ESERCITAZIONE #9

SIMULAZIONE D'ESAME 26-05-16

$L = 1\ m$

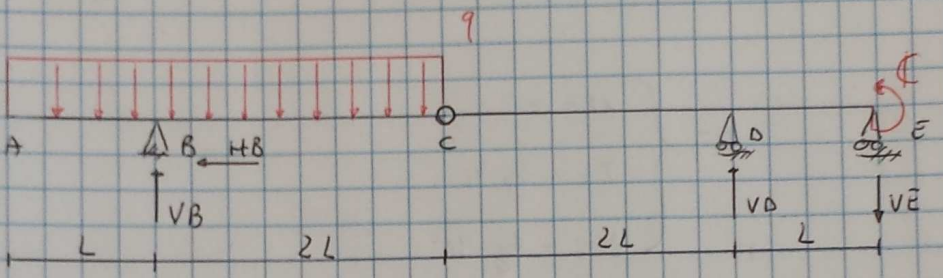
$q = 25\ kN/m$

$C = 25\ kNm$

$E = 210\ GPa$

$\sigma_{AMM} = 240\ MPa$

$\alpha = 10^{-5}\ C^{-1}$



$\rightarrow C = qL^2$

$\rightarrow \quad HB = 0$

$\uparrow \quad VB + VD = 3qL + VE$

$c)_{CBA} \quad VB \cdot 2L = 3qL \cdot \frac{3}{2}L \qquad \underline{VB = \frac{9}{4}qL}$

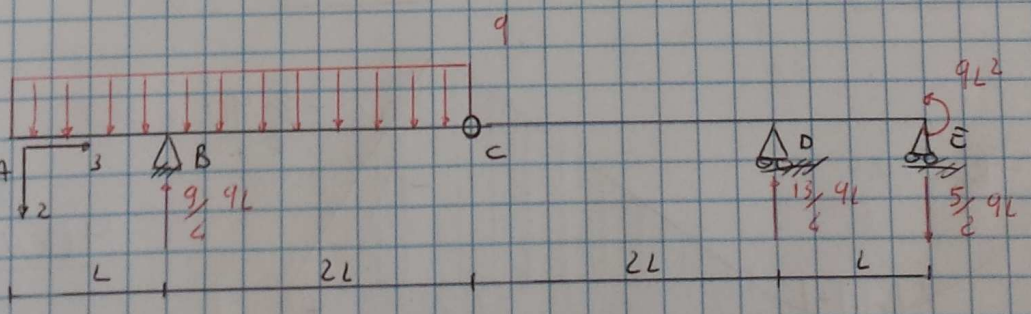
$c)_{CDE} \quad VD \cdot 2L + qL^2 = VE \cdot 3L \qquad VE = \frac{2}{3}VD + \frac{qL}{3}$

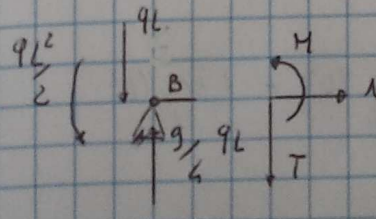
$\frac{9}{4}qL + VD = 3qL + \frac{2}{3}VD + \frac{qL}{3} \qquad \frac{1}{3}VD = \frac{10}{3}qL - \frac{9}{4}qL$

$\frac{1}{3}VD = \frac{40-27}{12} \qquad \underline{VD = \frac{13}{4}qL}$

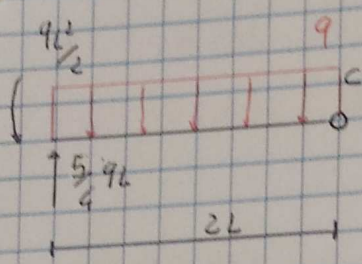
$VE = \frac{2}{3}\frac{13}{4}qL + \frac{qL}{3} \qquad VE = \frac{13}{6}qL + \frac{qL}{3} \qquad VE = \frac{15}{6}qL$

$\underline{VE = \frac{5}{2}qL}$

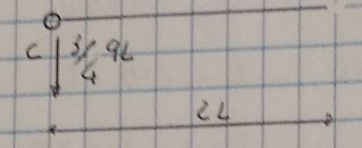




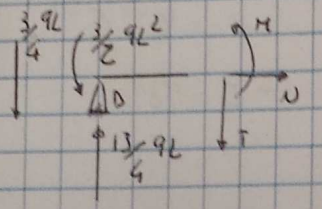
$B^+ \begin{cases} N = 0 \\ T = \frac{9}{4}qL - qL = \frac{5}{4}qL \\ M = -qL^2/2 \end{cases}$



