4ª ESERCITAZIONE

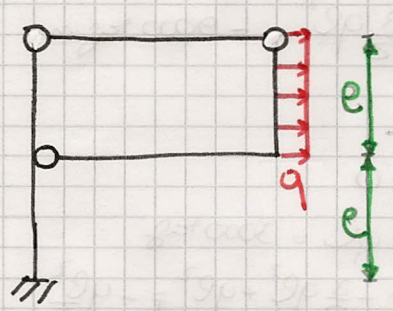
10/28

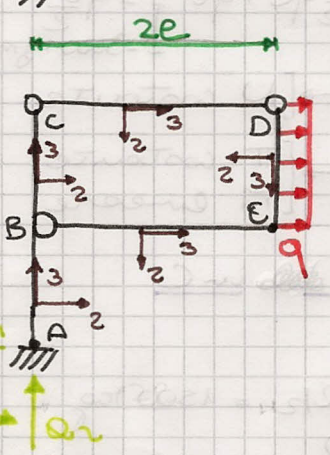
Disegnare i diagrammi dell'azione interna (N, T, M) nelle seguenti travature isostatiche

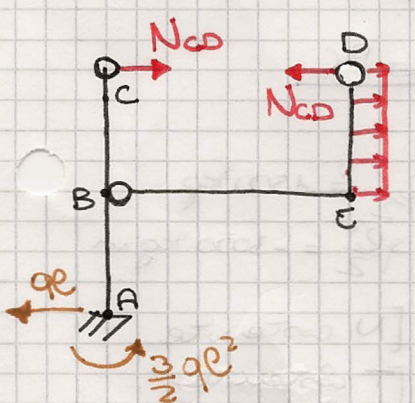
Esercizio 1

$q = 1500\ kg/m$

$\ell = 2m$







Svolgimento

① Calcolo delle reazioni vincolari (eseguito considerando la trave nella sua completezza)

Eq. cardinali della statica:

$$\begin{cases} e_1 = q\ell \\ e_2 = 0 \\ e_3 = q\ell\,\frac{3}{2}\ell \end{cases}$$

② Denominazione di tutti i nodi (A, ...)

③ Inserimento di un sistema di riferimento (2,3) su ogni tratto.

④ Essendoci una maglia chiusa occorre necessariamente seguire un taglio. Il punto + conveniente è sulla biella (o—o), poiché in tale elemento c'è una incognita, nel caso in cui esse sia scarica, è N in quanto M e T sono sempre nulli.

⑤ Eq. ausiliarie relative ad un elemento di connessione.

In questo caso facciamo riferimento alla cerniera B

$$(B)_{BED} \quad N_{CD}\,\ell = q\ell\,\frac{\ell}{2} \Rightarrow N_{CD} = q\frac{\ell}{2}$$

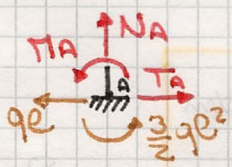
⑥ Applicazione del metodo delle equazioni indefinite

Analisi dei Tratti:

Tratto AB

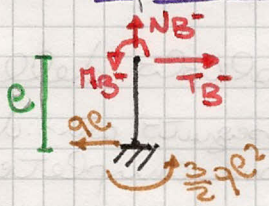
$\begin{cases} q_3 = 0 \\ q_2 = 0 \\ c = 0 \end{cases} \rightarrow \begin{cases} N \text{ costante} \\ T \text{ costante} \\ M \text{ lineare} \end{cases}$

• Equilibrio puntuale in A



$\begin{cases} N_A = 0 \\ T_A = q\ell = 3000 \text{ kg} \\ M_A = -\frac{3}{2}q\ell^2 = -9000 \text{ kg·m} \end{cases}$

