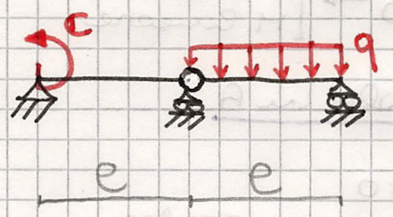
3ª ESERCITAZIONE

Disegnare i diagrammi dell'azione interna (N, T, M) nelle seguenti travature isostatiche.

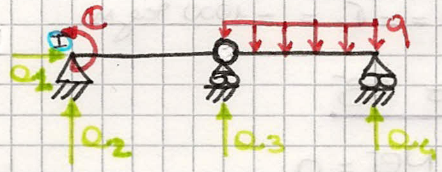
Esercizio 1



$q = 1500$ kg/m

$l = 2$ m

$C = 1000$ kg·m

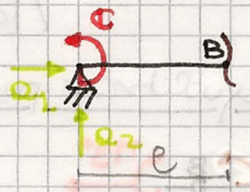


Svolgimento:

① Calcolo delle reazioni vincolari

$$\begin{cases} R_1 = 0 \\ R_2 + R_3 + R_4 = ql \\ (I) \quad C + R_3 l - ql \cdot \frac{3}{2} l + R_4 2l = 0 \end{cases}$$

Si nota che vi sono 3 equazioni in 4 incognite, per cui è necessario inserire un'equazione ausiliaria ottenuta spezzando l'asta nella cerniera.



$(B) \quad R_2 l = C$

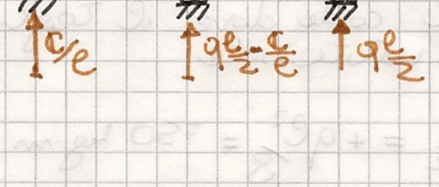
(Sapendo che il momento in corrispondenza di una cerniera è nullo)

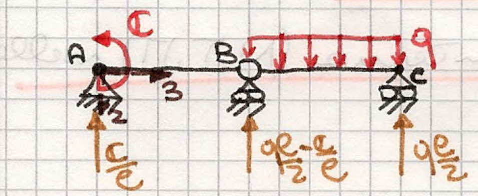
Inserisco l'equazione ausiliaria nel sistema precedente, ottenendo:

$$\begin{cases} R_1 = 0 \\ R_2 + R_3 + R_4 = ql \\ (I) \quad C + R_3 l - \frac{3}{2} q l^2 + 2 R_4 l = 0 \\ R_2 = C/l \end{cases}$$

Risolvendo il sistema ottengo:

$$\begin{cases} R_1 = 0 \\ R_2 = C/l \\ R_3 = q\frac{l}{2} - C/l \\ R_4 = ql/2 \end{cases}$$





Applico il metodo delle equazioni indefinite.

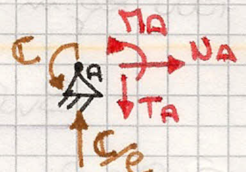
① Denominazione di tutti i nodi (A, ...)

② Inserimento di un sistema di riferimento (2,3) su ogni tratto.

③ Analisi dei tratti.

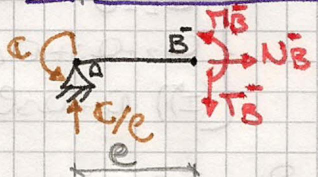
■ Tratto AB
$$\begin{cases} q_3=0 \rightarrow [N \text{ costante} \\ \left[\begin{matrix} q_2=0 \\ c=0 \end{matrix}\right. \rightarrow \left[\begin{matrix} T \text{ costante} \\ M \text{ lineare} \end{matrix}\right. \end{cases}$$

• Equilibrio puntuale in A



$$\begin{cases} N_A=0 \\ T_A=\mathbb{C}/\ell = 500 \text{ kg} \\ M_A=-\mathbb{C} = -1000 \text{ kg m} \end{cases}$$

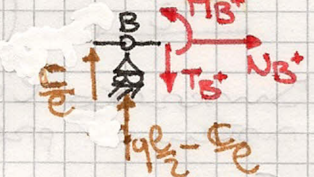
• Equilibrio tratto AB⁻



$$\begin{cases} N_{B^-}=0 \\ T_{B^-}=\mathbb{C}/\ell = 500 \text{ kg} \\ (A^\circ)\ M_{B^-}=-\mathbb{C}+\frac{\mathbb{C}}{\ell}\cdot\ell=0 \end{cases}$$

