$$a \frac{\partial^2 u}{\partial x^2} + b \frac{\partial^2 u}{\partial x \partial y} + c \frac{\partial^2 u}{\partial y^2} + d \frac{\partial u}{\partial x} + e \frac{\partial u}{\partial y} + f u = g(x,y)$$

$$\Delta = b^2 - 4ac$$

- $\Delta > \emptyset$ $\quad b^2 > 4ac \longleftrightarrow$ IPERBOLICA $\longleftrightarrow$ 2 radici reali $\longleftrightarrow$ EQ. ONDE $\leftarrow$
- $\Delta = \emptyset$ $\quad b^2 = 4ac \longleftrightarrow$ PARABOLICA $\longleftrightarrow$ 1 " " $\longleftrightarrow$ EQ. CALORE $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$
- $\Delta < \emptyset$ $\quad b^2 < 4ac \longleftrightarrow$ ELLITICA $\longleftrightarrow$ $\emptyset$ " "

$$\frac{\partial^2 f}{\partial t^2} - c^2 \frac{\partial^2 f}{\partial x^2} = 0$$

$$u = \frac{\partial f}{\partial t}$$

$$v = \frac{\partial f}{\partial x}$$

$$\frac{\partial^2 f}{\partial x \partial y} \longrightarrow \frac{\partial^2 f}{\partial x \partial t} = \frac{\partial^2 f}{\partial t \partial x}$$

$$v = \frac{\partial f}{\partial x}$$

$$u = \frac{\partial f}{\partial t}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial t}$$

$$\begin{cases} \dfrac{\partial u}{\partial t} - c^2 \dfrac{\partial v}{\partial x} = 0 \\[2ex] \dfrac{\partial v}{\partial t} - \dfrac{\partial u}{\partial x} = 0 \end{cases}$$

$$\begin{bmatrix} \dfrac{\partial u}{\partial t} \\[2ex] \dfrac{\partial v}{\partial t} \end{bmatrix} + \begin{bmatrix} 0 & -c^2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial u}{\partial x} \\[2ex] \dfrac{\partial v}{\partial x} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\underline{U}_t \qquad \underline{\underline{A}} \qquad \underline{U}_x = \underline{0}$$

AUTOVAL

- $\det\left(\underline{\underline{A}}^T - \alpha \underline{\underline{I}}\right) = \emptyset$

$$\det\begin{bmatrix} -\alpha & -1 \\ -c^2 & -\alpha \end{bmatrix} = \emptyset \rightarrow \alpha^2 - c^2 = \emptyset \longrightarrow \| \underline{\underline{\alpha = \pm c}} \|$$

AUTOVETT

- $\left(\underline{\underline{A}}^T - \alpha \underline{\underline{I}}\right) \underline{l} = \emptyset$

$$\begin{bmatrix} -\alpha & -1 \\ -c^2 & -\alpha \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix} \longrightarrow$$

$l_1 = 1$

- $\alpha = c \rightarrow -c\, l_1 - l_2 = \emptyset \longrightarrow l_2 = -c$
- $\alpha = -c \longrightarrow l_2 = c$