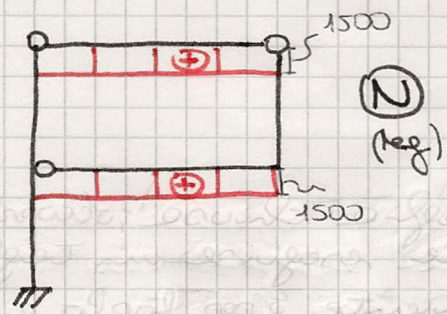
• Equilibrio di AB



$\begin{cases} N_{B^-} = 0 \\ T_{B^-} = q\ell = 3000 \text{ kg} \\ M_{B^-} = -\frac{3}{2}q\ell^2 + q\ell^2 = -\frac{q\ell^2}{2} = -3000 \text{ kg·m} \end{cases}$

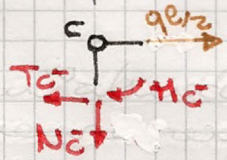
Tratto BC

$\begin{cases} q_3 = 0 \\ q_2 = 0 \\ c = 0 \end{cases} \rightarrow \begin{cases} N \text{ costante} \\ T \text{ costante} \\ M \text{ lineare} \end{cases}$



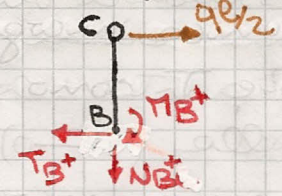
(N) (kg)

• Equilibrio puntuale in C

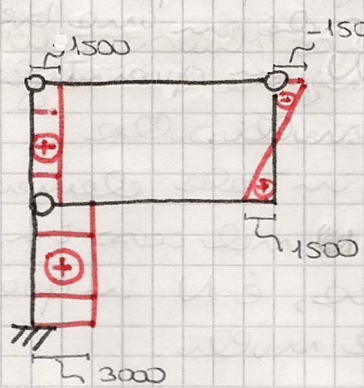


$\begin{cases} N_{C^-} = 0 \\ T_{C^-} = q\ell/2 = 1500 \text{ kg} \\ M_{C^-} = 0 \end{cases}$

• Equilibrio del Tratto BC



$\begin{cases} N_{B^+} = 0 \\ T_{B^+} = q\frac{\ell}{2} = 1500 \text{ kg} \\ M_{B^+} = -q\frac{\ell^2}{2} = -3000 \text{ kg·m} \end{cases}$

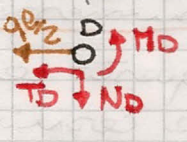


(T) (kg)

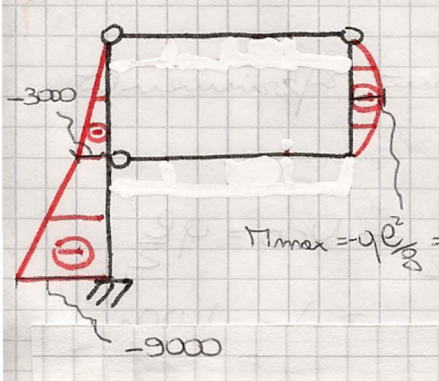
Tratto DE

$\begin{cases} q_3 = 0 \\ q_2 = q \\ c = 0 \end{cases} \rightarrow \begin{cases} N \text{ costante} \\ T \text{ lineare} \\ M \text{ parabolica} \end{cases}$

• Equilibrio puntuale in D



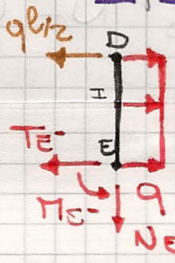
$\begin{cases} N_D = 0 \\ T_D = -q\frac{\ell}{2} = -1500 \text{ kg} \\ M_D = 0 \end{cases}$



(M) (kg·m)

$M_{max} = -q\frac{\ell^2}{8} = -750 \text{ kg·m}$

• Equilibrio tratto DE

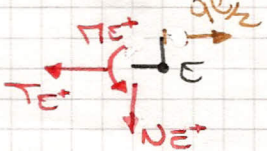


$\begin{cases} N_{E^-} = 0 \\ T_{E^-} = -\frac{1}{2}q\ell + q\ell = \frac{1}{2}q\ell = 1500 \text{ kg} \\ M_{E^-} = -q\frac{\ell^2}{2} + q\frac{\ell^2}{2} = 0 \end{cases}$