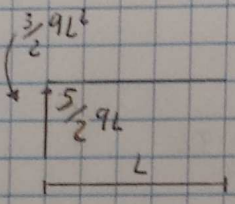
$$C\begin{cases} N=0 \\ T=5/4\,9L-2\,9L=-3/4\,9L \\ M=-9L^2/2+5/4\,9L\cdot 2L-2\,9L^2=\dfrac{(-2+10-8)}{4}9L^2=0 \end{cases}$$



$$D^-\begin{cases} N=0 \\ T=-3/4\,9L \\ M=-3/2\,9L^2 \end{cases}$$

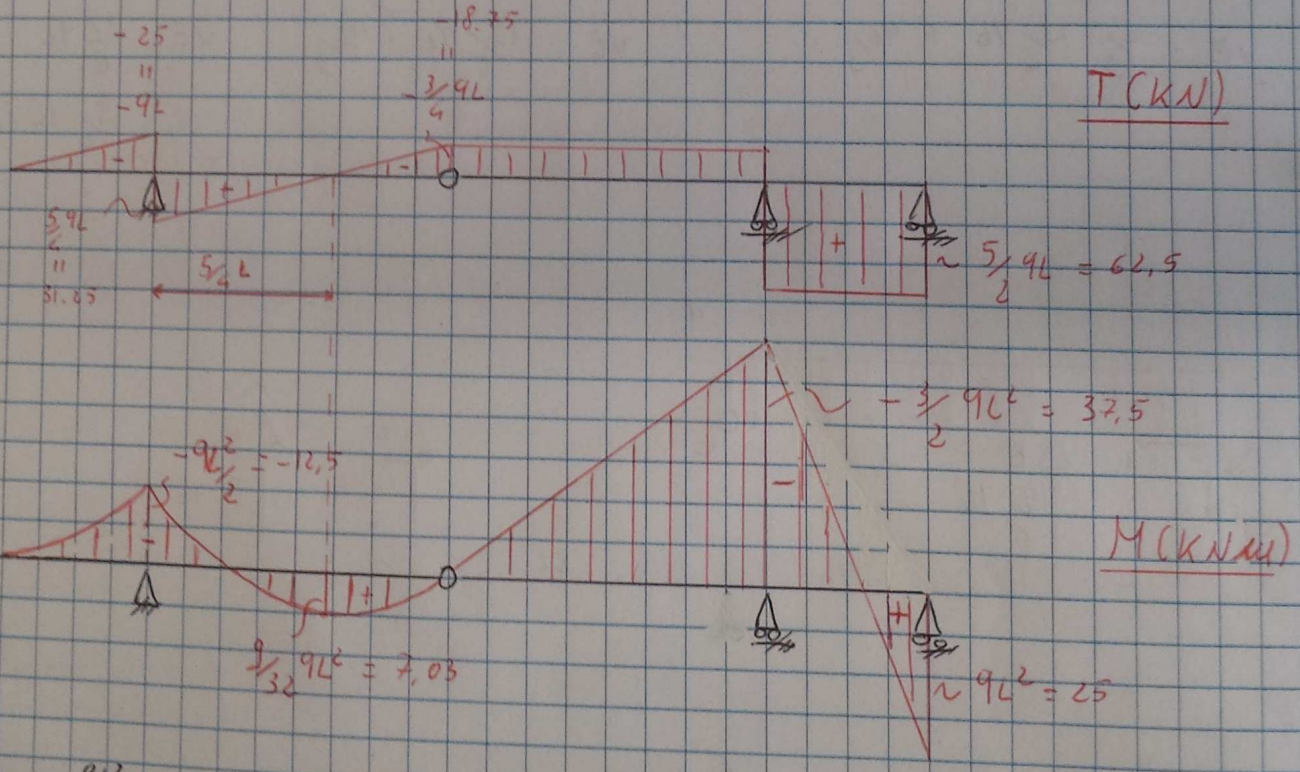


$$D^+\begin{cases} N=0 \\ T=13/4\,9L-3/4\,9L=\dfrac{10}{4}9L=5/2\,9L \\ M=-3/2\,9L^2 \end{cases}$$



