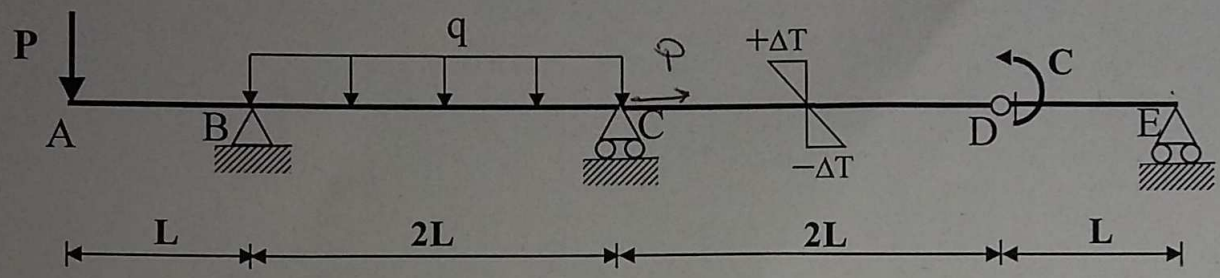
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
01/06/2016

A



L= 1 m, q = 20 kN/m, P= 20 kN, C = 40 kNm,
E = 210 GPa, σ_{amm} = 240 MPa, ΔT = +10° C , $\alpha = 10^{-5}$ °C^{-1}

La travatura isostatica in figura deve essere realizzata con profilati IPE.

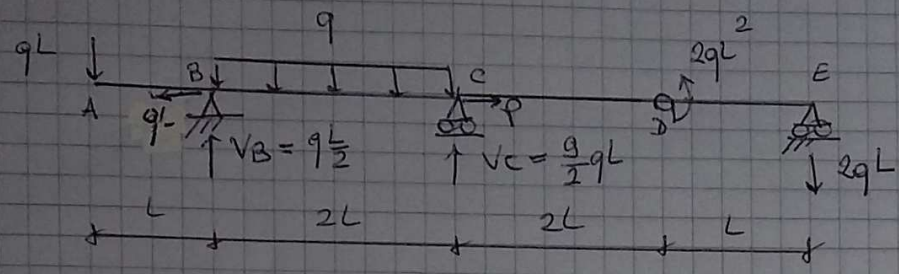
- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione.
- Dimensionare la travatura.
- Calcolare la rotazione relativa nella cerniera in D.
- Calcolare la rotazione relativa in D considerando in aggiunta al carico anche la presenza di un carico termico a farfalla ΔT sul tratto CDE.

SOLUZIONE FILA A

$qL = 20\ kN = P$

$qL^2 = 20\ kN\,m$

$C = 2qL^2$



$$\begin{cases} V_B + V_C = qL + 2qL + 2qL = 5qL \\ V_B\,4L + V_C\,2L - 5qL^2 - 2qL\,3L = 0 \end{cases}$$

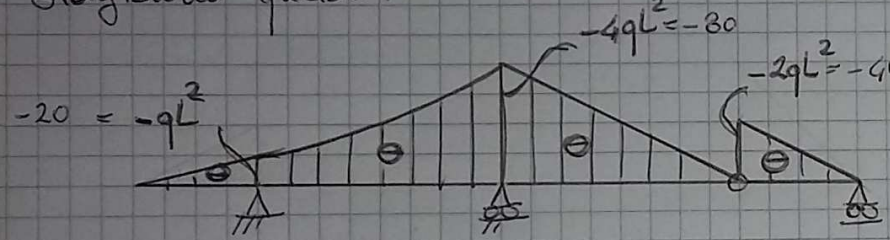
$$\begin{cases} V_B + V_C = 5qL \quad (\times(-2)) \\ 4V_B + 2V_C = 11qL \end{cases}$$

$$4V_B - 2V_B + 2V_C - 2V_C = 11qL - 10qL$$

$$2V_B = qL$$

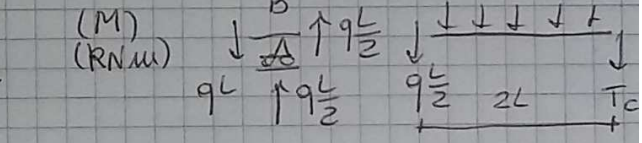
$$\begin{cases} V_B = qL/2 \\ V_C = 5qL - V_B = 5qL - q\frac{L}{2} = \frac{9}{2}qL \end{cases}$$

