DA "Modal Analysis Theory And Testing" by Heylen, Lammens, Sas.

A.6.
COMBINING NUMERICAL AND EXPERIMENTAL MODELS

A.6.0. INTRODUCTION.

Nowadays, in the design of mechanical structures, the dynamic behaviour gets more attention. Lifetime under cyclic loading, levels of vibration or noise radiation, interaction between control systems and structure vibration, ... are often important constraints for the designer. The analysis of the dynamic behaviour is however not straightforward. Designers determine modal parameters of a mechanical structure either by experimental or by numerical methods. Results of both investigations are expected to correlate closely. Experimental measurements on prototypes give information about the structure in the configuration of the test only. Finite element models allow to predict the dynamic behaviour of the structure under various loading and boundary conditions, but the reliability of the finite element models is often not guaranteed.

Model updating techniques verify and correct these finite element models by means of the experimental data. The result of a model updating analysis is a finite element model that is more reliable for further predictions. The model updating section (A.6.1) is primarily based upon reference a.6.26 by S.Lammens.

Secondly, the existence of finite element or experimental models of the structure to be tested, or of a similar structure, provides the test engineer with a lot of valuable information for the test to be performed. This may increase the quality of the data and/or reduce the testing time. Hence, this type of combining numerical and experimental models, called pre-test analysis, is a structured way of using the existing experience. Section A.6.2 covers the theory of some techniques that may be used for preparing a test set-up.

A.6.1. MODEL UPDATING.

A.6.1.1. Introduction and general scheme

Model updating aims at the development of a finite element model that yields accurate and reliable predictions of the dynamic behaviour of a mechanical structure. Fig.a.6.1 shows the general scheme of a model updating study.
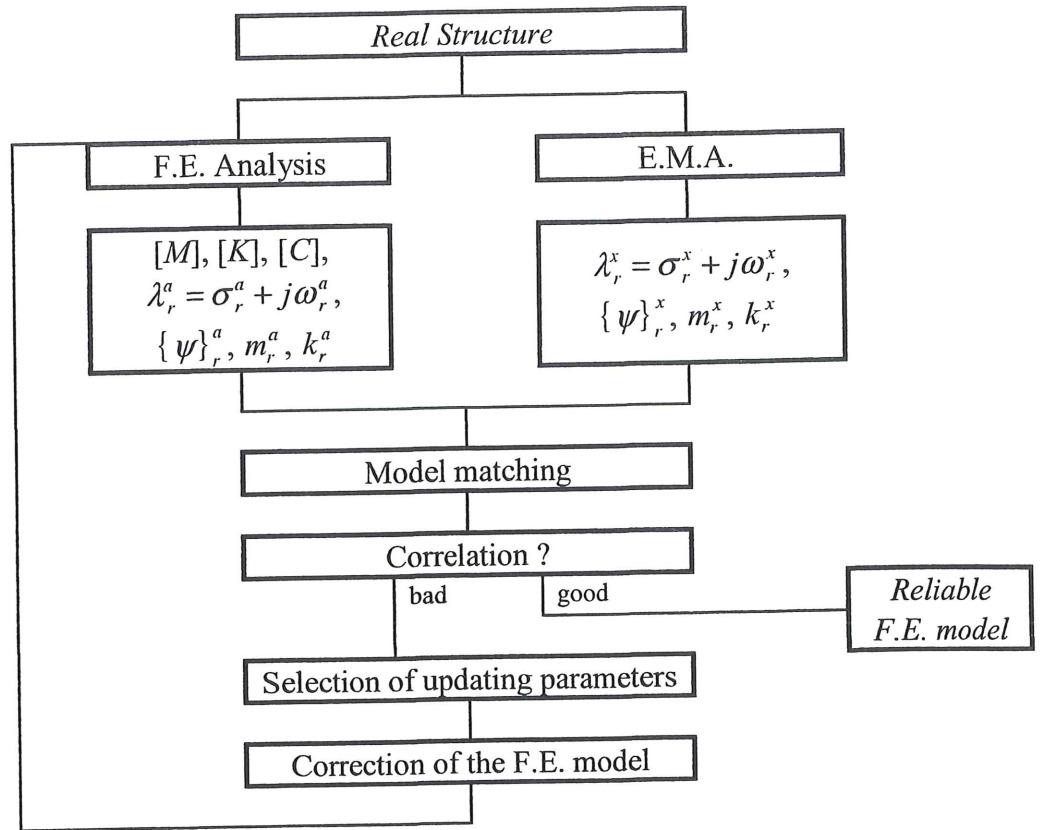
Fig.a.6.1: General scheme of a model updating study.

The updating procedure starts with the construction of a finite element model. The structure is divided in elements that are connected in nodes. Each node has one or more degrees of freedom. Degrees of freedom represent displacement and deformation of the structure in a discretised form. Every piece of structural material contributes to the overall mass, stiffness and damping through element matrices. Assembly of all element matrices yields the global system matrices: the mass matrix, $[M]$, the stiffness matrix, $[K]$, and the damping matrix, $[C]$. The damping matrix is often neglected, because the wide variety of (non-linear) damping mechanisms makes it very difficult to develop a reliable model of the damping behaviour. Furthermore the damping forces are often believed to be small in comparison with the inertial and elastic forces.

