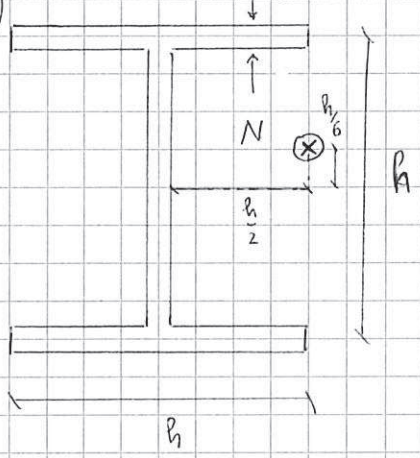
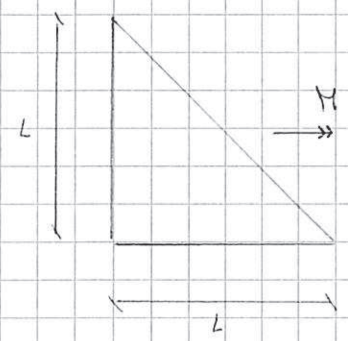
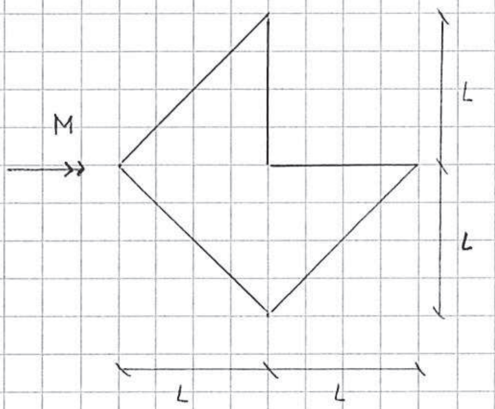
Es. 1



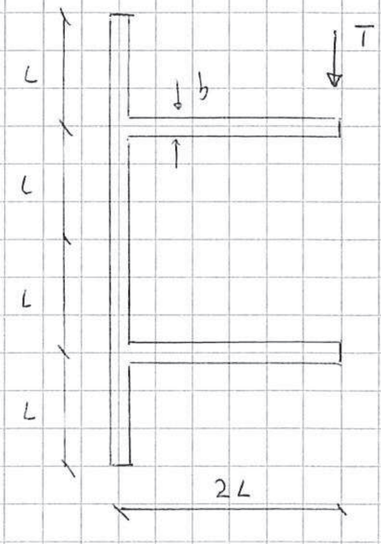
Es. 2



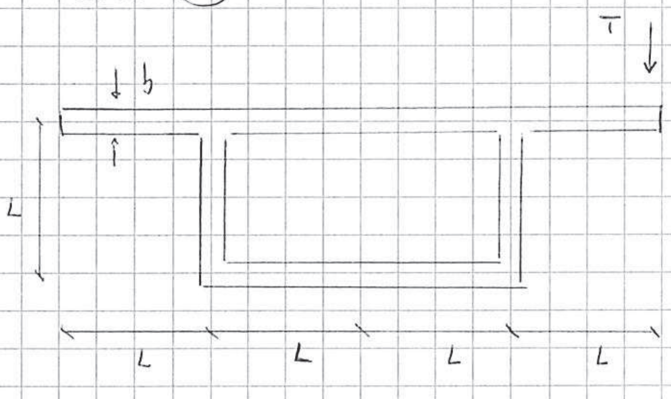
Es. 3



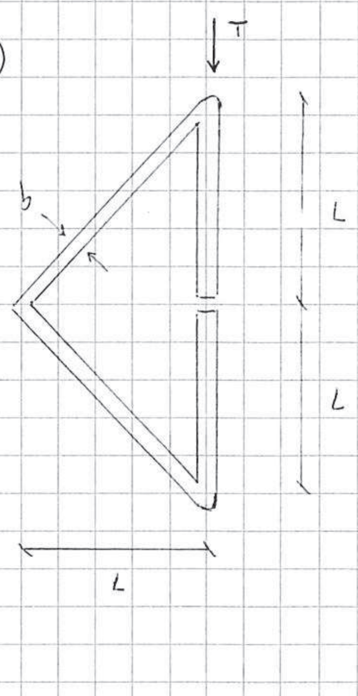
Es. 4



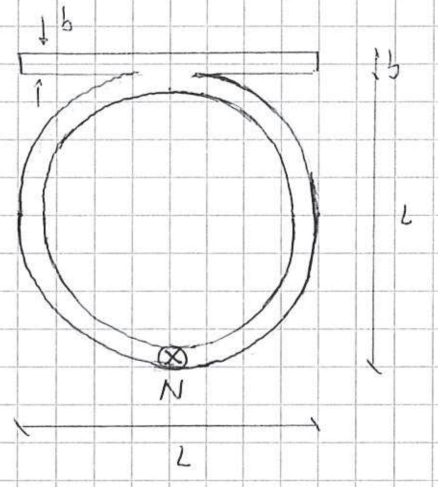
Es. 5



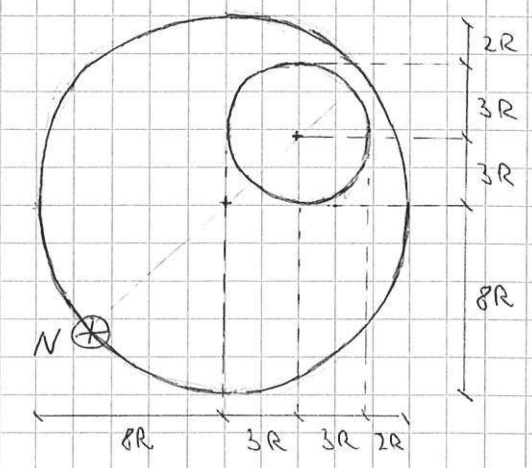
Es 6



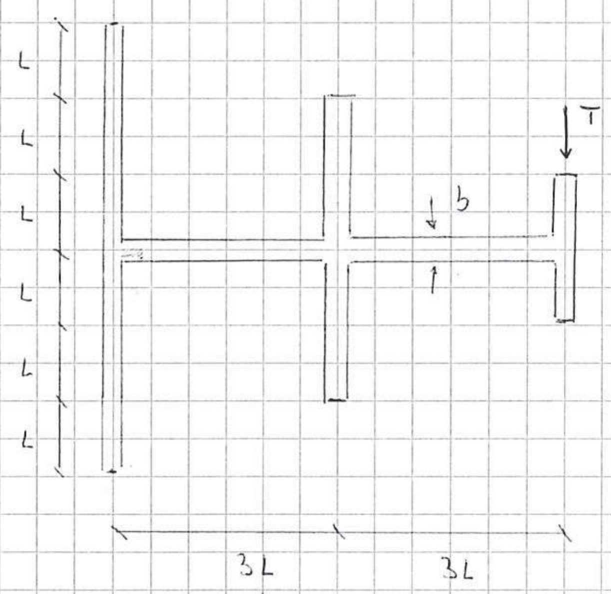
Es. 7



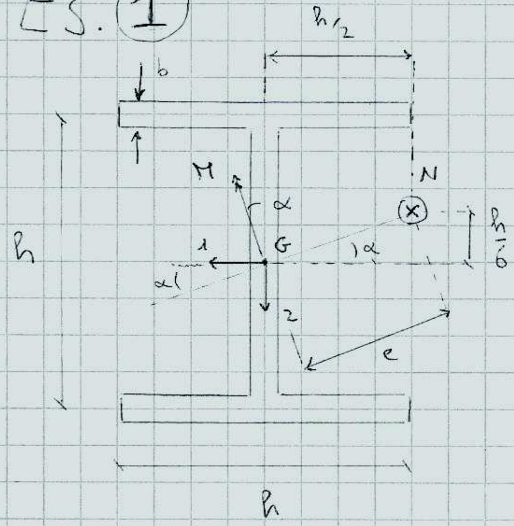
Es. 8



Es. 9



Es. 1



Sezione a 2 assi di simmetria $\rightarrow$ baricentro e terna principale univocamente definiti

$I_1 =$  $+$ 

$= \frac{1}{12} b h^3 + 2\left[\frac{1}{12} b^3 h + bh \cdot \left(\frac{h}{2}\right)^2\right]$

$= \frac{bh^3}{12} + \frac{bh^3}{2} = \frac{7}{12} bh^3$

$I_2 =$  $+$ 

$= \frac{1}{12} h b^3 + 2 \cdot \frac{1}{12} b h^3 = \frac{1}{6} b h^3$

eccentricità $:= e = \sqrt{(h/2)^2 + (h/6)^2} \sim 0.53\, h$

$\alpha = \tan^{-1}\left(\frac{h/6}{h/2}\right) = \tan^{-1}\left(1/3\right) = 18.43°$

$A = 3bh$

Caso di "pressoflessione deviata"

$$\begin{cases} N = -N \\ M_1 = M \sin\alpha = 0.32\, M \\ M_2 = -M\cos\alpha = -0.95\, M \end{cases}, \quad \text{con } |M| = Ne = 0.53\, Nh$$

eq. ne di Navier:

$$\sigma = -\frac{N}{3bh} + \frac{0.32 \cdot 0.53\, Nh}{\frac{7}{12} bh^3} x_2 + \frac{0.95 \cdot 0.53\, Nh}{\frac{1}{6} bh^3} x_1$$