Moltiplicando per le equazioni precedenti

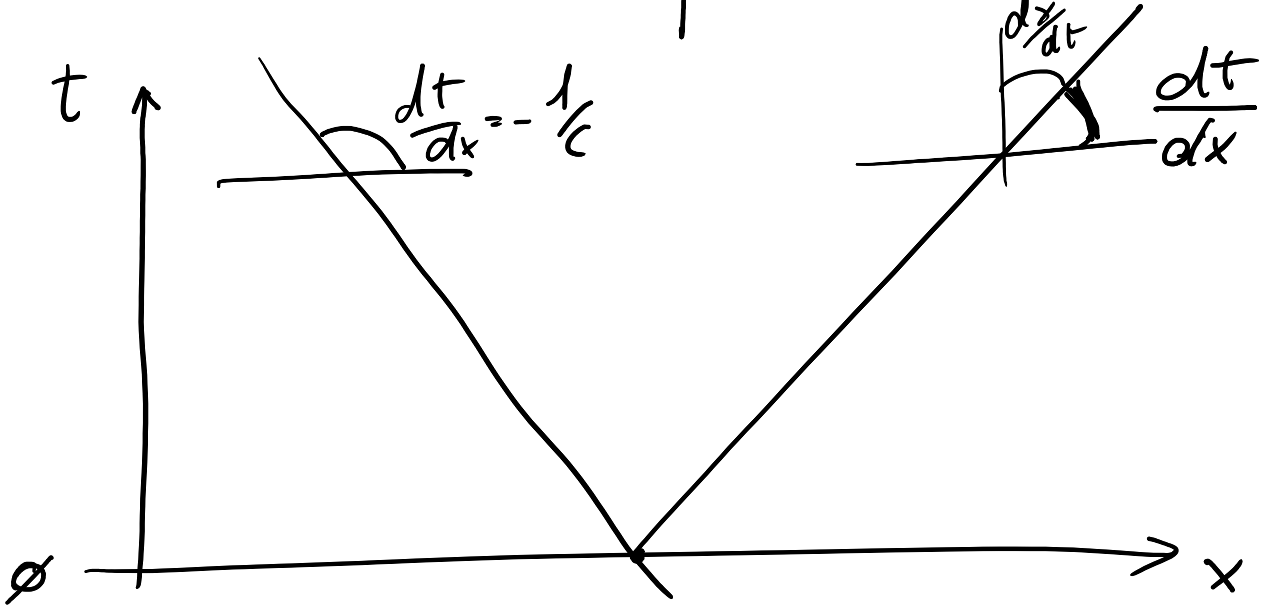
$$1 \cdot \left(\underline{\frac{\partial u}{\partial t} - c^2 \frac{\partial v}{\partial x}} = \emptyset \right) - c \cdot \left(\frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = \emptyset \right)$$

$c = \frac{\partial x}{\partial t}$ $\quad \frac{\partial u}{\partial t} - c \frac{\partial v}{\partial t} = \emptyset$ $\quad\Big|\quad$ $-c = \frac{\partial x}{\partial t}$ $\quad \frac{\partial u}{\partial t} + c \frac{\partial v}{\partial t} = \emptyset$

$$\frac{\partial u}{\partial t} - c\,\frac{\partial v}{\partial t} = \emptyset \qquad \text{su} \qquad \frac{dx}{dt} = c$$

$$\frac{\partial u}{\partial t} + c\,\frac{\partial v}{\partial t} = \emptyset \qquad \text{su} \qquad \frac{dx}{dt} = -c$$

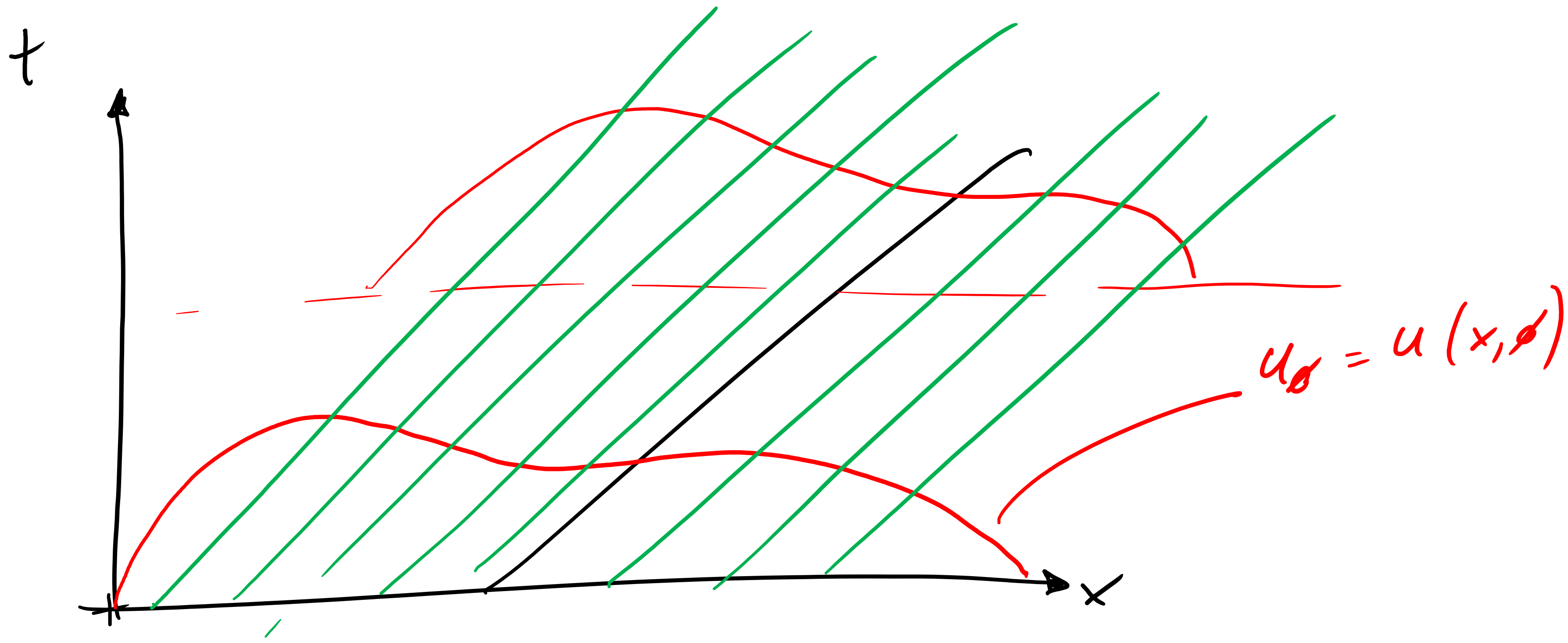
$$\frac{\partial u}{\partial t} - c\,\frac{\partial u}{\partial x} = \emptyset \longleftrightarrow du(x,t) = \frac{du}{dx}\frac{dx}{dt} + \frac{du}{dt} \qquad \longleftrightarrow \frac{dx}{dt} = -c \quad (+c)$$