$$E^-\begin{cases} N=0 \\ T=5/2\,9L \\ M=-3/2\,9L^2+5/2\,9L^2=9L^2 \end{cases}$$

$N=0$



$$M\left(\tfrac{5}{4}L\right)=-\frac{9L^2}{2}+\frac{5}{4}9L\cdot\frac{5}{4}L-\frac{5}{4}9L\cdot\frac{5}{8}L=-\frac{9L^2}{2}+\frac{25}{16}9L^2-\frac{25}{32}9L^2$$

$$=\frac{(-16+50-25)}{32}9L^2=\frac{9}{32}9L^2$$

2) DIMENSIONAMENTO

$\sigma_{AMM} = 240$ MPa

$M_{1MAX} / W_1 \leq \sigma_{AMM}$

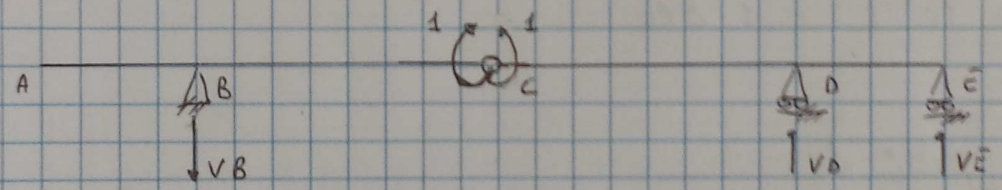
$M_{fMAX} = \frac{3}{2} q l^2 = \frac{3}{2} \cdot 25 \cdot 1 \cdot 10^6$ N mm

$W_1 \geq M_{fMAX} / \sigma_{AMM} = \frac{\frac{3}{2} \cdot 25 \cdot 10^6}{240} = 156250 \text{ mm}^3 = 156{,}25 \text{ cm}^3$

→ IPE 200 $\begin{cases} W_1 = 194{,}3 \text{ cm}^3 \\ I_1 = 1943 \text{ cm}^4 \end{cases}$

3) CALCOLARE $\Delta\varphi_C$

SISTEMA AUSILIARIO FORZE TENSIONI



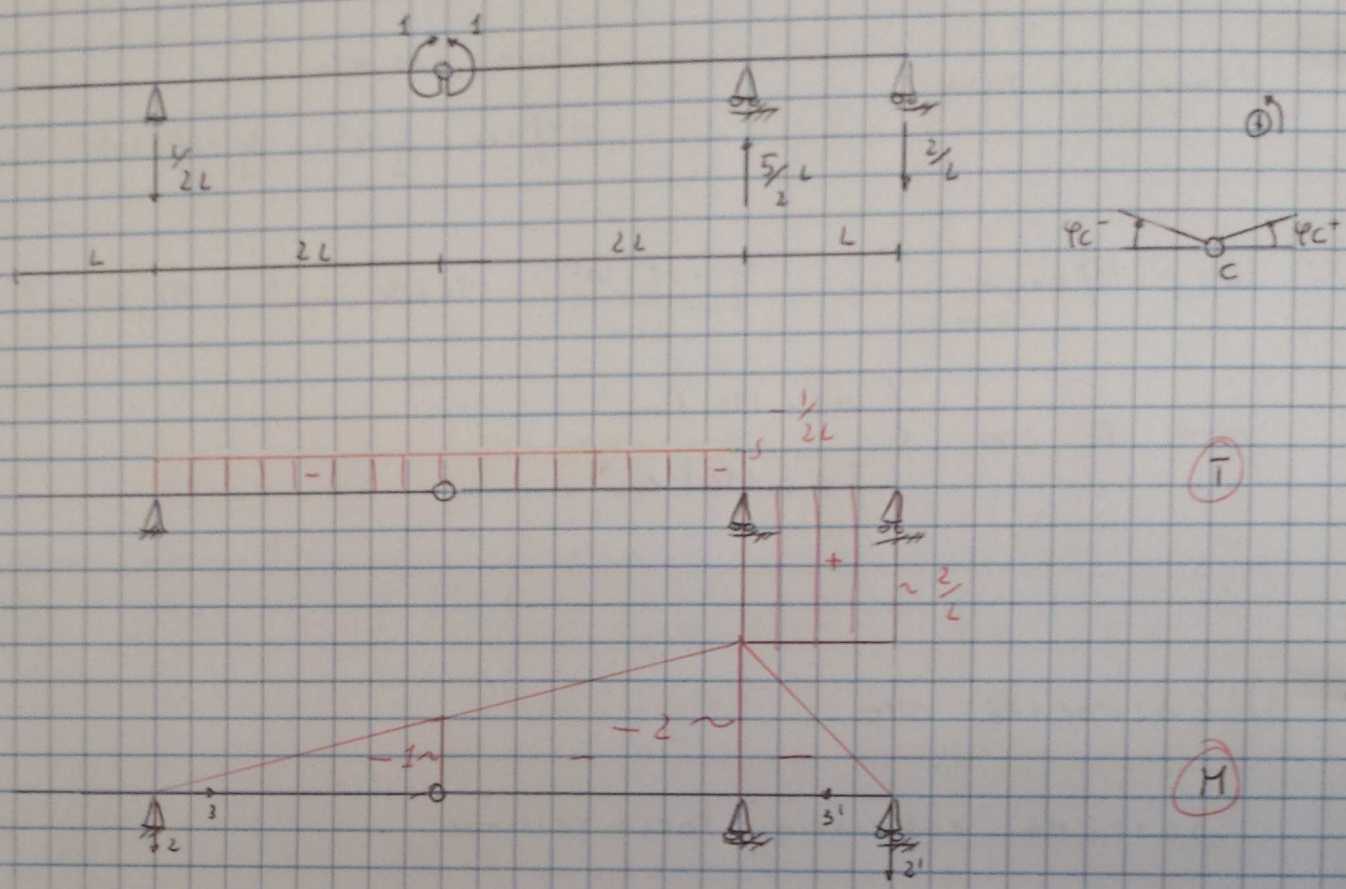
↻ CBA $VB \cdot 2l = 1$ $VB = \frac{1}{2l}$