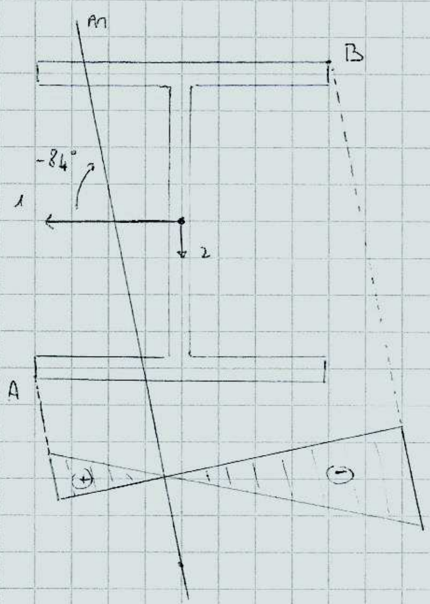
$$= -0.33 \frac{N}{bh} + 0.29 \frac{N}{bh^2} x_2 + 2.96 \frac{N}{bh^2} x_1$$

$$= \frac{N}{bh}\left(-0.33 + \frac{0.29}{h} x_2 + \frac{2.96}{h} x_1\right)$$

Asse neutro: $\sigma = 0 \iff -0.33 + \frac{0.29}{h} x_2 + \frac{2.96}{h} x_1 = 0$

$$\longrightarrow x_2 = \left(-\frac{2.96}{h} x_1 + 0.33\right) \cdot \frac{h}{0.29}$$

$$= \underbrace{-10.2}_{\tan^{-1}(-10.2) \sim -84°} x_1 + 1.14\, h$$

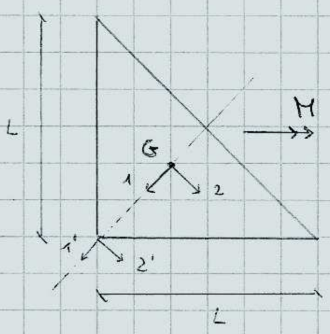


$$A = \left(\frac{h}{2}, \frac{h}{2}\right) \quad , \quad B = \left(-\frac{h}{2}, -\frac{h}{2}\right)$$

$$\sigma_A = \frac{N}{bh}\left(-0.33 + \frac{0.29}{h}\frac{h}{2} + \frac{2.96}{h}\frac{h}{2}\right) = 1.29\,\frac{N}{bh} > 0 \longrightarrow \sigma_A < \sigma^t_{adm}$$

$$\sigma_B = \frac{N}{bh}\left(-0.33 - \frac{0.29}{h}\frac{h}{2} - \frac{2.96}{h}\frac{h}{2}\right) = -1.96\,\frac{N}{bh} < 0 \longrightarrow \sigma_B > \sigma^c_{adm}$$

Es. 2



- Calcolo del baricentro

Definisco un riferimento (1', 2').
Poiché l'asse 1' è di simmetria
G ∈ asse 1' $\longrightarrow x_{2',G} = 0$

$$\boxed{x_{1',G} = -\frac{\sqrt{2}}{3}L}$$ (vedi baricentro di un triangolo)

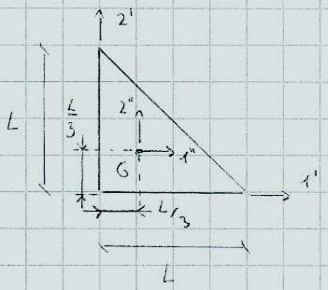
Definisco il riferimento centrale d'inerzia (1, 2) [assi principali d'inerzia e baricentrici]

- Calcolo dei momenti d'inerzia

Poiché la sezione quadrata ha ∞ assi principali, il mom. inerzia rispetto qualunque asse è: $I^{\square} = \frac{L^4}{12}$

Poiché l'asse 1 divide simmetricamente a metà il quadrato di lato L, si ha: $\boxed{I_1 = \frac{1}{2} I_1^{\square} = \frac{L^4}{24}}$

In alternativa: richiami:
inerzia triangolo isoscele



$$I_{1'} = \frac{1}{12} b h^3 = \frac{L^4}{12} \longrightarrow I_{1''} = I_{1'} - A \cdot d(1'-1'')^2$$

$$= \frac{L^4}{12} - \frac{L^2}{2} \cdot \left(\frac{L}{3}\right)^2 = \frac{L^4}{36}$$

analogamente:

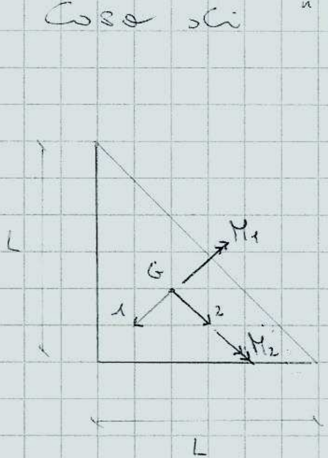
$$I_{2'} = \frac{L^4}{12} \longrightarrow I_{2''} = \frac{L^4}{36}$$

Nel nostro caso:
$$I_1 = 2 I_1^{\triangle} = 2 \cdot \frac{1}{12} \cdot \left(\frac{\sqrt{2}L}{2}\right)^4 = \frac{L^4}{24} \checkmark$$

$$I_2 = 2 I_2^{\triangle} = 2 \cdot \frac{1}{36}\left(\frac{\sqrt{2}L}{2}\right)^4 = \frac{L^4}{72}$$

- Calcolo delle tensioni

Caso di "flessione deviata"
$$\begin{cases} M_1 = (-) M \sqrt{2}/2 \\ M_2 = M\sqrt{2}/2 \end{cases}$$
opposto all'asse (1)



Eq.ne di Navier:

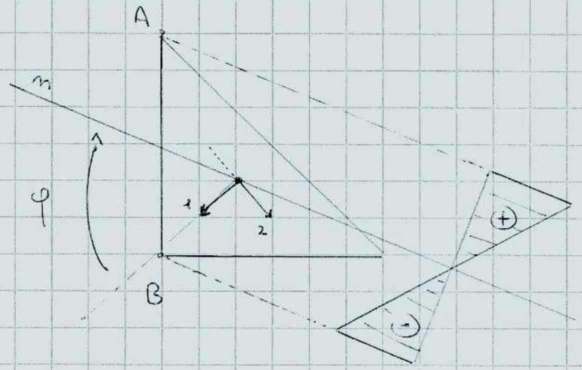
$$\sigma = \cancel{\frac{N}{A}} + \frac{M_1}{I_1} x_2 - \frac{M_2}{I_2} x_1$$

Eq.ne asse neutro: $\sigma = 0 \iff x_2 \frac{M_1}{I_1} = x_1 \frac{M_2}{I_2}$

$$\longrightarrow \frac{x_2}{x_1} = \tan(\varphi) = \frac{M_2}{M_1} \frac{I_1}{I_2} = \frac{M\sqrt{2}/2}{-M\sqrt{2}/2} \cdot \frac{\frac{L^4}{24}}{\frac{L^4}{72}} = -3$$

con φ = angolo che l'asse neutro forma con l'asse 1.

$\tan(\varphi) = -3 \longrightarrow \varphi \sim -71°$



Ricerco i punti più sollecitati.
Poiché σ è lineare, sono i punti più lontani dall'asse neutro.

$$A = \left(-\frac{\sqrt{2}}{6}L, -\frac{\sqrt{2}}{2}L\right), \quad B = \left(\frac{\sqrt{2}}{3}L, 0\right)$$

$$\sigma_A = \frac{M_1}{I_1} x_{2,A} - \frac{M_2}{I_2} x_{1,A}$$

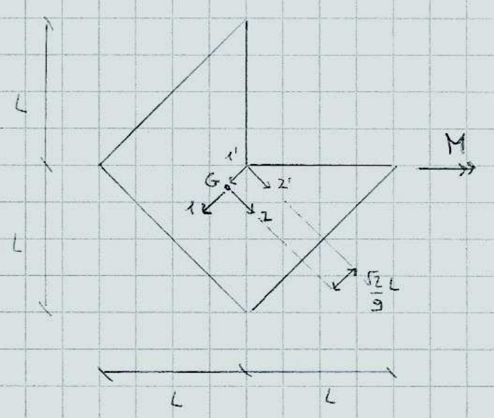
$$= -\frac{M\sqrt{2}}{2} \cdot \frac{\overset{12}{\cancel{24}}}{L^4}\left(-\frac{\sqrt{2}}{2}L\right) - \frac{M\sqrt{2}}{2} \cdot \frac{\overset{12}{\cancel{72}}}{L^4}\left(-\frac{\sqrt{2}}{6}L\right)$$

$$= 12\frac{M}{L^3} + 12\frac{M}{L^3} = 24\frac{M}{L^3} > 0 \rightarrow \sigma_A < \sigma^t_{adm}$$

$$\sigma_B = \frac{M_1}{I_1} x_{2,B} - \frac{M_2}{I_2} x_{1,B}$$

$$= -\frac{M\sqrt{2}}{2}\frac{24}{L^4} \cdot 0 - \frac{M\sqrt{2}}{2}\frac{\overset{24}{\cancel{72}}}{L^4} \cdot \frac{\sqrt{2}}{3}L = -24\frac{M}{L^3} < 0 \rightarrow \sigma_B > \sigma^c_{adm}$$

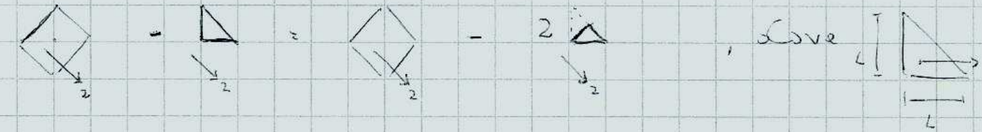
Es. 3



Asse (1') di simmetria $\rightarrow x_{2'G} = 0$

$$x_{1'G} = \frac{-\left(-\frac{\sqrt{2}}{3}L\right)\cdot\frac{L^2}{2}}{(\sqrt{2}L)^2-\frac{L^2}{2}} = \frac{\frac{\sqrt{2}}{6}L^3}{\frac{3}{2}L^2} = \frac{\sqrt{2}}{9}L$$

$$I_1 = \frac{1}{12}(\sqrt{2}L)^4 - \frac{1}{2}\cdot\frac{1}{12}\cdot L^4 = \left(\frac{1}{3}-\frac{1}{24}\right)L^4 = \frac{7}{24}L^4 \sim 0.29L^4$$



