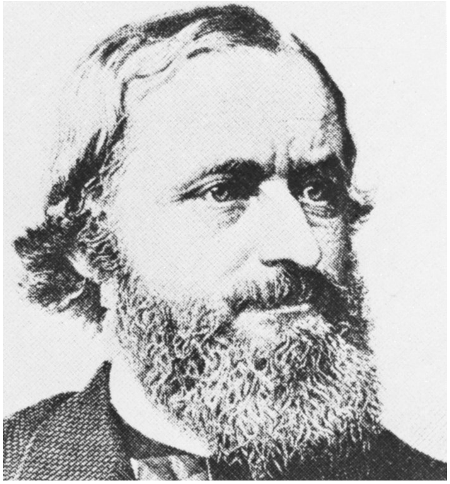

Gustav Robert Kirchhoff (1824–1887) was a German physicist and a friend of the famous chemist Robert Wilhelm Bunsen. Kirchhoff started teaching at Berlin University in 1848 and later moved to Heidelberg to occupy the chair of physics. There in 1859, he made his major contribution to physics, namely, the experimental discovery and theoretical analysis of a fundamental law of electromagnetic radiation. In addition, he made significant contributions to electrical circuits and the theory of elasticity. He published his important paper on the theory of plates in 1850 in which a satisfactory theory for the bending vibration of plates, along with the correct boundary conditions, was presented for the first time. In addition, he presented a paper on the vibration of bars of variable cross section. He moved to the University of Berlin in 1875 to occupy the chair of theoretical physics and published his famous book on mechanics in 1876. (Photo courtesy of *Applied Mechanics Reviews.*)

CHAPTER 10

Vibration Measurement and Applications

Chapter Outline

Chapter Outline 870
Learning Objectives 871
10.1 Introduction 871
10.2 Transducers 873
10.3 Vibration Pickups 879
10.4 Frequency-Measuring Instruments 890
10.5 Vibration Exciters 892
10.6 Signal Analysis 895
10.7 Dynamic Testing of Machines and Structures 900
10.8 Experimental Modal Analysis 900
10.9 Machine-Condition Monitoring and Diagnosis 915
10.10 Examples Using MATLAB 925
Chapter Summary 928
References 928
Review Questions 930
Problems 932
Design Projects 938

In some practical situations, it might be difficult to develop a mathematical model of the system and predict its vibration characteristics through an analytical study. In such cases, we can use experimental methods to measure the vibration response of the system to a known input. This helps in identifying the system in terms of its mass, stiffness, and damping. This chapter presents the various aspects of vibration measurement and applications. The basic scheme of vibration measurement is outlined first. Descriptions are given of transducers—devices which transform physical variables into equivalent electrical signals—and of vibration pickups and frequency measuring instruments used for vibration measurement. The working principles of mechanical and electrodynamic shakers or exciters, used to excite a machine or system to study its dynamic characteristics, are introduced. Signal analysis, which determines the response of a system under known excitation and presents it in a convenient form, is outlined along with descriptions of spectrum analyzer, bandpass filter, and bandwidth analyzers. The experimental modal analysis deals with the determination of natural frequencies, damping ratio, and mode shapes through vibration testing. The necessary equipment, digital signal processing, analysis of random signals, determination of modal data from observed peaks and Nyquist plot, and determination of mode shapes are described. Vibration severity criteria, machine maintenance techniques, machine-condition monitoring techniques, and instrumentation systems are presented for machine-condition monitoring and diagnosis. MATLAB programs are presented for plotting Nyquist circle and the acceleration equation.

Learning Objectives

After you have finished studying this chapter, you should be able to do the following:

- Understand the various types of transducers, vibration pickups, and frequency measuring instruments.
- Know the working principles of mechanical and electrodynamic shakers or exciters.
- Learn the process of signal analysis.
- Understand experimental modal analysis techniques to determine the natural frequencies, damping ratio, and mode shapes.
- Know the various aspects of machine-condition monitoring.
- Use MATLAB for plotting Nyquist circles and implementing methods of analysis discussed.

10.1 Introduction

In practice the measurement of vibration becomes necessary for the following reasons:

1. The increasing demands of higher productivity and economical design lead to higher operating speeds of machinery[1] and efficient use of materials through lightweight structures. These trends make the occurrence of resonant conditions more frequent during the operation of machinery and reduce the reliability of the system. Hence the periodic measurement of vibration characteristics of machinery and structures becomes essential to ensure adequate safety margins. Any observed shift in the natural

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[1] According to Eshleman, in reference [10.12], the average speed of rotating machines doubled—from 1800 rpm to 3600 rpm—during the period between 1940 and 1980.

frequencies or other vibration characteristics will indicate either a failure or a need for maintenance of the machine.

2. The measurement of the natural frequencies of a structure or machine is useful in selecting the operational speeds of nearby machinery to avoid resonant conditions.
3. The theoretically computed vibration characteristics of a machine or structure may be different from the actual values due to the assumptions made in the analysis.
4. The measurement of frequencies of vibration and the forces developed is necessary in the design and operation of active vibration-isolation systems.
5. In many applications, the survivability of a structure or machine in a specified vibration environment is to be determined. If the structure or machine can perform the expected task even after completion of testing under the specified vibration environment, it is expected to survive the specified conditions.
6. Continuous systems are often approximated as multidegree-of-freedom systems for simplicity. If the measured natural frequencies and mode shapes of a continuous system are comparable to the computed natural frequencies and mode shapes of the multidegree-of-freedom model, then the approximation will be proved to be a valid one.
7. The measurement of input and the resulting output vibration characteristics of a system helps in identifying the system in terms of its mass, stiffness, and damping.
8. The information about ground vibrations due to earthquakes, fluctuating wind velocities on structures, random variation of ocean waves, and road surface roughness are important in the design of structures, machines, oil platforms, and vehicle suspension systems.

Vibration Measurement Scheme. Figure 10.1 illustrates the basic features of a vibration measurement scheme. In this figure, the motion (or dynamic force) of the vibrating body is converted into an electrical signal by the vibration transducer or pickup. In general, a transducer is a device that transforms changes in mechanical quantities (such as displacement, velocity, acceleration, or force) into changes in electrical quantities (such as voltage or current). Since the output signal (voltage or current) of a transducer is too small to be recorded directly, a signal conversion instrument is used to amplify the signal to the required value. The output from the signal conversion instrument can be presented on a display unit for visual inspection, or recorded by a recording unit, or stored in a computer for later use. The data can then be analyzed to determine the desired vibration characteristics of the machine or structure.

Depending on the quantity measured, a vibration measuring instrument is called a vibrometer, a velocity meter, an accelerometer, a phase meter, or a frequency meter. If the instrument is designed to record the measured quantity, then the suffix "meter" is to be replaced by "graph" [10.1]. In some application, we need to vibrate a machine or structure to find its resonance characteristics. For this, electrodynamic vibrators, electrohydraulic vibrators, and signal generators (oscillators) are used.

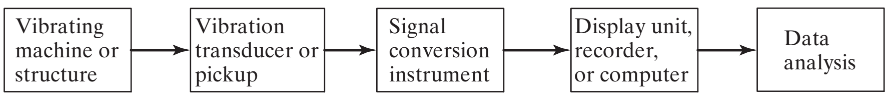


FIGURE 10.1 Basic vibration measurement scheme.

The following considerations often dictate the type of vibration-measuring instruments to be used in a vibration test: (1) expected ranges of the frequencies and amplitudes, (2) sizes of the machine/structure involved, (3) conditions of operation of the machine/equipment/structure, and (4) type of data processing used (such as graphical display or graphical recording or storing the record in digital form for computer processing).

10.2 Transducers

A transducer is a device that transforms values of physical variables into equivalent electrical signals. Several types of transducers are available; some of them are less useful than others due to their nonlinearity or slow response. Some of the transducers commonly used for vibration measurement are discussed below.

10.2.1 Variable-Resistance Transducers

In these transducers, a mechanical motion produces a change in electrical resistance (of a rheostat, a strain gage, or a semiconductor), which in turn causes a change in the output voltage or current. The schematic diagram of an electrical resistance strain gage is shown in Fig. 10.2. An electrical resistance strain gage consists of a fine wire whose resistance changes when it is subjected to mechanical deformation. When the strain gage is bonded to a structure, it experiences the same motion (strain) as the structure and hence its resistance change gives the strain applied to the structure. The wire is sandwiched between two sheets of thin paper. The strain gage is bonded to the surface where the strain is to be measured. The most common gage material is a copper-nickel alloy known as Advance. When the surface undergoes a normal strain (ϵ), the strain gage also undergoes the same strain and the resulting change in its resistance is given by [10.6]

$$K = \frac{\Delta R/R}{\Delta L/L} = 1 + 2\nu + \frac{\Delta r}{r}\frac{L}{\Delta L} \approx 1 + 2\nu \tag{10.1}$$

where

K = Gage factor for the wire

R = Initial resistance

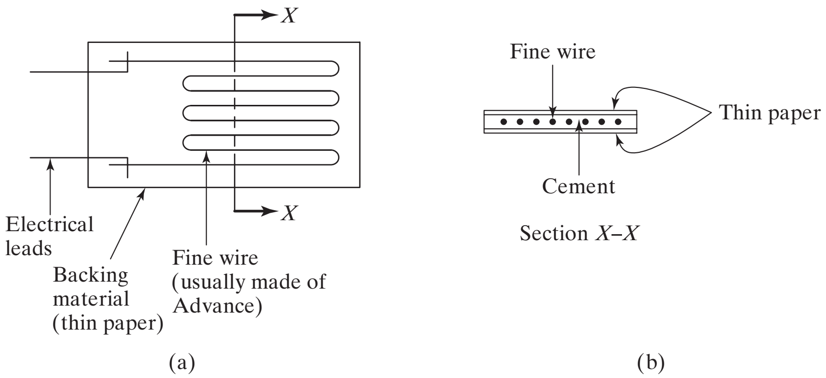


FIGURE 10.2 Electric resistance strain gage.

$$\begin{aligned}
\Delta R &= \text{Change in resistance} \\
L &= \text{Initial length of the wire} \\
\Delta L &= \text{Change in length of the wire} \\
v &= \text{Poisson's ratio of the wire} \\
r &= \text{Resistivity of the wire} \\
\Delta r &= \text{Change in resistivity of the wire} \approx 0 \text{ for Advance}
\end{aligned}$$

The value of the gage factor K is given by the manufacturer of the strain gage, hence the value of ϵ can be determined, once ΔR and R are measured, as

$$\epsilon = \frac{\Delta L}{L} = \frac{\Delta R}{RK} \tag{10.2}$$

In a vibration pickup[2] the strain gage is mounted on an elastic element of a spring-mass system, as shown in Fig. 10.3. The strain at any point on the cantilever (elastic member) is proportional to the deflection of the mass, $x(t)$, to be measured. Hence the strain indicated by the strain gage can be used to find $x(t)$. The change in resistance of the wire ΔR can be measured using a Wheatstone bridge, potentiometer circuit, or voltage divider. A typical Wheatstone bridge, representing a circuit which is sensitive to small changes in the resistance, is shown in Fig. 10.4. A d.c. voltage V is applied across the points a and c. The resulting voltage across the points b and d is given by [10.6]:

$$E = \left[\frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)}\right] V \tag{10.3}$$

Initially the resistances are balanced (adjusted) so that the output voltage E is zero. Thus, for initial balance, Eq. (10.3) gives

$$R_1 R_3 = R_2 R_4 \tag{10.4}$$

When the resistances (R_i) change by small amounts (ΔR_i), the change in the output voltage ΔE can be expressed as

$$\Delta E \approx V r_0 \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right) \tag{10.5}$$

where

$$r_0 = \frac{R_1 R_2}{(R_1 + R_2)^2} = \frac{R_3 R_4}{(R_3 + R_4)^2} \tag{10.6}$$

[2]When a transducer is used in conjunction with other components that permit the processing and transmission of the signal, the device is called a *pickup*.

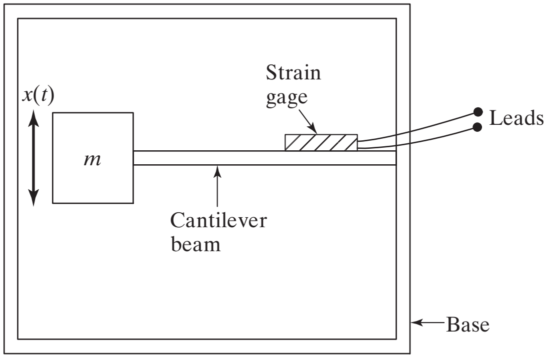


FIGURE 10.3 Strain gage as vibration pickup.

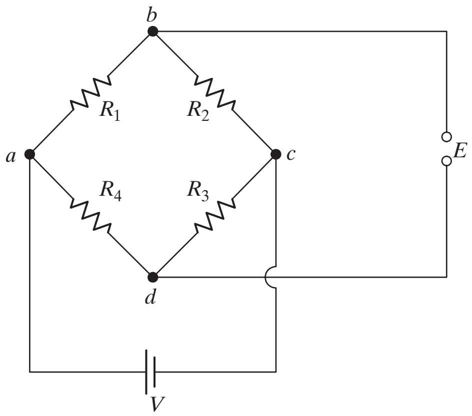


FIGURE 10.4 Wheatstone bridge.

If the strain gage leads are connected between the points a and b, $R_1 = R_g$, $\Delta R_1 = \Delta R_g$, and $\Delta R_2 = \Delta R_3 = \Delta R_4 = 0$, and Eq. (10.5) gives

$$\frac{\Delta R_g}{R_g} = \frac{\Delta E}{V r_0} \tag{10.7}$$

where R_g is the initial resistance of the gage. Equations (10.2) and (10.7) yield

$$\frac{\Delta R_g}{R_g} = \epsilon K = \frac{\Delta E}{V r_0}$$

or

$$\Delta E = KVr_0\epsilon \tag{10.8}$$

Since the output voltage is proportional to the strain, it can be calibrated to read the strain directly.

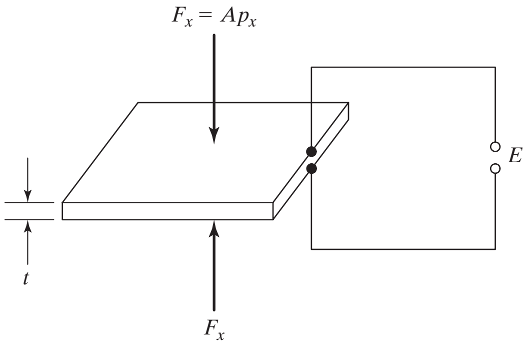
10.2.2 Piezoelectric Transducers

Certain natural and manufactured materials like quartz, tourmaline, lithium sulfate, and Rochelle salt generate electrical charge when subjected to a deformation or mechanical stress (see Fig. 10.5(a)). The electrical charge disappears when the mechanical loading is removed. Such materials are called piezoelectric materials and the transducers, which take advantage of the piezoelectric effect, are known as piezoelectric transducers. The charge generated in the crystal due to a force F_x is given by

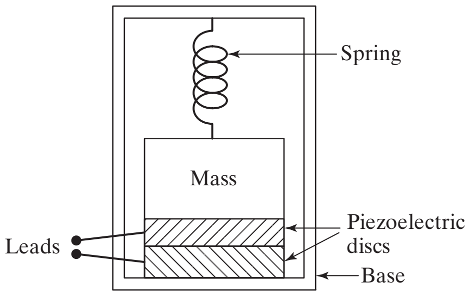
$$Q_x = kF_x = kAp_x \tag{10.9}$$

where k is called the piezoelectric constant, A is the area on which the force F_x acts, and p_x is the pressure due to F_x. The output voltage of the crystal is given by

$$E = vtp_x \tag{10.10}$$



(a)



(b)

FIGURE 10.5 Piezoelectric accelerometer.

where v is called the voltage sensitivity and t is the thickness of the crystal. The values of the piezoelectric constant and voltage sensitivity for quartz are 2.25×10^{-12} C/N and 0.055 volt-meter/N, respectively [10.6]. These values are valid only when the perpendicular to the largest face is along the x-axis of the crystal. The electric charge developed and the voltage output will be different if the crystal slab is cut in a different direction.

A typical piezoelectric transducer (accelerometer) is shown in Fig. 10.5(b). In this figure, a small mass is spring loaded against a piezoelectric crystal. When the base vibrates, the load exerted by the mass on the crystal changes with acceleration, hence the output voltage generated by the crystal will be proportional to the acceleration. The main advantages of the piezoelectric accelerometer include compactness, ruggedness, high sensitivity, and high frequency range [10.5, 10.8].

EXAMPLE 10.1 Output Voltage of a Piezoelectric Transducer

A quartz crystal having a thickness of 0.1 in. is subjected to a pressure of 50 psi. Find the output voltage if the voltage sensitivity is 0.055 V-m/N.

Solution: With $t = 0.1$ in. $= 0.00254$ m, $p_x = 50$ psi $= 344.738$ N/m^2, and $v = 0.055$ V-m/N, Eq. (10.10) gives

$$E = (0.055)(0.00254)(344.738) = 48.1599 \text{ volts}$$

■

10.2.3 Electrodynamic Transducers

When an electrical conductor, in the form of a coil, moves in a magnetic field as shown in Fig. 10.6, a voltage E is generated in the conductor. The value of E in volts is given by

$$E = Dlv \tag{10.11}$$

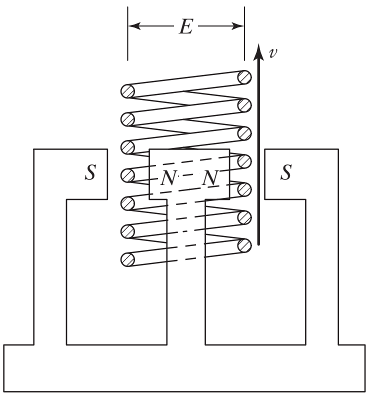


FIGURE 10.6 Basic idea behind electrodynamic transducer.

where D is the magnetic flux density (teslas), l is the length of the conductor (meters), and v is the velocity of the conductor relative to the magnetic field (meters/second). The magnetic field may be produced by either a permanent magnet or an electromagnet. Sometimes, the coil is kept stationary and the magnet is made to move. Since the voltage output of an electromagnetic transducer is proportional to the relative velocity of the coil, they are frequently used in "velocity pickups." Equation (10.11) can be rewritten as

$$Dl = \frac{E}{v} = \frac{F}{I} \tag{10.12}$$

where F denotes the force (newtons) acting on the coil while carrying a current I (amperes). Equation (10.12) shows that the performance of an electrodynamic transducer can be reversed. In fact, Eq. (10.12) forms the basis for using an electrodynamic transducer as a "vibration exciter" (see Section 10.5.2).

10.2.4 Linear Variable Differential Transformer Transducer

The schematic diagram of a *linear variable differential transformer* (*LVDT*) transducer is shown in Fig. 10.7. It consists of a primary coil at the center, two secondary coils at the ends, and a magnetic core that can move freely inside the coils in the axial direction. When an a.c. input voltage is applied to the primary coil, the output voltage will be equal to the difference of the voltages induced in the secondary coils. This output voltage depends on the magnetic coupling between the coils and the core, which in turn depends on the axial displacement of the core. The secondary coils are connected in phase opposition so that, when the magnetic core is in the exact middle position, the voltages in the two coils will be equal and 180° out of phase. This makes the output voltage of the LVDT as zero. When the core is moved to either side of the middle (zero) position, the magnetic coupling will be increased in one secondary coil and decreased in the other coil. The output polarity depends on the direction of the movement of the magnetic core.

The range of displacement for many LVDTs on the market is from 0.0002 cm to 40 cm. The advantages of an LVDT over other displacement transducers include insensitivity to

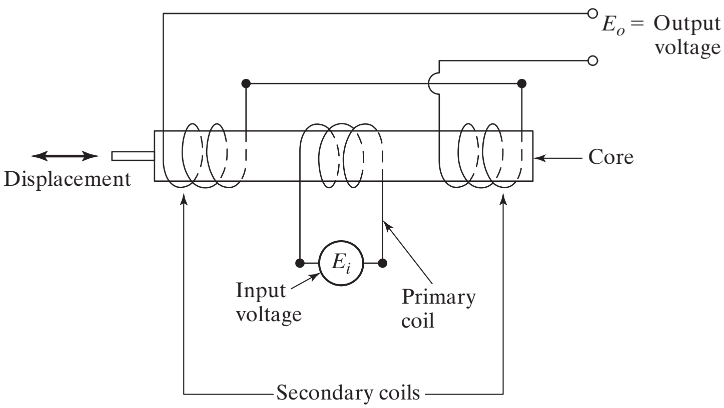


FIGURE 10.7 Schematic diagram of an LVDT transducer.