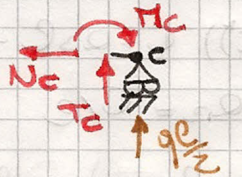
■ Tratto BC
$$\begin{cases} q_3=0 \rightarrow N \text{ costante} \\ \left[\begin{matrix} q_2=q \\ c=0 \end{matrix}\right. \rightarrow \left[\begin{matrix} T \text{ lineare} \\ M \text{ parabolico} \end{matrix}\right. \end{cases}$$

• Equilibrio puntuale in B

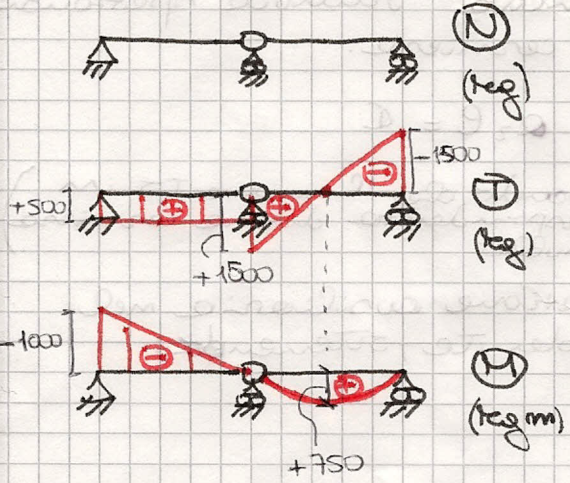


$$\begin{cases} N_{B^+}=0 \\ T_{B^+}=q\frac{\ell}{2}-\cancel{\frac{\mathbb{C}}{\ell}}+\cancel{\frac{\mathbb{C}}{\ell}}=1500 \text{ kg} \\ M_{B^+}=0 \end{cases}$$

• Equilibrio puntuale in C

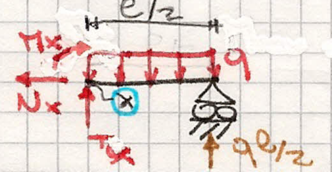


$$\begin{cases} N_C=0 \\ T_C=-q\frac{\ell}{2}=-1500 \text{ kg} \\ M_C=0 \end{cases}$$



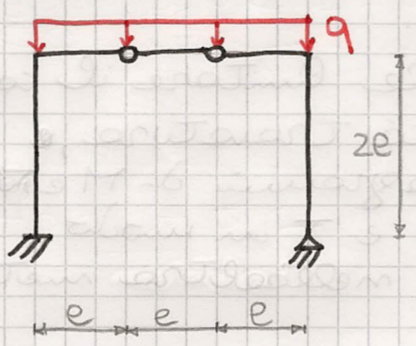
④ Costruzione dei grafici + quotature

Osservazione: Il max di M al tratto BC cade dove il taglio si ~~annulla~~ annulla e la sua entità è così valutabile:



$$M(x^*)\quad M_x=-q\frac{\ell}{2}\cdot\frac{\ell}{4}+q\frac{\ell}{2}\cdot\frac{\ell}{2}=+q\frac{\ell^2}{8}=750 \text{ kg m}$$

Esercizio 2



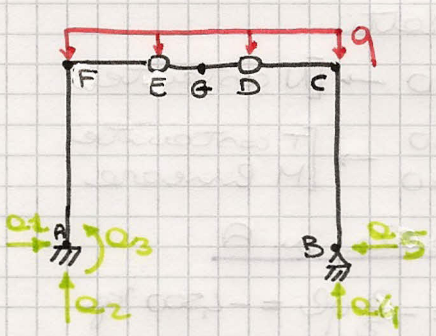
$q = 1500\ Kg/m$

$l = 2m$

Svolgimento

① Calcolo delle reazioni vincolari

Equaz. cardinali della statica



$$\begin{cases} Q_1 = Q_5 \\ Q_2 + Q_4 = q3l \\ (I↺)\ Q_3 = 3ql \cdot \frac{3}{2}l - Q_4 3l \end{cases}$$