, dove $I_j = \frac{L^4}{36}$

$$I_2 = \left[\frac{1}{12}(\sqrt{2}L)^4 + (\sqrt{2}L)^2\cdot\left(\frac{\sqrt{2}}{9}L\right)^2\right] - 2\left[\frac{1}{36}\left(\frac{\sqrt{2}}{2}L\right)^4 + \left(\frac{\sqrt{2}}{2}L\right)^2\frac{1}{2}\cdot\left(\frac{\sqrt{2}}{9}L+\frac{2}{3}\frac{\sqrt{2}}{2}L\right)^2\right]$$

$$= \frac{1}{3}L^4 + \frac{4}{81}L^4 - 2\left[\frac{L^4}{144}+\frac{8}{81}L^4\right] = \left(\frac{1}{3}+\frac{4}{81}-\frac{1}{72}-\frac{16}{81}\right)L^4 \sim 0.17L^4$$

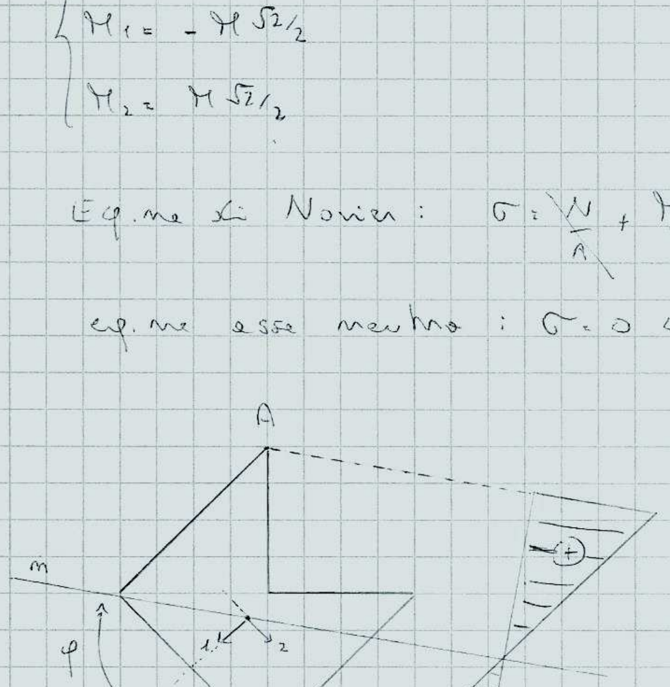
Caso di "flessione deviata"

$$\begin{cases} N = 0 \\ M_1 = -M\sqrt{2}/2 \\ M_2 = M\sqrt{2}/2 \end{cases}$$

Eq.ne di Navier: $\sigma = \cancel{\frac{N}{A}} + \frac{M_1}{I_1}x_2 - \frac{M_2}{I_2}x_1$

eq.ne asse neutro: $\sigma = 0 \iff \frac{x_2}{x_1} = \frac{M_2}{M_1}\frac{I_1}{I_2} = \frac{\frac{M\sqrt{2}}{2}}{-\frac{M\sqrt{2}}{2}}\cdot\frac{0.29L^4}{0.17L^4} = -1.7 = \tan(\varphi)$

$\rightarrow \varphi \sim -59.5°$



$$A = \left(-\frac{\sqrt{2}}{9}L - \frac{\sqrt{2}}{2}L\ ,\ -\frac{\sqrt{2}}{2}L\right) = \left(-\frac{11}{18}\sqrt{2}L\ ,\ -\frac{\sqrt{2}}{2}L\right)$$

$$B = \left(\frac{\sqrt{2}}{2}L - \frac{\sqrt{2}}{9}L\ ,\ \frac{\sqrt{2}}{2}L\right) = \left(\frac{7}{18}\sqrt{2}L\ ,\ \frac{\sqrt{2}}{2}L\right)$$

$$\sigma_A = \frac{M_1}{I_1}x_{2,A} - \frac{M_2}{I_2}x_{1,A}$$

$$= -M\frac{\sqrt{2}}{2}\frac{1}{0.29L^4}\left(-\frac{\sqrt{2}}{2}L\right) - M\frac{\sqrt{2}}{2}\frac{1}{0.17L^4}\left(-\frac{11}{18}\sqrt{2}L\right)$$

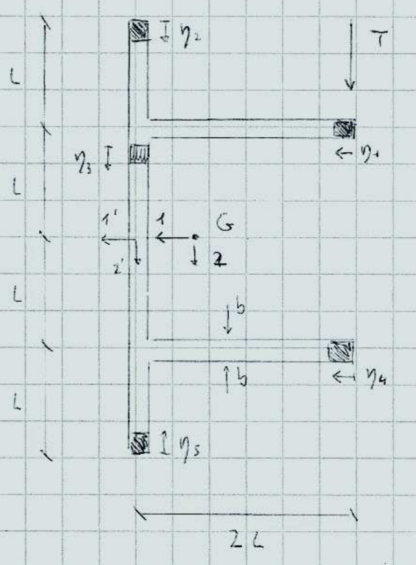
$$= \frac{1}{0.58}\frac{M}{L^3} + 3.6\frac{M}{L^3} = 5.32\frac{M}{L^3} > 0 \rightarrow \sigma_A < \sigma^t_{adm}$$

$$\sigma_B = \frac{M_1}{I_1}x_{2,B} - \frac{M_2}{I_2}x_{1,B}$$

$$= -M\frac{\sqrt{2}}{2}\frac{1}{0.29L^4}\left(\frac{\sqrt{2}}{2}L\right) - M\frac{\sqrt{2}}{2}\frac{1}{0.17L^4}\left(\frac{7}{18}\sqrt{2}L\right)$$

$$= -\frac{1}{0.58}\frac{M}{L^3} - 2.29\frac{M}{L^3} = -4.01\frac{M}{L^3} < 0 \rightarrow \sigma_B > \sigma^c_{adm}$$

Es. 4



Asse (1') di simmetria → $x_{2',G} = 0$

$$x_{1',G} = \frac{-2 \cdot 2L \cdot \frac{2bL}{2}}{2 \cdot 2Lb + 4Lb} = \frac{-4bL^2}{8bL} = -\frac{L}{2}$$

$$I_1 = \frac{1}{12} b (4L)^3 + 2\left[\frac{1}{12} b^3 (2L) + 2bL \cdot L^2\right] \simeq \frac{16}{3} bL^3 + 4bL^3 = \frac{28}{3} bL^3$$

N.B. $T \parallel$ asse **2** → non occorre I_2

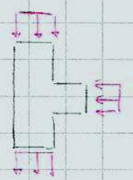
(η_1): $\tau_1 = -\frac{3}{28}\frac{T}{bL^3}\frac{1}{b}\eta_1 b \cdot (-L) = \frac{3}{28}\frac{T}{bL^2}\eta_1$

$\tau_1|_{\eta_1=0} = 0$, $\tau_1|_{\eta_1=2L} = \frac{3}{14}\frac{T}{bL}$

(η_2): $\tau_2 = -\frac{3}{28}\frac{T}{bL^3}\frac{1}{b}\eta_2 b \cdot \left[-\left(2L - \frac{\eta_2}{2}\right)\right] = \frac{3}{28}\frac{T}{bL^3}\eta_2\left(2L - \eta_2/2\right)$

$\tau_2|_{\eta_2=0} = 0$, $\tau_2|_{\eta_2=L} = \frac{3}{28}\frac{T}{bL^3} L\left(2L - L/2\right) = \frac{9}{56}\frac{T}{bL}$

equilibrio al nodo in corr. (3):



$\tau_{30} \cdot b \cdot 1 = \tau_1(2L)\, b \cdot 1 + \tau_2(L)\, b \cdot 1$

$\left(\frac{3}{14} + \frac{9}{56}\right)\frac{T}{bL} = \frac{3}{8}\frac{T}{bL}$

(η_3): $\tau_3 = \tau_{30} - \frac{3}{28}\frac{T}{bL^3}\frac{1}{b}\eta_3 b\left[-\left(L - \eta_3/2\right)\right] = \frac{3}{8}\frac{T}{bL} + \frac{3}{28}\frac{T}{bL^3}\eta_3\left(L - \eta_3/2\right)$

$\tau_3(L) = \frac{3}{8}\frac{T}{bL} + \frac{3}{28}\frac{T}{bL^3}\cdot\frac{L^2}{2} = \frac{3}{7}\frac{T}{bL}$

$\tau_3(2L) = \frac{3}{8}\frac{T}{bL} + \frac{3}{28}\frac{T}{bL^3}\, 2L\left(L - \frac{2L}{2}\right) = \frac{3}{8}\frac{T}{bL}$

(η_4): $\tau_4 = -\frac{3}{28}\frac{T}{bL^3}\frac{1}{b}\eta_4 b \cdot L = (-)\frac{3}{28}\frac{T}{bL^2}\eta_4$ dir. τ opposta a η_4

$\tau_4(2L) = -\frac{3}{14}\frac{T}{bL}$

(η_5): $\tau_5 = -\frac{3}{28}\frac{T}{bL^3}\frac{1}{b}\eta_5 b\left(2L - \eta_5/2\right) = -\frac{3}{28}\frac{T}{bL^3}\eta_5\left(2L - \frac{\eta_5}{2}\right)$