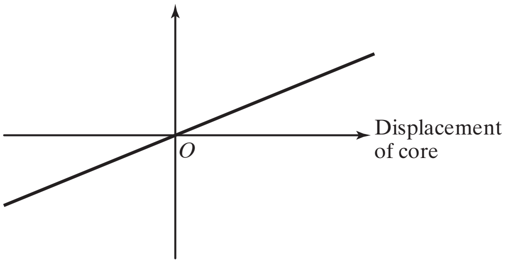


FIGURE 10.8 Linearity of voltage with displacement of core.

temperature and high output. The mass of the magnetic core restricts the use of the LVDT for high-frequency applications [10.4].

As long as the core is not moved very far from the center of the coil, the output voltage varies linearly with the displacement of the core, as shown in Fig. 10.8; hence the name linear variable differential transformer.

10.3 Vibration Pickups

When a transducer is used in conjunction with another device to measure vibrations, it is called a *vibration pickup*. The commonly used vibration pickups are known as seismic instruments. A seismic instrument consists of a mass-spring-damper system mounted on the vibrating body, as shown in Fig. 10.9. Then the vibratory motion is measured by finding the displacement of the mass relative to the base on which it is mounted.

The instrument consists of a mass m, a spring k, and a damper c inside a cage, which is fastened to the vibrating body. With this arrangement, the bottom ends of the spring and the dashpot will have the same motion as the cage (which is to be measured, y) and their vibration excites the suspended mass into motion. Then the displacement of the mass relative to the cage, $z = x - y$, where x denotes the vertical displacement of the suspended mass, can be measured if we attach a pointer to the mass and a scale to the cage, as shown in Fig. 10.9.[3]

The vibrating body is assumed to have a harmonic motion:

$$y(t) = Y \sin \omega t \tag{10.13}$$

The equation of motion of the mass m can be written as

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \tag{10.14}$$

[3]The output of the instrument shown in Fig. 10.9 is the relative mechanical motion of the mass, as shown by the pointer and the graduated scale on the cage. For high-speed operation and convenience, the motion is often converted into an electrical signal by a transducer.

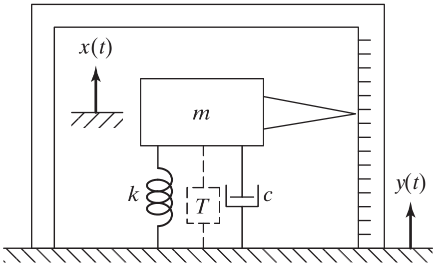


FIGURE 10.9 Seismic instrument.

By defining the relative displacement z as

$$z = x - y \tag{10.15}$$

Eq. (10.14) can be written as

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \tag{10.16}$$

Equations (10.13) and (10.16) lead to

$$m\ddot{z} + c\dot{z} + kz = m\omega^2 Y \sin \omega t \tag{10.17}$$

This equation is identical to Eq. (3.75); hence the steady-state solution is given by

$$z(t) = Z \sin(\omega t - \phi) \tag{10.18}$$

where Z and ϕ are given by (see Eqs. (3.76) and (3.77)):

$$Z = \frac{Y\omega^2}{[(k - m\omega^2)^2 + c^2\omega^2]^{1/2}} = \frac{r^2 Y}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}} \tag{10.19}$$

$$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right) = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right) \tag{10.20}$$

$$r = \frac{\omega}{\omega_n} \tag{10.21}$$

and

$$\zeta = \frac{c}{2m\omega_n} \tag{10.22}$$

The variations of Z and ϕ with respect to r are shown in Figs. 10.10 and 10.11. As will be seen later, the type of instrument is determined by the useful range of the frequencies, indicated in Fig. 10.10.

10.3.1 Vibrometer

A *vibrometer* or a *seismometer* is an instrument that measures the displacement of a vibrating body. It can be observed from Fig. 10.10 that $Z/Y \approx 1$ when $\omega/\omega_n \geq 3$ (range II). Thus the relative displacement between the mass and the base (sensed by the transducer) is essentially the same as the displacement of the base. For an exact analysis, we consider Eq. (10.19). We note that

$$z(t) \simeq Y \sin(\omega t - \phi) \tag{10.23}$$

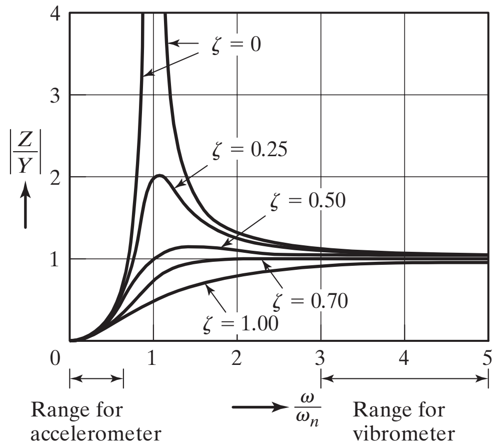


FIGURE 10.10 Response of a vibration-measuring instrument.

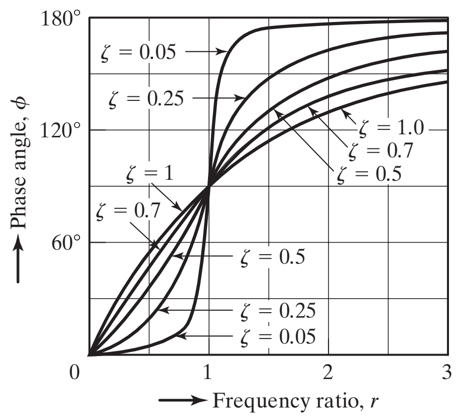


FIGURE 10.11 Variation of ϕ with r.

if

$$\frac{r^2}{[(1-r^2)^2+(2\zeta r)^2]^{1/2}} \approx 1 \tag{10.24}$$

A comparison of Eq. (10.23) with $y(t) = Y \sin \omega t$ shows that $z(t)$ gives directly the motion $y(t)$ except for the phase lag ϕ. This phase lag can be seen to be equal to 180° for $\zeta = 0$. Thus the recorded displacement $z(t)$ lags behind the displacement being measured $y(t)$ by time $t' = \phi/\omega$. This time lag is not important if the base displacement $y(t)$ consists of a single harmonic component.

Since $r = \omega/\omega_n$ has to be large and the value of ω is fixed, the natural frequency $\omega_n = \sqrt{k/m}$ of the mass-spring-damper must be low. This means that the mass must be large and the spring must have a low stiffness. This results in a bulky instrument, which is not desirable in many applications. In practice, the vibrometer may not have a large value of r and hence the value of Z may not be equal to Y exactly. In such a case, the true value of Y can be computed by using Eq. (10.19), as indicated in the following example.

EXAMPLE 10.2 Amplitude by Vibrometer

A vibrometer having a natural frequency of 4 rad/s and $\zeta = 0.2$ is attached to a structure that performs a harmonic motion. If the difference between the maximum and the minimum recorded values is 8 mm, find the amplitude of motion of the vibrating structure when its frequency is 40 rad/s.

Solution: The amplitude of the recorded motion Z is 4 mm. For $\zeta = 0.2$, $\omega = 40.0$ rad/s, and $\omega_n = 4$ rad/s, $r = 10.0$, and Eq. (10.19) gives

$$Z = \frac{Y(10)^2}{[(1-10^2)^2+\{2(0.2)(10)\}^2]^{1/2}} = 1.0093Y$$

Thus the amplitude of vibration of the structure is $Y = Z/1.0093 = 3.9631$ mm.

■

10.3.2 Accelerometer

An accelerometer is an instrument that measures the acceleration of a vibrating body (see Fig. 10.12). Accelerometers are widely used for vibration measurements [10.7] and also to record earthquakes. From the accelerometer record, the velocity and displacements are obtained by integration. Equations (10.18) and (10.19) yield

$$-z(t)\omega_n^2 = \frac{1}{[(1-r^2)^2+(2\zeta r)^2]^{1/2}}\{-Y\omega^2 \sin(\omega t - \phi)\} \tag{10.25}$$

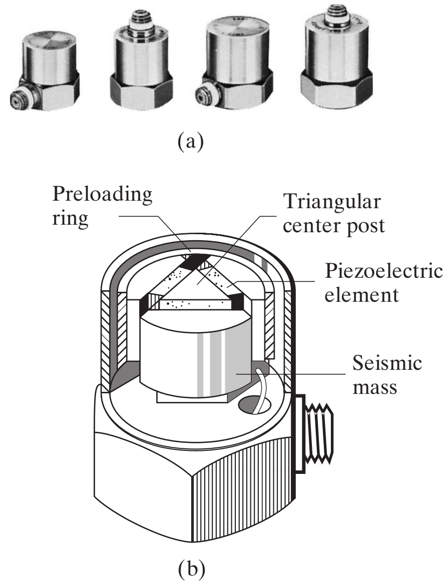


FIGURE 10.12 Accelerometers.
(Courtesy of Bruel and Kjaer Instruments, Inc., Marlborough, MA.)

This shows that if

$$\frac{1}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}} \simeq 1 \tag{10.26}$$

Eq. (10.25) becomes

$$-z(t)\omega_n^2 \simeq -Y\omega^2 \sin(\omega t - \phi) \tag{10.27}$$

By comparing Eq. (10.27) with $\ddot{y}(t) = -Y\omega^2 \sin \omega t$, we find that the term $z(t)\omega_n^2$ gives the acceleration of the base $\ddot{y}$, except for the phase lag ϕ. Thus the instrument can be made to record (give) directly the value of $\ddot{y} = -z(t)\omega_n^2$. The time by which the record lags the acceleration is given by $t' = \phi/\omega$. If $\ddot{y}$ consists of a single harmonic component, the time lag will not be of importance.

The value of the expression on the left-hand side of Eq. (10.26) is shown plotted in Fig. 10.13. It can be seen that the left-hand side of Eq. (10.26) lies between 0.96 and 1.04 for $0 \leq r \leq 0.6$ if the value of ζ lies between 0.65 and 0.7. Since r is small, the natural frequency of the instrument has to be large compared to the frequency of vibration to be measured. From the relation $\omega_n = \sqrt{k/m}$, we find that the mass needs to be small and the spring needs to have a large value of k (i.e., short spring), so the instrument will be small in size. Due

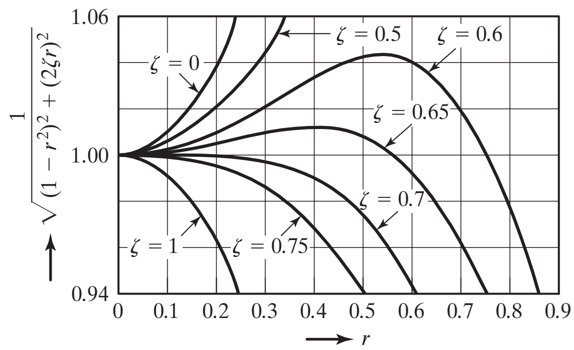


FIGURE 10.13 Variation of lefthand side of Eq. (10.26) with r.

to their small size and high sensitivity, accelerometers are preferred in vibration measurements. In practice, Eq. (10.26) may not be satisfied exactly; in such cases the quantity

$$\frac{1}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}}$$

can be used to find the correct value of the acceleration measured, as illustrated in the following example.

EXAMPLE 10.3 Design of an Accelerometer

An accelerometer has a suspended mass of 0.01 kg with a damped natural frequency of vibration of 150 Hz. When mounted on an engine undergoing an acceleration of $1g$ at an operating speed of 6000 rpm, the acceleration is recorded as 9.5 m/s^2 by the instrument. Find the damping constant and the spring stiffness of the accelerometer.

Solution: The ratio of measured to true accelerations is given by

$$\frac{1}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}} = \frac{\text{Measured value}}{\text{True value}} = \frac{9.5}{9.81} = 0.9684 \quad \text{(E.1)}$$

which can be written as

$$[(1 - r^2)^2 + (2\zeta r)^2] = (1/0.9684)^2 = 1.0663 \quad \text{(E.2)}$$

The operating speed of the engine gives

$$\omega = \frac{6000(2\pi)}{60} = 628.32 \text{ rad/s}$$

The damped natural frequency of vibration of the accelerometer is

$$\omega_d = \sqrt{1 - \zeta^2}\omega_n = 150(2\pi) = 942.48 \text{ rad/s}$$

Thus

$$\frac{\omega}{\omega_d} = \frac{\omega}{\sqrt{1 - \zeta^2}\omega_n} = \frac{r}{\sqrt{1 - \zeta^2}} = \frac{628.32}{942.48} = 0.6667 \tag{E.3}$$

Equation (E.3) gives

$$r = 0.6667\sqrt{1 - \zeta^2} \qquad \text{or} \qquad r^2 = 0.4444(1 - \zeta^2) \tag{E.4}$$

Substitution of Eq. (E.4) into (E.2) leads to a quadratic equation in ζ^2 as

$$1.5801\zeta^4 - 2.2714\zeta^2 + 0.7576 = 0 \tag{E.5}$$

The solution of Eq. (E.5) gives

$$\zeta^2 = 0.5260, 0.9115$$

or

$$\zeta = 0.7253, 0.9547$$

By choosing $\zeta = 0.7253$ arbitrarily, the undamped natural frequency of the accelerometer can be found as

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} = \frac{942.48}{\sqrt{1 - 0.7253^2}} = 1368.8889 \text{ rad/s}$$

Since $\omega_n = \sqrt{k/m}$, we have

$$k = m\omega_n^2 = (0.01)(1368.8889)^2 = 18738.5628 \text{ N/m}$$

The damping constant can be determined from

$$c = 2m\omega_n\zeta = 2(0.01)(1368.8889)(0.7253) = 19.8571 \text{ N-s/m}$$

■

10.3.3 Velometer

A velometer measures the velocity of a vibrating body. Equation (10.13) gives the velocity of the vibrating body

$$\dot{y}(t) = \omega Y \cos \omega t \tag{10.28}$$

and Eq. (10.18) gives

$$\dot{z}(t) = \frac{r^2 \omega Y}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}} \cos(\omega t - \phi) \tag{10.29}$$

If

$$\frac{r^2}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}} \simeq 1 \tag{10.30}$$

then

$$\dot{z}(t) \simeq \omega Y \cos(\omega t - \phi) \tag{10.31}$$

A comparison of Eqs. (10.28) and (10.31) shows that, except for the phase difference ϕ, $\dot{z}(t)$ gives directly $\dot{y}(t)$, provided that Eq. (10.30) holds true. In order to satisfy Eq. (10.30), r must be very large. In case Eq. (10.30) is not satisfied, then the velocity of the vibrating body can be computed using Eq. (10.29).

EXAMPLE 10.4

Design of a Velometer

Design a velometer if the maximum error is to be limited to 1 percent of the true velocity. The natural frequency of the velometer is to be 80 Hz and the suspended mass is to be 0.05 kg.

Solution: The ratio (R) of the recorded and the true velocities is given by Eq. (10.29):

$$R = \frac{r^2}{[(1 - r^2)^2 + (2\zeta r)^2]^{1/2}} = \frac{\text{Recorded velocity}}{\text{True velocity}} \tag{E.1}$$

The maximum of (E.1) occurs when (see Eq. (3.82))

$$r = r^* = \frac{1}{\sqrt{1 - 2\zeta^2}} \tag{E.2}$$

Substitution of Eq. (E.2) into (E.1) gives

$$\frac{\left(\dfrac{1}{1-2\zeta^2}\right)}{\sqrt{\left[1-\left(\dfrac{1}{1-2\zeta^2}\right)\right]^2+4\zeta^2\left(\dfrac{1}{1-2\zeta^2}\right)}}=R$$

which can be simplified as

$$\frac{1}{\sqrt{4\zeta^2-4\zeta^4}}=R \tag{E.3}$$

For an error of 1 percent, $R = 1.01$ or 0.99, and Eq. (E.3) leads to

$$\zeta^4-\zeta^2+0.245075=0 \tag{E.4}$$

and

$$\zeta^4-\zeta^2+0.255075=0 \tag{E.5}$$

Equation (E.5) gives imaginary roots and Eq. (E.4) gives

$$\zeta^2 = 0.570178,\ 0.429821$$

or

$$\zeta = 0.755101,\ 0.655607$$

We choose the value $\zeta = 0.755101$ arbitrarily. The spring stiffness can be found as

$$k = m\omega_n^2 = 0.05(502.656)^2 = 12633.1527 \text{ N/m}$$

since

$$\omega_n = 80(2\pi) = 502.656 \text{ rad/s}$$

The damping constant can be determined from

$$c = 2\zeta\omega_n m = 2(0.755101)(502.656)(0.05) = 37.9556 \text{ N-s/m}$$

■

10.3.4 Phase Distortion

As shown by Eq. (10.18), all vibration-measuring instruments exhibit phase lag. Thus the response or output of the instrument lags behind the motion or input it measures. The time lag is given by the phase angle divided by the frequency ω. The time lag is not important if we measure a single harmonic component. But, occasionally, the vibration to be recorded is not harmonic but consists of the sum of two or more harmonic components. In such a case, the recorded graph may not give an accurate picture of the vibration, because different harmonics may be amplified by different amounts and their phase shifts may also be different. The distortion in the waveform of the recorded signal is called the *phase distortion* or *phase-shift error*. To illustrate the nature of the phase-shift error, we consider a vibration signal of the form shown in Fig. 10.14(a) [10.10]:

$$y(t) = a_1 \sin \omega t + a_3 \sin 3\omega t \tag{10.32}$$

Let the phase shift be 90° for the first harmonic and 180° for the third harmonic of Eq. (10.32). The corresponding time lags are given by $t_1 = \theta_1/\omega = 90°/\omega$ and $t_2 = \theta_2/(3\omega) = 180°/(3\omega)$. The output signal is shown in Fig. 10.14(b). It can be seen that the output signal is quite different from the input signal due to phase distortion.

As a general case, let the complex wave being measured be given by the sum of several harmonics as

$$y(t) = a_1 \sin \omega t + a_2 \sin 2\omega t + \cdots \tag{10.33}$$

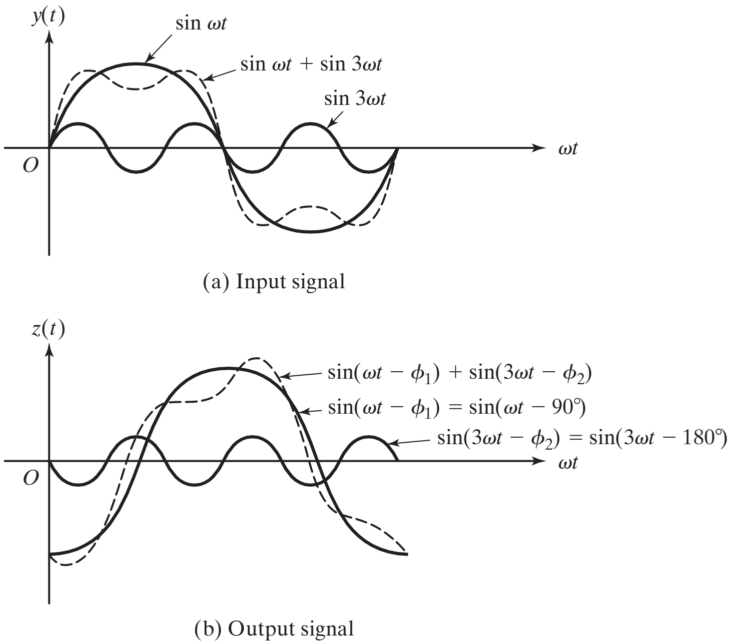


FIGURE 10.14 Phase-shift error.

If the displacement is measured using a vibrometer, its response to each component of the series is given by an equation similar to Eq. (10.18), so that the output of the vibrometer becomes

$$z(t) = a_1 \sin(\omega t - \phi_1) + a_2 \sin(2\omega t - \phi_2) + \cdots \tag{10.34}$$

where

$$\tan \phi_j = \frac{2\zeta\left(j \dfrac{\omega}{\omega_n}\right)}{1 - \left(j \dfrac{\omega}{\omega_n}\right)^2}, \qquad j = 1, 2, \ldots \tag{10.35}$$

Since ω/ω_n is large for this instrument, we can find from Fig. 10.11 that $\phi_j \simeq \pi$, $j = 1, 2, \ldots$, and Eq. (10.34) becomes

$$z(t) \simeq -[a_1 \sin \omega t + a_2 \sin 2\omega t + \cdots] \simeq -y(t) \tag{10.36}$$

Thus the output record will be simply opposite to the motion being measured. This is unimportant and can easily be corrected.