Equazioni ausiliarie
(scritte ∀ elemento di connessione)

$$(D↺)_{BCD} \quad -Q_5 \cdot 2l + Q_4 \cdot l = ql \cdot \frac{l}{2}$$

$$(E↺)_{BCDE} \quad -Q_5 \cdot 2l + Q_4 \cdot 2l = 2ql \cdot l$$

Combinandole ottengo:

$$Q_4 = \frac{3}{2}ql$$

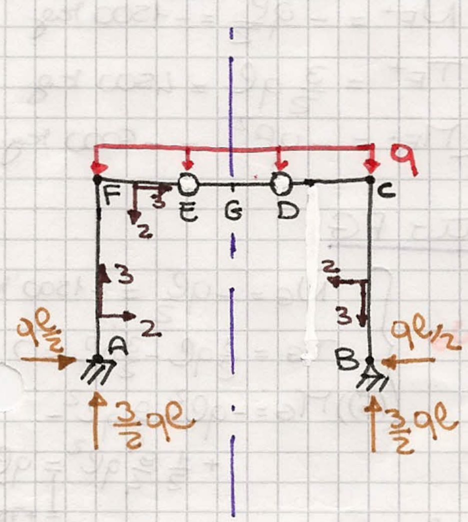
$$Q_5 = ql/2$$

Inserisco le relazioni nel sistema di equazioni cardinali, ottenendo:

$$\begin{cases} Q_1 = Q_5 \\ Q_2 + Q_4 = 3ql \\ (I↺)\ Q_3 = 3ql\ \frac{3}{2}l - 3Q_4 l \\ Q_4 = \frac{3}{2}ql \\ Q_5 = ql/2 \end{cases}$$

Risolvendo il sistema ottengo:

$$\begin{cases} Q_1 = Q_5 = ql/2 \\ Q_2 = 3/2\, ql \\ Q_3 = \frac{9}{2}ql^2 - \frac{9}{2}ql^2 = 0 \\ Q_4 = 3/2\, ql \end{cases}$$



Applico il metodo delle eq. indefinite

① Denominazione di tutti i nodi (A, ...)

② Inserimento di un sistema di riferimento (2;3) su ogni tratto.

Osservazione

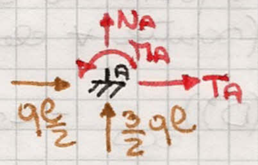
Le reazioni vincolari ed il carico sono simmetrici rispetto all'asse segnato in figura.
È quindi possibile limitare il calcolo di M, T e N su metà travatura, riportando i diagrammi di M e N in modo simmetrico e T in modo antisimmetrico nella metà opposta.

3) Analisi dei Tratti

■ **Tratto AF**

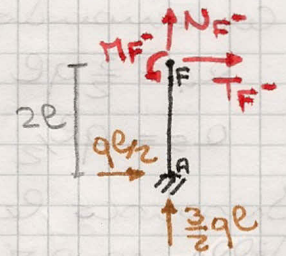
$$\begin{cases} q_3 = 0 \rightarrow [\text{N costante} \\ \left[\begin{matrix} q_2 = 0 \\ c = 0 \end{matrix}\right. \rightarrow \left[\begin{matrix} \text{T costante} \\ \text{M lineare} \end{matrix}\right. \end{cases}$$

• Equilibrio puntuale in A



$$\begin{cases} N_A = -\frac{3}{2}q\ell = -4500 \text{ kg} \\ T_A = -q\ell/2 = -1500 \text{ kg} \\ M_A = 0 \end{cases}$$

• Equilibrio tratto AF^-

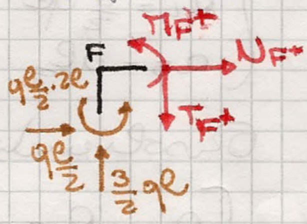


$$\begin{cases} N_{F^-} = -\frac{3}{2}q\ell = -4500 \text{ kg} \\ T_{F^-} = -q\ell/2 = -1500 \text{ kg} \\ M_{F^-} = -\frac{q\ell}{2}\cdot 2\ell = -q\ell^2 \quad (F')) \\ \qquad = -6000 \text{ kg·m} \end{cases}$$

■ **Tratto FC**

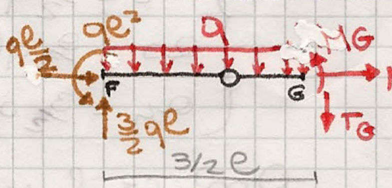
$$\begin{cases} q_3 = 0 \rightarrow [\text{N costante} \\ q_2 = q \\ c = 0 \end{cases} \rightarrow \left[\begin{matrix} \text{T lineare} \\ \text{M parabolico} \end{matrix}\right.$$