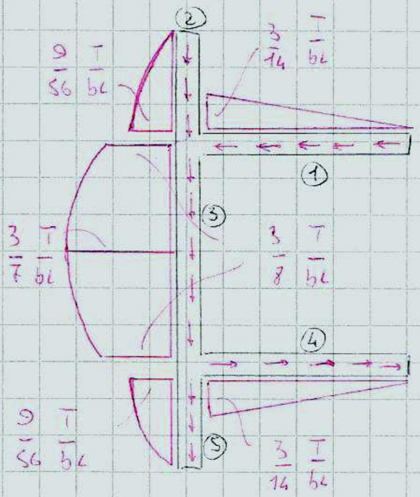
$\tau_5(L) = -\frac{3}{28}\frac{T}{bL^3} L\left(2L - \frac{L}{2}\right) = -\frac{9}{56}\frac{T}{bL}$

verifica l'equilibrio al nodo:

cambiati di segno

$$\tau_3(2L) = \tau_4(2L) + \tau_5(L)$$

$$\frac{3}{8}\frac{T}{bL} = \left(\frac{3}{14}+\frac{9}{56}\right)\frac{T}{bL} = \frac{3}{8}\frac{T}{bL} \quad ✓$$



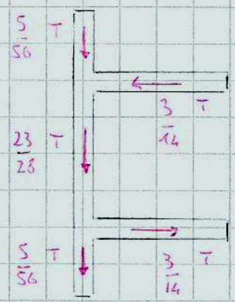
$$R_1 = R_4 = \int_0^{2L} \frac{3}{28}\frac{T}{bL^2}\eta_1 b\, d\eta = \frac{3}{28}\frac{T}{bL^2} b \frac{1}{2} 4L^2 = \frac{3}{14}T$$

$$R_2 = R_5 = \int_0^{L} \frac{3}{28}\frac{T}{bL^3}\left(\eta_2\cdot 2L - \frac{\eta_2^2}{2}\right) b\, d\eta = \frac{3}{28}\frac{T}{L^3}\left[2L\cdot\frac{L^2}{2} - \frac{1}{2}\frac{L^3}{3}\right] = \frac{5}{56}T$$

$$R_3 = \int_0^{2L}\left(\frac{3}{28}\frac{T}{bL^3}\left(\eta_3 L - \frac{\eta_3^2}{2}\right) + \frac{3}{28}\frac{T}{bL}\right) b\, d\eta = \frac{3}{28}\frac{T}{bL^3}\left[L\frac{1}{2}4L^2 - \frac{1}{2}\frac{1}{3}8L^3\right]\cancel{b} + \frac{3}{28}\frac{T}{bL}\cdot 2L = \frac{23}{28}T$$

verifica: $(\uparrow)$: $R_2 + R_3 + R_5 = 2\cdot\frac{5}{56}T + \frac{23}{28}T = T$ ✓

$(\rightarrow)$: $R_1 - R_5 = \frac{3}{14}T - \frac{3}{14}T = 0$ ✓



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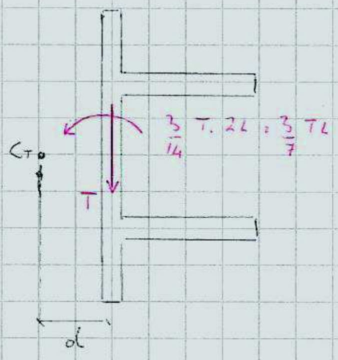
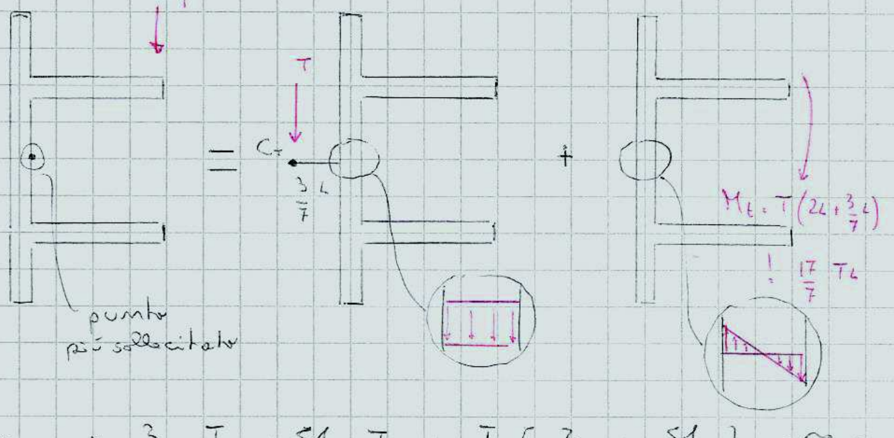


$$M_t(C_T) = 0 = Td - \frac{3}{7}TL$$

$$\rightarrow d = \frac{3}{7}L$$

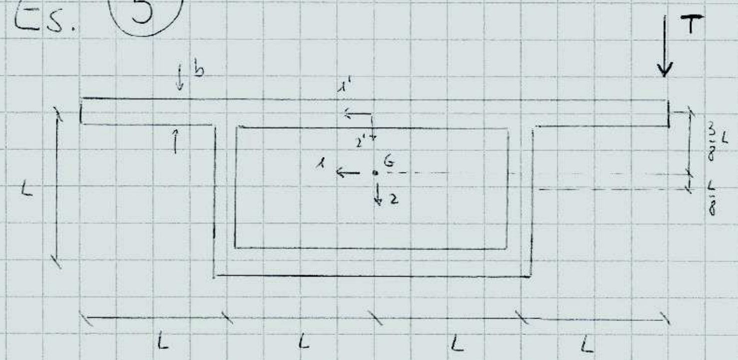
Per cui posso scomporre il sistema iniziale come:



$$\tau_{tors} = \frac{3 M_t b}{\int_\Sigma b^3(s)\, ds} = \frac{3\cdot\frac{17}{7}TLb}{b^3\cdot 8L} = \frac{51}{56}\frac{T}{b^2}$$

$$\tau_{max} = \frac{3}{7}\frac{T}{bL} + \frac{51}{56}\frac{T}{b^2} = \frac{T}{b}\left(\frac{3}{7L} + \frac{51}{56b}\right) \leq \tau_{adm}$$

Es. 5



Asse (2') di simmetria

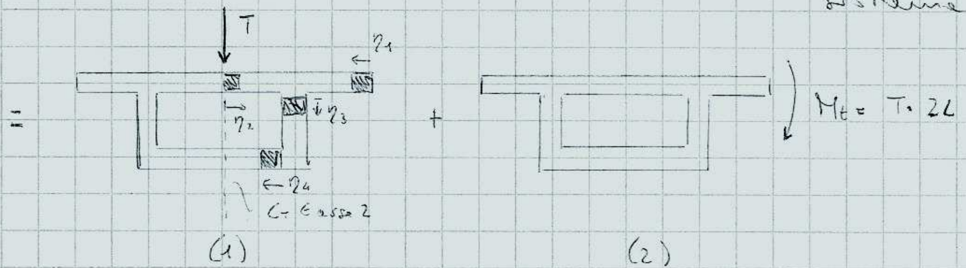
$\rightarrow x_{1',G} = 0$

$$x_{2',G} = \frac{2\frac{L}{2}bL + L\cdot 2bL}{4bL + 2bL + 2bL} = \frac{3}{8}L$$

$$I_1 = \frac{1}{12}b^3(4L) + 4bL\left(\frac{3}{8}L\right)^2 + 2\left[\frac{1}{12}bL^3 + bL\left(\frac{L}{8}\right)^2\right] + \frac{1}{12}b^3(2L) + 2bL\left(\frac{L}{8}+\frac{L}{2}\right)^2$$

$$= \frac{9}{16}bL^3 + \frac{1}{6}bL^3 + \frac{1}{32}bL^3 + \frac{25}{32}bL^3 = \frac{37}{24}bL^3 \sim 1.54\,bL^3$$

Per simmetria, $C_T \in$ asse 2 $\rightarrow$ posso scomporre il mio sistema come:



• Sistema (1)

(η_1): $\tau_1 = -\frac{T}{1.54bL^3}\frac{1}{b}\eta_1 b\cdot\left(-\frac{3}{8}L\right) \sim 0.243\frac{T}{bL^2}\eta_1$

$$\tau_1\big|_{\eta_1=L} = 0.243\frac{T}{bL}$$

(η_2): per simmetria τ_{20} (τ sull'asse di simmetria) $= 0$

$$\tau_2 = -\frac{T}{1.54bL^3}\frac{1}{b}\eta_2 b\left(-\frac{3}{8}L\right) \sim 0.243\frac{T}{bL^2}\eta_2$$

$$\tau_2\big|_{\eta_2=L} = 0.243\frac{T}{bL}$$

equilibrio al nodo:

$$\tau_{30}\,b\cdot 1 = \tau_1(L)\,b\cdot 1 + \tau_2(L)\,b\,1 = 2\cdot 0.243\frac{T}{bL} = 0.486\frac{T}{bL}$$