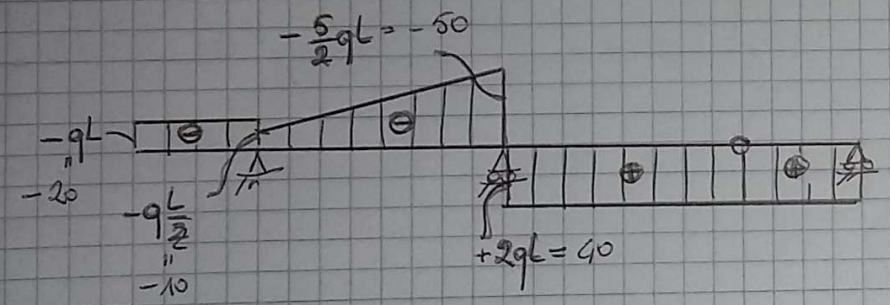
Diagrammi quotati:



$M_C^+ = -2qL\,3L + 2qL^2 = -4qL^2$

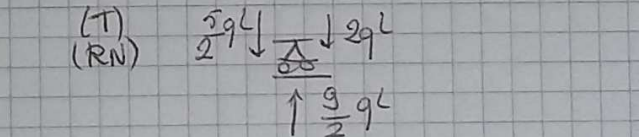
(M)
(RN m)

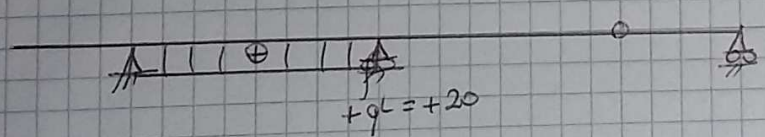




$T_C^- = -q\frac{L}{2} - 2qL = -\frac{5}{2}qL$

(T)
(RN)





(N)
(RN)

Dimensionamento:

$$W_1 \geq \frac{qL^2}{\sigma_{AMM}} = \frac{80 \cdot 10^3}{\frac{240}{3} \cdot 10^6}\ m^3 = 333\ cm^3 \rightarrow \text{IPE 270}$$

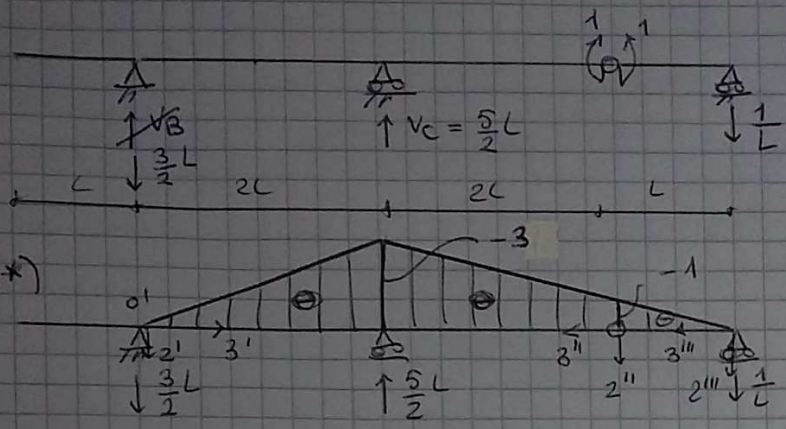
$W_1 = 428,9\ cm^3$

$I_1 = 5790\ cm^4$

$H = 27\ cm$

$A = 45,95\ cm^2$

Rotazione relativa in D:



$$\begin{cases} V_B + V_C = \frac{1}{L} \\ V_B 4L + V_C 2L + 1 = 0 \end{cases}$$

$$\begin{cases} V_B + V_C = 1/L \\ 4V_B + 2V_C = -1/L \end{cases} \quad \times(-2)$$

$$4V_B - 2V_B + 2V_C - 2V_C = -\frac{1}{L} - \frac{2}{L}$$

$$2V_B = -\frac{3}{L}$$

$$V_B = -\frac{3}{2L}$$

$$V_C = \frac{1}{2L} + \frac{3}{2}L = \frac{5}{2}L$$

$$1 \cdot \Delta\varphi_D^q = \frac{1}{EI_1}\int_0^{2L}\left(-q\frac{x_3'^2}{2} - q\frac{L}{2}x_3' - qL^2\right)\left(-\frac{3}{2L}x_3'\right)dx_3' + \frac{1}{EI_1}\int_0^{2L}\left(-1-\frac{x_3''}{L}\right)\left(-2qLx_3''\right)dx_3''$$

$$+\frac{1}{EI_1}\int_0^{L}\left(-2qLx_3'''\right)\left(-\frac{x_3'''}{L}\right)dx_3'''$$

$$= \frac{1}{EI_1}\left(8qL^3 + \frac{28}{3}qL^3 + \frac{2}{3}qL^3\right) = \frac{18qL^3}{EI_1} = \frac{18 \cdot 20 \cdot 10^3}{210 \cdot 10^8 \cdot 5790 \cdot 10^{-8}} = 0,0296 \text{ rad} \simeq 1,7°$$

Carico termico:

$$1 \cdot \Delta\varphi_D^{\Delta T} = \int_{CDE} M^* \chi_t \, dx_3 = \chi_t \int_{CDE} M^* \, dx_3 = \left(-\frac{2\alpha\Delta T}{H}\right)\left(-3L \cdot \frac{3}{2}\right) = \frac{9\alpha\Delta TL}{H}$$

$$= \frac{9 \cdot 10^{-5} \cdot 10^4}{0,27} = 0,0033 \text{ rad}$$

$$= 0,19°$$

$$\Delta\varphi_D^{TOT} = \frac{9\alpha\Delta TL}{H} + \frac{18qL^3}{EI_1} \simeq 1,89°$$

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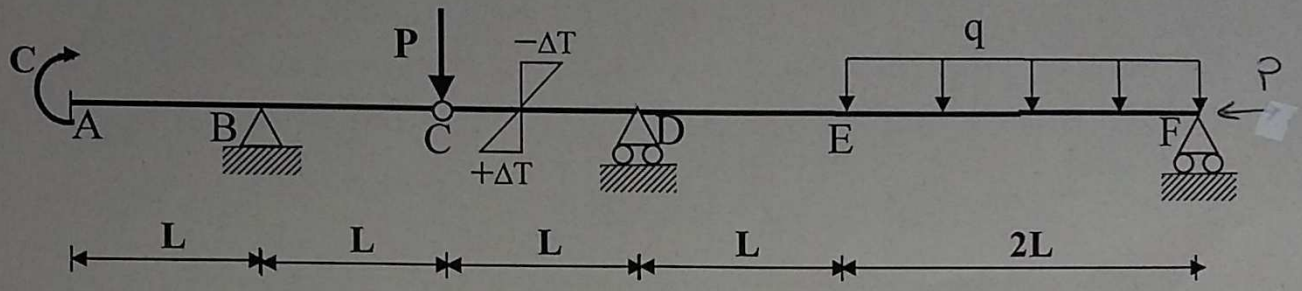
B



L= 1 m, q = 25 kN/m, P= 25 kN, C = 50 kNm,
E = 210 GPa, σ_{amm} = 240 MPa, ΔT = +10° C , $\alpha = 10^{-5}$ °C^{-1}

La travatura isostatica in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione.
- Dimensionare la travatura.
- Calcolare la rotazione relativa nella cerniera in C.
- Calcolare la rotazione relativa in C considerando in aggiunta al carico anche la presenza di un carico termico a farfalla ΔT sul tratto ABCD.

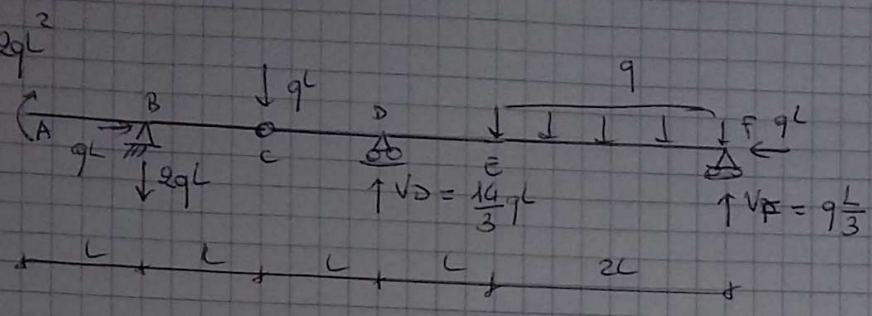
...UZIONE FILA B

$qL = 25\ kN$

$qL^2 = 25\ kN\sqrt{m}$

$P = qL$

$\mathcal{C} = 2qL$



$$\begin{cases} V_D + V_F = 2qL + qL + 2qL = 5qL \\ V_D 4L + V_F 4L = 2qL\,3L \end{cases}$$

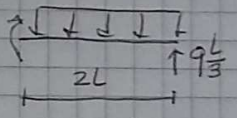
$$\begin{cases} V_D + V_F = 5qL \quad (\times(-1)) \\ V_D + 4V_F = 6qL \end{cases}$$

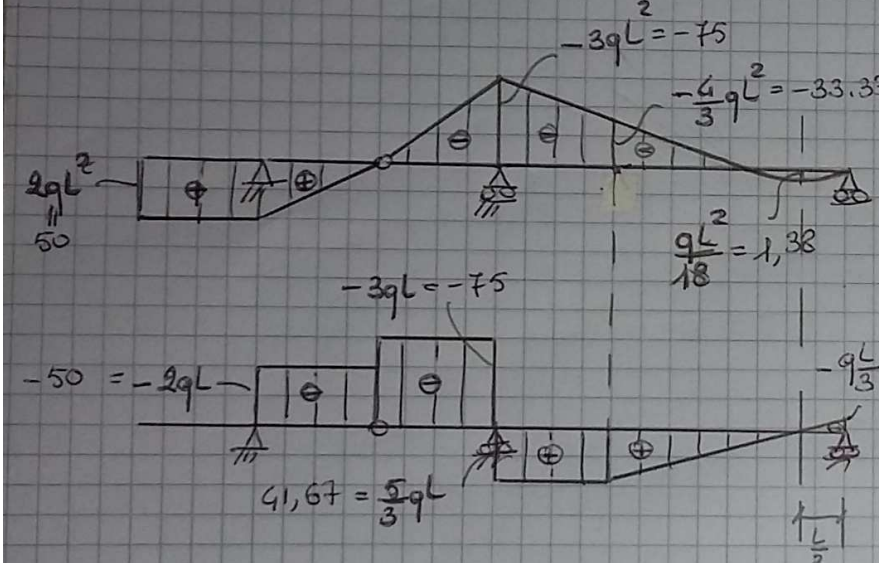
$$V_D + 4V_F - V_D - V_F = 6qL - 5qL = qL$$

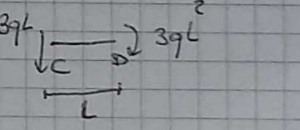
$$\begin{cases} 3V_F = qL \\ V_D = 5qL - V_F \end{cases} \qquad \begin{cases} V_F = qL/3 \\ V_D = \frac{14}{3} qL \end{cases}$$

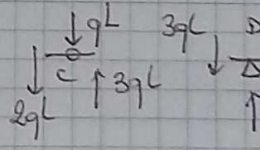
Diagrammi quotati:

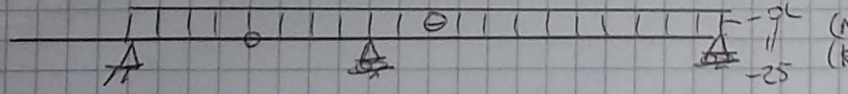
$M_E = q\frac{L}{3}\,2L - 2qL^2 = -\frac{4}{3}qL^2$

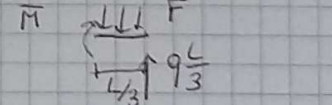












$\bar{M} = q\frac{L}{3}\frac{L}{3} - \frac{1}{2}q\frac{L}{3}\frac{L}{3} = q\frac{L^2}{18}$

Dimensionamento:

$$W_1 \geq \frac{3qL^2}{6A\sigma_{amm}} = \frac{75 \cdot 10^3 \cdot 10^6}{240 \cdot 10^6}\ cm^3 = 312{,}5\ cm^3 \rightarrow \text{IPE 240}$$

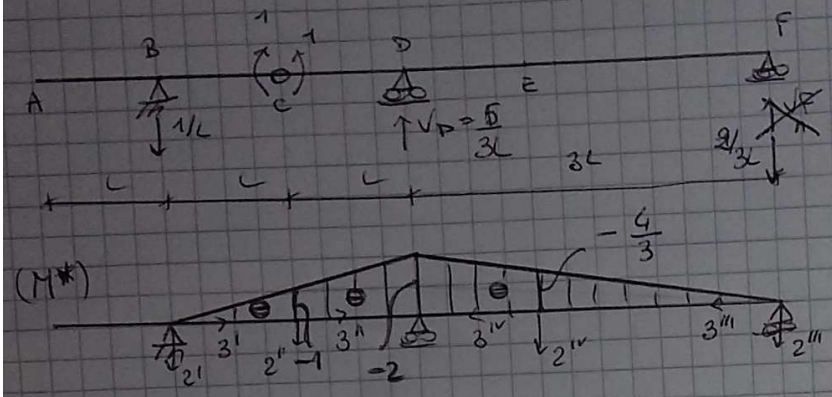
$W_1 = 324{,}3\ cm^3$

$I_1 = 3892\ cm^4$

$H = 24\ cm$

$A = 39{,}12\ cm^2$

stazione relativa in C:



$$\begin{cases} V_D + V_F = 1/L & (x(-L)) \\ V_D L + V_F 4L + 1 = 0 \end{cases}$$

$$V_D L + V_F 4L - V_D L - V_F L + 1 = -1$$

$$3 V_F L = -2$$

$$V_F = -2/3L$$

$$V_D = 1/L + \frac{2}{3L} = \frac{5}{3L}$$

$$1 \cdot \Delta\varphi_C^q = \frac{1}{EI_1}\int_0^L (2qL^2 - 2qLx_3')\left(-\frac{1}{L}x_3'\right)dx_3' + \frac{1}{EI_1}\int_0^L (-3qLx_3'')\left(-1-\frac{x_3''}{L}\right)dx_3''$$

$$+ \frac{1}{EI_1}\int_0^{2L}\left(-q\frac{x_3'''^2}{2} + q\frac{L}{3}x_3'''\right)\left(-\frac{2}{3L}x_3'''\right)dx_3''' + \frac{1}{EI_1}\int_0^L \left(-\frac{4}{3}qL^2 - \frac{5}{3}qLx_3''''\right)\left(-\frac{4}{3}-\frac{2}{3L}x_3''''\right)dx$$

$$= \frac{1}{EI_1}\left[-q\frac{L^3}{3} + \frac{5}{2}qL^3 + \frac{20}{27}qL^3 + \frac{100}{27}qL^3\right] = \frac{119}{18}\frac{qL^3}{EI_1} = \frac{119 \cdot 25 \cdot 10^{32}}{18 \cdot 210 \cdot 10^{8} \cdot 3892 \cdot 10^{8}}$$

$$\cong 0{,}02 = 1{,}16°$$

Carico termico:

$$1 \cdot \Delta\varphi_C^{\Delta T} = \int_{ABCD} \chi_t M^* dx_3' = \chi_t \cdot \int_{ABCD} M^* dx_3' = \frac{2\alpha\Delta T}{H}\left(-\frac{2L \cdot 2}{2}\right)$$

$$= -\frac{4\alpha\Delta T L}{H} = -\frac{4 \cdot 10^{-5} \cdot 10}{0{,}25} = -0{,}0016 = -0{,}095°$$

$$\Delta\varphi_C^{TOT} = \Delta\varphi_C^{q} + \Delta\varphi_C^{\Delta T} = 1{,}16° - 0{,}095° = 1{,}065°$$

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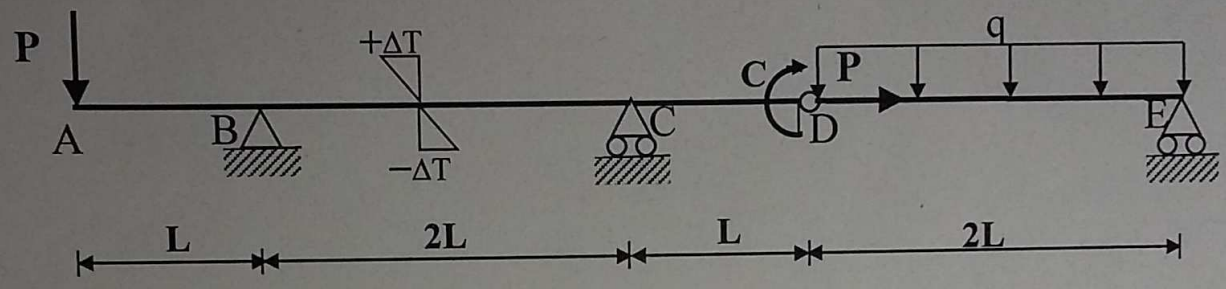
C



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La travatura isostatica in figura deve essere realizzata con profilati IPE.

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- Dimensionare la travatura.
- Calcolare la rotazione relativa nella cerniera in D.
- Calcolare la rotazione relativa in D considerando in aggiunta al carico anche la presenza di un carico termico a farfalla ΔT sul tratto ABC.

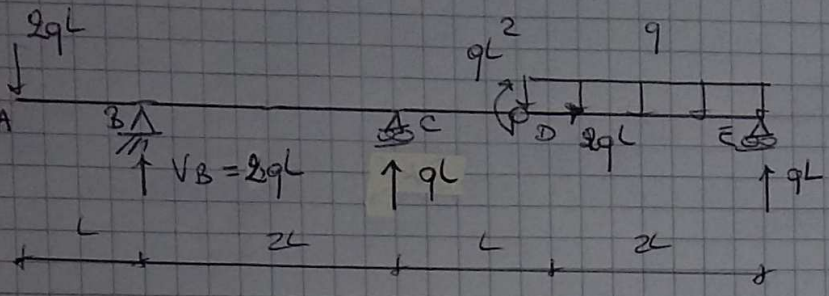
OLUZIONE FILAC

$qL = 20$ kN

$qL^2 = 20$ kN

$P = 2qL$

$C = qL^2$



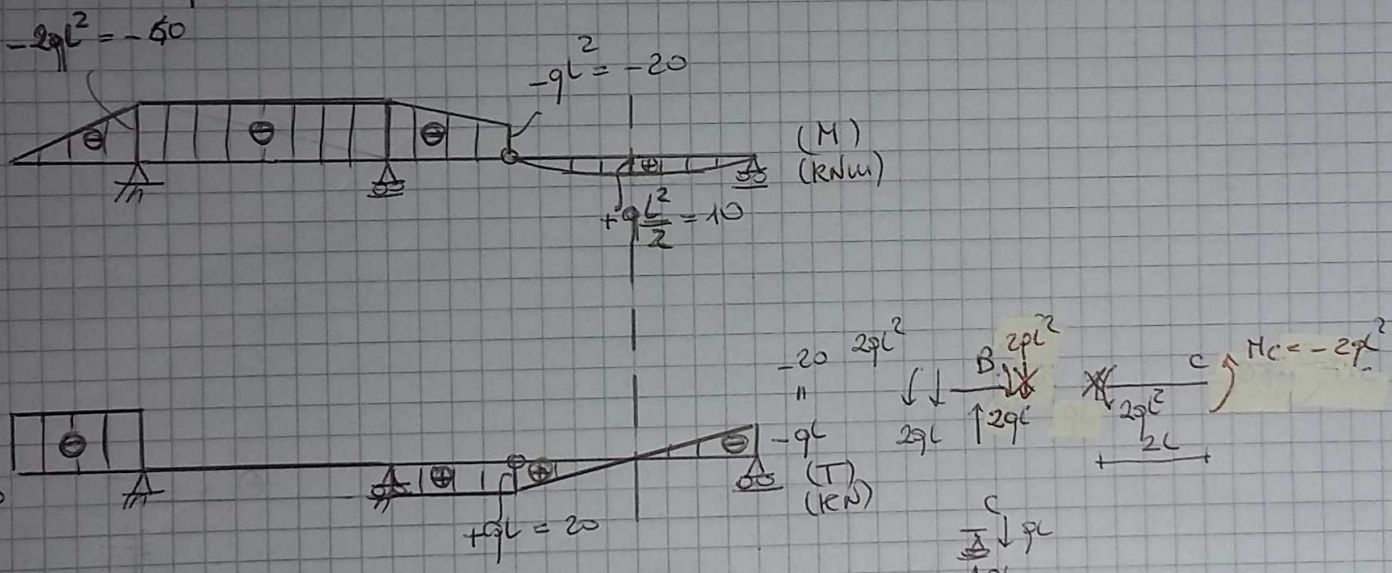
$$\begin{cases} V_B + V_C = 2qL + 2qL - qL = 3qL \\ V_B\,3L + V_C\,L + qL^2 - 2qL\,4L = 0 \end{cases}$$

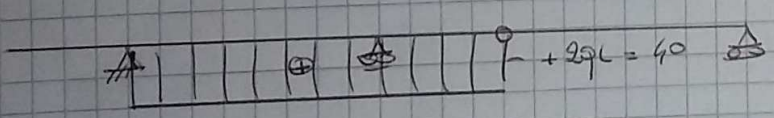
$$\begin{cases} V_B + V_C = 3qL \quad (\times(-1)) \\ 3\cdot V_B + V_C = 7qL \end{cases}$$

$$3V_B + V_C - V_B - V_C = (7-3)qL = 4qL$$

$$\begin{cases} V_B = 2qL \\ V_C = qL \end{cases}$$

Diagrammi quotati:





Dimensionamento:

$$W_1 \geq \frac{2qL^2}{\sigma_{AMM}} = \frac{40 \cdot 10^3}{206 \cdot 10^6}\ cm^3 = 167\ cm^3 \rightarrow IPE\ 200$$