Starting from the system matrices modal parameters are calculated by solving the equation of motion in the Laplace domain:

a.6.1
$$\left(- p^2 [M] + p[C] + [K]\right)\{X\} = \{0\}.$$

This equation can be rewritten as an eigenvalue problem (see section A.1.2.2):

a.6.2
$$\left(p\begin{bmatrix} [M] & [0] \\ [0] & -[M] \end{bmatrix} + \begin{bmatrix} [C] & [K] \\ [M] & [0] \end{bmatrix} \right) \begin{Bmatrix} \dot{X} \\ X \end{Bmatrix} = \{0\}.$$

This yields the eigenvalues $\lambda_r = \sigma_r + j\omega_r$ and the corresponding eigenvectors $\{\psi\}_r$. Scaling of the eigenvectors determines the modal masses m_r and the modal stiffnesses k_r (see section A.1.2.7. and A.1.2.8).

Finite element analysis provides following input to the model updating procedure:
      the system matrices: $[M]$, $[K]$, and $[C]$ or frequency response functions,
      the modal parameters: $\lambda_r = \sigma_r + j\omega_r$, $\{\psi\}_r$, m_r and k_r for $r=1, N_m$.

As already mentioned, damping is often neglected: $[C]=[0]$ and $\sigma_r=0$.

To avoid confusion with experimental data the analytical modal parameters will be denoted with a superscript [a] throughout this chapter:

$$\lambda_r^a = \sigma_r^a + j\omega_r^a, \{\psi\}_r^a, m_r^a, k_r^a.$$

These finite element analysis results will be updated during the model updating process. Experimentally identified modal parameters are the reference data to be matched by the analytical modal parameters.

In an experimental test frequency response functions of the structure are measured. Then parameter estimation techniques identify the modal parameters (see section A.3). These experimental parameters will be denoted in this chapter with a superscript [x]:

$$\lambda_r^x = \sigma_r^x + j\omega_r^x, \{\psi\}_r^x, m_r^x, k_r^x.$$

The updating procedure starts with the model matching step. Generally, the mesh of measurement points does not correspond completely with the set of nodes of the finite element model. First, a measurement point does not necessarily coincide with a node of the finite element model. This problem can easily be avoided by a good communication between the person that builds the analytical model and the person that designs the measurement set-up. Next, and more important, finite element models are normally composed of far more degrees of freedom than the number of degrees of freedom that are measured during the test. For some of the correlation techniques and for most of the correction techniques, the analytical and the experimental model must show a one-to-one correspondence of the degrees of freedom. In order to solve this mesh incompatibility, either the analytical system matrices must be reduced to the number of degrees of freedom of the experimental set-up, or the experimental data must be expanded to the number of degrees of freedom of the finite element model. Section A.6.1.2. gives an overview of some commonly used reduction and expansion techniques. Reduction of system matrices or expansion of experimental data is an important obstacle for a successful model updating.

After the model matching step, the updating procedure continues with a correlation check. Analytical modal parameters are compared in various ways with experimental modal parameters. If correlation is good, the updating process stops here and the finite element model is considered sufficiently reliable for further calculations and predictions. If, most likely, correlation is bad, the finite element model must be corrected by the updating procedure in order to improve correlation with experimental data. Section A.6.1.3. gives an overview of the most often used correlation techniques.

In case of bad correlation the finite element model must be corrected. A first step in the correction of the analytical model is the selection of the updating parameters. This step identifies which parameters of the finite element model are inaccurate and have to be corrected. The selection of the updating parameters is a crucial step for the success of the updating procedure. Section A.6.1.4. gives an overview of the different sorts of updating parameters.

The final step of the updating procedure is the correction step. The correction step uses the experimental data in order to find new values for the updating parameters. After the introduction of the updated parameters in the finite element model, this model yields, in case of a successful correction step, analytical modal parameters that correlate well with the experimental modal data. Section A.6.1.5. gives an overview of state of the art correction techniques.

Data obtained with experimental techniques and data from the finite element model are often not compatible, mainly for four reasons:

- <u>measured degrees of freedom do not coincide with degrees of freedom in the finite element model</u>: The model matching step in the updating procedure provides an, approximate, solution for this problem.

- <u>the set of experimental modal data is incomplete</u>: Not only the number of experimental degrees of freedom is limited, but also the set of experimental modal data is limited. The measurement of frequency response functions is not performed but in a limited bandwidth. Hence, the set of identified modal data is incomplete: Mode shapes outside this frequency range cannot be identified. Since these incomplete experimental data are used as reference data in the updating procedure, the solution of the updating process will not be unique.