By using a similar reasoning, we can show, in the case of a velometer, that

$$\dot{z}(t) \simeq -\dot{y}(t) \tag{10.37}$$

for an input signal consisting of several harmonics. Next we consider the phase distortion for an accelerometer. Let the acceleration curve to be measured be expressed, using Eq. (10.33), as

$$\ddot{y}(t) = -a_1\omega^2 \sin \omega t - a_2(2\omega)^2 \sin 2\omega t - \cdots \tag{10.38}$$

The response or output of the instrument to each component can be found as in Eq. (10.34), and so

$$\ddot{z}(t) = -a_1\omega^2 \sin(\omega t - \phi_1) - a_2(2\omega)^2 \sin(2\omega t - \phi_2) - \cdots \tag{10.39}$$

where the phase lags ϕ_j are different for different components of the series in Eq. (10.39). Since the phase lag ϕ varies almost linearly from 0° at $r = 0$ to 90° at $r = 1$ for $\zeta = 0.7$ (see Fig. 10.11), we can express ϕ as

$$\phi \simeq \alpha r = \alpha \frac{\omega}{\omega_n} = \beta\omega \tag{10.40}$$

where α and $\beta = \alpha/\omega_n$ are constants. The time lag is given by

$$t' = \frac{\phi}{\omega} = \frac{\beta\omega}{\omega} = \beta \tag{10.41}$$

This shows that the time lag of the accelerometer is independent of the frequency for any component, provided that the frequency lies in the range $0 \leq r \leq 1$. Since each component of the signal has the same time delay or phase lag, we have, from Eq. (10.39),

$$\begin{aligned} -\omega^2 \ddot{z}(t) &= -a_1\omega^2 \sin(\omega t - \omega\beta) - a_2(2\omega)^2 \sin(2\omega t - 2\omega\beta) - \cdots \\ &= -a_1\omega^2 \sin \omega\tau - a_2(2\omega)^2 \sin 2\omega\tau - \cdots \end{aligned} \tag{10.42}$$

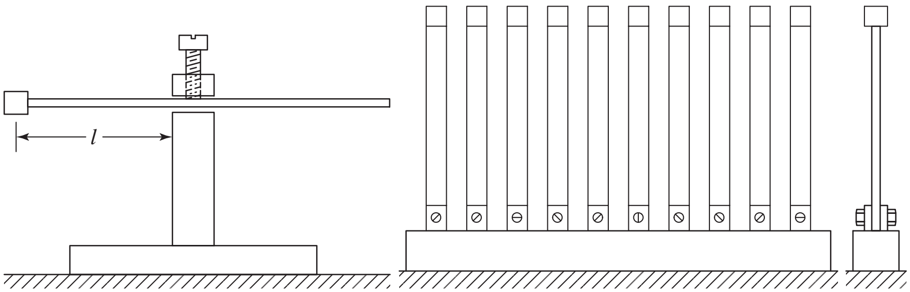
where $\tau = t - \beta$. Note that Eq. (10.42) assumes that $0 \leq r \leq 1$—that is, even the highest frequency involved, $n\omega$, is less than ω_n. This may not be true in practice. Fortunately, no significant phase distortion occurs in the output signal, even when some of the higher-order frequencies are larger than ω_n. The reason is that, generally, only the first few components are important to approximate even a complex waveform; the amplitudes of the higher harmonics are small and contribute very little to the total waveform. Thus the output record of the accelerometer represents a reasonably true acceleration being measured [10.7, 10.11].

10.4 Frequency-Measuring Instruments

Most frequency-measuring instruments are of the mechanical type and are based on the principle of resonance. Two kinds are discussed in the following paragraphs: the Fullarton tachometer and the Frahm tachometer.

Single-Reed Instrument or Fullarton Tachometer. This instrument consists of a variable-length cantilever strip with a mass attached at one of its ends. The other end of the strip is clamped, and its free length can be changed by means of a screw mechanism (see Fig. 10.15(a)). Since each length of the strip corresponds to a different natural frequency, the reed is marked along its length in terms of its natural frequency. In practice, the clamped end of the strip is pressed against the vibrating body, and the screw mechanism is manipulated to alter its free length until the free end shows the largest amplitude of vibration. At that instant, the excitation frequency is equal to the natural frequency of the cantilever; it can be read directly from the strip.

Multireed-Instrument or Frahm Tachometer. This instrument consists of a number of cantilevered reeds carrying small masses at their free ends (see Fig. 10.15(b)). Each reed has a different natural frequency and is marked accordingly. Using a number of reeds makes it possible to cover a wide frequency range. When the instrument is mounted on a vibrating body, the reed whose natural frequency is nearest the unknown frequency of the



(a) (b)

FIGURE 10.15 Frequency-measuring instruments.

body vibrates with the largest amplitude. The frequency of the vibrating body can be found from the known frequency of the vibrating reed.

Stroboscope. A stroboscope is an instrument that produces light pulses intermittently. The frequency at which the light pulses are produced can be altered and read from the instrument. When a specific point on a rotating (vibrating) object is viewed with the stroboscope, it will appear to be stationary only when the frequency of the pulsating light is equal to the speed of the rotating (vibrating) object. The main advantage of the stroboscope is that it does not make contact with the rotating (vibrating) body. Due to the persistence of vision, the lowest frequency that can be measured with a stroboscope is approximately 15 Hz. A typical stroboscope is shown in Fig. 10.16.

FIGURE 10.16 A stroboscope. (Courtesy of Bruel and Kjaer Instruments, Inc., Marlborough, MA.)

10.5 Vibration Exciters

The vibration exciters or shakers can be used in several applications such as determination of the dynamic characteristics of machines and structures and fatigue testing of materials. The vibration exciters can be mechanical, electromagnetic, electrodynamic, or hydraulic type. The working principles of mechanical and electromagnetic exciters are described in this section.

10.5.1 Mechanical Exciters

As indicated in Section 1.10 (Fig. 1.46), a Scotch yoke mechanism can be used to produce harmonic vibrations. The crank of the mechanism can be driven either by a constant- or a variable-speed motor. When a structure is to be vibrated, the harmonic force can be applied either as an inertia force, as shown in Fig. 10.17(a), or as an elastic spring force, as shown

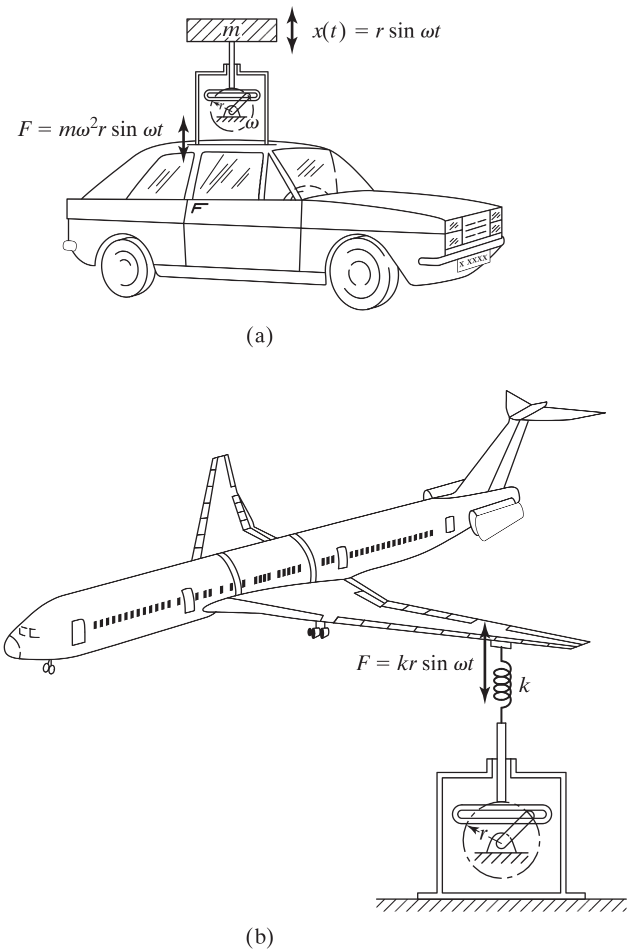


FIGURE 10.17 Vibration of a structure through (a) an inertia force and (b) an elastic spring force.

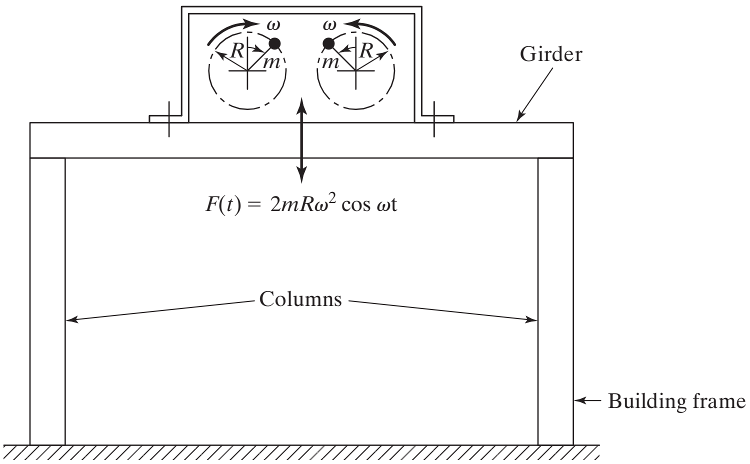


FIGURE 10.18 Vibration excitation due to unbalanced force.

in Fig. 10.17(b). These vibrators are generally used for frequencies less than 30 Hz and loads less than 700 N [10.1].

The unbalance created by two masses rotating at the same speed in opposite directions (see Fig. 10.18) can be used as a mechanical exciter. This type of shaker can be used to generate relatively large loads between 250 and 25,000 N. If the two masses, of magnitude m each, rotate at an angular velocity ω at a radius R, the vertical force $F(t)$ generated is given by

$$F(t) = 2mR\omega^2 \cos \omega t \tag{10.43}$$

The horizontal components of the two masses cancel, hence the resultant horizontal force will be zero. The force $F(t)$ will be applied to the structure to which the exciter is attached.

10.5.2 Electrodynamic Shaker

The schematic diagram of an *electrodynamic shaker*, also known as the *electromagnetic exciter*, is shown in Fig. 10.19(a). As stated in Section 10.2.3, the electrodynamic shaker can be considered as the reverse of an electrodynamic transducer. When current passes through a coil placed in a magnetic field, a force F (in Newtons) proportional to the current I (in amperes) and the magnetic flux intensity D (in teslas) is produced which accelerates the component placed on the shaker table:

$$F = DIl \tag{10.44}$$

where l is the length of the coil (in meters). The magnetic field is produced by a permanent magnet in small shakers while an electromagnet is used in large shakers. The magnitude of acceleration of the table or component depends on the maximum current and the masses of

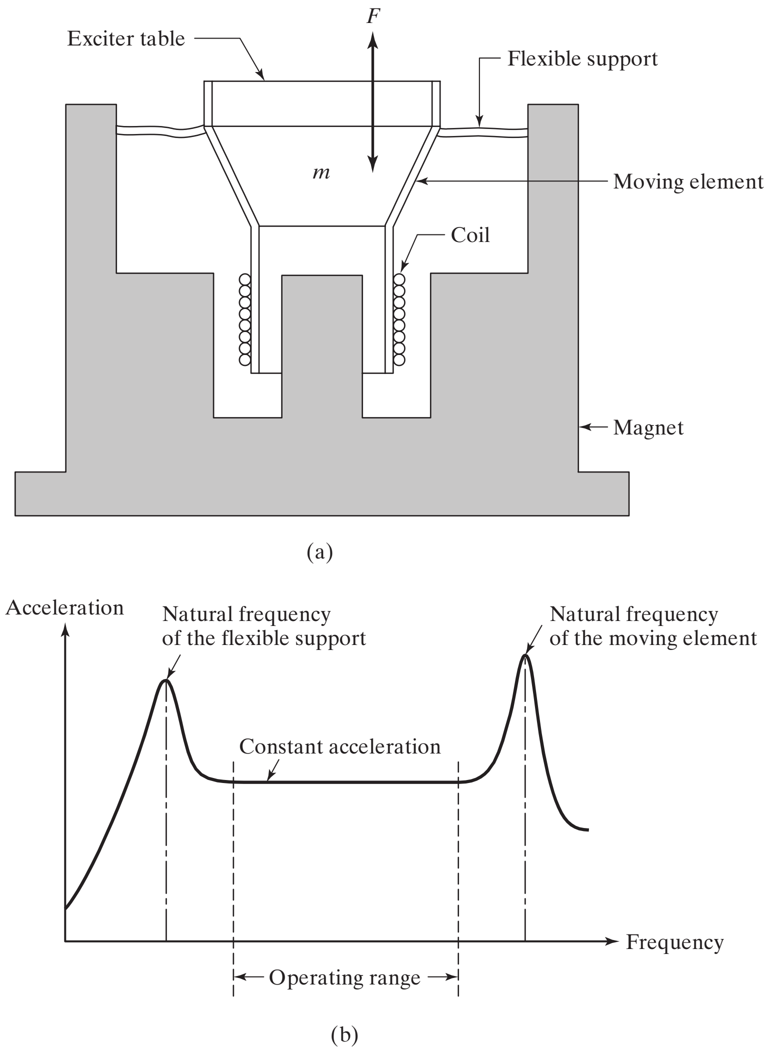


FIGURE 10.19 (a) Electrodynamic shaker. (b) Typical resonance characteristics of an electrodynamic exciter.

the component and the moving element of the shaker. If the current flowing through the coil varies harmonically with time (a.c. current), the force produced also varies harmonically. On the other hand, if direct current is used to energize the coil, a constant force is generated at the exciter table. The electrodynamic exciters can be used in conjunction with an inertia or a spring as in the case of Figs. 10.17(a) and (b) to vibrate a structure.

Since the coil and the moving element should have a linear motion, they are suspended from a flexible support (having a very small stiffness) as shown in Fig. 10.19(a). Thus the

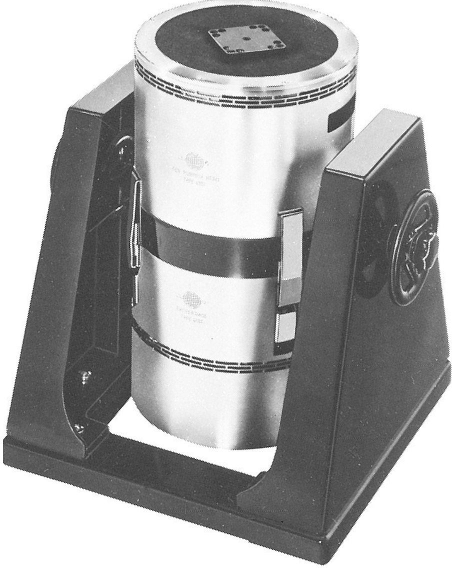


FIGURE 10.20 An exciter with a general-purpose head. (Courtesy of Bruel and Kjaer Instruments, Inc., Marlborough, MA.)

electromagnetic exciter has two natural frequencies—one corresponding to the natural frequency of the flexible support and the other corresponding to the natural frequency of the moving element, which can be made very large. These two resonant frequencies are shown in Fig. 10.19(b). The operating-frequency range of the exciter lies between these two resonant frequencies, as indicated in Fig. 10.19(b) [10.7].

The electrodynamic exciters are used to generate forces up to 30,000 N, displacements up to 25 mm, and frequencies in the range of 5 to 20 kHz [10.1]. A practical electrodynamic exciter is shown in Fig. 10.20.

10.6 Signal Analysis

In signal analysis, we determine the response of a system under a known excitation and present it in a convenient form. Often, the time response of a system will not give much useful information. However, the frequency response will show one or more discrete frequencies around which the energy is concentrated. Since the dynamic characteristics of individual components of the system are usually known, we can relate the distinct frequency components (of the frequency response) to specific components [10.3].

For example, the acceleration-time history of a machine frame that is subjected to excessive vibration might appear as shown in Fig. 10.21(a). This figure cannot be used to identify the cause of vibration. If the acceleration-time history is transformed to the

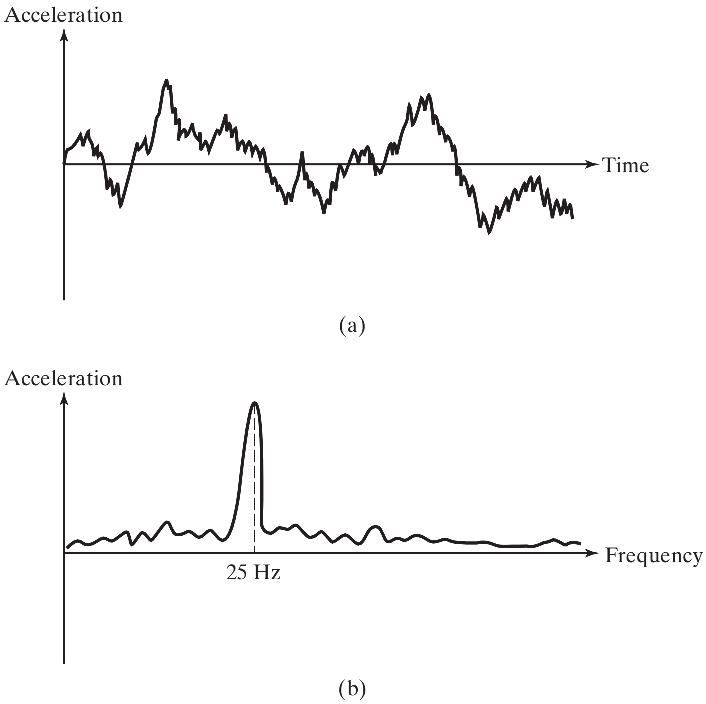


FIGURE 10.21 Acceleration history.

frequency domain, the resulting frequency spectrum might appear as shown in Fig. 10.21(b), where, for specificness, the energy is shown concentrated around 25 Hz. This frequency can easily be related, for example, to the rotational speed of a particular motor. Thus the acceleration spectrum shows a strong evidence that the motor might be the cause of vibration. If the motor is causing the excessive vibrations, changing either the motor or its speed of operation might avoid resonance and hence the problem of excessive vibrations.

10.6.1 Spectrum Analyzers

Spectrum or frequency analyzers can be used for signal analysis. These devices analyze a signal in the frequency domain by separating the energy of the signal into various frequency bands. The separation of signal energy into frequency bands is accomplished through a set of filters. The analyzers are usually classified according to the type of filter employed. For example, if an octave band filter is used, the spectrum analyzer is called an *octave band analyzer.*

In recent years, digital analyzers have become quite popular for real-time signal analysis. In a real-time frequency analysis, the signal is continuously analyzed over all the frequency bands. Thus the calculation process must not take more time than the time taken to collect the signal data. Real-time analyzers are especially useful for machinery health monitoring, since a change in the noise or vibration spectrum can be observed at the same time that change in the machine occurs. There are two types of real-time analysis procedures: the digital filtering method and the fast Fourier transform (FFT) method [10.13].

The digital filtering method is best suited for constant-percent bandwidth analysis, the FFT method for constant-bandwidth analysis. Before we consider the difference between those two approachers, we first discuss the basic component of a spectrum analyzer—namely, the bandpass filter.

10.6.2 Bandpass Filter

A bandpass filter is a circuit that permits the passage of frequency components of a signal over a frequency band and rejects all other frequency components of the signal. A filter can be built by using, for example, resistors, inductors, and capacitors. Figure 10.22 illustrates the response characteristics of a filter whose lower and upper cutoff frequencies are f_l and f_u, respectively. A practical filter will have a response characteristic deviating from the ideal rectangle, as shown by the full line in Fig. 10.22. For a good bandpass filter, the ripples within the band will be minimum and the slopes of the filter skirts will be steep to maintain the actual bandwidth close to the ideal value, $B = f_u - f_l$. For a practical filter, the frequencies f_l and f_u at which the response is 3 dB[4] below its mean bandpass response are called the cutoff frequencies.

There are two types of bandpass filters used in signal analysis: the constant-percent bandwidth filters and constant-bandwidth filters. For a constant-percent bandwidth filter, the ratio of the bandwidth to the center (tuned) frequency, $(f_u - f_l)/f_c$, is a constant. The octave,[5]

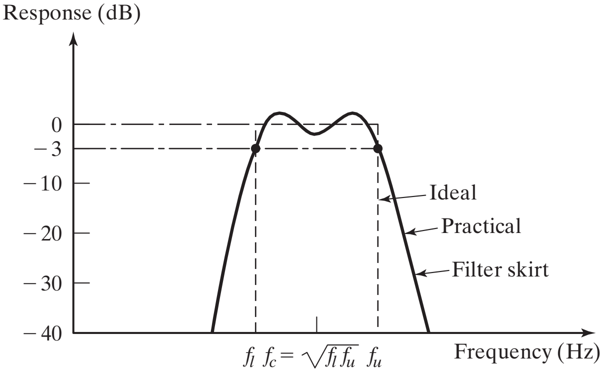


FIGURE 10.22 Response of a filter.