(!) $M_{max} = -q\frac{\ell}{2}\cdot\frac{\ell}{2} + q\frac{\ell}{2}\cdot\frac{\ell}{4} = -q\frac{\ell^2}{8}$

- Tratto EB $\begin{cases} q_3 = 0 \rightarrow [N \text{ costante} \\ q_2 = 0 \\ c = 0 \end{cases} \rightarrow \begin{cases} T \text{ costante} \\ M \text{ lineare} \end{cases}$

- Equilibrio in E

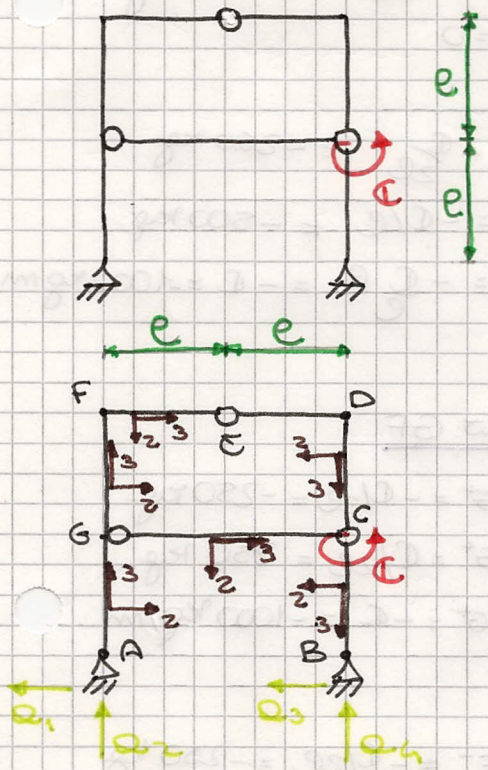


$\begin{cases} TE^+ = 0 \\ NE^- = q\frac{\ell}{2} = 1500 \text{ kg} \\ ME^- = 0 \end{cases}$

Esercizio 2

$\ell = 2m$

$C = 1T \cdot m$



Svolgimento

① Calcolo delle reazioni vincolari

Eq. cardinali della statica:

$$\begin{cases} Q_1 + Q_3 = 0 \\ Q_2 + Q_4 = 0 \\ (A) \quad C + Q_4 2\ell = 0 \end{cases} \Rightarrow \begin{cases} Q_1 = -Q_3 \\ Q_2 = C/2\ell \\ Q_4 = -C/2\ell \end{cases}$$

Eq. della rotazione in C

$$(C)_{CB} \quad Q_3 \ell = C \Rightarrow Q_3 = C/\ell$$

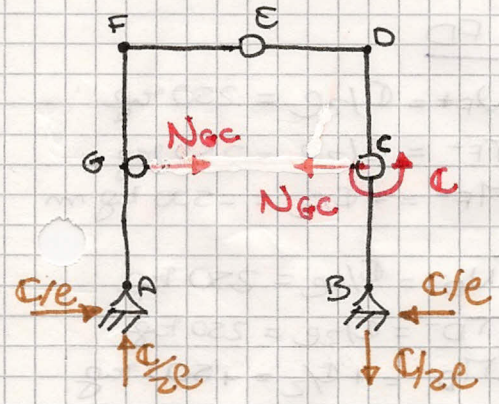
a cui le reazioni vincolari diventano:

$$\begin{cases} Q_1 = -C/\ell \\ Q_2 = C/2\ell \\ Q_3 = C/\ell \\ Q_4 = -C/2\ell \end{cases}$$

② Denominazione di tutti i nodi (A,...)

③ Inserimento di un sistema di riferimento (2,3) su ogni tratto

④ Taglio della biella GC, in modo tale da avere come unica incognita, sul tratto GC, [illegible] N



⑤ Eq. ausiliaria relativa ad un elemento di connessione.