(η_3): $\tau_3 = \tau_{30} - \frac{T}{1.54bL^3}\frac{1}{b}\eta_3 b\left[-\left(\frac{3}{8}L - \frac{\eta_3}{2}\right)\right]$

$$= 0.486\frac{T}{bL} + 0.65\frac{T}{bL^3}\eta_3\left(\frac{3}{8}L - \frac{\eta_3}{2}\right)$$

$$\tau_3\big|_{\eta_3=L} = 0.486\frac{T}{bL} + 0.65\frac{T}{bL^3}L\left(\frac{3}{8}L - \frac{L}{2}\right)$$

$$= (0.486 - 0.081)\frac{T}{bL} = 0.405\frac{T}{bL}$$

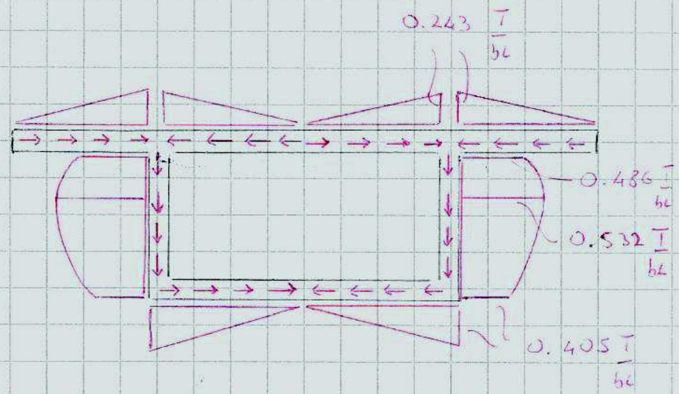
$$\frac{d\tau_3}{d\eta_3} = \frac{3}{8}L - \eta_3 = 0 \iff \eta_3 = \frac{3}{8}L \quad \text{(fibra baricentrica)}$$

$$\tau_3\Big|_{\eta_3=\frac{3}{8}L} = 0.486\,\frac{T}{bL} + 0.65\,\frac{T}{bL^3}\cdot\frac{3}{8}L\left(\frac{3}{8}L - \frac{3}{16}L\right) = (0.486 + 0.046)\,\frac{T}{bL} = 0.532\,\frac{T}{bL}$$

per equilibrio: $\tau_{40} = \tau_3(L) = 0.405\,\frac{T}{bL}$

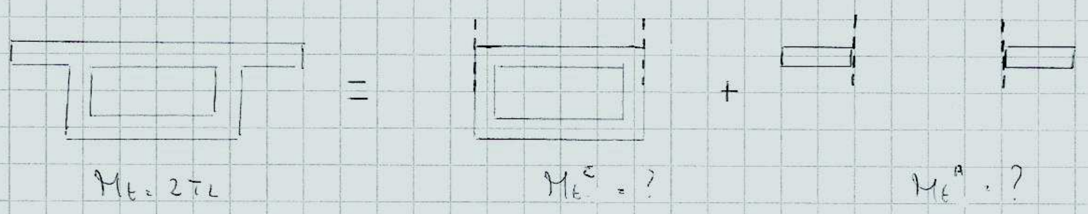
$$(\eta_4):\ \tau_4 = \tau_{40} - \frac{T}{1.54\,bL^3}\,\frac{1}{5}\,\eta_4 b\cdot\frac{5}{8}L = 0.405\,\frac{T}{bL} - 0.405\,\frac{T}{bL^2}\,\eta_4$$

$$\tau_4\big|_{\eta_4=L} = 0 \ \checkmark \ \text{(asse simmetria)}$$



- Sistema (2)

Poiché la sezione ha linea media in parte chiusa, in parte aperta, posso così scomporre il sistema:



per congruenza la rotazione torsionale (unitaria) θ' deve essere la stessa per entrambe le sezioni: ↳ $\theta'_C = \theta'_A$

Inoltre la coppia torcente applicata M_t si ripartisce sulle due sezioni, chiusa e aperta, in funzione delle relative rigidezze torsionali.
Ossia vale il sistema:

$$\begin{cases} M_t = M_t^A + M_t^C \\ \theta'_A = \theta'_C \end{cases}$$

eq.ne costitutiva della torsione:

$$M_t = G \frac{I_0}{q} \theta' \quad , \text{ con: } \quad \frac{I_0}{q} = \frac{1}{3} \int_{\mathcal{L}} b^3(s)\, ds \quad , \times \text{ sez. aperte}$$

$$\frac{I_0}{q} = \frac{4 A_0^2}{\int_{\mathcal{L}} \frac{1}{b(s)}\, ds} \quad , \times \text{ sez. chiuse}$$

$$\theta'_A = \frac{M_t^A}{G\left(\frac{I}{q}\right)^A} = \frac{M_t^A}{G \frac{1}{3} \int_{\mathcal{L}} b^3(s)\, ds} = \frac{3 M_t^A}{G\, b^3 \cdot 2L} = \frac{3}{2} \frac{M_t^A}{G b^3 L}$$

$$\theta'_C = \frac{M_t^C}{G\left(\frac{I_0}{q}\right)^C} = \frac{M_t^C}{G \frac{4 A_0^2}{\int_{\mathcal{L}} \frac{1}{b(s)} ds}} = \frac{\frac{1}{b} \cdot (2L \cdot 2 + L \cdot 2)\, M_t^C}{G \cdot 4 \cdot (2L \cdot L)^2} = \frac{3}{8} \frac{M_t^C}{G b L^3}$$

$$\text{ma } \theta'_A = \theta'_C \longrightarrow \frac{\cancel{3}}{\cancel{2}} \frac{M_t^A}{\cancel{G} b^3 L} = \frac{\cancel{3}}{\cancel{8}_4} \frac{M_t^C}{\cancel{G} b L^3}$$

$$\frac{M_t^A}{b^2} = \frac{1}{4} \frac{M_t^C}{L^2} \longrightarrow M_t^C = 4 \frac{L^2}{b^2} M_t^A$$

per equilibrio:

$$M_t = M_t^A + M_t^C = M_t^A \left(1 + 4\frac{L^2}{b^2}\right)$$

$$\longrightarrow M_t^A = \frac{b^2}{4L^2 + b^2} M_t = \frac{M_t}{1 + 4\frac{L^2}{b^2}}$$

$$\longrightarrow M_t^C = M_t - M_t^A = M_t\left(1 - \frac{b^2}{4L^2 + b^2}\right) = M_t \cdot \frac{4L^2}{4L^2 + b^2} = \frac{M_t}{1 + \frac{1}{4}\frac{b^2}{L^2}}$$

N.B. se $b \ll L$, $M_t^C \sim M_t$ (ossia la sezione chiusa è molto più rigida e assorbe la quasi totalità del carico)

Si ricorda che:

- per sezioni aperte, le τ sono lineari sullo spessore (con valore nullo sulla linea media), per cui assumono il valore massimo sulla superficie, e si ha:

$$\tau_{max} = \frac{3 M_t\, b}{\int_{\mathcal{L}} b^3(s)\, ds}$$

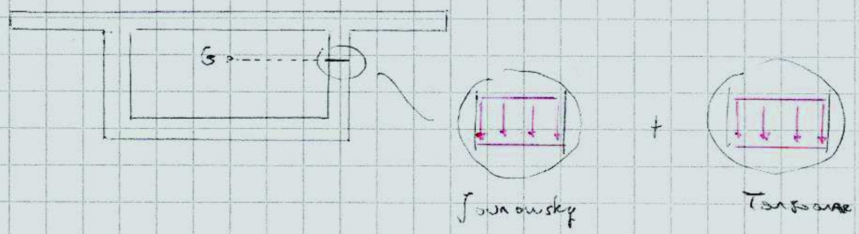
- per sezioni chiuse, le τ sono uniformi sullo spessore e tangenti alla linea media, assumono il valore massimo dove è minimo lo spessore.
  Vale la formula di Bredt: $\tau(s) = \dfrac{M_t}{2 A_0 b(s)}$

$$\longrightarrow \tau_A = \frac{3 M_t^A b}{\int_\Gamma b^3(s)\,ds} = \frac{3b}{b^3 2L} \frac{M_t}{1+4\frac{L^2}{b^2}} = \frac{M_t}{\frac{2}{3} b^2 L \left(1+4\frac{L^2}{b^2}\right)}$$

$$\longrightarrow \tau_C = \frac{M_t^C}{2 A_o b} = \frac{1}{2 \cdot 2L^2 \cdot b} \frac{M_t}{1+\frac{1}{4}\frac{b^2}{L^2}} = \frac{M_t}{4L^2 b \left(1+\frac{1}{4}\frac{b^2}{L^2}\right)}$$

$$\frac{\tau_C}{\tau_A} = \frac{1}{6}\frac{b}{L} \frac{1+4\frac{L^2}{b^2}}{1+\frac{1}{4}\frac{b^2}{L^2}}$$, se $L \sim 10b \longrightarrow \frac{\tau_C}{\tau_A} \sim 6.68$, ossia la sezione chiusa è più sollecitata a torsione

Poiché il valore massimo di τ calcolato con Jourawsky si ha sulla sezione chiusa e poiché risulta $\tau_C > \tau_A$ il punto della sezione più sollecitato è univocamente definito.



$$\tau_{max} = 0.532 \frac{T}{bL} + \frac{2TL}{4L^2 b \left(1+\frac{1}{4}\frac{b^2}{L^2}\right)}$$

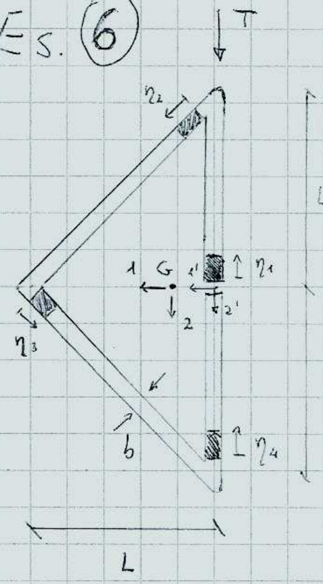
se $L \sim 10b \longrightarrow$

$$\tau_{max} = 0.532 \frac{T}{10b^2} + \frac{2T \cdot 10b}{4(10b)^2 \cdot b \left(1+\frac{1}{4}\frac{b^2}{100b^2}\right)}$$

$$= 0.053 \frac{T}{b^2} + 0.05 \frac{T}{b^2}$$

$$= 0.103 \frac{T}{b^2} \leq \tau_{adm}$$

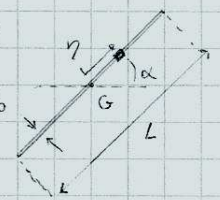
Es. 6



Asse (1)' di simmetria → $x_{2',G} = 0$

$$x_{1',G} = \frac{2 \cdot \frac{L}{2} \cdot L\sqrt{2}b}{2L\sqrt{2}b + 2Lb} = \frac{\sqrt{2}L^2 b}{(2\sqrt{2}+2)Lb} \sim 0.3L$$