[4]A decibel (dB) of a quantity (such as power, P) is defined as

$$\text{Quantity in dB} = 10 \log_{10}\left(\frac{P}{P_{\text{ref}}}\right)$$

where P is the power and P_{ref} is a reference value of the power.

[5]An octave is the interval between any two frequencies $(f_2 - f_1)$, whose frequency ratio (f_2/f_1), is 2. Two frequencies f_1 and f_2 are said to be separated by a number of octaves N when

$$\frac{f_2}{f_1} = 2^N \qquad \text{or} \qquad N\,(\text{in octaves}) = \log_2\left(\frac{f_2}{f_1}\right)$$

where N can be an integer or a fraction. If $N = 1$, we have an octave; if $N = 1/3$, we get a one-third octave, and so on.

TABLE 10.1

Lower cutoff limit (Hz)	5.63	11.2	22.4	44.7	89.2	178	355	709	1410
Center frequency (Hz)	8.0	16.0	31.5	63.0	125	250	500	1000	2000
Upper cutoff limit (Hz)	11.2	22.4	44.7	89.2	178	355	709	1410	2820

one-half-octave, and one-third-octave band filters are examples of constant-percent bandwidth filters. Some of the cutoff limits and center frequencies of octave bands used in signal analysis are shown in Table 10.1. For a constant-bandwidth filter, the bandwidth, $f_u - f_l$, is independent of the center (tuned) frequency, f_c.

10.6.3 Constant-Percent Bandwidth and Constant-Bandwidth Analyzers

The primary difference between the constant-percent bandwidth and constant-bandwidth analyzers lies in the detail provided by the various bandwidths. The octave band filters, whose upper cutoff frequency is twice the lower cutoff frequency, give a less detailed (too coarse) analysis for practical vibration and noise encountered in machines. The one-half-octave band filter gives twice the information but requires twice the amount of time to obtain the data. A spectrum analyzer with a set of octave and one-third-octave filters can be used for noise (signal) analysis. Each filter is tuned to a different center frequency to cover the entire frequency range of interest. Since the lower cutoff frequency of a filter is equal to the upper cutoff frequency of the previous filter, the composite filter characteristic will appear as shown in Fig. 10.23. Figure 10.24 shows a real-time octave and fractional-octave digital frequency analyzer. A constant-bandwidth analyzer is used to obtain a more detailed analysis than in the case of a constant-percent bandwidth analyzer, especially in the high-frequency range of the signal. The constant-bandwidth filter, when used with a continuously varying center frequency, is called a wave or heterodyne analyzer. Heterodyne analyzers are available with constant filter bandwidths ranging from one to several hundred hertz. A practical heterodyne analyzer is shown in Fig. 10.25.

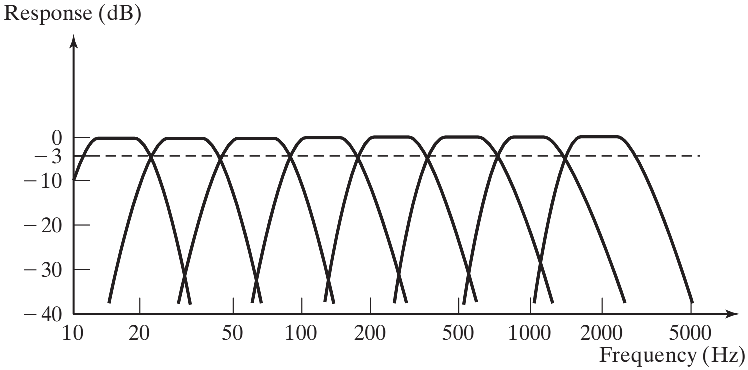


FIGURE 10.23 Response characteristic of a typical octave band filter set.

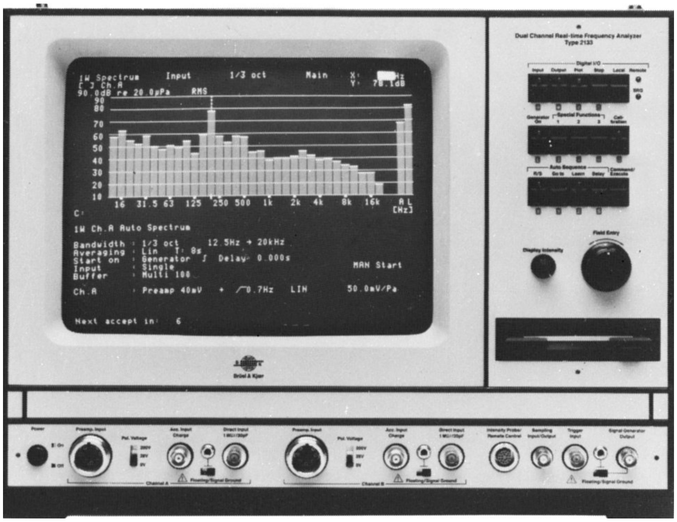


FIGURE 10.24 Octave and fractional-octave digital frequency analyzer. (Courtesy of Bruel and Kjaer Instruments, Inc., Marlborough, MA.)

FIGURE 10.25 Heterodyne analyzer. (Courtesy of Bruel and Kjaer Instruments, Inc., Marlborough, MA.)

10.7 Dynamic Testing of Machines and Structures

The dynamic testing of machines (structures) involves finding their deformation at a critical frequency. This can be done using the following two approaches [10.3].

10.7.1 Using Operational Deflection-Shape Measurements

In this method, the forced dynamic deflection shape is measured under the steady-state (operating) frequency of the system. For the measurement, an accelerometer is mounted at some point on the machine (structure) as a reference, and another moving accelerometer is placed at several other points, and in different directions, if necessary. Then the magnitudes and the phase differences between the moving and reference accelerometers at all the points under steady-state operation of the system are measured. By plotting these measurements, we can find how the various parts of the machine (structure) move relative to one another and also absolutely.

The deflection shape measured is valid only for the forces/frequency associated with the operating conditions; as such, we cannot get information about deflections under other forces and/or frequencies. However, the measured deflection shape can be quite useful. For example, if a particular part or location is found to have excessive deflection, we can stiffen that part or location. This, in effect, increases the natural frequency beyond the operational frequency range of the system.

10.7.2 Using Modal Testing

Since any dynamic response of a machine (structure) can be obtained as a combination of its modes, a knowledge of the mode shapes, modal frequencies, and modal damping ratios constitutes a complete dynamic description of the machine (structure). The experimental modal analysis procedure is described in the following section.

10.8 Experimental Modal Analysis

10.8.1 The Basic Idea

Experimental modal analysis, also known as *modal analysis* or *modal testing*, deals with the determination of natural frequencies, damping ratios, and mode shapes through vibration testing. Two basic ideas are involved:

1. When a structure, machine, or any system is excited, its response exhibits a sharp peak at resonance when the forcing frequency is equal to its natural frequency when damping is not large.
2. The phase of the response changes by 180° as the forcing frequency crosses the natural frequency of the structure or machine, and the phase will be 90° at resonance.

10.8.2 The Necessary Equipment

The measurement of vibration requires the following hardware:

1. An exciter or source of vibration to apply a known input force to the structure or machine.
2. A transducer to convert the physical motion of the structure or machine into an electrical signal.

3. A signal conditioning amplifier to make the transducer characteristics compatible with the input electronics of the digital data acquisition system.
4. An analyzer to perform the tasks of signal processing and modal analysis using suitable software.

Exciter. The exciter may be an electromagnetic shaker or an impact hammer. As explained in Section 10.5.2, the electromagnetic shaker can provide large input forces so that the response can be measured easily. Also the output of the shaker can be controlled easily if it is of electromagnetic type. The excitation signal is usually of a swept sinusoidal or a random type signal. In the swept sinusoidal input, a harmonic force of magnitude F is applied at a number of discrete frequencies over a specific frequency range of interest. At each discrete frequency, the structure or machine is made to reach a steady state before the magnitude and phase of the response are measured. If the shaker is attached to the structure or machine being tested, the mass of the shaker will influence the measured response (known as the *mass loading effect*). As such, care is to be taken to minimize the effect of the mass of the shaker. Usually the shaker is attached to the structure or machine through a short thin rod, called a *stringer,* to isolate the shaker, reduce the added mass, and apply the force to the structure or machine along the axial direction of the stringer. This permits the control of the direction of the force applied to the structure or machine.

The impact hammer is a hammer with a built-in force transducer in its head, as indicated in Examples 4.7 and 4.8. The impact hammer can be used to hit or impact the structure or machine being tested to excite a wide range of frequencies without causing the problem of mass loading. The impact force caused by the impact hammer, which is nearly proportional to the mass of the hammer head and the impact velocity, can be found from the force transducer embedded in the head of the hammer. As shown in Section 6.15, the response of the structure or machine to an impulse is composed of excitations at each of the natural frequencies of the structure or machine.

Although the impact hammer is simple, portable, inexpensive, and much faster to use than a shaker, it is often not capable of imparting sufficient energy to obtain adequate response signals in the frequency range of interest. It is also difficult to control the direction of the applied force with an impact hammer. A typical frequency response of a structure or machine obtained using an impact hammer is shown in Fig. 10.26. The shape of the frequency response is dependent on the mass and stiffness of both the hammer and the structure or machine. Usually, the useful range of frequency excitation is limited by a cutoff frequency, ω_c, which implies that the structure or machine did not receive sufficient energy to excite modes beyond ω_c. The value of ω_c is often taken as the frequency where the amplitude of the frequency response reduces by 10 to 20 dB from its maximum value.

Transducer. Among the transducers, the piezoelectric transducers are most popular (see Section 10.2.2). A piezoelectric transducer can be designed to produce signals proportional to either force or acceleration. In an accelerometer, the piezoelectric material acts as a stiff spring that causes the transducer to have a resonant or natural frequency. Usually, the maximum measurable frequency of an accelerometer is a fraction of its natural

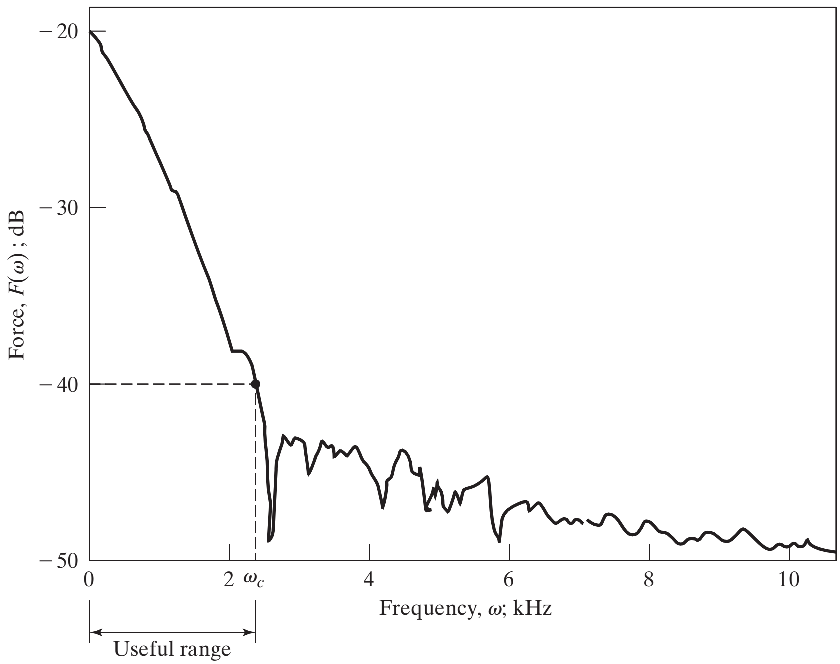


FIGURE 10.26 Frequency response of an impulse created by an impact hammer.

frequency. Strain gages can also be used to measure the vibration response of a structure or machine, as discussed in Section 10.2.1.

Signal Conditioner. Since the output impedance of transducers is not suitable for direct input into the signal analysis equipment, signal conditioners, in the form of charge or voltage amplifiers, are used to match and amplify the signals before signal analysis.

Analyzer. The response signal, after conditioning, is sent to an analyzer for signal processing. A type that is commonly used is the *fast Fourier transform (FFT) analyzer*. Such an analyzer receives analog voltage signals (representing displacement, velocity, acceleration, strain, or force) from a signal-conditioning amplifier, filter, and digitizer for computations. It computes the discrete frequency spectra of individual signals as well as cross-spectra between the input and the different output signals. The analyzed signals can be used to find the natural frequencies, damping ratios, and mode shapes in either numerical or graphical form.

The general arrangement for the experimental modal analysis of a structural or mechanical system is shown in Fig. 10.27. Note that all the equipment is to be calibrated before it is used. For example, an impact hammer is use more frequently in experimental stress analysis. The reason is that it is more convenient and faster to use than a shaker. An impact hammer consists of a force transducer or load cell built into the head (or tip) of the hammer. The built in force transducer is to be calibrated dynamically whenever the head or tip is changed. Similarly, the transducers, along with the signal conditioners, are to be calibrated with respect to magnitude and phase over the frequency range of interest.

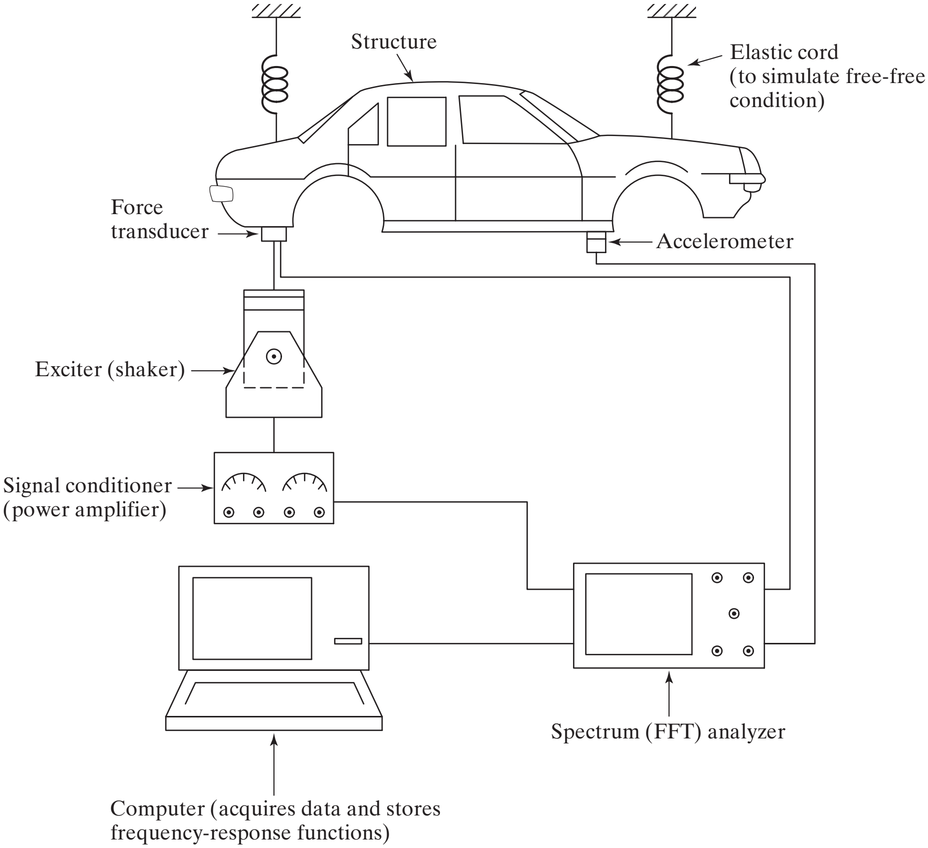


FIGURE 10.27 Experimental modal analysis.

10.8.3 Digital Signal Processing

The analyzer converts the analog time-domain signals, $x(t)$, into digital frequency-domain data using Fourier series relations, given by Eqs. (1.97) to (1.99), to facilitate digital computation. Thus the analyzer accepts the analog output signals of accelerometers or force transducers, $x(t)$, and computes the spectral coefficients of these signals a_0, a_n, and b_n using Eqs. (1.97) to (1.99) in the frequency domain. The process of converting analog signals into digital data is indicated in Fig. 10.28 for two representative signals. In Fig. 10.28, $x(t)$ denotes the analog signal and $x_i = x(t_i)$ represents the corresponding digital record, with t_i indicating the ith discrete value of time. This process is performed by an analog-to-digital (A/D) converter, which is part of a digital analyzer. If N samples of $x(t)$ are collected at discrete values of time, t_i, the data $[x_1(t_i), x_2(t_i), \ldots, x_N(t_i)]$ can be used to obtain the discrete form of Fourier transform as

$$x_j = x(t_j) = \frac{a_0}{2} + \sum_{i=1}^{N/2}\left(a_i \cos\frac{2\pi i t_j}{T} + b_i \sin\frac{2\pi i t_j}{T}\right); \qquad j = 1, 2, \ldots, N \quad (10.45)$$

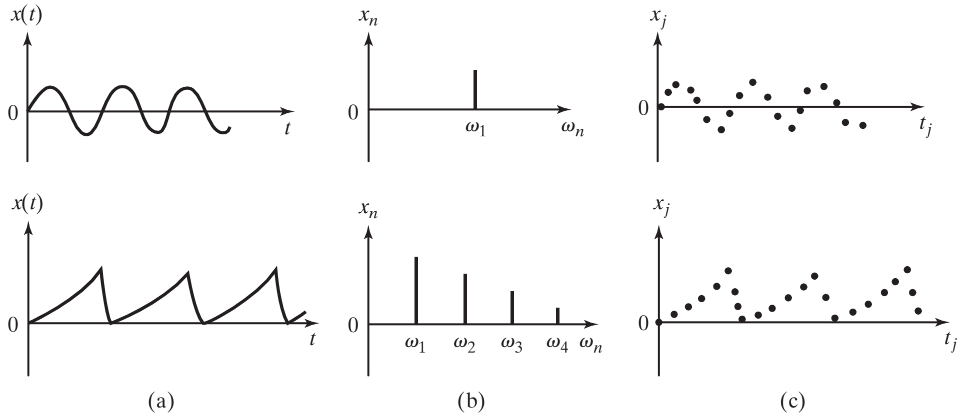


FIGURE 10.28 Representation of signals in different forms: (a) Signals in time domain. (b) Signals in frequency domain. (c) Digital records of $x(t)$.

where the digital spectral coefficients a_0, a_i, and b_i are given by (see Eqs. (1.97) to (1.99))

$$a_0 = \frac{1}{N}\sum_{j=1}^{N} x_j \tag{10.46}$$

$$a_i = \frac{1}{N}\sum_{j=1}^{N} x_j \cos\frac{2\pi i t_j}{N} \tag{10.47}$$

$$b_i = \frac{1}{N}\sum_{j=1}^{N} x_j \sin\frac{2\pi i t_j}{N} \tag{10.48}$$

with the number of samples N equal to some power of 2 (such as 256, 512, or 1024) which is fixed for a given analyzer. Equations (10.46) to (10.48) denote N algebraic equations for each of the N samples. The equations can be expressed in matrix form as

$$\vec{X} = [A]\vec{d} \tag{10.49}$$

where $\vec{X} = \{x_1\ x_2\ \cdots\ x_N\}^T$ is the vector of samples, $\vec{d} = \{a_0\ a_1\ a_2 \cdots a_{N/2}\ b_1\ b_2 \cdots b_{N/2}\}^T$ is the vector of spectral coefficients, and $[A]$ is the matrix composed of the coefficients $\cos\frac{2\pi i t_j}{T}$ and $\sin\frac{2\pi i t_j}{T}$ of Eqs. (10.46)–(10.48). The frequency content of the signal or response of the system can be determined from the solution

$$\vec{d} = [A]^{-1}\vec{X} \tag{10.50}$$

where $[A]^{-1}$ is computed efficiently using fast Fourier transform (FFT) by the analyzer.

10.8.4 Analysis of Random Signals

The input and output data measured by the transducers usually contain some random component or noise that makes it difficult to analyze the data in a deterministic manner. Also, in some cases random excitation is used in vibration testing. Thus random signal analysis becomes necessary in vibration testing. If $x(t)$ is a random signal, as shown in Fig. 10.29, its average or mean, denoted as $\overline{x}$, is defined as[6]

$$\overline{x}(t) = \lim_{T \to \infty} \frac{1}{N} \int_0^T x(t)\, dt \tag{10.51}$$

which, for a digital signal, can be expressed as

$$\overline{x} = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} x(t_j) \tag{10.52}$$

Corresponding to any random signal $y(t)$, we can always define a new variable $x(t)$ as $x(t) = y(t) - \overline{y}(t)$, so that the mean value of $x(t)$ is zero. Hence, without loss of generality, we can assume the signal $x(t)$ to have a zero mean and define the mean square value or variance of $x(t)$, denoted as $\overline{x}^2(t)$, as

$$\overline{x}^2(t) = \lim_{N \to \infty} \frac{1}{T} \int_0^T x^2(t)\, dt \tag{10.53}$$

which, for a digital signal with N samples of $x(t)$ at $t = t_1, t_2, \ldots, t_N$, can be expressed as

$$\overline{x}^2 = \lim_{n \to \infty} \frac{1}{N} \sum_{j=1}^{N} x^2(t_j) \tag{10.54}$$

The root mean square (RMS) value of $x(t)$ is given by

$$x_{\text{RMS}} = \sqrt{\overline{x}^2} \tag{10.55}$$

FIGURE 10.29 A random signal, $x(t)$.

[6]A detailed discussion of random signals (processes) and random vibration is given in Chapter 14.

The autocorrelation function of a random signal $x(t)$, denoted as $R(t)$, gives a measure of the speed with which the signal changes in the time domain and is defined as

$$R(t) = x^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(\tau) x(\tau + t) \, d\tau \tag{10.56}$$

which, for a digital signal, can be written as

$$R(n, \Delta t) = \frac{1}{N - n} \sum_{j=0}^{N-n} x_j x_{j+n} \tag{10.57}$$

where N is the number of samples, Δt is the sampling interval, and n is an adjustable parameter that can be used to control the number of points used in the computation. It can be seen that $R(0)$ denotes the mean square value, $\overline{x}^2$, of $x(t)$. The autocorrelation function can be used to identify the presence of periodic components present (buried) in a random signal. If $x(t)$ is purely random, then $R(t) \to 0$ as $T \to \infty$. However, if $x(t)$ is periodic or has a periodic component, then $R(t)$ will also be periodic.

The power spectral density (PSD) of a random signal $x(t)$, denoted as $S(\omega)$, gives a measure of the speed with which the signal changes in the frequency domain and is defined as the Fourier transform of $R(t)$:

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} \, d\tau \tag{10.58}$$

which, in digital form, can be expressed as

$$S(\Delta\omega) = \frac{|x(\omega)|^2}{N \, \Delta t} \tag{10.59}$$

where $|x(\omega)|^2$ represents the magnitude of the Fourier transform of the sampled data of $x(t)$. The definitions of autocorrelation and PSD functions can be extended for two different signals, such as a displacement signal $x(t)$ and an applied force signal $f(t)$. This leads to the cross-correlation function, $R_{xf}(t)$ and the cross-PSD $S_{xf}(\omega)$:

$$R_{xf}(t) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(\tau) f(\tau + t) \, d\tau \tag{10.60}$$

$$S_{xf}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xf}(\tau) e^{-i\omega\tau} \, d\tau \tag{10.61}$$

Equations (10.60) and (10.61) permit the determination of the transfer functions of the structure or machine being tested. In Eq. (10.60), if $f(\tau + t)$ is replaced by $x(\tau + t)$, we

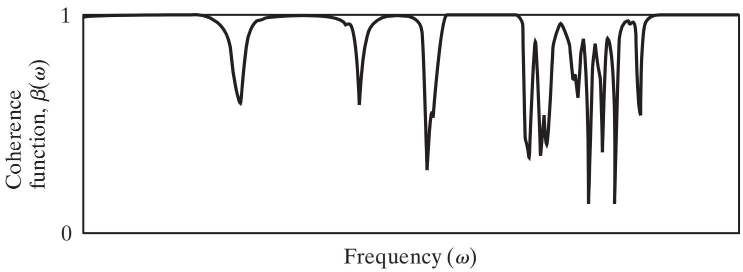


FIGURE 10.30 A typical coherence function.

obtain $R_{xx}(t)$, which when used in Eq. (10.61), leads to $S_{xx}(\omega)$. The frequency-response function, $H(i\omega)$, is related to the PSD functions as

$$S_{xx}(\omega) = |H(i\omega)|^2 S_{ff}(\omega) \tag{10.62}$$

$$S_{fx}(\omega) = H(i\omega) S_{ff}(\omega) \tag{10.63}$$

$$S_{xx}(\omega) = H(i\omega) S_{xf}(\omega) \tag{10.64}$$

with $f(t)$ and $x(t)$ denoting the random force input and the resulting output response, respectively. $S_{xx}(\omega)$, given by Eq. (10.62), contains information about the magnitude of the transfer function of the system (structure or machine), while $S_{xf}(\omega)$ and $S_{xx}(\omega)$, given by Eqs. (10.63) and (10.64), contain information about both magnitude and phase. In vibration testing, the spectrum analyzer first computes different spectral density functions from the transducer output, and then computes the frequency-response functuion $H(i\omega)$ of the system using Eqs. (10.63) and (10.64).

Coherence Function. A function, known as *coherence function* (β), is defined as a measure of the noise present in the signals as

$$\beta(\omega) = \left(\frac{S_{fx}(\omega)}{S_{ff}(\omega)}\right)\left(\frac{S_{xf}(\omega)}{S_{xx}(\omega)}\right) = \frac{|S_{xf}(\omega)|^2}{S_{xx}(\omega)\,S_{ff}(\omega)} \tag{10.65}$$

Note that if the measurements of x and f are pure noises, then $\beta = 0$, and if the measurements of x and f are not contaminated at all with noise, then $\beta = 1$. The plot of a typical coherence function is shown in Fig. 10.30. Usually, $\beta \approx 1$ near the natural frequency of the system because the signals are large and are less influenced by the noise.

10.8.5 Determination of Modal Data from Observed Peaks

The frequency-response function, $H(i\omega)$, computed from Eq. (10.63) or (10.64), can be used to find the natural frequencies, damping ratios, and mode shapes corresponding to all resonant peaks observed in the plot of $H(i\omega)$. Let the graph of the frequency-response function be as shown in Fig. 10.31, with its four peaks or resonances suggesting that the system being tested can be modeled as a four-degree-of-freedom system. Sometimes it

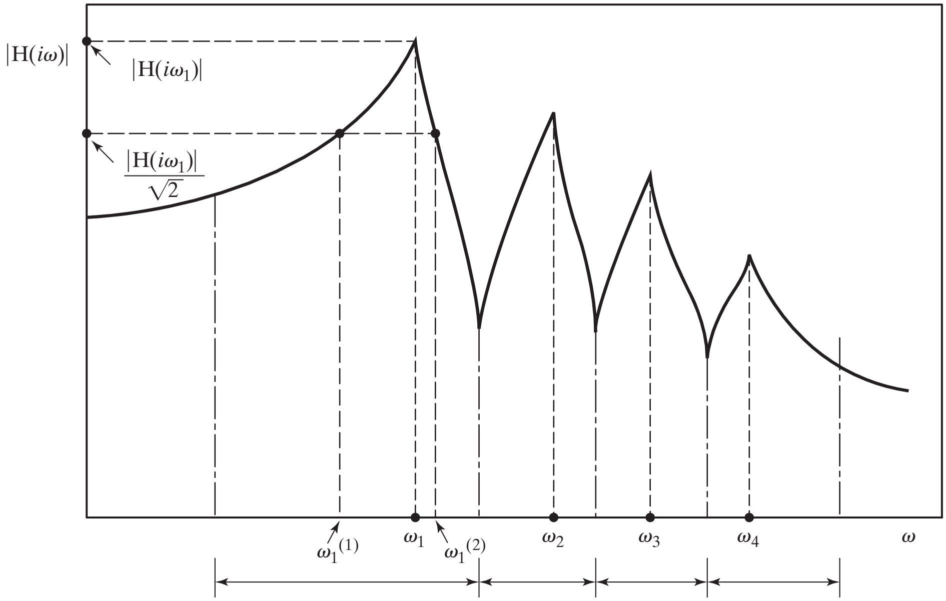


FIGURE 10.31 A typical graph of the frequency-response function of a structure or machine, obtained using Eq. (10.63) or (10.64).

becomes difficult to assign the number of degrees of freedom to the system, especially when the resonant peaks are closely spaced in the graph of $H(i\omega)$, which can be plotted by applying a harmonic force of adjustable frequency at a specific point of the structure or machine, measuring the response (for example, displacement) at another point, and finding the value of the frequency-response function using Eq. (10.63) or (10.64). The graph of $H(i\omega)$, similar to Fig. 10.31, can be plotted by finding the values of $H(i\omega)$ at a number of frequencies of the applied harmonic force.

A simple method of finding the modal data involves the use of a single-degree-of-freedom approach. In this method, the graph of $H(i\omega)$ is partitioned into several frequency ranges with each range bracketing one peak, as shown in Fig. 10.31. Each partitioned frequency range is then considered as the frequency-response function of a single-degree-of-freedom system. This implies that the frequency-response function in each frequency range is dominated by that specific single mode. As observed in Section 3.4, a peak denotes a resonance point corresponding to a phase angle of 90°. Thus the resonant frequencies can be identified as the peaks in the graph of $H(i\omega)$, which can be confirmed from an observation of the values of the phase angle to be 90° at each of the peaks. The damping ratio corresponding to peak j, with resonant frequency ω_j, in Fig. 10.31 denotes the modal damping ratio, ζ_j. This ratio can be found, using Eq. (3.45), as

$$\zeta_j = \frac{\omega_j^{(2)} - \omega_j^{(1)}}{2\,\omega_j} \tag{10.66}$$

where $\omega_j^{(1)}$ and $\omega_j^{(2)}$, known as half-power points, lie on either side of the resonant frequency ω_j and satisfy the relation

$$|H(i\omega_j^{(1)})| = |H(i\omega_j^{(2)})| = \frac{|H(i\omega_j)|}{\sqrt{2}} \tag{10.67}$$

Note that ω_j actually represents the damped natural frequency of the system being tested. However, when damping is small, ω_j can be considered approximately equal to the undamped natural frequency of the system. When the system being tested is approximated as a k-degree-of-freedom system ($k = 4$ for the system corresponding to Fig. 10.31), each peak observed in the graph of $H(i\omega)$ is assumed to be a single-degree-of-freedom system, and the k resonant frequencies (peaks) and the corresponding damping ratios are determined by repeating the above procedure (and using an application of Eq. (10.66)) k times.

EXAMPLE 10.5 Determination of Damping Ratio from Bode Diagram

The graphs showing the variations of the magnitude of the response and its phase angle with the frequency of a single-degree-of-freedom system, as indicated in Fig. 3.11, provides the frequency response of the system. Instead of dealing with the magnitude curves directly, if the logarithms of the magnitude ratios (in decibels) are used, the resulting plots are called Bode diagrams. Find the natural frequency and the damping ratio of a system whose Bode diagram is shown in Fig. 10.32.

Solution: The natural frequency, which corresponds approximately to the peak response of the system, can be seen to be 10 Hz and the peak response to be −35 dB. The half-power points

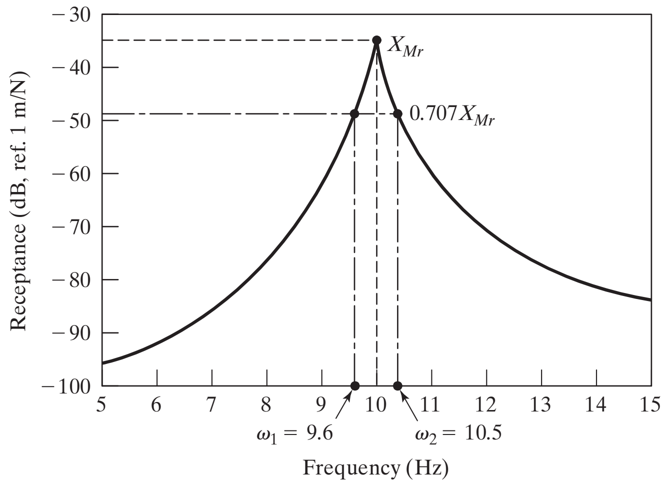


FIGURE 10.32 Bode diagram.

correspond to frequencies ω_1 and ω_2, where the response amplitude is equal to 0.707 times the peak response. From Fig. 10.32, the half-power points can be identified as $\omega_1 = 9.6$ Hz and $\omega_2 = 10.5$ Hz; thus the damping ratio can be determined by using Eq. (10.66) as

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n} = \frac{10.5 - 9.6}{2(10.0)} = 0.045$$

■

The procedure described in this section for finding the modal parameters is basically a visual approach. A more systematic, computer-based approach that can be implemented by the analyzer in conjunction with suitable programming is presented in the next section.

10.8.6 Determination of Modal Data from Nyquist Plot

According to in this method, a single mode is also assumed to dominate in the neighborhood of its natural frequency in the frequency-response function. When the real and imaginary parts of the frequency-response function of a single-degree-of-freedom system (given by Eq. (3.54)) are plotted along the horizontal and vertical axes of a graph for a range of frequencies, the resulting graph will be in the form of a circle, known as the *Nyquist circle* or *Nyquist plot*. The frequency-response function, given by Eq. (3.54), can be written as

$$\alpha(i\omega) = \frac{1}{1 - r^2 + i2\zeta r} = u + iv \tag{10.68}$$

where

$$r = \frac{\omega}{\omega_n} \tag{10.69}$$

$$u = \text{Real part of } \alpha(i\omega) = \frac{1 - r^2}{(1 - r^2)^2 + 4\zeta^2 r^2} \tag{10.70}$$

$$v = \text{Imaginary part of } \alpha(i\omega) = \frac{-2\zeta r}{(1 - r^2)^2 + 4\zeta^2 r^2} \tag{10.71}$$

During vibration testing, the analyzer has the driving frequency values ω and the corresponding computed values of $u = \text{Re}(\alpha)$ and $v = \text{Im}(\alpha)$ from the measured data. The graph between u and v resembles a circle for large values of damping (ζ), while it increasingly assumes the shape of a circle as the damping becomes smaller and smaller, as shown in Fig. 10.33.

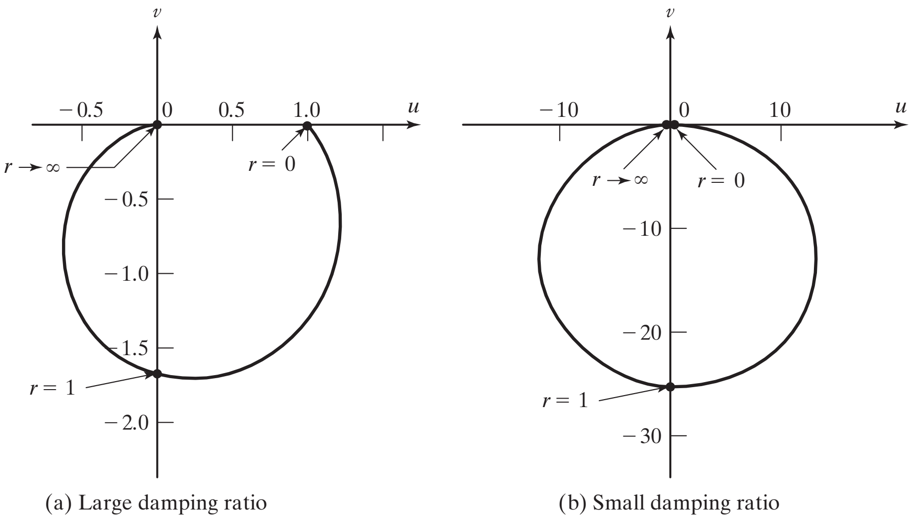


FIGURE 10.33 Nyquist circle.

Properties of Nyquist Circle. To identify the properties of the Nyquist circle, we first observe that large values of u and v are attained in the vicinity of resonance, $r = 1$. In that region, we can replace $1 - r^2$ in Eqs. (10.70) and (10.71) as

$$1 - r^2 = (1 + r)(1 - r) \approx 2(1 - r) \qquad \text{and} \qquad 2\zeta r \approx 2\zeta$$

so that

$$u = \text{Re}(\alpha) \approx \frac{1 - r}{2[(1 - r)^2 + \zeta^2]} \tag{10.72}$$

$$v = \text{Im}(\alpha) \approx \frac{-\zeta}{2[(1 - r)^2 + \zeta^2]} \tag{10.73}$$

It can be easily verified that u and v, given by Eqs. (10.72) and (10.73), satisfy the relation

$$u^2 + \left(v + \frac{1}{4\zeta}\right)^2 = \left(\frac{1}{4\zeta}\right)^2 \tag{10.74}$$

which denotes the equation of a circle with its center at $\left(u = 0, v = -\frac{1}{4\zeta}\right)$ and radius $\frac{1}{4\zeta}$. The half-power points occur at $r = 1 \pm \zeta$, which correspond to $u = \pm\frac{1}{4\zeta}$ and $v = \frac{1}{4\zeta}$. These points are located at the two ends of the horizontal diameter of the circle, at which point u has its maximum magnitude.

These observations can be used to find $\omega_n\,(r = 1)$ and ζ. Once the measured values of the frequency-response function $H(i\omega)$ are available (with the applied force magnitude fixed) for a range of driving frequencies ω, instead of searching for the peak in the plot of $H(i\omega)$ versus ω, we can construct the Nyquist plot of Re $(H(i\omega))$ against Im $(H(i\omega))$ by using a least squares approach to fit a circle. This process also averages out the experimental errors. The intersection of the fitted circle with the negative imaginary axis will then correspond to $H(i\omega_n)$. The bandwidth, $(\omega^{(2)} - \omega^{(1)})$, is given by the difference of the frequencies at the two horizontal diametral points, from which ζ can be found as $\zeta = \left(\dfrac{\omega^{(2)} - \omega^{(1)}}{2\omega_n}\right)$.

10.8.7 Measurement of Mode Shapes

To determine the mode shapes from vibration testing, we need to express the equations of motion of the multidegree-of-freedom system in modal coordinates [10.18]. For this, we first consider an undamped system.

Undamped Multidegree-of-Freedom System. The equations of motion of an undamped multidegree-of-freedom system in physical coordinates are given by

$$[m]\ddot{\vec{x}} + [k]\vec{x} = \vec{f} \tag{10.75}$$

For free harmonic vibration, Eq. (10.75) becomes

$$\left[[k] - \omega_i^2[m]\right]\vec{y}_i = \vec{0} \tag{10.76}$$

where ω_i is the ith natural frequency and $\vec{y}_i$ is the corresponding mode shape. The orthogonality relations for the mode shapes can be expressed as

$$[Y]^T[m][Y] = \text{diag}[M] \equiv \left[{}^{\nwarrow}M_{i\searrow}\right] \tag{10.77}$$

$$[Y]^T[k][Y] = \text{diag}[K] \equiv \left[{}^{\nwarrow}K_{i\searrow}\right] \tag{10.78}$$

where $[Y]$ is the modal matrix containing the modes $\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_N$ as columns (N denotes the number of degrees of freedom of the system, also equal to the number of measured natural frequencies or peaks), M_i and K_i are the elements of diag $[M]$ and diag $[K]$, also called the *modal mass* and *modal stiffness,* respectively, corresponding to mode i, and

$$\omega_i^2 = \frac{K_i}{M_i} \tag{10.79}$$

When the forcing functions are harmonic, $\vec{f}(t) = \vec{F}e^{\tilde{i}\omega t}$, with $\tilde{i} = \sqrt{-1}$, Eq. (10.75) yields

$$\vec{x}(t) = \vec{X}e^{\tilde{i}\omega t} = \left[[k] - \omega^2[m]\right]^{-1}\vec{F}e^{\tilde{i}\omega t} \equiv [\alpha(\omega)]\vec{F}e^{\tilde{i}\omega t} \tag{10.80}$$

where $[\alpha(\omega)]$ is called the *frequency-response function* or *receptance matrix* of the system. Using the orthogonality relations of Eqs. (10.77) and (10.78), $[\alpha(\omega)]$ can be expressed as

$$[\alpha(\omega)] = [Y]\Big[[K] - \omega^2[M]\Big]^{-1}[Y]^T \tag{10.81}$$

An individual element of the matrix $[\alpha(\omega)]$ lying in row p and column q denotes the harmonic response of one coordinate, X_p, caused by a harmonic force applied at another coordinate, F_q (with no other forces), and can be written as

$$\alpha_{pq}(\omega) = [\alpha(\omega)]_{pq} = \left.\frac{X_p}{F_q}\right|_{\text{with} \quad F_j=0;\, j=1,\,2,\,\ldots,\,N;\; j\neq q}$$

$$= \sum_{i=1}^{N} \frac{\left(\vec{y}_i\right)_p \left(\vec{y}_i\right)_q}{K_i - \omega^2 M_i} \tag{10.82}$$

where $(\vec{y}_i)_j$ denotes the jth component of mode $\vec{y}_i$. If the modal matrix $[Y]$ is further normalized (rescaled or mass-normalized) as

$$[\Phi] \equiv \left[\vec{\phi}_1\, \vec{\phi}_2 \cdots \vec{\phi}_N\right] = [Y][M]^{-1/2} \tag{10.83}$$

the shape of the modes $\vec{\phi}_1, \vec{\phi}_2, \ldots, \vec{\phi}_N$ will not change, but Eq. (10.82) becomes

$$\alpha_{pq}(\omega) = \sum_{i=1}^{N} \frac{\left(\vec{\phi}_i\right)_p \left(\vec{\phi}_i\right)_q}{\omega_i^2 - \omega^2} \tag{10.84}$$

Damped Multidegree-of-Freedom System. The equations of motion of a damped multidegree-of-freedom system in physical coordinates are given by

$$[m]\ddot{\vec{x}} + [c]\dot{\vec{x}} + [k]\vec{x} = \vec{f} \tag{10.85}$$

For simplicity, we assume proportional damping, so that the damping matrix $[c]$ can be expressed as

$$[c] = a[k] + b[m] \tag{10.86}$$

where a and b are constants. Then the undamped mode shapes of the system, $\vec{y}_i$ and $\vec{\phi}_i$, diagonalize not only the mass and stiffness matrices, as indicated in Eqs. (10.77) and (10.78), but also the damping matrix:

$$[Y]^T[c][Y] = \text{diag}\,[C] = \left[\nwarrow C_i \searrow\right] \tag{10.87}$$

Thus the mode shapes of the damped system will remain the same as those of the undamped system, but the natural frequencies will change and in general become complex. When the forcing vector $\vec{f}$ is assumed to be harmonic in Eq. (10.85), the frequency-response function or receptance can be derived as

$$\alpha_{pq}(\omega) = [\alpha(\omega)]_{pq} = \sum_{i=1}^{N} \frac{(\vec{y}_i)_p (\vec{y}_i)_q}{K_i - \omega^2 M_i + \tilde{i}\omega C_i} \tag{10.88}$$

When mass-normalized mode shapes are used (see Eq. 10.83), $\alpha_{pq}(\omega)$ becomes

$$\alpha_{pq}(\omega) = \sum_{i=1}^{N} \frac{(\vec{\phi}_i)_p (\vec{\phi}_i)_q}{\omega_i^2 - \omega^2 + 2\tilde{i}\zeta_i \omega_i \omega} \tag{10.89}$$

where ζ_i is the damping ratio in mode i.

As indicated earlier, the element of the matrix $[\alpha(\omega)]$ in row p and column q, $\alpha_{pq}(\omega) = [\alpha(\omega)]_{pq}$, denotes the transfer function between the displacement or response at point $p(X_p)$ and the input force at point $q(F_q)$ of the system being tested (with all other forces equal to zero). Since this transfer function denotes the ratio $\frac{X_p}{F_q}$, it is given by $H_{pq}(\omega)$. Thus

$$\alpha_{pq}(\omega) = H_{pq}(\omega) \tag{10.90}$$

If the peaks or resonant (natural) frequencies of the system are well separated, then the term corresponding to the particular peak (ith peak) dominates all other terms in the summation of Eq. (10.88) or (10.89). By substituting $\omega = \omega_i$ in Eq. (10.89), we obtain

$$\alpha_{pq}(\omega_i) = H_{pq}(\omega_i) = \frac{(\vec{\phi}_i)_p (\vec{\phi}_i)_q}{\omega_i^2 - \omega_i^2 + \tilde{i}2\zeta_i \omega_i^2}$$

or

$$|\alpha_{pq}(\omega_i)| = |H_{pq}(\omega_i)| = \frac{|(\vec{\phi}_i)_p (\vec{\phi}_i)_q|}{2\zeta_i \omega_i^2}$$

or

$$|(\vec{\phi}_i)_p (\vec{\phi}_i)_q| = 2\zeta_i \omega_i^2 |H_{pq}(\omega_i)| \tag{10.91}$$

It can be seen that Eq. (10.91) permits the computation of the absolute value of $(\vec{\phi}_i)_p (\vec{\phi}_i)_q$ using the measured values of the natural frequency (ω_i), damping ratio (ζ_i), and the transfer function $|H_{pq}(\omega_i)|$ at peak i. To determine the sign of the element $(\vec{\phi}_i)_p (\vec{\phi}_i)_q$, the phase plot of $H_{pq}(\omega_i)$ can be used. Since there are only N independent unknown components of $\vec{\phi}_i$ in the N^2 elements of the matrix $\left[(\vec{\phi}_i)_p(\vec{\phi}_i)_q\right] = \left[\vec{\phi}_i\,\vec{\phi}_i^{\mathrm{T}}\right]_{pq}$, N measurements of $|H_{pq}(\omega_i)|$ are required to determine the mode shape $\vec{\phi}_i$ corresponding to the modal frequency ω_i. This can be achieved by measuring the displacement or response of the system at point q with input at point 1 first, at point 2 next, ..., and at point N last.

10.9 Machine-Condition Monitoring and Diagnosis

Most machines produce low levels of vibration when designed properly. During operation, all machines are subjected to fatigue, wear, deformation, and foundation settlement. These effects cause an increase in the clearances between mating parts, misalignments in shafts, initiation of cracks in parts, and unbalances in rotors—all leading to an increase in the level of vibration, which causes additional dynamic loads on bearings. As time progresses, the vibration levels continue to increase, leading ultimately to the failure or breakdown of the machine. The common types of faults or operating conditions that lead to increased levels of vibration in machines include bent shafts, eccentric shafts, misaligned components, unbalanced components, faulty bearings, faulty gears, impellers with faulty blades, and loose mechanical parts.

10.9.1 Vibration Severity Criteria

The vibration severity charts, given by standards such as ISO 2372, can be used as a guide to determine the condition of a machine. In most cases, the root mean square (RMS) value of the vibratory velocity of the machine is compared against the criteria set by the standards. Although it is very simple to implement this procedure, the overall velocity signal used for comparison may not give sufficient warning of the imminent damage of the machine.

10.9.2 Machine Maintenance Techniques

The life of a machine follows the classic *bathtub curve* shown in Fig. 10.34. Since the failure of a machine is usually characterized by an increase in vibration and/or noise level, the vibration level also follows the shape of the same bathtub curve. The vibration level decreases during the initial running-in period, then increases very slowly during the normal operating period due to the normal wear, and finally increases rapidly due to excessive wear until failure or breakdown in the wearout period.

Three types of maintenance schemes can be used in practice:

1. *Breakdown maintenance.* The machine is allowed to fail, at which time the failed machine is replaced by a new one. This strategy can be used if the machine is inexpensive to replace and the breakdown does not cause any other damage. Otherwise, the cost of lost production, safety risks, and additional damage to other machines make this scheme unacceptable.
2. *Preventive maintenance.* Maintenance is performed at fixed intervals such as every 3000 operating hours or once a year. The maintenance intervals are usually determined

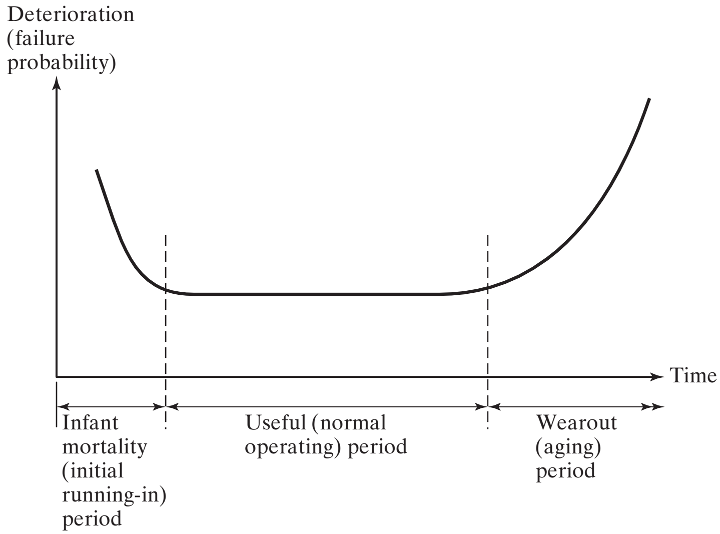


FIGURE 10.34 The bathtub curve for the life of a machine.

statistically from past experience. Although this method reduces the chance of unexpected breakdowns, it has been found to be uneconomical. The stoppage for maintenance involves not only lost production time but also a high risk of introducing imperfections due to human error. In addition, the probability of failure of a machine component cannot be reduced by replacing it with a new one during the normal wearout period.

3. *Condition-based maintenance.* The fixed-interval overhauls are replaced by fixed-interval measurements that permit the observation of changes in the running condition of the machine regularly. Thus the onset of fault conditions can be detected and their developments closely followed. The measured vibration levels can be extrapolated in order to predict when the vibration levels reach unacceptable values and when the machine must be serviced. Hence this scheme is also known as predictive maintenance. In this method, the maintenance costs are greatly reduced due to fewer catastrophic failures, better utilization of spare parts, and elimination of the unnecessary preventive maintenance. The vibration level (and hence the failure probability) of the machine due to condition-based maintenance follows the shape indicated in Fig. 10.35.

10.9.3 Machine-Condition Monitoring Techniques

Several methods can be used to monitor the condition of a machine, as indicated in Fig. 10.36. Aural and visual methods are the basic forms of monitoring techniques in which a skilled technician, having an intimate knowledge of machines, can identify a failure simply by listening to the sounds and/or visually observing the large amplitudes of vibration produced by a damaged machine. Sometimes a microphone or a stroboscope is used to hear the machine noise. Similarly, devices ranging from magnifying glasses to

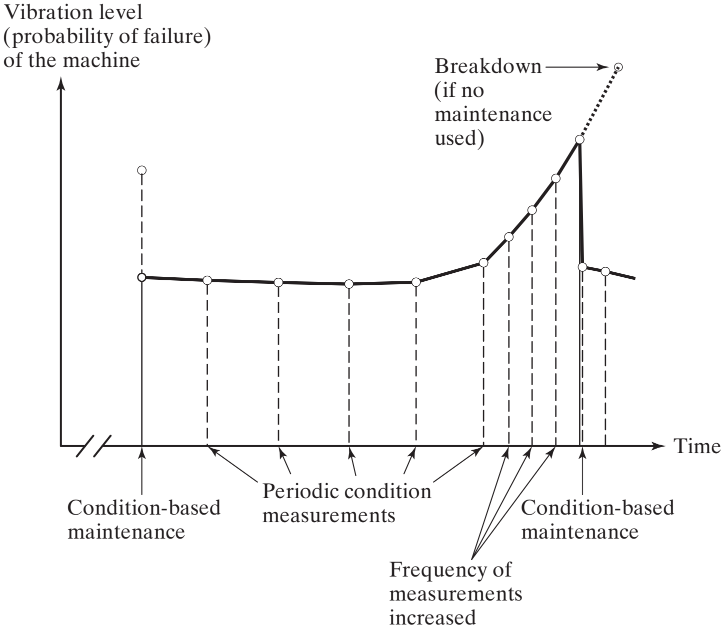


FIGURE 10.35 Condition-based maintenance.

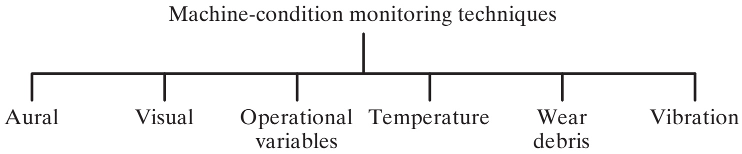


FIGURE 10.36 Machine-condition monitoring techniques.

stroboscopes are used to visually monitor the condition of a machine. Current and voltage monitoring can be used for the condition monitoring of electrical drives such as large generators and motors.

In the operational-variables method of monitoring, also known as performance or duty-cycle monitoring, the performance of a machine is observed with regard to its intended duty. Any deviation from the intended performance denotes a malfunction of the machine. Temperature monitoring involves measuring the operational or surface temperature of a machine. This method can be considered as a kind of operational-variables method. A rapid increase in the temperature of a component, occurring mostly due to wear, is an indication of a malfunction such as inadequate lubricant in journal bearings. Temperature monitoring uses such devices as optical pyrometers, thermocouples, thermography, and resistance thermometers. In some cases, dye penetrants are used to identify cracks occurring

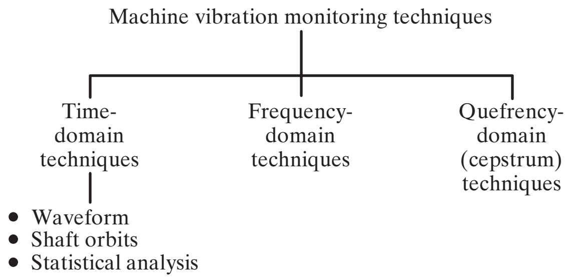


FIGURE 10.37 Machine vibration monitoring techniques.

on the surface of a machine. This procedure requires the use of heat-sensitive paints, known as thermographic paints, to detect surface cracks on hot surfaces. In such cases, the most suitable paint matching the expected surface temperature is selected.

Wear debris is generated at relative moving surfaces of load-bearing machine elements. The wear particles that can be found in the lubricating oils or grease can be used to assess the extent of damage. As wear increases, the particles of the material used to construct machine components such as bearings and gears can be found in increasing concentration. Thus the severity of the wear can be assessed by observing the concentration (quantity), size, shape, and color of the particles. Note that the color of the particles indicates how hot they have been.

Vibration analysis is most commonly used for machine-condition monitoring. Vibration in machines is caused by cyclic excitation forces arising from imbalances, wear, or failure of parts. What type of changes occur in the vibration level, how these changes can be detected, and how the condition of the machine is interpreted has been the topic of several research studies in the past. The available vibration monitoring techniques can be classified as shown in Fig. 10.37. These techniques are described in the following section.

10.9.4 Vibration Monitoring Techniques

Time-Domain Analysis

Time Waveforms. Time-domain analysis uses the time history of the signal (waveform). The signal is stored in an oscilloscope or a real-time analyzer and any nonsteady or transient impulses are noted. Discrete damages such as broken teeth in gears and cracks in inner or outer races of bearings can be identified easily from the waveform of the casing of a gearbox. As an example, Fig. 10.38 shows the acceleration signal of a single-stage gearbox. The pinion of the gear pair is coupled to a 5.6-kW, 2865-rpm, AC electric motor. Since the pinion (shaft) speed is 2865 rpm or 47.75 Hz, the period can be noted as 20.9 ms. The acceleration waveform indicates that pulses occur periodically with a period of 20 ms approximately. Noting that this period is the same as the period of the pinion, the origin of the pulses in the acceleration signal can be attributed to a broken gear tooth on the pinion.

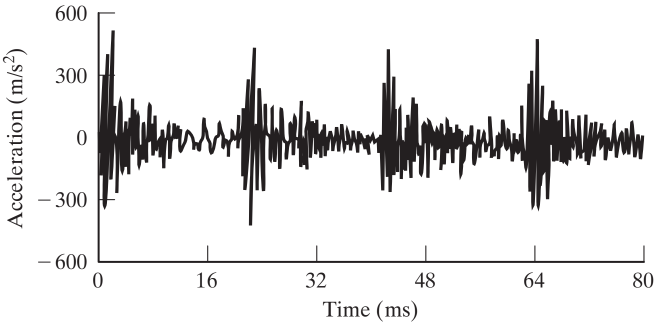


FIGURE 10.38 Time-domain waveform of a faulty gearbox [10.23].

Indices. In some cases, indices such as the peak level, the root mean square (RMS) level, and the crest factor are used to identify damage in machine-condition monitoring. Since the peak level occurs only once, it is not a statistical quantity and hence is not a reliable index to detect damage in continuously operating systems. Although the RMS value is a better index to detect damage in steady-state applications, it may not be useful if the signal contains information from more than one component, as in the case of vibration of a complete gearbox that consists of several gears, shafts, and bearings. The crest factor, defined as the ratio of the peak to RMS level, includes information from both the peak and the RMS levels. However, it may also not be able to identify failure in certain cases. For example, if the failure occurs progressively, the RMS level of the signal might be increasing gradually, although the crest factor might be showing a decreasing trend.

Orbits. Sometimes, certain patterns known as Lissajous figures can be obtained by displaying time waveforms obtained from two transducers whose outputs are shifted by 90° in phase. Any change in the pattern of these figures or orbits can be used to identify faults such as misalignment in shafts, unbalance in shafts, shaft rub, wear in journal bearings, and hydrodynamic instability in lubricated bearings. Figure 10.39 illustrates a change in orbit caused by a worn bearing. The enlarged orbit diameter in the vertical direction indicates that the bearing has become stiffer in the horizontal direction—that is, it has more bearing clearance in the vertical direction.

Statistical Methods

Probability Density Curve. Every vibration signal will have a characteristic shape for its probability density curve. The probability density of a signal can be defined as the probability of finding its instantaneous amplitude within a certain range, divided by the range. Usually, the waveform corresponding to good components will have a bell-shaped probability density curve similar to normal distribution. Thus any significant deviation from the bell shape can be associated with the failure of a component. Since the use of the

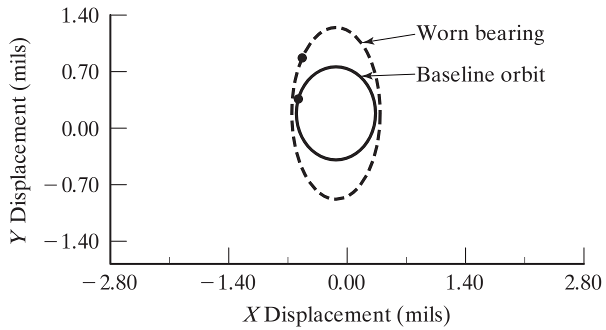


FIGURE 10.39 Change in orbit due to a bearing failure [10.23].

probability density curve involves the comparison of variations in shape rather than variations in amplitudes, it is very useful in the diagnosis of faults in machines.

Moments. In some cases, the moments of the probability density curve can be used for the machine-condition monitoring. The moments of the curve are similar to mechanical moments about the centroidal axis of the area. The first four moments of a probability density curve (with proper normalization) are known as the mean, standard deviation, skewness, and kurtosis, respectively. For practical signals, the odd moments are usually close to zero and the even moments denote the impulsiveness of the signal. The fourth-order moment, kurtosis, is commonly used in machine-condition monitoring. The kurtosis is defined as

$$k = \frac{1}{\sigma^4} \int_{-\infty}^{\infty} (x - \overline{x})^4 f(x)\, dx \tag{10.92}$$

where $f(x)$ is the probability density function of the instantaneous amplitude, $x(t)$, at time t, $\overline{x}$ is the mean value, and σ is the standard deviation of $x(t)$. Faults such as cracked races and spalling of rollers and balls in bearings cause relatively large pulses in the time-domain waveform of the signal, which in turn lead to large values of kurtosis. Thus an increase in the value of kurtosis can be attributed to the failure of a machine component.

Frequency-Domain Analysis

Frequency Spectrum. The frequency-domain signal or frequency spectrum is a plot of the amplitude of vibration response versus the frequency and can be derived by using the digital fast Fourier analysis of the time waveform. The frequency spectrum provides valuable information about the condition of a machine. The vibration response of a machine is governed not only by its components but also by its assembly, mounting, and installation. Thus the vibration characteristics of any machine are somewhat unique to that particular machine; hence the vibration spectrum can be considered as the vibration signature of that machine. As long as the excitation forces are constant or vary by small amounts, the measured vibration level of the machine also remains constant or varies by small amounts. However, as the machine starts developing faults, its vibration level and hence the shape of the frequency spectrum change. By comparing the frequency spectrum of the machine in damaged condition with the reference frequency spectrum corresponding to the machine in good condition, the nature and location of the fault can be detected. Another important characteristic of a spectrum is that each rotating element in a machine generates identifiable frequency, as illustrated in Fig. 10.40; thus the changes in the spectrum at a given frequency can be attributed directly to the corresponding machine component. Since such changes can be detected more easily compared to changes in the overall vibration levels, this characteristic will be very valuable in practice.

Since the peaks in the spectrum relate to various machine components, it is necessary to be able to compute the fault frequencies. A number of formulas can be derived to find the fault frequencies of standard components like bearings, gearboxes, pumps, fans, and pulleys. Similarly, certain standard fault conditions can be described for standard faults such as unbalance, misalignment, looseness, oil whirl, and resonance.

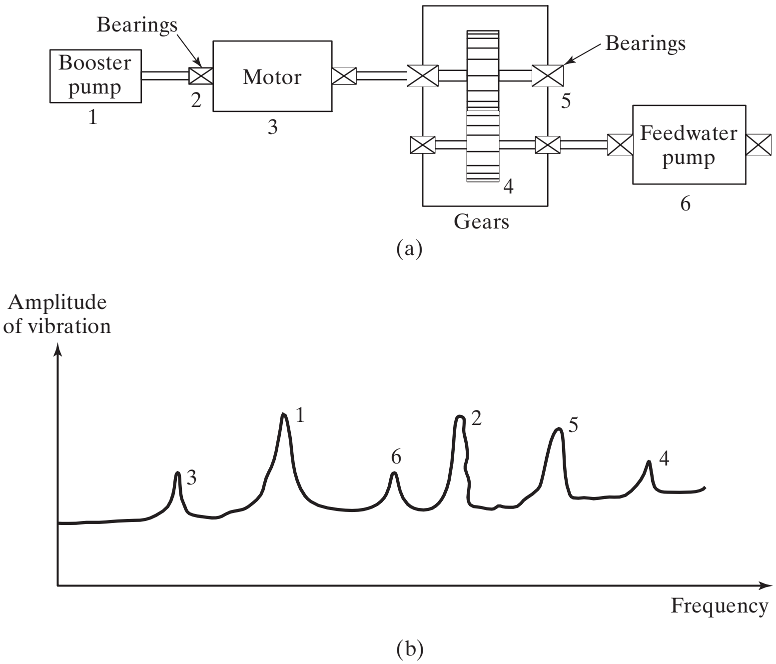


FIGURE 10.40 Relationship between machine components and the vibration spectrum.

Quefrency-Domain Analysis. Quefrency serves as the abscissa (x-axis) for a parameter known as cepstrum, similar to frequency, that serves as the abscissa for the parameter spectrum. Several definitions are available for the term *cepstrum* in the literature. Originally, cepstrum was defined as the power spectrum of the logarithm of the power spectrum. If $x(t)$ denotes a time signal, its power spectrum, $S_X(\omega)$, is given by

$$S_X(\omega) = |F\{x(t)\}|^2 \tag{10.93}$$

where $F\{\}$ denotes the Fourier transform of $\{\}$:

$$F\{x(t)\} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{i\omega t}\,dt \tag{10.94}$$

Thus the cepstrum, $c(\tau)$, is given by

$$c(\tau) = |F\{\log S_X(\omega)\}|^2 \tag{10.95}$$

Later, the cepstrum was defined as the inverse Fourier transform of the logarithm of the power spectrum, so that $c(\tau)$ becomes

$$c(\tau) = F^{-1}\{\log S_X(\omega)\} \tag{10.96}$$

The word *cepstrum* is derived by rearranging the letters in the word *spectrum*. The reason for this link is that the cepstrum is basically the spectrum of a spectrum. In fact, many of the terms used in spectrum analysis have been modified for use in cepstrum analysis. A few examples are given below:

Quefrency—Frequency
Rahmonics—Harmonics
Gamnitude—Magnitude
Saphe—Phase

From this, it is logical to see why quefrency serves as the abscissa of the cepstrum.

In practice, the choice of the definition of cepstrum is not critical, since both definitions—Eqs. (10.95) and (10.96)—show distinct peaks in the same location if there is strong periodicity in the (logarithmic) spectrum. The cepstrum is useful in machine-condition monitoring and diagnosis, since it can detect any periodicity in the spectrum caused by the failure of components, such as a blade in a turbine and a gear tooth in a gearbox. As an example, the spectra and cepstra of two truck gearboxes, one in good condition and the other in bad condition, running on a test stand with first gear in engagement, are shown in Figs. 10.41(a) to (d). Note that in Fig. 10.41(a), the good gearbox shows no marked periodicity in its spectrum while the bad gearbox indicates a large number of sidebands with an approximate spacing of 10 Hz in its spectrum (Fig. 10.41(b)). This spacing cannot be determined more accurately from Fig. 10.41(b). Similarly, the cepstrum of the good gearbox does not indicate any quefrencies prominently (Fig. 10.41(d)). However, the cepstrum of the bad gearbox (Fig. 10.41(c)) indicates three prominant quefrencies at 28.1 ms (35.6 Hz), 95.9 ms (10.4 Hz), and 191.0 ms (5.2 Hz). The first series of rahmonics corresponding to 35.6 Hz has been identified to correspond

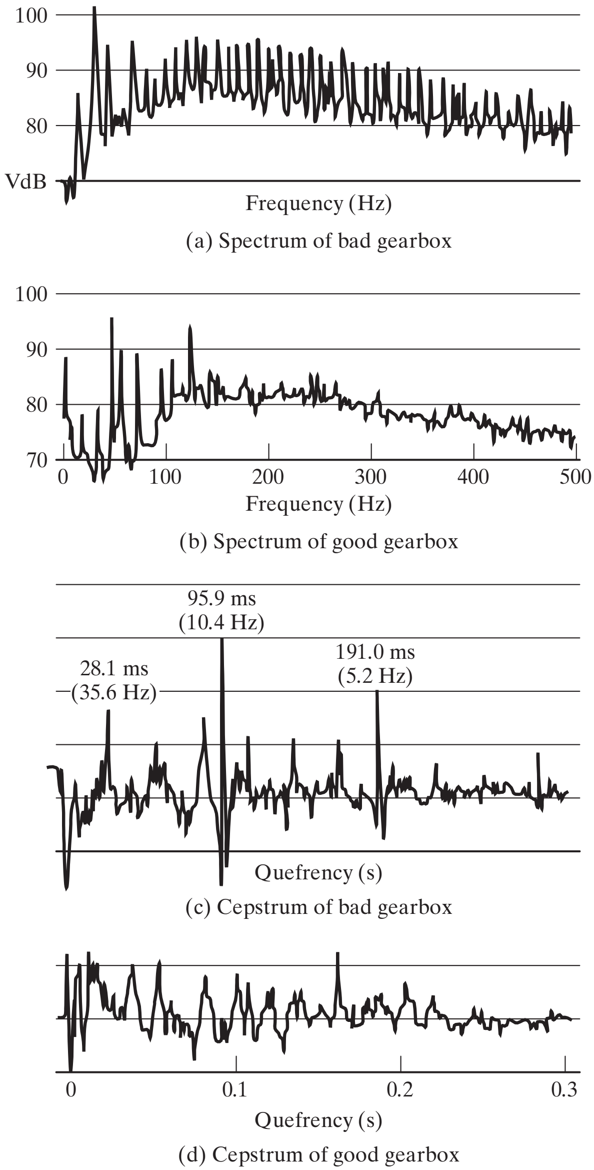


FIGURE 10.41 Spectrum and cepstrum of a gearbox [10.24].

to the input speed of the gearbox. The theoretical output speed is 5.4 Hz. Thus the rahmonics corresponding to 10.4 Hz are not expected to be same as the second harmonic of the output speed, which would be 10.8 Hz. A careful examination revealed that the rahmonics corresponding to the frequency 10.4 Hz are same as the speed of the second gear. This indicates that the second gear was at fault although the first gear was in engagement.

10.9.5 Instrumentation Systems

Based on their degree of sophistication, three types of instrumentation systems can be used for condition monitoring of machines—the basic system, the portable system, and the computer-based system. The first type, which can be labeled as the *basic system*, consists of a simple pocket-sized vibration meter, a stroboscope, and a headset. The vibration meter measures the overall vibration levels (RMS or peak values of acceleration or velocity) over suitable frequency ranges, the stroboscope indicates the speed of the machine, and the headset aids in hearing the machine vibration. The overall RMS velocity readings can be compared with published severity charts and any need for condition-based maintenance can be established. The overall vibration levels can also be plotted against time to find how rapidly the condition of the machine is changing. The vibration meter can also be used in conjunction with a pocket computer to collect and store the measurements. Sometimes, an experienced operator can hear the vibration (sound) of a machine over a period of time and find its condition. In some cases, faults such as misalignment, unbalance, or looseness of parts can be observed visually.

The *portable condition-monitoring system* consists of a portable fast Fourier transform (FFT) vibration analyzer based on battery power. This vibration analyzer can be used for fault detection by recording and storing vibration spectra from each of the measurement points. Each newly recorded spectrum can be compared with a reference spectrum that was recorded at that particular measurement point when the machine was known to be in good condition. Any significant increase in the amplitudes in the new spectrum indicates a fault that needs further investigation. The vibration analyzer also has certain diagnostic capability to identify problems such as faulty belt drives and gearboxes and loose bearings. When the fault diagnosed requires a replacement of parts, it can be done by the operator. If a rotor requires balancing, the vibration analyzer can be used to compute the locations and magnitudes of the correction masses necessary to rebalance the rotor.

The *computer-based condition-monitoring system* is useful and economical when the number of machines, the number of monitoring points, and the complexity of fault detection increases. It consists of an FFT vibration analyzer coupled with a computer for maintaining a centralized database that can also provide diagnostic capabilities. The data are stored on a disk, allowing them to be used for spectrum comparison or for three-dimensional plots (see Fig. 10.42). Certain computer-based systems use tape recorders to record vibration signals from each machine at all the measurement points. These measurements can be played back into the computer for storage and postprocessing.

10.9.6 Choice of Monitoring Parameter

Piezoelectric accelerometers are commonly used for measuring the vibration of machines. They are preferred because of their smaller size, superior frequency and dynamic range, reliability over long periods, and robustness. When an accelerometer is used as the vibration pickup, the velocity and displacements can be obtained from the integrators built into the analyzer. Thus the user can choose between acceleration, velocity, and displacement as the monitoring parameter. Although any of these three spectra can be used for the condition monitoring of a machine, usually the velocity spectrum will be the flattest one (indicating that the range of velocity amplitudes is the smallest). Since a change in the amplitude of velocity can be observed easily in a flatter spectrum, velocity is commonly used as the parameter for monitoring the condition of machines.

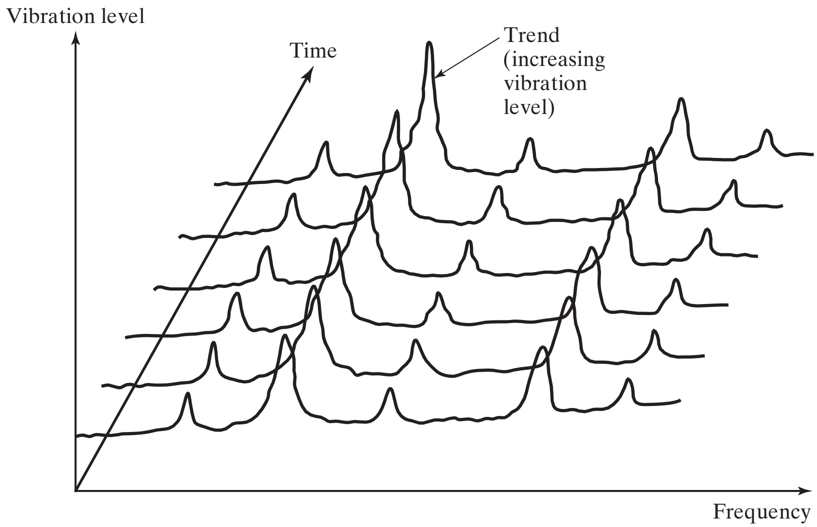


FIGURE 10.42 Three-dimensional plot of data.

10.10 Examples Using MATLAB

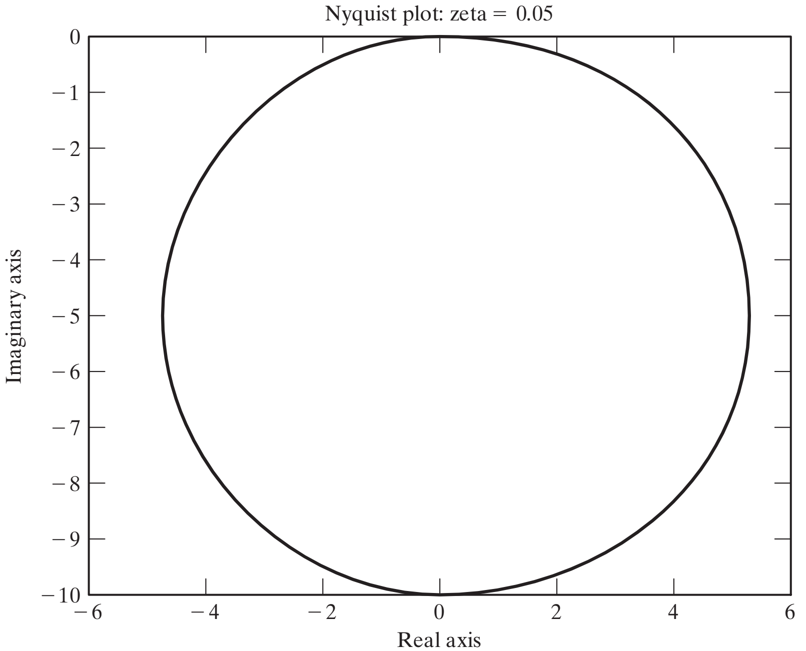
EXAMPLE 10.6 Plotting of Nyquist Circle

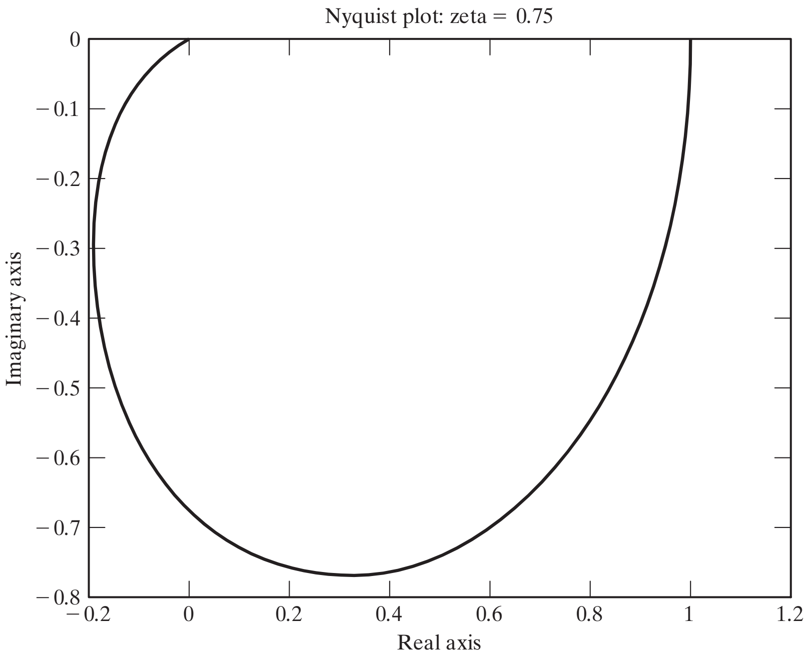
Using MATLAB, plot the Nyquist circle for the following data:

a. $\zeta = 0.75$
b. $\zeta = 0.05$

Solution: Equations (10.70) and (10.71) are plotted along the horizontal and vertical axes. The MATLAB program to plot the Nyquist circle is given below.

```
%Ex10_6.m
zeta = 0.05;
for i = 1: 10001
    r(i) = 50 * (i-1) / 10000;
    Re1(i) = ( 1-r(i)^2 )/( (1-r(i)^2)^2 + 4*zeta^2*r(i)^2 );
    Im1(i) = -2*zeta*r(i)/( (1-r(i)^2)^2 + 4*zeta^2*r(i)^2 );
end
zeta = 0.75;
for i = 1: 10001
    r(i) = 50 *(i-1) / 10000;
    Re2(i) = ( 1-r(i)^2 )/( (1-r(i)^2)^2 + 4*zeta^2*r(i)^2 );
    Im2(i) = -2*zeta*r(i)/( (1-r(i)^2)^2 + 4*zeta^2*r(i)^2 );
end
plot(Re1, Im1);
title('Nyquist plot: zeta = 0.05');
ylabel('Imaginary axis');
xlabel('Real axis');
pause;
plot(Re2, Im2);
title('Nyquist plot: zeta = 0.75');
ylabel('Imaginary axis');
xlabel('Real axis');
```


Nyquist plot: zeta = 0.05
Imaginary axis
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
−6
−4
−2
0
2
4
6
Real axis


Nyquist plot: zeta = 0.75
Imaginary axis
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.2
0
0.2
0.4
0.6
0.8
1
1.2
Real axis

■

EXAMPLE 10.7 Plotting of Accelerometer Equation

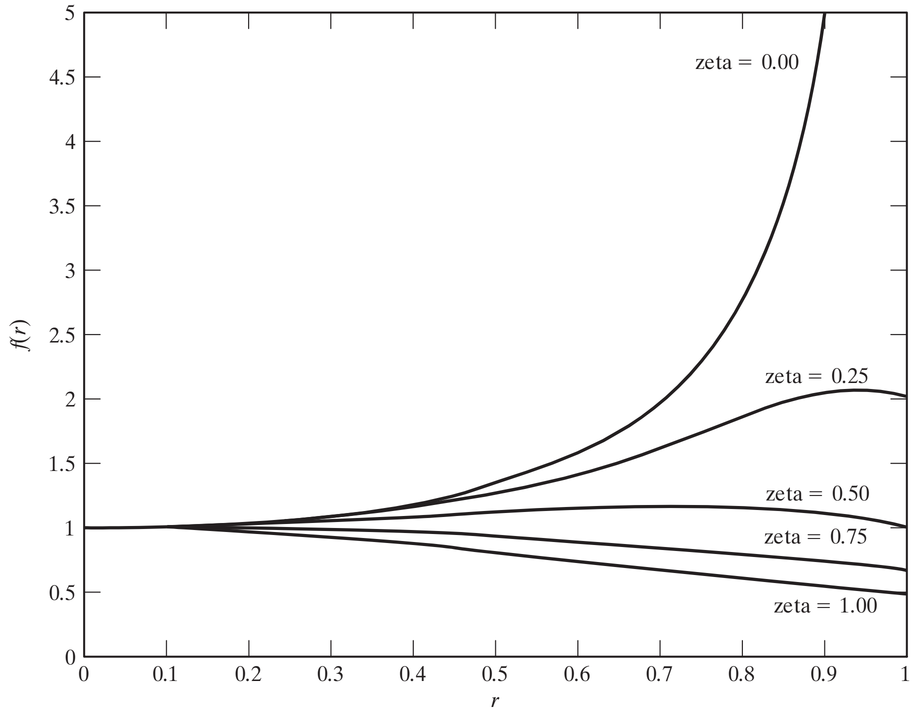
Using MATLAB, plot the ratio of measured to true accelerations, given by

$$f(r) = \frac{1}{\{(1 - r^2)^2 + (2\zeta r)^2\}^{1/2}} \tag{E.1}$$

for $\zeta = 0.0, 0.25, 0.5, 0.75$, and 1.0.

Solution: The MATLAB program to plot Eq. (E.1) in the range $0 \leq r \leq 1$ is given below.

```
%Ex10_7.m
zeta = 0.0;
for i = 1: 101
   r(i) = (i-1)/100;
   f1(i) = 1/sqrt((1-r(i)^2)^2 + (2*zeta*r(i))^2);
end
zeta = 0.25;
for i = 1: 101
   r(i) = (i-1)/100;
   f2(i) = 1/sqrt( (1-r(i)^2)^2 + (2*zeta*r(i))^2 );
end
zeta = 0.5;
for i = 1: 101
   r(i) = (i-1)/100;
   f3(i) = 1/sqrt( (1-r(i)^2)^2 + (2*zeta*r(i))^2 );
end
zeta = 0.75;
for i = 1: 101
   r(i) = (i-1)/100;
   f4(i) = 1/sqrt( (1-r(i)^2)^2 + (2*zeta*r(i))^2 );
end
zeta = 1.0;
for i = 1: 101
   r(i) = (i-1)/100;
   f-(i) = 1/sqrt( (1-r(i)^2)^2 + (2*zeta*r(i))^2 );
end
plot(r,f1);
axis([0 1 0 5]);
gtext('zeta = 0.00');
hold on;
plot(r,f2);
gtext('zeta = 0.25');
hold on;
plot(r,f3);
gtext('zeta = 0.50');
hold on;
plot(r,f4);
gtext('zeta = 0.75');
hold on;
plot(r,f5);
gtext('zeta = 1.00');
xlabel('r');
ylabel('f(r)');
```



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CHAPTER SUMMARY

In some practical applications, it might be difficult to develop a mathematical model, derive the governing equations, and conduct analysis to predict the vibration characteristics of the system. In such cases, we can measure the vibration characteristics of the system under known input conditions and develop a mathematical model of the system. We presented the various aspects of vibration measurement and applications. We discussed the various types of transducers, vibration pickups, frequency measuring instruments, and shakers (exciters) available for vibration measurement. We described signal analysis and experimental modal analysis and determination of natural frequencies, damping ratio, and mode shapes. We presented machine-condition monitoring and diagnosis techniques. Finally, we presented MATLAB solutions for vibration-measurement-related analysis problems.

Now that you have finished this chapter, you should be able to answer the review questions and solve the problems given below.

