_1}\int_0^{2L}\left(\frac{x_3'}{2L}\right)\left(-\frac{qL^3}{6} + \frac{7}{12}qLx_3'\right)dx_3'$$

$$= \frac{q}{24EI_1}\int_0^{2L}\left(-2Lx_3' + 7x_3'^2\right)dx_3'$$

$$= \frac{q}{24EI_1}\left[-2L\frac{8L^2}{2} + \frac{7}{3}8L^3\right] = \frac{44}{3}\frac{qL^3}{24EI_1} = \frac{11}{18}\frac{qL^3}{EI_1}$$

$$= \frac{11}{18}\frac{20\cdot 10^3}{210\cdot 10^9 \cdot 1943\cdot 10^{-8}} = 0{,}003 = 0{,}17°$$

Carico termico:

$$\gamma_{1t} + \gamma_{10} + \gamma_{11}X_1 = 0$$

$$\gamma_{1t} = -\frac{1}{L}\alpha\Delta T\,2L = -2\alpha\Delta T$$

$$X_1 = -\frac{qL^2}{6} + \frac{EI_1\,2\alpha\Delta T}{L} = \left(-3.33 + \frac{210\cdot 10^9 \cdot 1943\cdot 10^{-8}\cdot 2\cdot 10^{-5}\cdot 20}{1}\cdot 10^{-3}\right)\text{kNm}$$

$$= (-3.33 + 1.63)\,\text{kNm} = -1.7\ \text{kNm}$$

Occorre anche i diagrammi.