• Equilibrio nodo F



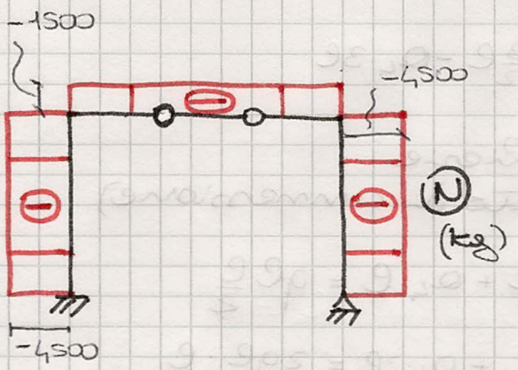
$$\begin{cases} N_{F^+} = -\frac{q\ell}{2} = -1500 \text{ kg} \\ T_{F^+} = \frac{3}{2}q\ell = 4500 \text{ kg} \\ M_{F^+} = -q\ell^2 = -6000 \text{ kg·m} \end{cases}$$

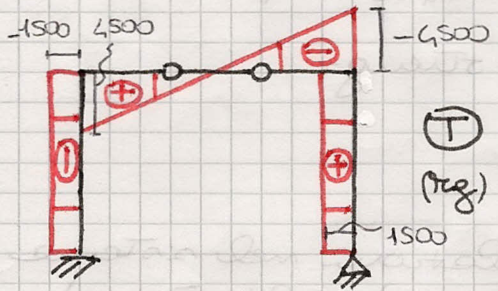
• Equilibrio tratto FG

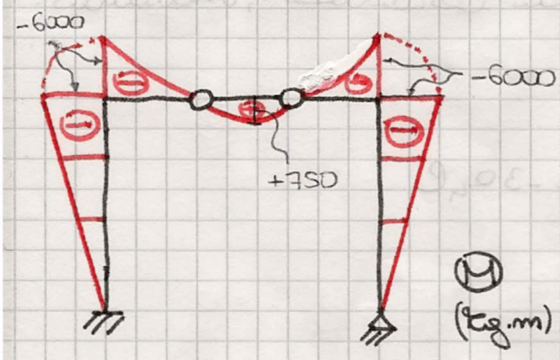


$$\begin{cases} N_G = -\frac{q\ell}{2} = -1500 \text{ kg} \\ T_G = \frac{3}{2}q\ell - \frac{3}{2}q\ell = 0 \\ (G)) \; M_G = -q\ell^2 + \frac{9}{4}q\ell^2 - \\ \qquad + \frac{1}{2}\frac{9}{4}q\ell^2 = \frac{q\ell^2}{8} \\ \qquad = +750 \text{ kg·m} \end{cases}$$

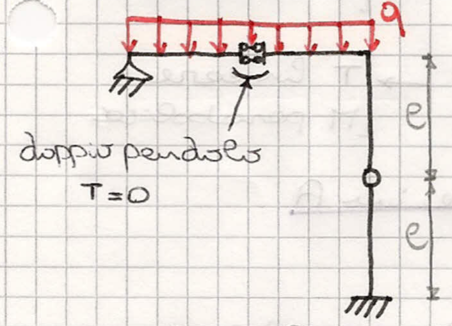
4) Costruzione grafici + quotature







Esercizio 3

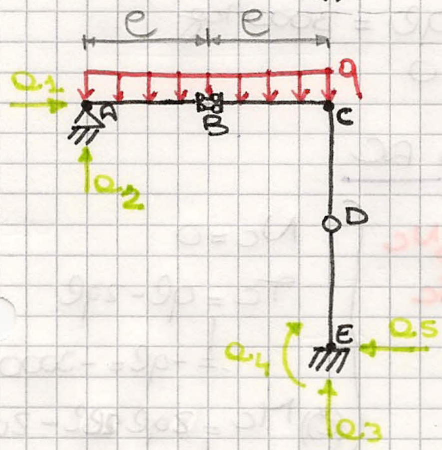


$q = 1500\ kg/m$

$l = 2\ m$

Svolgimento

① Calcolo delle reazioni vincolari

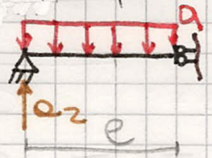


Equaz. cardinali della statica:

$$\begin{cases} R_1 = R_5 \\ R_3 + R_2 = 2ql \\ (I^*))\ -R_4 - R_5 \cdot 2l + R_3 \cdot 2l - 2ql \cdot l = 0 \end{cases}$$

Equazioni ausiliarie

① doppio pendolo (T = 0, non trasmette la forza di taglio)