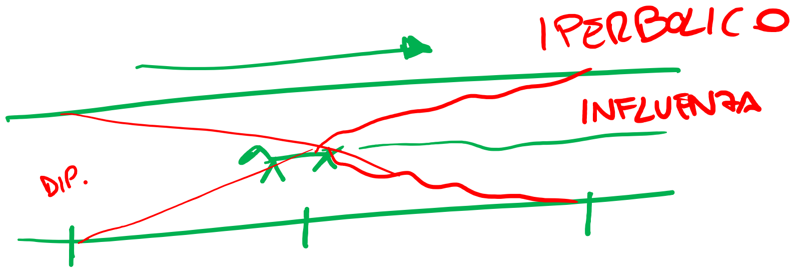


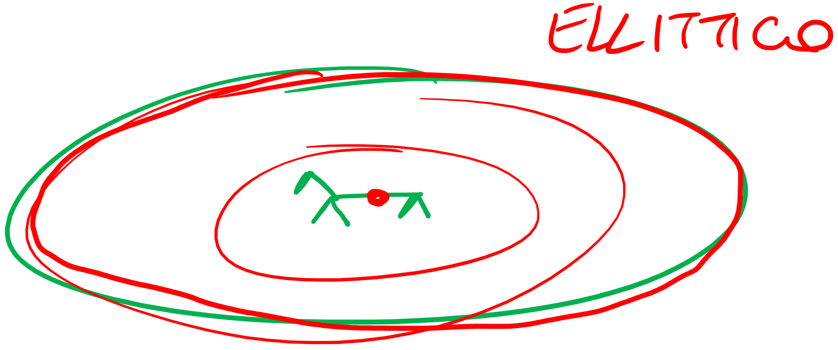
$$\frac{dt}{dx} = \frac{1}{c}$$

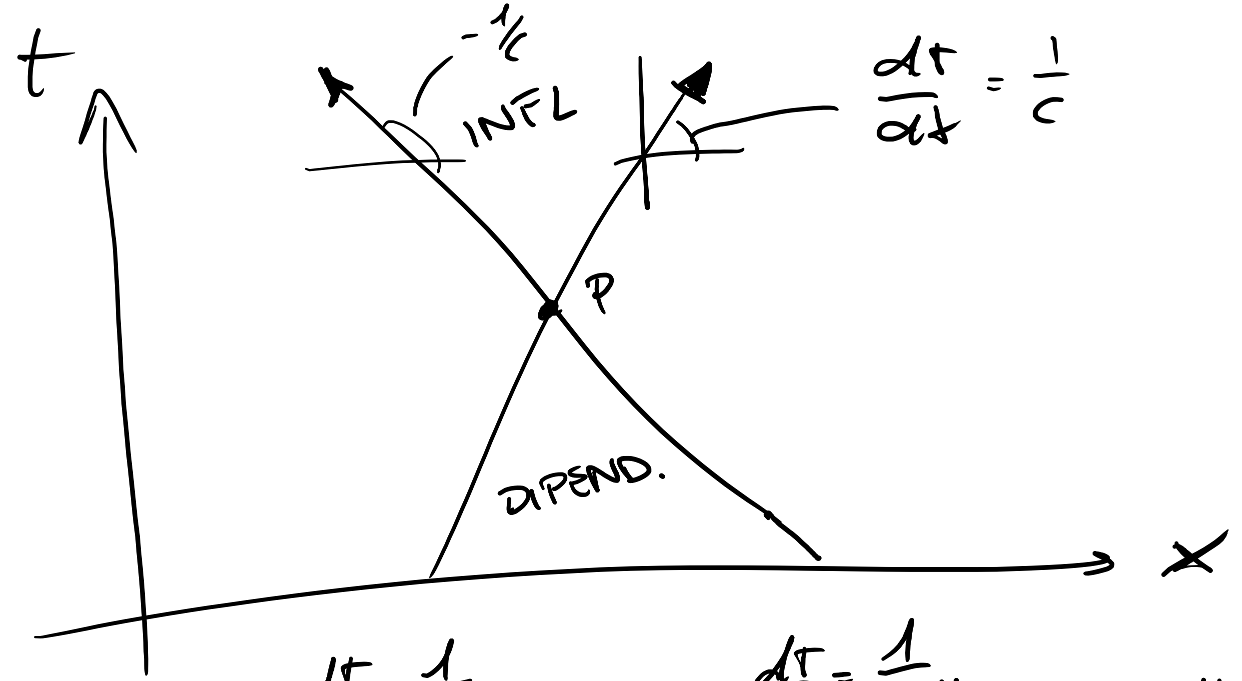