In tal caso facciamo riferimento alla cerniera E

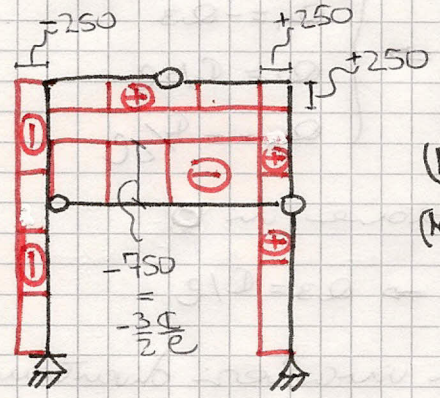
$$(E)_{AGFC} \quad N_{GC} \cdot \ell - \frac{C}{2\ell} \cdot \ell + \frac{C}{\ell} \cdot 2\ell = 0$$

$$\Rightarrow N_{GC} = -\frac{3}{2} C/\ell$$

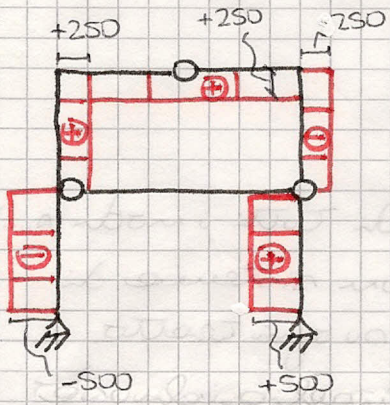
⑥ Applicazione del metodo delle equazioni indefinite

Analisi dei tratti:

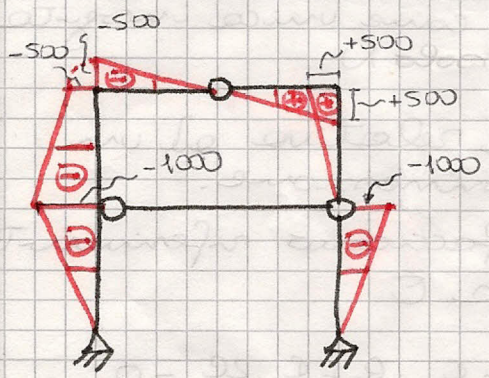
In tutti i tratti $\begin{cases} q_3 = 0 \rightarrow [N \text{ costante} \\ q_2 = 0 \rightarrow [T \text{ costante} \\ C = 0 \rightarrow [M \text{ lineare} \end{cases}$



(N) (kg)

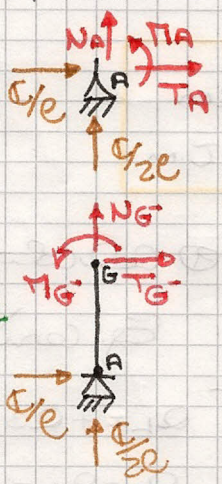


(T) (kg)



(M) (kg·m)

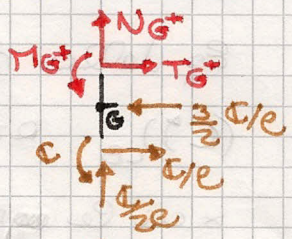
- Equilibrio tratto AG



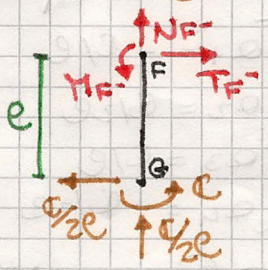
$$\begin{cases} N_A = -C/2e = -250\ kg \\ T_A = -C/e = -500\ kg \\ M_A = 0 \end{cases}$$

$$\begin{cases} N_{G^-} = -C/2e = -250\ kg \\ T_{G^-} = -C/e = -500\ kg \\ M_{G^-} = -\frac{C}{e}e = -C = -1000\ kg\cdot m \end{cases}$$