Facciamo solo l'equilibrio alla traslazione verticale:

$$R_2 = ql$$

② cerniera

$$(D^*))_{ABCD} \Rightarrow R_1 l + 2ql \cdot l - R_2 2l = 0$$

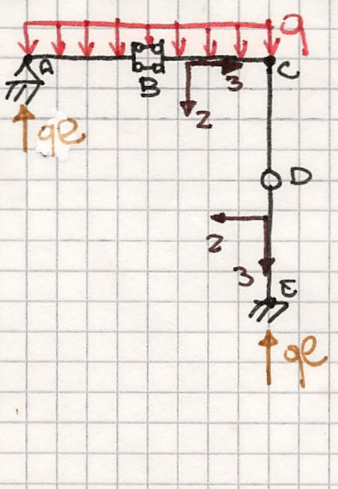
$$R_1 = 2R_2 - 2ql \underset{R_2 = ql}{=} 2ql - 2ql = 0$$

Inserisco le relazioni ottenute nel sistema di equazioni cardinali, ottenendo:

$$\begin{cases} R_1 = R_5 \\ R_3 + R_2 = 2ql \\ R_4 + 2R_5 l + 2R_3 l = 2ql^2 \\ R_2 = ql \\ R_1 = 0 \end{cases}$$

Risolvendo il sistema ottengo:

$$\begin{cases} R_1 = R_5 = 0 \\ R_3 = R_2 = ql \\ R_4 = 0 \end{cases}$$



Applico il metodo delle equazioni indefinite.

① Denominazione di tutti i nodi

② Inserimento di un sistema di riferimento (2;3) su ogni tratto.

Esercizio 4 (Es 1 fila A, I° parziale, 3/11/2008)

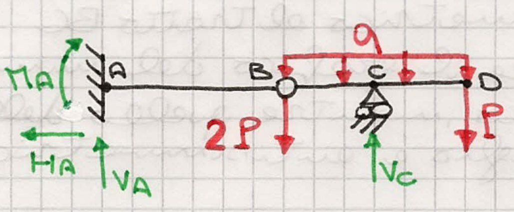
$l = 1\,m$

$q = 2000\ kg/m$

$P = 1T = q\frac{l}{2}$

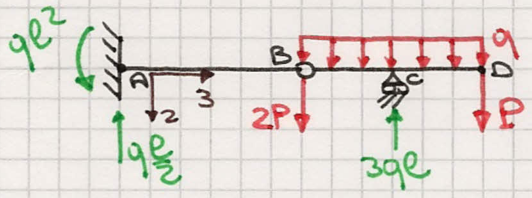
Svolgimento

① Calcolo delle reazioni vincolari



$$\begin{cases} (\rightarrow)\ H_A = 0 \\ (B\circlearrowleft)_{BCD}\ V_C\, l = q\frac{l}{2}\,2l + 2ql\, l = 3ql^2 \Rightarrow V_C = 3ql = 6T \\ (\uparrow)\ V_A = 2ql + \frac{3}{2}ql - 3ql = q\frac{l}{2} = 1T \\ (B\circlearrowleft)_{AB}\ M_A = -V_A\cdot 2l = -ql^2 = -2T\cdot m \end{cases}$$

ottenendo la seguente travatura:

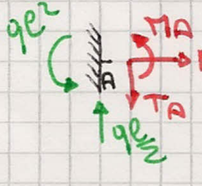


② Applico il metodo delle eq. indefinite:

- Inserimento di un sistema di riferimento (2,3) su ogni tratto
- Analisi dei tratti

Tratto AB
$$\begin{cases} q_3 = 0 \longrightarrow [N \text{ costante} \\ \begin{bmatrix} q_2 = 0 \\ C = 0 \end{bmatrix} \longrightarrow \begin{bmatrix} T \text{ costante} \\ M \text{ lineare} \end{bmatrix} \end{cases}$$

- Equilibrio puntuale in A



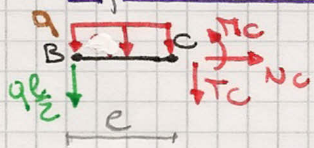
$$\begin{cases} N_A = 0 \\ T_A = ql/2 = 1000\,kg \\ M_A = -ql^2 = -2000\ kg\cdot m \end{cases}$$

- Equilibrio puntuale in B

$$\begin{cases} M_B = 0 \text{ (c'è la cerniera)} \\ N_B = 0 \\ T_B = ql/2 - ql = -ql/2 = -1000\,kg \end{cases}$$

Tratto BC
$$\begin{cases} q_3 = 0 \longrightarrow [N \text{ costante} \\ \begin{bmatrix} q_2 = q \\ C = 0 \end{bmatrix} \longrightarrow \begin{bmatrix} T \text{ lineare} \\ M \text{ parabolico} \end{bmatrix} \end{cases}$$