↻ CDE $VD \cdot 2l + VE \cdot 3l + 1 = 0$ $VD = -\frac{3}{2} VE - \frac{1}{2l}$

↑ $VB = VE + VD$ $\frac{1}{2l} = VE - \frac{3}{2} VE - \frac{1}{2l}$ $\frac{1}{l} = -\frac{1}{2} VE$

$VE = -\frac{2}{l}$

$\frac{1}{2l} = -\frac{2}{l} + VD$ $VD = \frac{5}{2l}$



$$L_e = -\varphi_C^- \cdot 1 + \varphi_C^+ \cdot 1 = 1 \cdot \Delta\varphi_C$$

$$L_{ik} = \frac{1}{EI} \int_{Strut} M M^* \, dx_3$$

$$= \frac{1}{EI}\left[\int_0^{2L}\left(-\frac{9L^2}{2} + \frac{5}{4}9Lx_3 - \frac{9x_3^2}{2}\right)\left(-\frac{1}{2L}x_3\right)dx_3 + \int_0^{2L}\left(-\frac{3}{4}9Lx_3\right)\left(-1-\frac{x_3}{2L}\right)dx_3\right.$$

$$\left. + \int_0^{L}\left(9L^2 - \frac{5}{2}9Lx_3'\right)\left(-\frac{2}{L}x_3'\right)dx_3'\right]$$

$$= \frac{1}{EI}\left[\int_0^{2L}\left(+\frac{9L}{4}x_3 - \frac{5}{8}9x_3^2 + \frac{9x_3^3}{4L}\right)dx_3 + \int_0^{2L}\left(+\frac{3}{4}9Lx_3 + \frac{3}{8}9x_3^2\right)dx_3\right.$$

$$\left. + \int_0^{L}\left(-29Lx_3' + 59x_3'^2\right)dx_3'\right]$$

$$= \frac{1}{EI}\left[\left(\frac{9Lx_3^2}{8} - \frac{5}{24}9x_3^3 + \frac{9x_3^4}{16L}\right)_0^{2L} + \left(\frac{3}{8}9Lx_3^2 + \frac{3}{24}9x_3^3\right)_0^{2L} + \left[-9Lx_3'^2 + \frac{5}{3}9x_3'^3\right]_0^{L}\right]$$