$$u(x, 0) = u_0$$

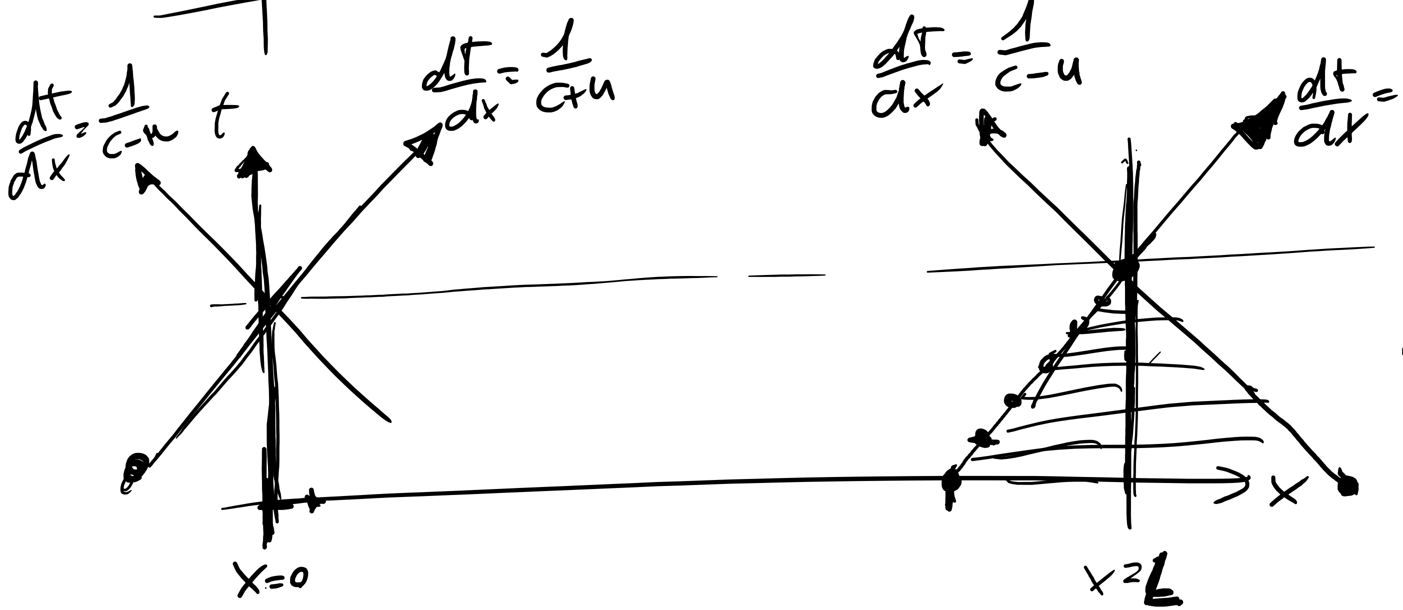
Metodo delle caratteristiche











$c =$ speed of sound

$$\frac{1}{c} \qquad \frac{1}{c-u}$$