- Equilibrio tratto GF

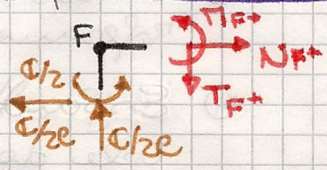


$$\begin{cases} N_{G^+} = -C/2e = -250\ kg \\ T_{G^+} = C/2e = 250\ kg \\ M_{G^+} = -C = -1000\ kg\cdot m \end{cases}$$

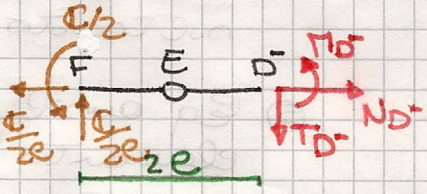


$$\begin{cases} N_{F^-} = -C/2e = -250\ kg \\ T_{F^-} = C/2e = 250\ kg \\ M_{F^-} = -C + \frac{C}{2e}e = -\frac{C}{2} = -500\ kg\cdot m \end{cases}$$

- Equilibrio tratto FD

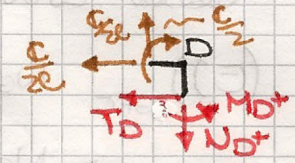


$$\begin{cases} N_{F^+} = C/2e = 250\ kg \\ T_{F^+} = C/2e = 250\ kg \\ M_{F^+} = -C/2 = -500\ kg\cdot m \end{cases}$$



$$\begin{cases} N_{D^-} = C/2e = 250\ kg \\ T_{D^-} = C/2e = 250\ kg \\ M_{D^-} = +C/2 = +500\ kg \end{cases}$$

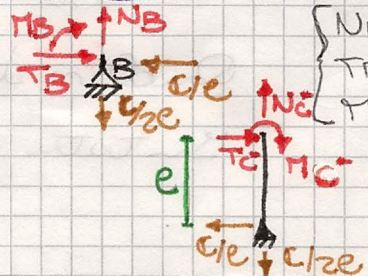
- Equilibrio tratto DC



$$\begin{cases} N_{D^+} = C/2e = 250\ kg \\ T_{D^+} = -C/2e = -250\ kg \\ M_{D^+} = C/2 = 500\ kg\cdot m \end{cases}$$

Sul "C" $M_C = 0$

- Equilibrio tratto CB



$$\begin{cases} N_B = C/2e = 250\ kg \\ T_B = C/e = 500\ kg \\ M_B = 0 \end{cases}$$

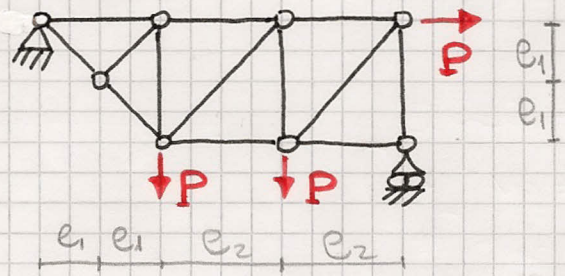
$$\begin{cases} N_{C^-} = C/2e \\ T_{C^-} = C/e \\ M_{C^-} = -\frac{C}{e}e = -C \end{cases}$$

valore della coppia concentrata in C

Esercitio 3 (Es. 3 I° parziale del 3/11/2003)

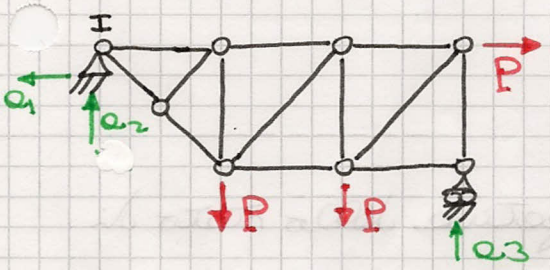
Calcolare lo stato di sollecitazione per