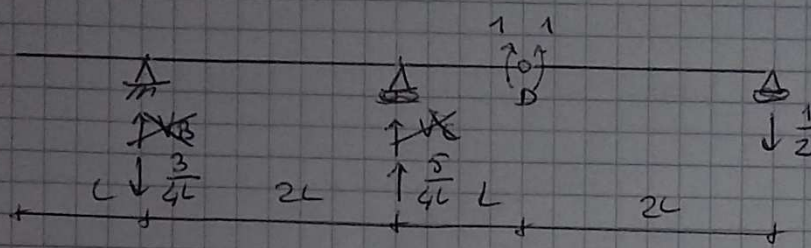
$W_1 = 194{,}3\ cm^3$

$I_1 = 1943\ cm^4$

$H = 20\ cm$

$A = 28{,}48\ cm^2$

Rotazione relativa in D:



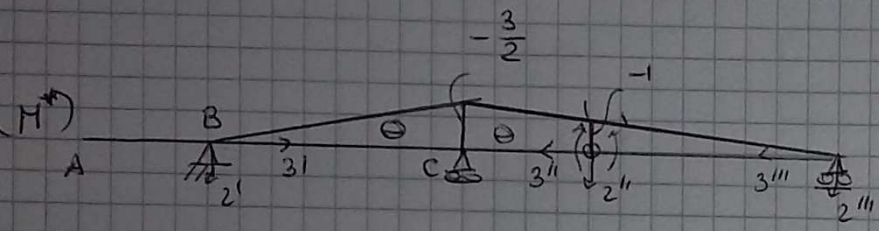
$$\begin{cases} V_B\,3L + V_C\,L + 1 = 0 \\ V_B + V_C = \frac{1}{2L} \end{cases}$$

$$\begin{cases} V_B + V_C = \frac{1}{2L} \\ 3V_B + V_C = -\frac{1}{L} \end{cases} \quad (\times(-1))$$

$$3V_B + V_C - V_B - V_C = -\frac{1}{L} - \frac{1}{2L} = -\frac{3}{2L}$$

$$V_B = -\frac{3}{4L}$$

$$V_C = \frac{1}{2L} - V_B = \frac{1}{2L^2} + \frac{3}{4L} = \frac{5}{4L}$$



$$1 \cdot \Delta\varphi_D^q = \frac{1}{EI_1}\int_0^{2L}(-2qL^2)\left(-\frac{3}{4L}x_3'\right)dx_3' + \frac{1}{EI_1}\int_0^{L}\left(-qL^2 - qLx_3''\right)\left(-1-\frac{1}{2L}x_3''\right)dx_3''$$

$$+\frac{1}{EI_1}\int_0^{2L}\left(-q\frac{x_3'''^2}{2} + qLx_3'''\right)\left(-\frac{1}{2L}x_3'''\right)dx_3'''$$

$$= \frac{1}{EI_1}\left[3qL^3 + \frac{23}{12}qL^3 - \frac{qL^3}{3}\right] = \frac{55qL^3}{12EI_1} = \frac{55\cdot 20\cdot 10^3}{12\cdot 210\cdot 10^5\cdot 1943\cdot 10^{-8}} = 0{,}022 = 1{,}29°$$

Carico termico:

$$1 \cdot \Delta\varphi_D^{\Delta T} = \int_{ABC} \chi_t M^* dx_3 = \chi_t \frac{3L}{2}\left(-\frac{3}{2}\right) = \left(-\frac{2\alpha\Delta T}{H}\right)\left(-\frac{3}{2}L\right) = \frac{3\alpha\Delta TL}{H} = \frac{3\cdot 10^{-5}\cdot 10}{9{,}20}$$

$$= 0{,}0015 = 9{,}086°$$

$$\varphi_B^{TOT} = \Delta\varphi_D^q + \Delta\varphi_D^{\Delta T} = 1{,}29° + 9{,}086° = 1{,}37°$$