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REVIEW QUESTIONS

10.1 Give brief answers to the following:

1. What is the importance of vibration measurement?
2. What is the difference between a vibrometer and a vibrograph?
3. What is a transducer?
4. Discuss the basic principle on which a strain gage works.
5. Define the gage factor of a strain gage.
6. What is the difference between a transducer and a pickup?
7. What is a piezoelectric material? Give two examples of such material.
8. What is the working principle of an electrodynamic transducer?
9. What is an LVDT? How does it work?
10. What is a seismic instrument?
11. What is the frequency range of a seismometer?
12. What is an accelerometer?
13. What is phase-shift error? When does it become important?
14. Give two examples of a mechanical vibration exciter.
15. What is an electromagnetic shaker?
16. Discuss the advantage of using operational deflection shape measurement.
17. What is the purpose of experimental modal analysis?
18. Describe the use of the frequency-response function in modal analysis.
19. Name two frequency-measuring instruments.
20. State three methods of representing the frequency-response data.
21. How are Bode plots used?
22. How is a Nyquist diagram constructed?
23. What is the principle of mode superposition? What is its use in modal analysis?
24. State the three types of maintenance schemes used for machinery.
25. How are orbits used in machine diagnosis?
26. Define the terms *kurtosis* and *cepstrum*.

10.2 Indicate whether each of the following statements is true or false:

1. A strain gage is a variable-resistance transducer.
2. The value of the gage factor of a strain gage is given by the manufacturer.
3. The voltage output of an electromagnetic transducer is proportional to the relative velocity of the coil.
4. The principle of the electrodynamic transducer can be used in vibration exciters.
5. A seismometer is also known as a vibrometer.
6. All vibration-measuring instruments exhibit phase lag.
7. The time lag is important when measuring harmonic motion of frequency ω.
8. The Scotch yoke mechanism can be used as a mechanical shaker.
9. The time response of a system gives better information on energy distribution than does the frequency response.
10. A spectrum analyzer is a device that analyzes a signal in the frequency domain.
11. The complete dynamic response of a machine can be determined through modal testing.
12. The damping ratio of a vibrating system can be found from the Bode diagram.
13. The spectrum analyzers are also known as fast Fourier transform (FFT) analyzers.

14. In breakdown maintenance, the machine is run until failure.
15. Time-domain waveforms can be used to detect discrete damages of machinery.

10.3 Fill in each of the following blanks with the appropriate word:

1. A device that transforms values of physical variables into equivalent electrical signals is called a _____.
2. Piezoelectric transducers generate electrical _____ when subjected to mechanical stress.
3. A seismic instrument consists of a _____ system mounted on the vibrating body.
4. The instrument that measures the acceleration of a vibrating body is called _____.
5. _____ can be used to record earthquakes.
6. The instrument that measures the velocity of a vibrating body is called a _____.
7. Most mechanical frequency-measuring instruments are based on the principle of _____.
8. The Frahm tachometer is a device consisting of several _____ carrying masses at free ends.
9. The main advantage of a stroboscope is that it can measure the speed without making _____ with the rotating body.
10. In real-time frequency analysis, the signal is continuously analyzed over all the _____ bands.
11. Real-time analyzers are useful for machinery _____ monitoring, since a change in the noise or vibration spectrum can be observed immediately.
12. An _____ is the interval between any two frequencies $(f_2 - f_1)$ whose frequency ratio $\left(\frac{f_2}{f_1}\right)$ is 2.
13. The dynamic testing of a machine involves finding the _____ of the machine at a critical frequency.
14. For vibration testing, the machine is supported to simulate a _____ condition of the system so that rigid body modes can also be observed.
15. The excitation force is measured by a _____ cell.
16. The response of a system is usually measured by _____.
17. The frequency response of a system can be measured using _____ analyzers.
18. The condition of a machine can be determined using _____ severity charts.
19. The life of a machine follows the classic _____ curve.
20. The _____ observed in Lissajous figures can be used to identify machinery faults.
21. Cepstrum can be defined as the power spectrum of the logarithm of the _____.

10.4 Select the most appropriate answer out of the choices given:

1. When a transducer is used in conjunction with another device to measure vibration, it is called a
   a. vibration sensor **b.** vibration pickup **c.** vibration actuator
2. The instrument that measures the displacement of a vibrating body is called a
   a. seismometer **b.** transducer **c.** accelerometer
3. The circuit that permits the passage of frequency components of a signal over a frequency band and rejects all other frequency components is called a
   a. bandpass filter **b.** frequency filter **c.** spectral filter
4. A decibel (dB) is a quantity, such as power (P), defined in terms of a reference value (P_{ref}), as
   a. $10 \log_{10}\left(\frac{P}{P_{\text{ref}}}\right)$ **b.** $\log_{10}\left(\frac{P}{P_{\text{ref}}}\right)$ **c.** $\frac{1}{P_{\text{ref}}} \log_{10}(P)$

5. The following function plays an important role in the experimental modal analysis:
   a. time-response function
   b. modal-response function
   c. frequency-response function
6. The method of subjecting a system to a known force as an initial condition and then releasing is known as
   a. step relaxation
   b. excitation by electromagnetic shaker
   c. impactor
7. The process of using an electrical signal, generalized by a spectrum analyzer, for applying a mechanical force on a system is known as
   a. step relaxation
   b. excitation by electromagnetic shaker
   c. impactor
8. The procedure of using a hammer with a built-in load cell to apply load at different points of a system is known as
   a. step relaxation
   b. excitation by electromagnetic shaker
   c. impactor
9. During the initial running-in period, usually the deterioration of a machine
   a. decreases **b.** increases **c.** remains constant
10. During the normal operating period, the deterioration of a machine usually
   a. decreases **b.** increases **c.** remains constant
11. During the aging or wearout period, the deterioration of a machine usually
   a. decreases **b.** increases **c.** remains constant

10.5 Match the items in the two columns below:

1. Piezoelectric accelerometer	**a.** produces light pulses intermittently
2. Electrodynamic transducer	**b.** has high output and is insensitive to temperature
3. LVDT transducer	**c.** frequently used in velocity pickups
4. Fullarton tachometer	**d.** has high sensitivity and frequency range
5. Stroboscope	**e.** variable-length cantilever with a mass at its free end

PROBLEMS

Section 10.2 Transducers

10.1 A Rochelle salt crystal, having a voltage sensitivity of 0.098 V-m/N and thickness 2 mm, produced an output voltage of 200 volts under pressure. Find the pressure applied to the crystal.

Section 10.3 Vibration Pickups

10.2 A spring-mass system with $m = 0.5$ kg and $k = 10{,}000$ N/m, with negligible damping, is used as a vibration pickup. When mounted on a structure vibrating with an amplitude of 4 mm, the total displacement of the mass of the pickup is observed to be 12 mm. Find the frequency of the vibrating structure.

10.3 The vertical motion of a machine is measured by using the arrangement shown in Fig. 10.43. The motion of the mass m relative to the machine body is recorded on a drum. If the damping constant c is equal to $c_{cri}/\sqrt{2}$, and the vertical vibration of the machine body is given by $y(t) = Y \sin \omega t$, find the amplitude of motion recorded on the drum.

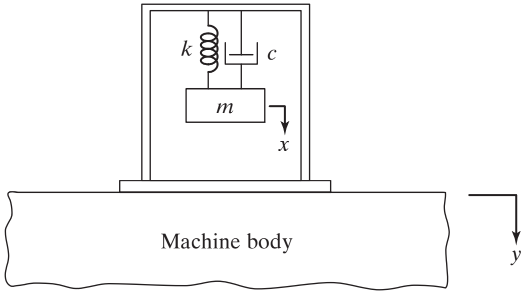

FIGURE 10.43

10.4 It is proposed that the vibration of the foundation of an internal combustion engine be measured over the speed range 500 rpm to 1500 rpm using a vibrometer. The vibration is composed of two harmonics, the first one caused by the primary inertia forces and the second one by the secondary inertia forces in the engine. Determine the maximum natural frequency of the vibrometer in order to have an amplitude distortion less than 2 percent.

10.5 Determine the maximum percent error of a vibrometer in the frequency-ratio range $4 \le r < \infty$ with a damping ratio of $\zeta = 0$.

10.6 Solve Problem 10.5 with a damping ratio of $\zeta = 0.67$.

10.7 A vibrometer is used to measure the vibration of an engine whose operating-speed range is from 500 to 2000 rpm. The vibration consists of two harmonics. The amplitude distortion must be less than 3 percent. Find the natural frequency of the vibrometer if (a) the damping is negligible and (b) the damping ratio is 0.6.

10.8 A spring-mass system, having a static deflection of 10 mm and negligible damping, is used as a vibrometer. When mounted on a machine operating at 4000 rpm, the relative amplitude is recorded as 1 mm. Find the maximum values of displacement, velocity, and acceleration of the machine.

10.9 A vibration pickup has a natural frequency of 5 Hz and a damping ratio of $\zeta = 0.5$. Find the lowest frequency that can be measured with a 1 percent error.

10.10 A vibration pickup has been designed for operation above a frequency level of 100 Hz without exceeding an error of 2 percent. When mounted on a structure vibrating at a frequency of 100 Hz, the relative amplitude of the mass is found to be 1 mm. Find the suspended mass of the pickup if the stiffness of the spring is 4000 N/m and damping is negligible.

10.11 A vibrometer has an undamped natural frequency of 10 Hz and a damped natural frequency of 8 Hz. Find the lowest frequency in the range to infinity at which the amplitude can be directly read from the vibrometer with less than 2 percent error.

10.12 Determine the maximum percent error of an accelerometer in the frequency-ratio range $0 < r \le 0.65$ with a damping ratio of $\zeta = 0$.

10.13 Solve Problem 10.12 with a damping ratio of 0.75.

10.14 Determine the necessary stiffness and the damping constant of an accelerometer if the maximum error is to be limited to 3 percent for measurements in the frequency range of 0 to 100 Hz. Assume that the suspended mass is 0.05 kg.

10.15 An accelerometer is constructed by suspending a mass of 0.1 kg from a spring of stiffness 10,000 N/m with negligible damping. When mounted on the foundation of an engine, the peak-to-peak travel of the mass of the accelerometer has been found to be 10 mm at an engine speed of 1000 rpm. Determine the maximum displacement, maximum velocity, and maximum acceleration of the foundation.

10.16 A spring-mass-damper system, having an undamped natural frequency of 100 Hz and a damping constant of 20 N-s/m, is used as an accelerometer to measure the vibration of a machine operating at a speed of 3000 rpm. If the actual acceleration is 10 m/s^2 and the recorded acceleration is 9 m/s^2, find the mass and the spring constant of the accelerometer.

10.17 A machine shop floor is subjected to the following vibration due to electric motors running at different speeds:

$$x(t) = 20 \sin 4\pi t + 10 \sin 8\pi t + 5 \sin 12\pi t \text{ mm}$$

If a vibrometer having an undamped natural frequency of 0.5 Hz, and a damped natural frequency of 0.48 Hz is used to record the vibration of the machine shop floor, what will be the accuracy of the recorded vibration?

10.18 A machine is subjected to the vibration

$$x(t) = 20 \sin 50t + 5 \sin 150t \text{ mm} \qquad (t \text{ in sec})$$

An accelerometer having a damped natural frequency of 80 rad/s and an undamped natural frequency of 100 rad/s is mounted on the machine to read the acceleration directly in mm/s^2. Discuss the accuracy of the recorded acceleration.

Section 10.4 Frequency-Measuring Instruments

10.19 A variable-length cantilever beam of rectangular cross section $\frac{1}{16}$ in. $\times$ 1 in., made of spring steel, is used to measure the frequency of vibration. The length of the cantilever can be varied between 2 in. and 10 in. Find the range of frequencies that can be measured with this device.

Section 10.8 Experimental Modal Analysis

10.20 Show that the real component of the harmonic response of a viscously damped single-degree-of-freedom system (from X in Eq. 3.54) attains a maximum at

$$R_1 = \frac{\omega_1}{\omega_n} = \sqrt{1 - 2\zeta}$$

and a minimum at

$$R_2 = \frac{\omega_2}{\omega_n} = \sqrt{1 + 2\zeta}$$

10.21 Find the value of the frequency at which the imaginary component of the harmonic response of a viscously damped single-degree-of-freedom system (from X in Eq. 3.54) attains a minimum.

10.22 Construct the Nyquist diagram for a single-degree-of-freedom system with hysteretic damping.

10.23 The Bode plot of shaft vibration of a turbine obtained during coast-down is shown in Fig. 10.44. Determine the damping ratio of the system when the static deflection of the shaft is equal to 0.05 mil.

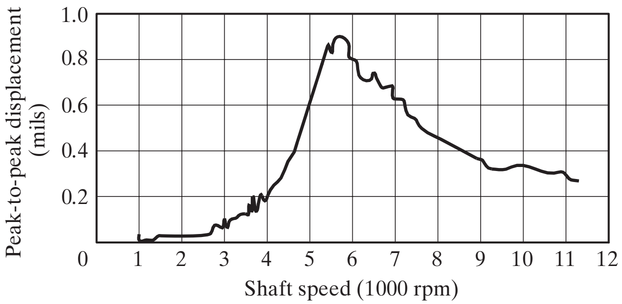

FIGURE 10.44

10.24 The vibratory response at the bearing of an internal combustion engine is shown in Fig. 10.45. Determine the equivalent viscous damping ratio of the system.

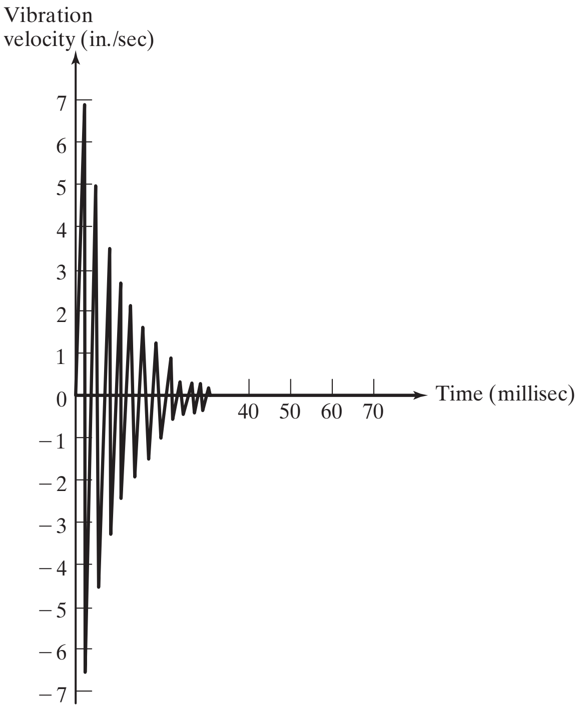

FIGURE 10.45 Response in time domain.

10.25 Suggest a method of using the Bode plot of phase angle versus frequency (Fig. 3.11(b)) to identify the natural frequency and the damping ratio of the system.

Section 10.9 Machine-Condition Monitoring and Diagnosis

10.26 Two ball bearings, each with 16 balls, are used to support the shaft of a fan that rotates at 750 rpm. Determine the frequencies, in hertz, corresponding to the following defects:* cage, inner race, outer race, and ball. Assume that $d = 15$ mm, $D = 100$ mm, and $\alpha = 30°$.

10.27 Determine the defect frequencies in hertz* corresponding to roller, inner race, outer race, and cage defects for a roller bearing with 18 rollers when installed in a machine that runs at a speed of 1000 rpm. Assume $d = 2$ cm, $D = 15$ cm, and $\alpha = 20°$.

10.28 An angular contact thrust bearing consists of 18 balls, each of diameter 10 mm, and is mounted on a shaft that rotates at 1500 rpm. If the contact angle of the bearing is 40° with a pitch diameter 80 mm, find the frequencies corresponding to cage, ball, inner race, and outer race faults.*

10.29 Find the value of kurtosis for a vibration signal that is uniformly distributed in the range 1–5 mm;

$$f(x) = \frac{1}{4}; \qquad 1 \le x \le 5 \text{ mm}$$

10.30 Find the value of kurtosis for a vibration amplitude that can be approximated as a discrete random variable with the following probability mass function:

x (mm)	1	2	3	4	5	6	7
$f(x)$	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{3}{16}$	$\frac{6}{16}$	$\frac{3}{16}$	$\frac{3}{32}$	$\frac{1}{32}$

Section 10.10 MATLAB Problems

10.31 Figure 10.46 shows the experimental transfer function of a structure. Determine the approximate values of ω_i and ζ_i.

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*Each type of failure in ball and roller bearings generates frequency of vibration f(impact rate per minute) as follows. Inner race defect: $f = \frac{1}{2}nN(1 + c)$; outer race defect: $f = \frac{1}{2}nN(1 - c)$; ball or roller defect: $f = \frac{DN}{d}c(2 - c)$; cage defect: $f = \frac{1}{2}N(1 - c)$, where d = ball or roller diameter, D = pitch diameter, α = contact angle, n = number of balls or rollers, N = speed (rpm), and $c = \frac{d}{D}\cos\alpha$.

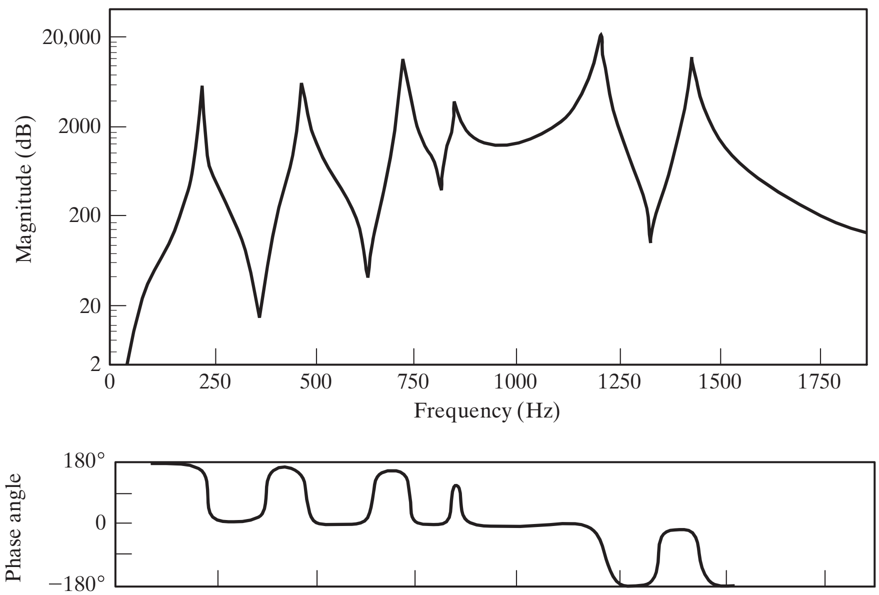

FIGURE 10.46

10.32 The experimental Nyquist circle of a structure is shown in Fig. 10.47. Estimate the modal damping ratio corresponding to this circle.

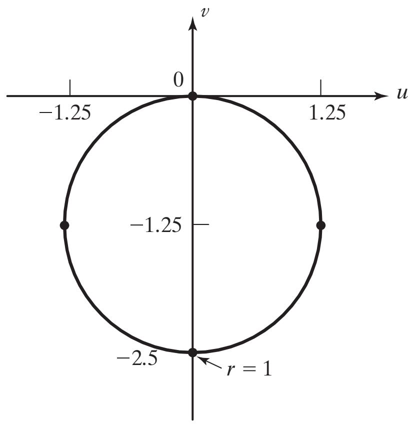

FIGURE 10.47

DESIGN PROJECTS

10.33 Design a vibration exciter to satisfy the following requirements:

a. Maximum weight of the test specimen = 10 N
b. Range of operating frequency = 10 to 50 Hz
c. Maximum acceleration level = 20 g
d. Maximum vibration amplitude = 0.5 cm peak to peak.

10.34 Frahm tachometers are particularly useful to measure the speeds of engines whose rotating shafts are not easily accessible. When the tachometer is placed on the frame of a running engine, the vibration generated by the engine will cause one of the reeds to vibrate noticeably when the engine speed corresponds to the resonant frequency of a reed. Design a compact and lightweight Frahm tachometer with 12 reeds to measure engine speeds in the range 300–600 rpm.

10.35 A cantilever beam with an end mass m is fixed at the top of a multistory building to measure the acceleration induced at the top of the building during wind and earthquake loads (see Fig. 10.48). Design the beam (that is, determine the material, cross-sectional dimensions, and the length of the beam) such that the stress induced in the beam should not exceed the yield stress of the material under an acceleration of 0.2 g at the top of the building. Assume that the end mass m is equal to one-half of the mass of the beam.

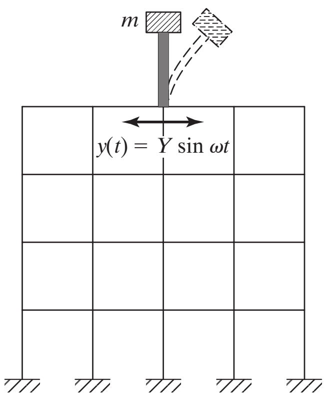

FIGURE 10.48