$e_1 = 0{,}5\,m$; $e_2 = 1\,m$; $P = 1\,T$



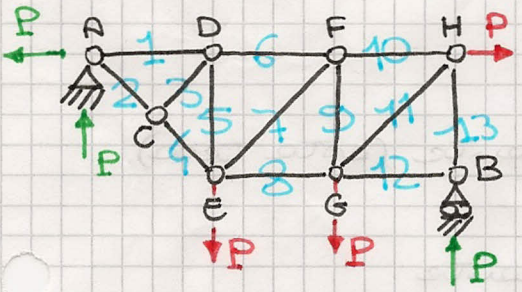
Svolgimento

① Calcolo delle reazioni vincolari:



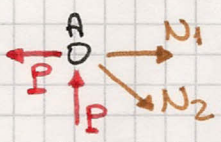
$$\begin{cases} (\rightarrow) & Q_1 = P \\ (\uparrow) & Q_2 + Q_3 = 2P \rightarrow Q_2 = 2P - P = P \\ (\circlearrowright) & Q_3 \cdot 6e_1 = P\,4e_1 + 2P e_1 \Rightarrow Q_3 = P \end{cases}$$

② Numerazione di tutte le aste

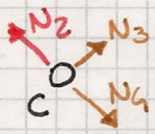


③ Partendo dal nodo in cui convergono solo due aste (forze) si passa sequenzialmente a gli altri nodi calcolando le rispettive forze, attraverso due equazioni: una per le forze verticali e una per quelle orizzontali.

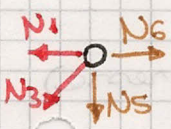
Equilibrio ai nodi:



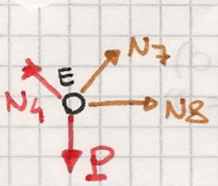
$$\begin{cases} N_1 = P - N_2 \frac{\sqrt{2}}{2} = P - P = 0 \\ \frac{\sqrt{2}}{2} N_2 = P \rightarrow N_2 = \sqrt{2}P \end{cases}$$



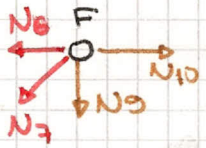
$$\begin{cases} N_3 = 0 \\ N_4 = N_2 = \sqrt{2}P \end{cases}$$



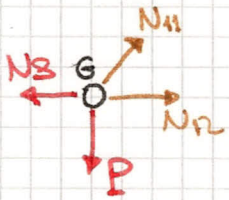
$$\begin{cases} N_6 = N_1 + N_3 \frac{\sqrt{2}}{2} = 0 \\ N_5 = \frac{\sqrt{2}}{2} N_3 = 0 \end{cases}$$



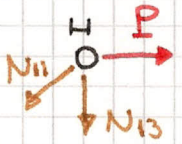
$$\begin{cases} N_7 \frac{\sqrt{2}}{2} = P - N_4 \frac{\sqrt{2}}{2} = 0 \\ N_8 = N_4 \frac{\sqrt{2}}{2} = P\sqrt{2}\frac{\sqrt{2}}{2} = 0 \end{cases}$$



$$\begin{cases} N_9 = N_7 \frac{\sqrt{2}}{2} = 0 \\ N_{10} = N_6 + N_7 \frac{\sqrt{2}}{2} = 0 \end{cases}$$

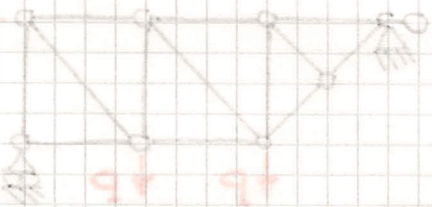


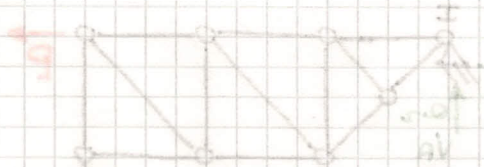
$$\begin{cases} N_{11} \frac{\sqrt{2}}{2} = P \Rightarrow N_{11} = P\sqrt{2} \\ N_{12} = P - N_{11} \frac{\sqrt{2}}{2} = 0 \end{cases}$$



$$\begin{cases} N_{13} = -N_{11} \frac{\sqrt{2}}{2} = -P \\ N_{11} \frac{\sqrt{2}}{2} = P \Rightarrow P\sqrt{2}\frac{\sqrt{2}}{2} = P \text{ ok} \end{cases}$$

ok

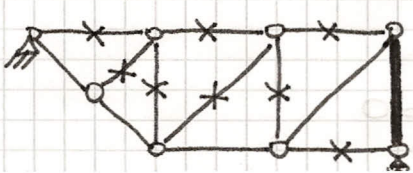




⑤ Tabella riepilogativa dello stato di sollecitazione:

ASTA	N	~~kg~~	
1	0	0	
2	$P\sqrt{2}$	1414	tirante (asta tesa)
3	0	0	
4	$P\sqrt{2}$	1414	tirante
5	0	0	
6	0	0	
7	0	0	
8	P	1000	tirante
9	0	0	
10	0	0	
11	$P\sqrt{2}$	1414	tirante
12	0	0	
13	$-P$	-1000	puntone (asta compressa)

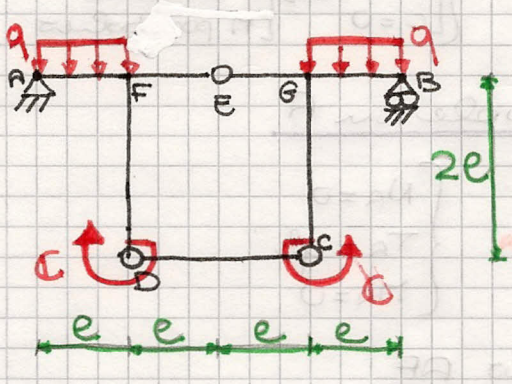
⑤ Schema riepilogativo



— TIRANTE ($N>0$)
▬ PUNTONE ($N<0$)
✱ SCARICA

Esercizio 4

(Esercizio 4, I° prova parziale, 3/11/2008)



$\ell = 0{,}5\ m$

$C = 1\ T\cdot m = 2q\ell^2$

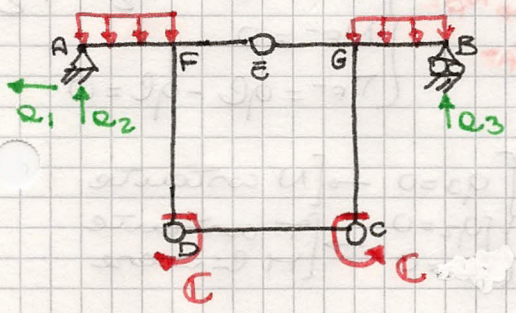
$q = 2000\ kg/m$

Svolgimento

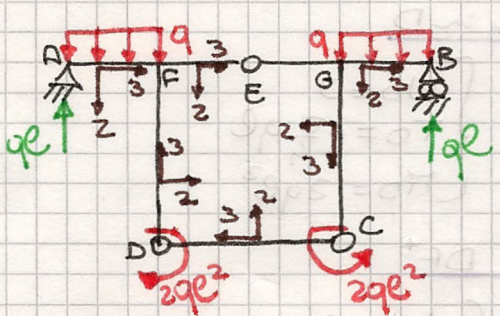
① Calcolo delle reazioni vincolari

Eq. cardinali della statica

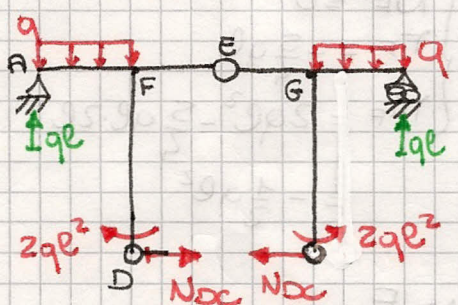
$$\begin{cases} (\rightarrow) \quad Q_1 = 0 \\ (\uparrow) \quad Q_2 + Q_3 = 2q\ell \\ \text{a seguito della simmetria } Q_2 = Q_3 = q\ell \end{cases}$$



② Inserimento di un s. di riferimento (2;3) su ogni tratto.