- Equilibrio tratto BC

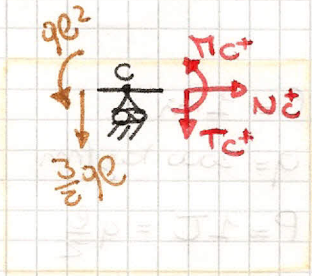


$N_C = 0$

$T_C = -ql/2 - ql = -\frac{3}{2}ql = -3000\,kg$

$M_C = -q\frac{l^2}{2} - q\frac{l^2}{2} = -ql^2 = -2000\ kg\cdot m$

• Equilibrio in C

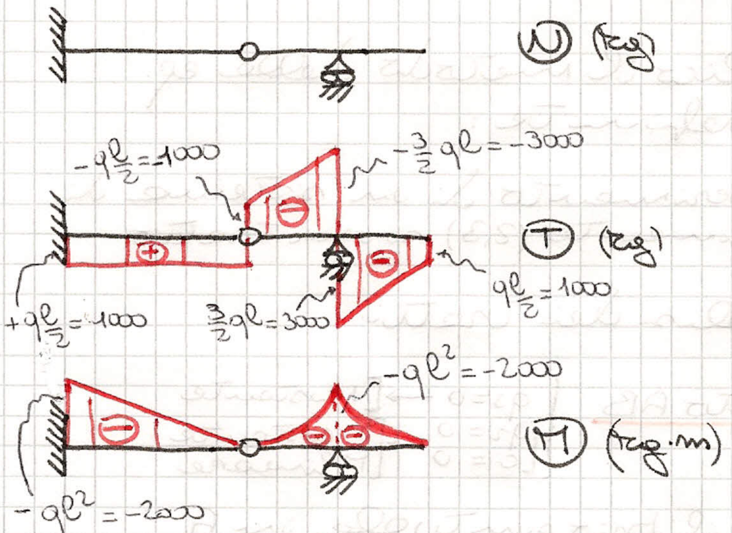


$$\begin{cases} M_C^+ = -ql^2 = -2000 \text{ kg}\cdot\text{m} \\ N_C^+ = 0 \\ T_C^+ = -\frac{3}{2}ql \end{cases}$$

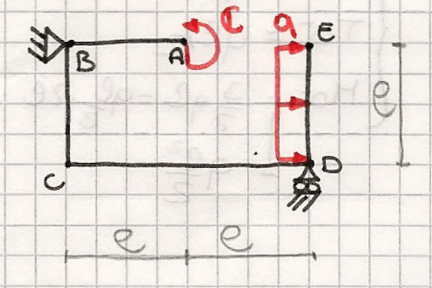
■ Tratto CD

Essendo simmetrico al tratto BC allora, anche il grafico del momento è simmetrico, mentre quello dello sforzo di taglio è antisimmetrico.

⑥ Costruzione dei grafici + quotature



Esercizio 5

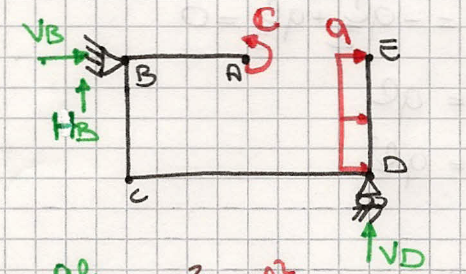


$l = 1\,m$

$q = 2000\ kg/m$

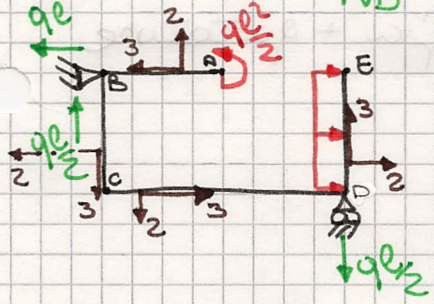
$C = 1\ t\,m = ql^2/2$

Svolgimento



① Calcolo delle reazioni vincolari

$$\begin{cases} (\rightarrow) & V_B = -ql \\ (\uparrow) & H_B + V_D = 0 \longrightarrow H_B = ql/2 \\ (B\circlearrowright) & V_D \cdot 2l + ql^2/2 + \overset{C}{ql^2/2} = 0 \Rightarrow V_D = -ql/2 \end{cases}$$



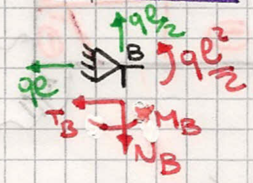
② Applicazione del metodo delle eq. indefinite.

• Inserimento di un sistema di riferimento (2;3) su ogni tratto.

③ Analisi dei tratti

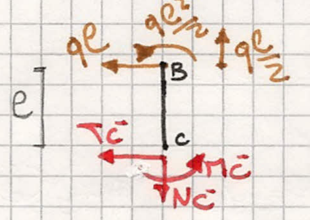
▪ Tratto AB, BC, CD
$$\begin{cases} q_3 = 0 \longrightarrow [N \text{ costante} \\ \begin{bmatrix} q_2 = 0 \\ c = 0 \end{bmatrix} \longrightarrow \begin{bmatrix} T \text{ costante} \\ M \text{ lineare} \end{bmatrix} \end{cases}$$

• Equilibrio puntuale in B



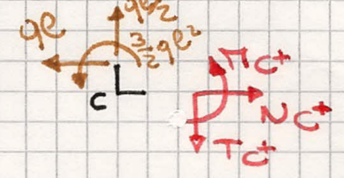
$$\begin{cases} N_B = ql/2 \\ T_B = -ql \\ M_B = -ql^2/2 \end{cases}$$

• Equilibrio di BC^-



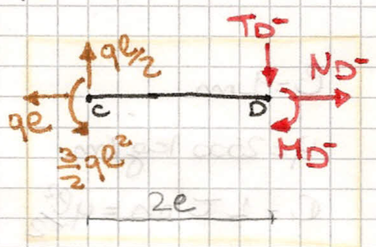
$$\begin{cases} N_C^- = ql/2 \\ T_C^- = -ql \\ M_C^- = -ql^2/2 - ql \cdot l = -\frac{3}{2} ql^2 \end{cases}$$

• Equilibrio in C



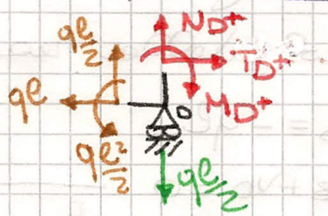
$$\begin{cases} M_{C^+} = \frac{3}{2} ql^2 \\ N_{C^+} = ql \\ T_{C^+} = -\frac{3}{2} ql^2 \end{cases}$$

• Equilibrio di CD⁻



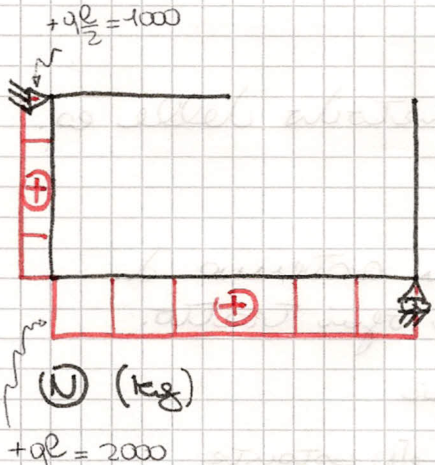
$$\begin{cases} N_{D^-} = q\ell \\ T_{D^-} = q\ell/2 \\ M_{D^-} = \frac{3}{2}q\ell - q\frac{\ell}{2}\cdot 2\ell = q\frac{\ell^2}{2} \end{cases}$$

• Equilibrio in D

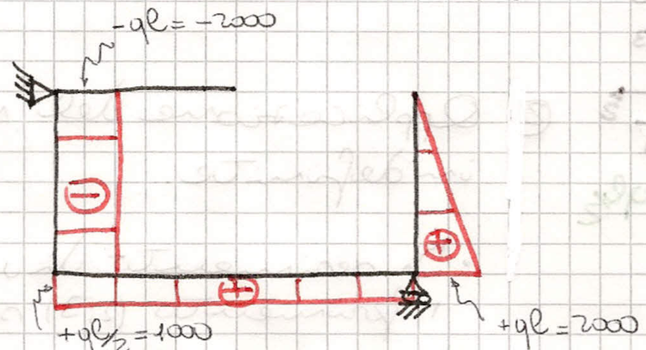


$$\begin{cases} N_{D^+} = -q\frac{\ell}{2} + q\frac{\ell}{2} = 0 \\ T_{D^+} = q\ell \\ M_{D^+} = q\frac{\ell^2}{2} \end{cases}$$

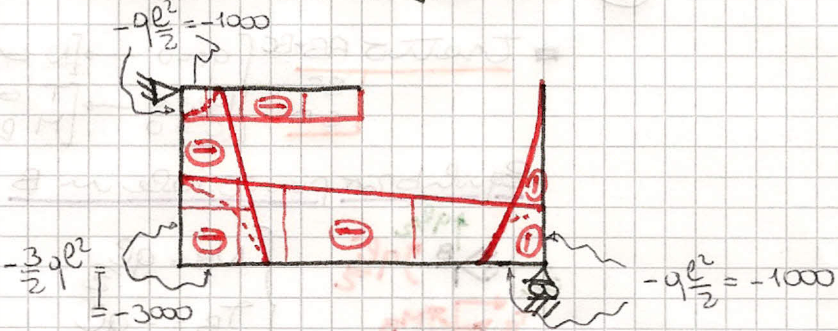
(4) Costruzione dei grafici + quotature



(N) (kg)



(T) (kg)



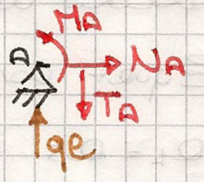
(M) (kg·m)

③ Analisi dei Tratti

■ Tratto AC

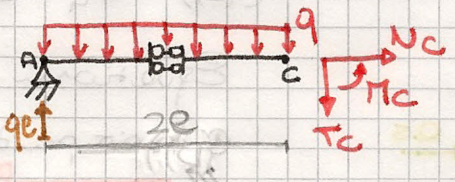
$$\begin{cases} q_3 = 0 \longrightarrow [\,N \text{ costante} \\ q_2 = q \longrightarrow [\,T \text{ lineare},\ M \text{ parabolico} \\ c = 0 \end{cases}$$

• Equilibrio puntuale in A



$$\begin{cases} N_A = 0 \\ T_A = ql = 3000 \text{ kg} \\ M_A = 0 \end{cases}$$

• Equilibrio tratto AC

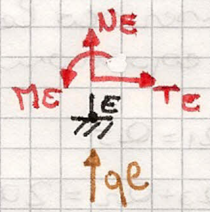


$$\begin{cases} N_C = 0 \\ T_C = ql - 2ql = -ql = -3000 \text{ kg} \\ (C)\ M_C = ql \cdot 2l - 2ql \cdot l = 0 \end{cases}$$

■ Tratto CE

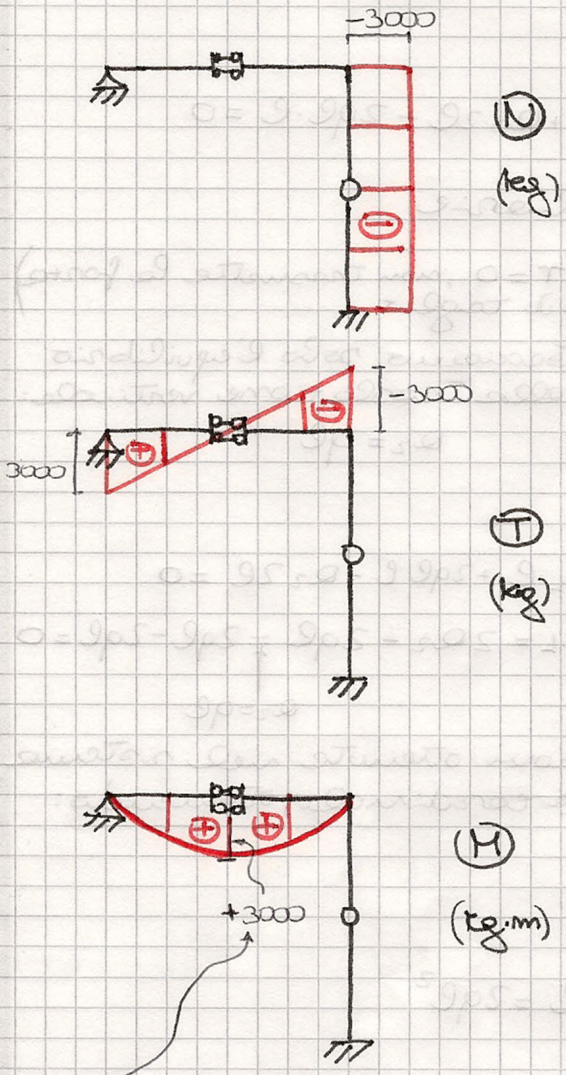
$$\begin{cases} q_3 = 0 \longrightarrow [\,N \text{ costante} \\ q_2 = 0 \longrightarrow [\,T \text{ costante},\ M \text{ lineare (e si annulla in D)} \\ c = 0 \end{cases}$$

• Equilibrio in E



$$\begin{cases} N_E = -ql = -3000 \text{ kg} \\ M_E = 0 \\ T_E = 0 \end{cases}$$

④ Costruzione grafici + quotature



$$M_{max} = ql^2 - ql \cdot \frac{l}{2} = +\frac{ql^2}{2} = +3000 \text{ kg·m}$$

(B*)