N.B. Ricordare che:



$$I_\alpha = 2\int_0^{L/2} (\eta \sin\alpha)^2 \, b \, d\eta = 2b\sin^2\alpha \, \frac{1}{3}\frac{L^3}{8} = \frac{bL^3}{12}\sin^2\alpha$$

$I_1 = 1$  $+ 1$ 

$$= 2\left[\frac{1}{12} bL^3 + bL\left(\frac{L}{2}\right)^2\right] + 2\left[\frac{1}{12} b(\sqrt{2}L)^3\left(\frac{\sqrt{2}}{2}\right)^2 + \sqrt{2}Lb\cdot\left(\frac{L}{2}\right)^2\right]$$

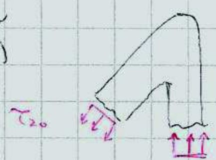
$$= \frac{1}{6} bL^3 + \frac{1}{2} bL^3 + \frac{\sqrt{2}}{6} bL^3 + \frac{\sqrt{2}}{2} bL^3 \sim 1.61\, bL^3$$

(η_1) : $\tau_1 = -\frac{T}{1.61bL^3}\frac{1}{b}\eta_1 b\left(-\eta_1/2\right) \sim 0.31\frac{T}{bL^3}\eta_1^2$

$\tau_1|_{\eta_1=L} = 0.31\frac{T}{bL}$ N.B. $\frac{d\tau_1}{d\eta_1} = 2\cdot 0.31\frac{T}{bL^3}\eta_1$ $\begin{cases} =0 \text{ se } \eta_1=0 \\ \max \text{ se } \eta_1 = L \end{cases}$

→ pendenza crescente (vedi diagramma)

equilibrio in sez. (3) al nodo:

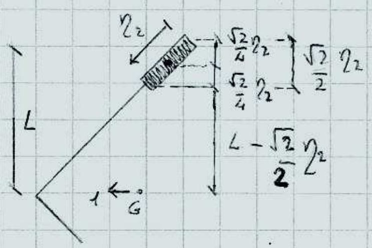


$\tau_{20}\, b\cdot 1 = \tau_1(L)\, b 1$

→ $\tau_{20} = 0.31\frac{T}{bL}$

(η_2) : $\tau_2 = \tau_{20} - \frac{T}{1.61bL^3}\frac{1}{b}\eta_2 b\left[-\left(L - \frac{\sqrt{2}}{4}\eta_2\right)\right]$

$= 0.31\frac{T}{bL} + 0.62\frac{T}{bL^3}\eta_2\left(L - \frac{\sqrt{2}}{4}\eta_2\right)$



$\tau_2|_{\eta_2=\sqrt{2}L} = 0.31\frac{T}{bL} + 0.62\frac{T}{bL^3}\sqrt{2}L\left(L - \frac{\sqrt{2}}{4}\sqrt{2}L\right)$

$= 0.31\frac{T}{bL} + 0.62\frac{T}{bL^3} L^2\frac{\sqrt{2}}{2} = 0.748\frac{T}{bL}$

per equilibrio al nodo : $\tau_{30} = \tau_2(\sqrt{2}L) = 0.748\frac{T}{bL}$

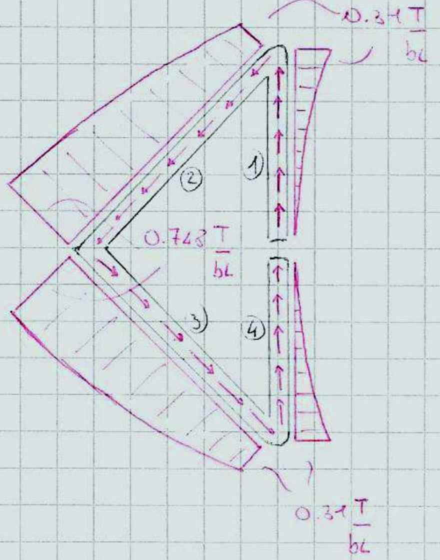
(η_3) : $\tau_3 = \tau_{30} - \frac{T}{1.61bL^3}\frac{1}{b}\eta_3 b\cdot\frac{\sqrt{2}}{4}\eta_3 = 0.748\frac{T}{bL} - 0.22\frac{T}{bL^3}\eta_3^2$

$\tau_3|_{\eta_3=\sqrt{2}L} = 0.748\frac{T}{bL} - 0.22\frac{T}{bL^3}\cdot 2L^2 \sim 0.31\frac{T}{bL}$ $(= \tau_1(L))$

per equilibrio al nodo: $\tau_{40} = \tau_3(\sqrt{2}L) = 0.31\,\frac{T}{bL}$

(η_4): $\tau_4 = \tau_{40} - \frac{T}{1.61\,bL^3}\,\frac{1}{b}\,\eta_4 b\left(L - \frac{\eta_4}{2}\right) = 0.31\,\frac{T}{bL} - 0.62\,\frac{T}{bL^3}\,\eta_4\left(L - \frac{\eta_4}{2}\right)$

$\tau_4\big|_{\eta_4 = L} = 0.31\,\frac{T}{bL} - 0.62\,\frac{T}{bL^3}\,\frac{L^2}{2} = 0$ ✓

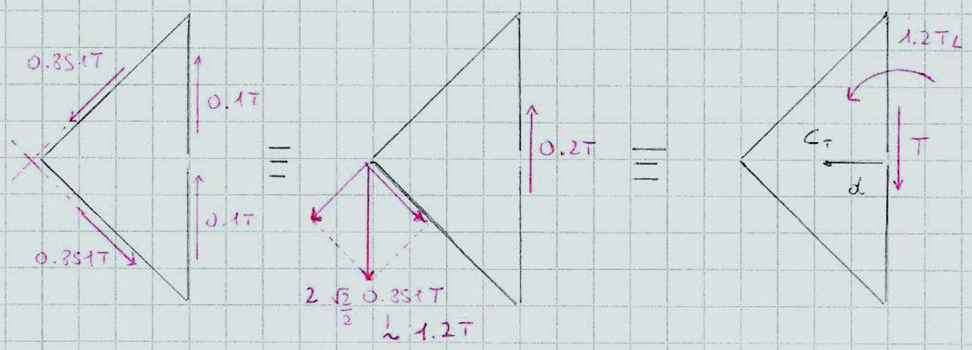


$R_1 = R_4 = \int_0^L 0.31\,\frac{T}{bL^3}\,\eta_1^2\, b\, d\eta = 0.31\,\frac{T}{bL^3}\, b\,\frac{L^3}{3} \sim 0.1\,T$

$R_2 = R_3 = \int_0^{\sqrt{2}L}\left(0.31\,\frac{T}{bL} + 0.62\,\frac{T}{bL^3}\left(\eta_2 L - \frac{\sqrt{2}}{4}\eta_2^2\right)\right) b\, d\eta$

$= 0.31\,\frac{T}{bL}\cdot b\cdot\sqrt{2}L + 0.62\,\frac{T}{bL^3}\, b\left(L\cdot\frac{1}{2}(\sqrt{2}L)^2 - \frac{\sqrt{2}}{4}\cdot\frac{1}{3}(\sqrt{2}L)^3\right)$

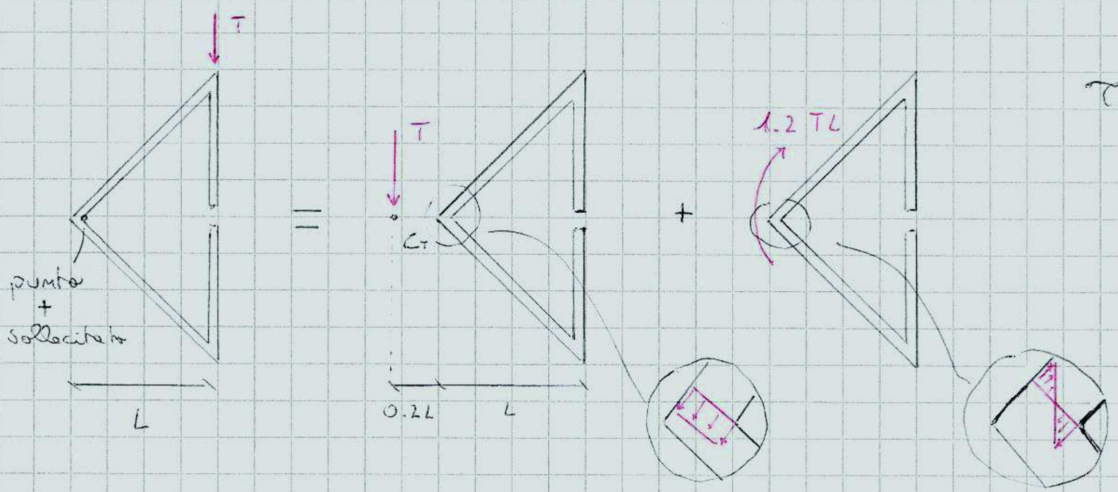
$\sim 0.851\,T$



$M_t(C_T) = 0$

$= T d - 1.2\,TL$

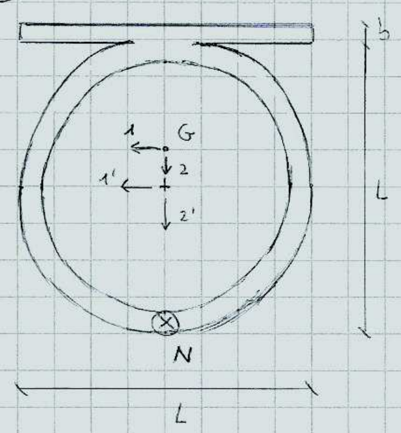
$\rightarrow d = 1.2\,L$



$\tau_{t\,as} = \frac{3 M_t b}{\int_\Sigma b^3(s)\,ds} = \frac{3\cdot 1.2\,TL\,b}{b^3\cdot(2L + 2\sqrt{2}L)} \sim 0.746\,\frac{T}{b^2}$