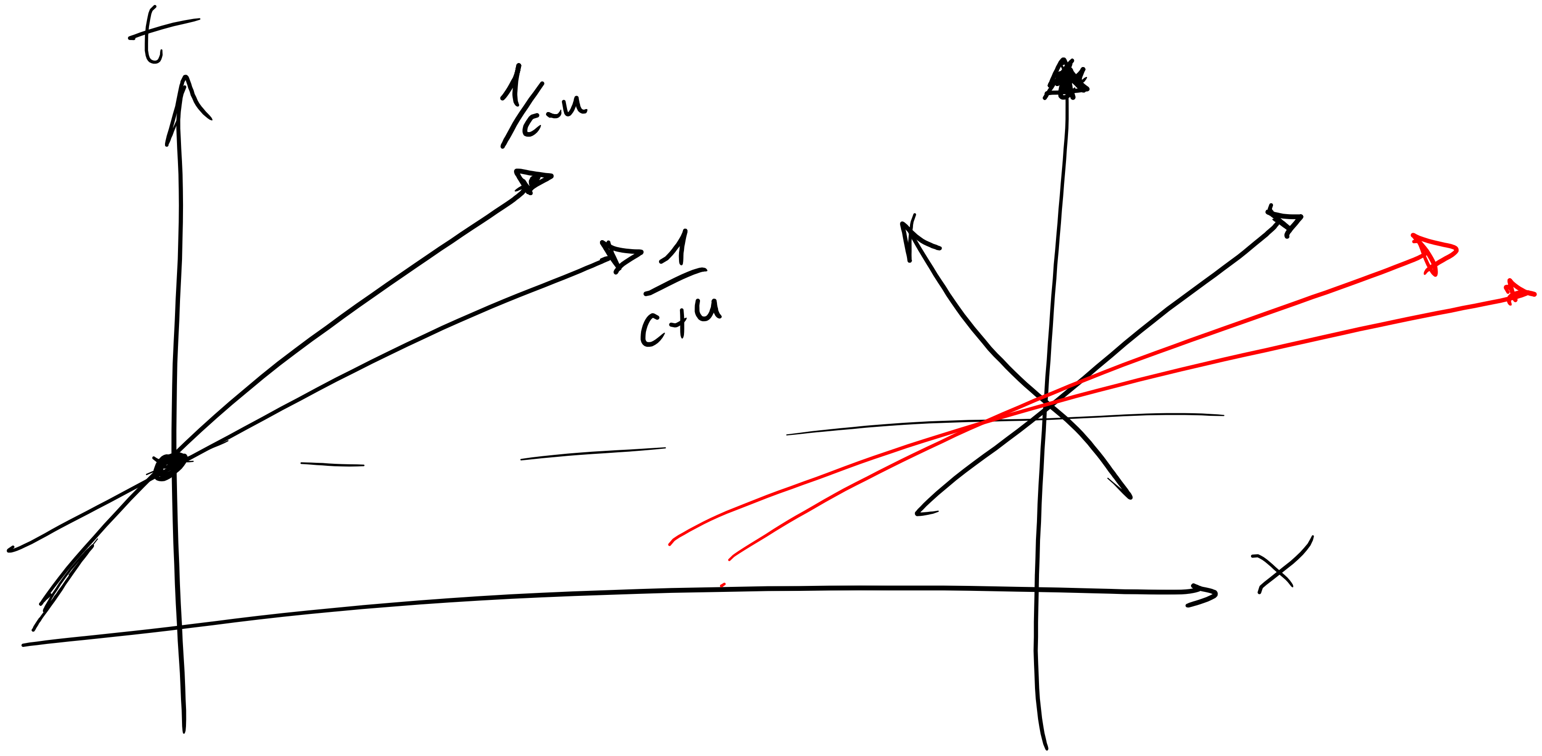
$$-\frac{1}{c} \qquad \frac{1}{c+u}$$

$x \in (0, L)$

$P = \emptyset$

$c+u > 0$

$c-u < \emptyset$


t
1/c-u
1
c+u
x