③ Escludo una maglia chiusa occorre necessariamente eseguire un taglio. Il taglio + conveniente è quello della biella DC, ottenendo come unica incognita N_{DC}



④ Eq. ausiliaria, relativa ad un elemento di connessione:

$$(E\vartheta)_{ADE} \quad N_{DC}\,2\ell = 2q\ell^2 - q\ell\,\frac{3}{2}\ell + q\ell\,2\ell$$

$$\perp \tfrac{5}{2}q\ell^2$$

$$\Rightarrow N_{DC} = 5/4\ q\ell$$

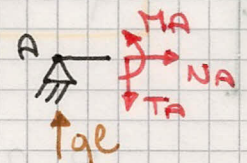
⑤ Applicazione del metodo delle equazioni indefinite

- Quadro dei tratti

■ Tratto AF

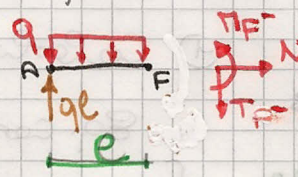
$\begin{cases} q_3 = 0 \rightarrow [N \text{ costante} \\ [q_2 = q \\ [c = 0 \end{cases} \rightarrow \begin{cases} [T \text{ lineare} \\ [M \text{ parabolica} \end{cases}$

• Eq. puntuale in A



$\begin{cases} N_A = 0 \\ T_A = q\ell \\ M_A = 0 \end{cases}$

• Eq. tratto AF

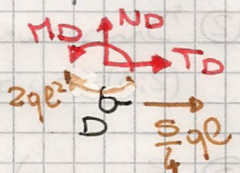


$\begin{cases} M_{F^-} = q\ell^2 - q\frac{\ell^2}{2} = q\frac{\ell^2}{2} \\ N_{F^-} = 0 \\ T_{F^-} = q\ell - q\ell = 0 \end{cases}$

■ Tratto DF

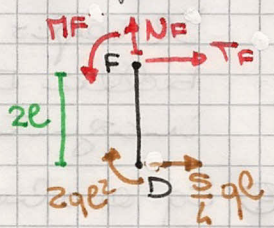
$\begin{cases} q_3 = 0 \rightarrow [N \text{ costante} \\ [q_2 = 0 \\ [c = 0 \end{cases} \rightarrow \begin{cases} [T \text{ costante} \\ [M \text{ lineare} \end{cases}$

• Eq. puntuale in D



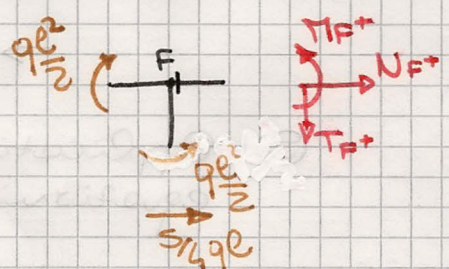
$\begin{cases} N_D = 0 \\ T_D = -\frac{5}{4}q\ell \\ M_D = 2q\ell^2 \end{cases}$

• Eq. tratto DF



$\begin{cases} N_F = 0 \\ T_F = -\frac{5}{4}q\ell \\ M_F = 2q\ell^2 - \frac{5}{4}q\ell \cdot 2\ell \\ \quad = -\frac{1}{2}q\ell^2 \end{cases}$

• Equilibrio in F



$\begin{cases} N_{F^+} = -\frac{5}{4}q\ell \\ T_{F^+} = 0 \\ M_{F^+} = q\frac{\ell^2}{2} - q\frac{\ell^2}{2} = 0 \end{cases}$

⑥ Costruzione dei diagrammi finali + quotature