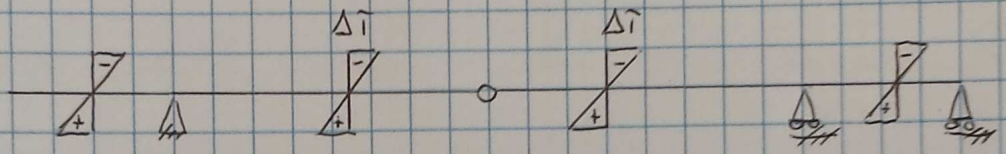
$$= \frac{1}{EI}\left[\left(\frac{1}{2}9L^3 - \frac{5}{3}9L^3 + 9L^3\right) + \left(\frac{3}{2}9L^3 + 9L^3\right) + \left[-9L^3 + \frac{5}{3}9L^3\right)\right]$$

$$= \frac{1}{EI}\left[-\frac{9L^3}{6} + \frac{5}{2}9L^3 + \frac{2}{3}9L^3\right] = \frac{1}{EI} \cdot 39L^3 = 39L^3/EI$$

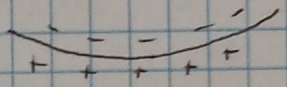
$$\Delta\varphi_C = 3\,qL^3/_{EI}$$

$$= \frac{3 \cdot 25 \cdot 10^9}{210 \cdot 10^3 \cdot 1943 \cdot 10^4} = \underline{0{,}0184\ rad = 1{,}05°}$$

c) ASSUMERE ΔT TALE DA ANNULLARE $\Delta\varphi_C$



$$L_e = 1 \cdot \Delta\varphi^T = L_i$$



CURVATURA POSITIVA

$$L_i = \int_{STRUTT} M^* \chi_T$$

$$\chi_T = \frac{2\alpha\Delta T}{H} = \cos T.$$

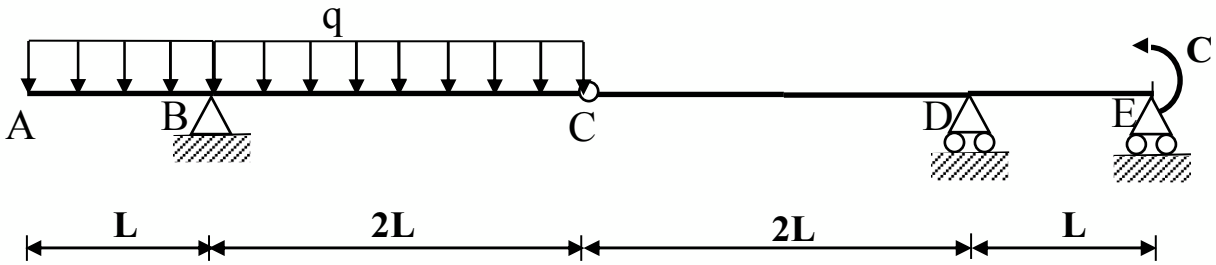
$$L_i = \chi_T \left[\underbrace{4L \cdot \left(-{}^2/_2\right)}_{\text{AREA TRIANGOLO LUNGO } 4L \text{ DEL SISTEMA}} * \left(b - {}^h/_2\right)\right] + \chi_T \left[L\left(-{}^2/_2\right)\right] = -5L\chi_T$$

$$0 = \Delta\varphi_C + \Delta\varphi_C^T \qquad \Delta\varphi_C{}^T = -\Delta\varphi_C$$

$$0{,}0184 = -\left(-10\alpha\Delta T\, L/H\right) \qquad \frac{\Delta\varphi_C H}{10\alpha L} = +\Delta T$$

$$\frac{0{,}0184 \cdot 200}{10 \cdot 10^{-5} \cdot 10^3} = \underline{36{,}8\ °C}$$

CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
ESERCITAZIONE SCRITTA DI STATICA
26/05/2016



L= 1 m, q = 25 kN/m, C = 25 kNm,
E = 210 GPa, σ_{amm} = 240 MPa, $\alpha = 10^{-5}$ °C^{-1}

La travatura isostatica in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione.
- Dimensionare la travatura.
- Calcolare la rotazione relativa nella cerniera in C.
- Assumendo di applicare un carico termico a farfalla su tutta la trave, calcolare la variazione di temperatura necessaria per annullare la rotazione relativa calcolata al punto precedente.