$\tau_{max} = 0.748\,\frac{T}{bL} + 0.746\,\frac{T}{b^2} \leq \tau_{adm}$

Es. 7

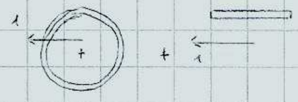


$b \ll L$

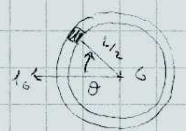
Asse 2' di simmetria

$\rightarrow x_{1',G} = 0$

$$x_{2',G} = \frac{-\frac{L}{2}\, bL}{\pi L b + bL} = \frac{-\frac{1}{2}\, bL^2}{bL(\pi+1)} = \frac{-L}{2(\pi+1)} \sim -\frac{L}{8}$$

$I_1 =$ 

$$= \left[\int_0^{2\pi} \left(\frac{L}{2}\sin\theta\right)^2 b\frac{L}{2}\,d\theta + \pi bL\left(\frac{L}{8}\right)^2\right] + \left[\frac{1}{12} b^3 L + bL\left(\frac{L}{2}-\frac{L}{8}\right)^2\right]$$



$$= \frac{L^3}{8}\, b\, \frac{2\pi}{2} + \frac{\pi}{64} bL^3 + \frac{9}{64} bL^3 \sim 0.58\, bL^3$$

N.B. In questo caso non occorre calcolare I_2, poiché il carico N genera momento solo attorno all'asse (1).

Caso di "pressoflessione retta"

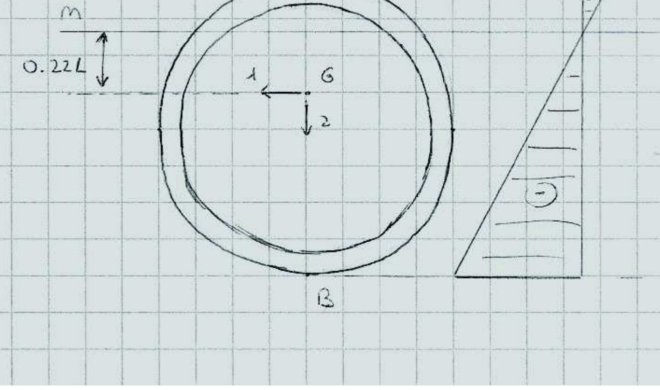
$$\begin{cases} N = -N \\ M_1 = -N\left(\frac{L}{2}+\frac{L}{8}\right) = -\frac{5}{8} NL \\ M_2 = 0 \end{cases} \rightarrow$$

Eq.ne Navier:

$$\sigma = \frac{N}{A} + \frac{M_1}{I_1} x_2 - \cancel{\frac{M_2}{I_2} x_1} = -\frac{N}{A} - \frac{5}{8} \frac{NL}{I_1} x_2$$

asse neutro:

$$\sigma = 0 \iff \frac{5}{8}\frac{NL}{I_1} x_2 = -\frac{N}{A} \rightarrow x_2 = -\frac{N}{A}\cdot\frac{8}{5}\frac{I_1}{NL} = -\frac{8}{5}\,\frac{0.58\, bL^3}{bL(\pi+1)L} \sim -0.22\, L$$



$$A = \left(0, -\left(\frac{L}{2} - \frac{L}{8}\right)\right) = \left(0, -\tfrac{3}{8}L\right)$$

$$B = \left(0, \frac{L}{2} + \frac{L}{8}\right) = \left(0, \tfrac{5}{8}L\right)$$

$$\sigma_A = -\frac{N}{A} - \frac{5}{8}\frac{NL}{I_1}\left(-\frac{3}{8}L\right)$$

$$= -\frac{N}{bL(\pi+1)} + \frac{15}{64}\frac{NL^2}{0.58bL^3} = \frac{N}{bL}\left(-\frac{1}{\pi+1} + \frac{15}{64\cdot 0.58}\right) \sim 0.16\frac{N}{bL} > 0$$

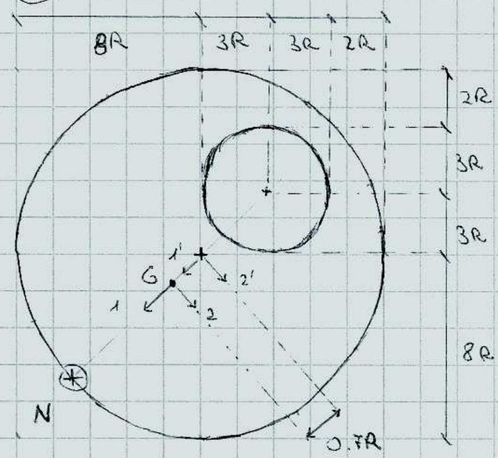
$$\rightarrow \sigma_A < \sigma^{t}_{adm}$$

$$\sigma_B = -\frac{N}{A} - \frac{5}{8}\frac{NL}{I_1}\cdot\frac{5}{8}L$$

$$= -\frac{N}{bL(\pi+1)} - \frac{25}{64}\frac{NL^2}{0.58bL^3} = -\frac{N}{bL}\left(\frac{1}{\pi+1} + \frac{25}{64\cdot 0.58}\right) \sim -0.91\frac{N}{bL} < 0$$

$$\rightarrow \sigma_B > \sigma^{c}_{adm}$$

Es. 8



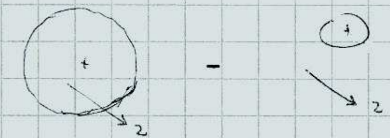
Asse 1' di simmetria $\longrightarrow x_{2',G} = 0$

$$x_{1',G} = \frac{-(-3\sqrt{2}R)\pi(3R)^2}{\pi(8R)^2 - \pi(3R)^2} \sim 0.7R$$

N.B. N genera momento solo intorno all'asse 2

$I_1 =$ 

$$= \frac{\pi}{4}(8R)^4 - \frac{\pi}{4}(3R)^4 \sim 3152\,R^4$$

$I_2 =$ 

$$= \left[\frac{\pi}{4}(8R)^4 + \pi(8R)^2\cdot(0.7R)^2\right] - \left[\frac{\pi}{4}(3R)^4 + \pi(3R)^2\cdot(0.7R + 3\sqrt{2}R)^2\right]$$

$$\sim 2560\,R^4$$

Caso di "pressoflessione retta":

$$\begin{cases} N = -N \\ M_2 = N\cdot e = N(8R - 0.7R) = 7.3\,NR \\ M_1 = 0 \end{cases}$$

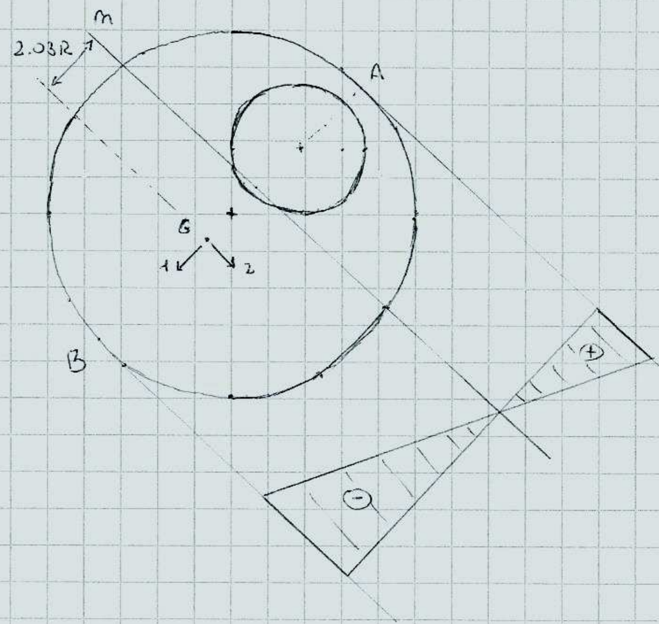
Eq.ne di Navier:

$$\sigma = \frac{N}{A} + \cancel{\frac{M_1}{I_1}x_2} - \frac{M_2}{I_2}x_1$$

$$= -\frac{N}{55\pi R^2} - \frac{7.3\,NR}{2560\,R^4}x_1$$

Asse neutro:

$$\sigma = 0 \iff x_1 = -\frac{N}{55\pi R^2}\cdot\frac{2560\,R^4}{7.3\,NR} = -2.03\,R$$



$$A = (-0.7R - 8R, 0) = (-8.7R, 0)$$

$$B = (8R - 0.7R, 0) = (7.3R, 0)$$

$$\sigma_A = \frac{N}{A} - \frac{M_2}{I_2} x_{1,A}$$

$$= -\frac{N}{55\pi R^2} - \frac{7.3NR}{2560R^4}(-8.7R)$$

$$= 0.019\frac{N}{R^2} > 0$$

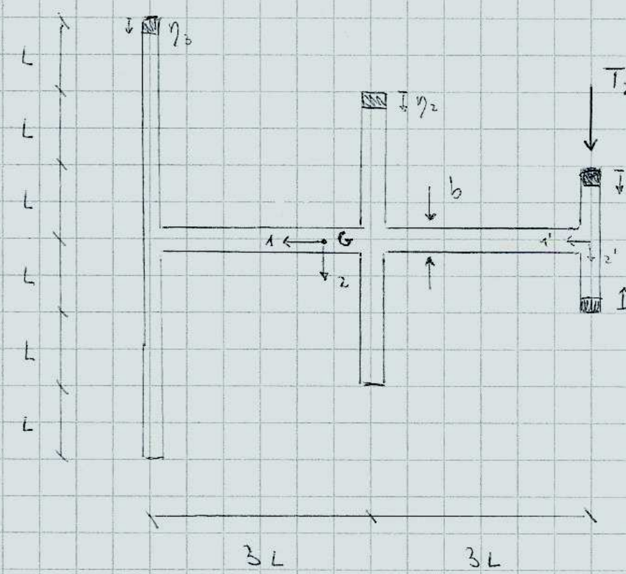
$$\longrightarrow \sigma_A < \sigma^t_{adm}$$

$$\sigma_B = \frac{N}{A} - \frac{M_2}{I_2} x_{1,B}$$

$$= -\frac{N}{55\pi R^2} - \frac{7.3NR}{2560R^4}(7.3R)$$

$$= -0.027\frac{N}{R^2} < 0 \qquad \longrightarrow \sigma_B < \sigma^c_{adm}$$

ES. 9



Calcolo del baricentro:

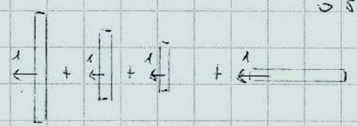
asse (1') di simmetria $\longrightarrow x_{2',G} = 0$

$$x_{1',G} = \frac{\frac{3}{2}L\cdot 3Lb + 3L\cdot 4Lb + \frac{9}{2}L\cdot 3Lb + 6L\cdot 6Lb}{2Lb + 3Lb + 4Lb + 3Lb + 6Lb} = \frac{66}{18}L \sim 3.66L$$

Formula di Jourawsky:

$$\tau = -\frac{T_2}{I_1}\frac{S_1(A^*)}{b} \qquad (i)$$

N.B. Dalla relazione (i) vedo subito che l'anima centrale è scarica, poiché baricentrica, ossia per essa è $S_1(A^*) = 0$ (si verifica avanti)

$I_1 =$ 

$$= \frac{1}{12}b(6L)^3 + \frac{1}{12}b(4L)^3 + \frac{1}{12}b(2L)^3 + \frac{1}{12}b^3\cdot 6L = 24\,bL^3$$

con: $S_1(A^*) = A^*\cdot x_{2,G}(A^*)$

Calcolo delle τ con Jourawsky

(η_1): $\tau_1 = -\frac{T_2}{24bL^3}\cdot\frac{1}{b}\cdot\eta_1 b\cdot\left[-\left(L-\frac{\eta_1}{2}\right)\right] = \frac{T_2}{24bL^3}\eta_1\left(L-\frac{\eta_1}{2}\right)$

$\tau_1|_{\eta_1=0} = 0$

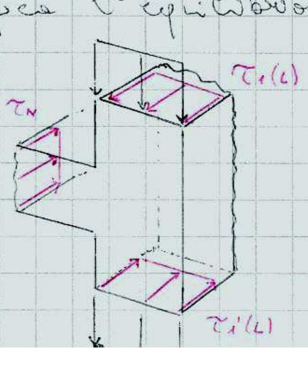
$\tau_1|_{\eta_1=L} = \frac{T_2}{24bL^3}L\left(L-\frac{L}{2}\right) = \frac{T_2}{48bL}$

(η_1'): $\tau_1' = -\frac{T_2}{24bL^3}\frac{1}{b}\eta_1' b\left(L-\frac{\eta_1'}{2}\right) = -\frac{T_2}{24bL^3}\eta_1'\left(L-\frac{\eta_1'}{2}\right)$

$\tau_1'|_{\eta_1'=0} = 0$

$\tau_1'|_{\eta_1'=L} = (-)\frac{T_2}{48bL}$ verso opposto a η_1'

verifica l'equilibrio al nodo in dir. (3):



lunghezza unitaria in dir (3)

$\tau_N\, b\cdot 1 = \tau_1(L)\cdot b\cdot 1 - \tau_1'(L)\cdot b\cdot 1$ (equilibrio tra forze)

$= \frac{T_2}{48bL} - \frac{T_2}{48bL} = 0$ ✓

corretto x analogia idrodinamica

(η_2): $\tau_2 = -\dfrac{T_2}{24bL^3}\dfrac{1}{b}\cdot \eta_2 b\left[-\left(2L-\dfrac{\eta_2}{2}\right)\right] = \dfrac{T_2}{24bL^3}\,\eta_2\left(2L-\eta_2/2\right)$

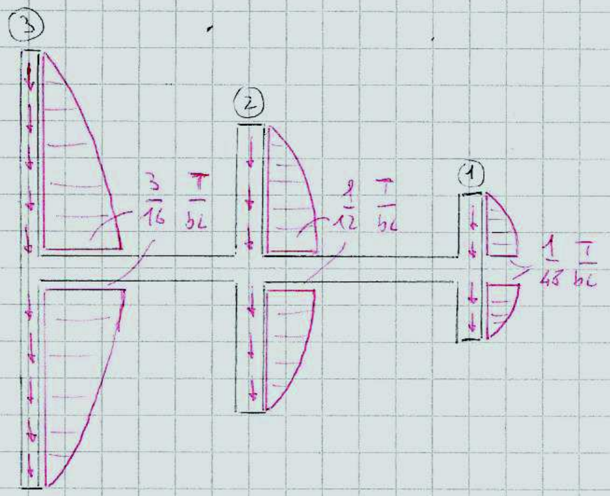
$\tau_2|_{\eta_2=0} = 0$

$\tau_2|_{\eta_2=2L} = \dfrac{T_2}{24bL^3}\,2L\left(2L-2L/2\right) = \dfrac{T}{12bL}$ (per η_2' vedere η_1')

(η_3): $\tau_3 = -\dfrac{T_2}{24bL^3}\dfrac{1}{b}\,\eta_3\left[-\left(3L-\dfrac{\eta_3}{2}\right)\right] = \dfrac{T_2}{24bL^3}\,\eta_3\left(3L-\eta_3/2\right)$

$\tau_3|_{\eta_3=0} = 0$

$\tau_3|_{\eta_3=3L} = \dfrac{T_2}{24bL^3}\,3L\left(3L-3L/2\right) = \dfrac{3}{16}\dfrac{T}{bL}$



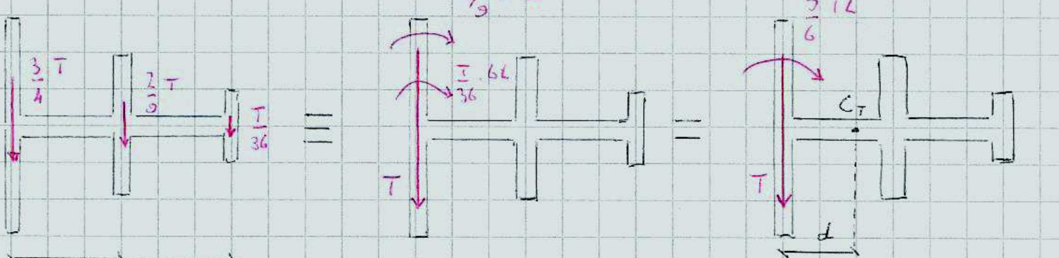
$$R_1 = 2\int_0^L \frac{T_2}{24bL^3}\left(\eta_1 L - \frac{\eta_1^2}{2}\right) b\, d\eta = 2\cdot\frac{T_2}{24bL^3}\left[L\frac{L^2}{2} - \frac{1}{2}\frac{L^3}{3}\right] = \frac{T_2}{36}$$

$$R_2 = 2\int_0^{2L} \frac{T_2}{24bL^3}\left(2L\eta_2 - \eta_2^2/2\right) b\, d\eta = 2\frac{T_2}{24bL^3}\left[2L\cdot\frac{1}{2}4L^2 - \frac{1}{2}\frac{1}{3}8L^3\right] = \frac{2}{9}T_2$$

$$R_3 = 2\int_0^{3L} \frac{T_2}{24bL^3}\left(3L\eta_3 - \eta_3^2/2\right) b\, d\eta = 2\frac{T_2}{24bL^3}\left[3L\cdot\frac{1}{2}9L^2 - \frac{1}{2}\cdot\frac{1}{3}27L^3\right] = \frac{3}{4}T_2$$

verifica:

$R_1 + R_2 + R_3 = \left(\dfrac{1}{36} + \dfrac{2}{9} + \dfrac{3}{4}\right)T_2 = T_2$ ✓

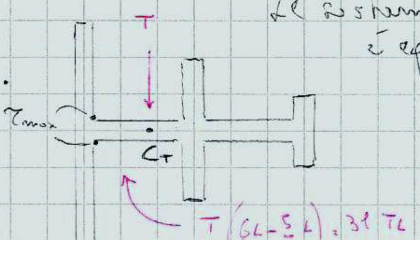


$\dfrac{2}{9}T\cdot 3L + \dfrac{T}{36}\cdot 6L = \dfrac{5}{6}TL$

Creo un sistema equivalente spostando le risultanti e introducendo i relativi momenti di trasporto

Ricerca del "centro di taglio": $M_t(C_T) = 0 = Td - \frac{5}{6}TL \longrightarrow d = \dfrac{5}{6}L$

Il sistema iniziale è equivalente a:



Calcolo le τ da torsione:

$$\tau_{tors} = \frac{3M_t\cdot b}{\int_\gamma b^3(s)\,ds} = \frac{3\cdot\frac{31}{6}TL\cdot b}{b^3\cdot 18L} = \frac{31}{36}\frac{T}{b^2}$$

$\longrightarrow \tau_{max} = 3\ T + 31\ T\ (\tau_{tors}$