- <u>noise contaminates measurements</u>: Relative errors of 3% on resonance frequencies, relative errors of 10% on the components of the mode shapes and relative errors of 30% on modal masses and stiffnesses are considered to be acceptable. It is obvious that the use of erroneous experimental data gives misleading indications in the correlation and the error localisation step and causes the correction step to converge to inaccurate and meaningless values. Most often, the experimental modal masses and stiffnesses are not used in the updating procedure. The inaccuracy of the experimental data will not be further discussed in this chapter, but it should be stressed that a successful updating study is only possible with carefully measured data.

- <u>damping cannot be included accurately in the finite element model</u>: Damping information is inherently present in the experimental data but often neglected in the finite element model. As a consequence, an undamped finite element model is updated by means of experimental data of a damped structure. It is obvious that this discrepancy can cause errors in the updating process. Mode normalisation techniques are an approximate way to avoid the difficulty. The damping incompatibility will not be further discussed.

The example of appendix AA.6.1 illustrates many of the concepts introduced in this model updating section.

A.6.1.2. Model matching.

A.6.1.2.1. Introduction.

The number of degrees of freedom of a finite element model most often exceeds by far the number of measured degrees of freedom. Finite element analyses require fine meshes of nodes in order to provide accurate predictions. It is not practical, and often even not possible, to measure all the corresponding degrees of freedom on the real structure:

- many finite element nodes are internal to the structure and cannot be accessed for measurement,
- rotational degrees of freedom are difficult to measure,
- for the purpose of an experimental modal analysis study, it is not necessary to have a fine mesh of measurement points.

Most updating methods, however, require a one-to-one correspondence between the analytical and the experimental degrees of freedom. The number and the location of the degrees of freedom must be identical for both data sets. The model matching step in the updating procedure provides an approximate solution for this mesh incompatibility.

In a first phase, the corresponding analytical degree of freedom for every experimental degree of freedom is identified. This action defines the set of "active" degrees of freedom. All other analytical degrees of freedom are called "deleted" degrees of freedom:

a.6.3 $$\{ X_F \} = \begin{Bmatrix} \{ X_A \} \\ \{ X_D \} \end{Bmatrix},$$

where: $\{X_A\}$ = active degrees of freedom,
$\{X_D\}$ = deleted degrees of freedom,
$\{X_F\}$ = full set of degrees of freedom.

After the creation of pairs of corresponding analytical and experimental degrees of freedom, experimental mode shapes on the one hand and analytical mode shapes and system matrices on the other hand still have a different dimension. In the second phase of the model matching step this size incompatibility is solved by reduction of the analytical model or by expansion of the experimental model.

Reduction of the analytical model eliminates the deleted degrees of freedom from analytical mode shapes and system matrices. The reduction of mode shapes is quite straightforward. Deleted degrees of freedom are omitted from the mode shapes. Reduction of system matrices is less straightforward. So called reduction techniques must be used. All different reduction techniques have a restricted validity. Reduced system matrices give exact descriptions of the dynamic behaviour of the active degrees

of freedom for one or a few specific frequencies only. Furthermore reduction of system matrices destroys connectivity and as a consequence also the physical intrepretability of the system matrix elements. This fact disables almost completely the use of reduction methods in combination with error localisation and correction methods that are based on local differences between the analytical and experimental data. When used in methods based on more global measures and at specific frequencies where the reduction can be believed to be physically meaningful, reduction of analytical system matrices is however often more favourable than expansion of experimental mode shapes. Section A.6.1.2.2. discusses reduction methods in more detail.

Expansion techniques expand the measured mode shapes to the full set of degrees of freedom of the finite element model. Most of the expansion techniques use data of the finite element model to expand the measured mode shapes. Since the differences between the experimental modal data and the analytical modal data are the base of most of the updating procedures, the expansion of mode shapes should be made with great care in order not to bias the information that is given by the experimental mode shapes. Section A.6.1.2.3. gives a detailed overview of the most often used expansion techniques

A.6.1.2.2. Reduction techniques.

General remarks.

Reduction techniques express the analytical system matrices in terms of the degrees of freedom that correspond with the experimentally measured degrees of freedom only. These degrees of freedom are called "active". The other degrees of freedom ("deleted" degrees of freedom) are eliminated (see equation a.6.3).

Reduction techniques define a relation between the active and the deleted degrees of freedom through a "transformation" matrix $[T_D]$ or $[T_F]$:

a.6.4 $\qquad \{X_D\} = [T_D]\{X_A\},$

a.6.5 $\qquad \{X_F\} = [T_F^*]\{X_A\},$

a.6.6 $\qquad [T_F] = \begin{bmatrix} [\diagdown I_\diagdown] \\ [T_D] \end{bmatrix}.$

$[T_F]$ is used to create the reduced mass $[M^R]$ and reduced stiffness matrix $[K^R]$. For an undamped system following energy relations hold:

a.6.7 $\qquad \{X_F\}'[M]\{X_F\} = \{X_A\}'[M^R]\{X_A\}$ and

a.6.8 $\qquad \{X_F\}'[K]\{X_F\} = \{X_A\}'[K^R]\{X_A\}.$

Substitution of $\{X_F\}$ by $[T_F]\{X_A\}$ gives: