Caratteristiche di una PDE

Fluidodinamica delle machine

03/11/2020

Equazione delle onde

- Pura avvezione

$$\frac{\partial^2 f}{\partial t^2} - c^2 \frac{\partial^2 f}{\partial x^2} = 0$$

- Sostituendo possiamo scrivere un sistema di equazioni differenziali alle derivate parziali del prim'ordine

$$u = \frac{\partial f}{\partial t}; \qquad \mathrm{v} = \frac{\partial f}{\partial x}; \qquad \frac{\partial u}{\partial t} - c^2 \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = 0$$

ricordando che

$$\frac{\partial^2 f}{\partial x \partial t} = \frac{\partial^2 f}{\partial t \partial x}$$

Soluzione del Sistema

$$\frac{\partial u}{\partial t} - c^2 \frac{\partial v}{\partial x} = 0$$
$$\frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = 0$$

$$\rightarrow \quad \begin{bmatrix} \frac{\partial u}{\partial t} \\ \frac{\partial v}{\partial t} \end{bmatrix} + \begin{bmatrix} 0 & -c^2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial x} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

U_t + A U_x = 0

- Cerchiamo gli autovalori del Sistema imponendo

$$\det(\boldsymbol{A}^T - \alpha \boldsymbol{I}) = 0 \rightarrow det \begin{bmatrix} -\alpha & -1 \\ -c^2 & -\alpha \end{bmatrix} = (\alpha^2 - c^2) = 0 \rightarrow \alpha = \pm c$$

- Gli autovettori invece, si trovano imponendo

$$(\boldsymbol{A}^T - \alpha_{\boldsymbol{1,2}} \boldsymbol{I})\boldsymbol{l} = \boldsymbol{0} \rightarrow \begin{bmatrix} -\alpha & -1 \\ -c^2 & -\alpha \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \rightarrow$$

$l_1 = 1$

$$if\ \alpha = +c \ \rightarrow -c\, l_1 - l_2 = 0 \rightarrow l_2 = -c$$
$$if\ \alpha = -c \ \rightarrow +c\, l_1 - l_2 = 0 \rightarrow l_2 = +c$$

Sostituendo nel Sistema di partenza

$\alpha = +c \, , l_1 = 1 \, , l_2 = -c \;\rightarrow \qquad 1 \cdot \left(\frac{\partial u}{\partial t} - c^2 \frac{\partial v}{\partial x} = 0 \right) - c \cdot \left(\frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} = 0 \right)$

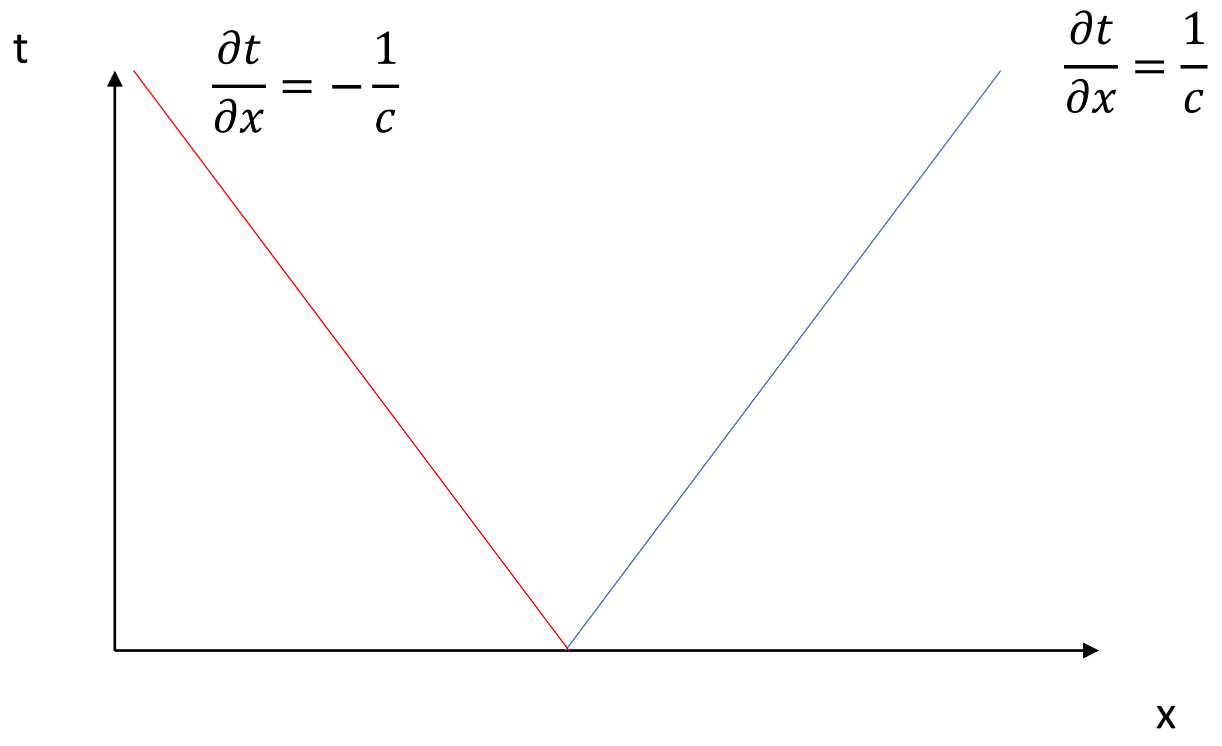
- Da cui

$$\frac{\partial u}{\partial t} - c \frac{\partial v}{\partial t} = 0 \;\; su \;\; \frac{\partial x}{\partial t} = c$$

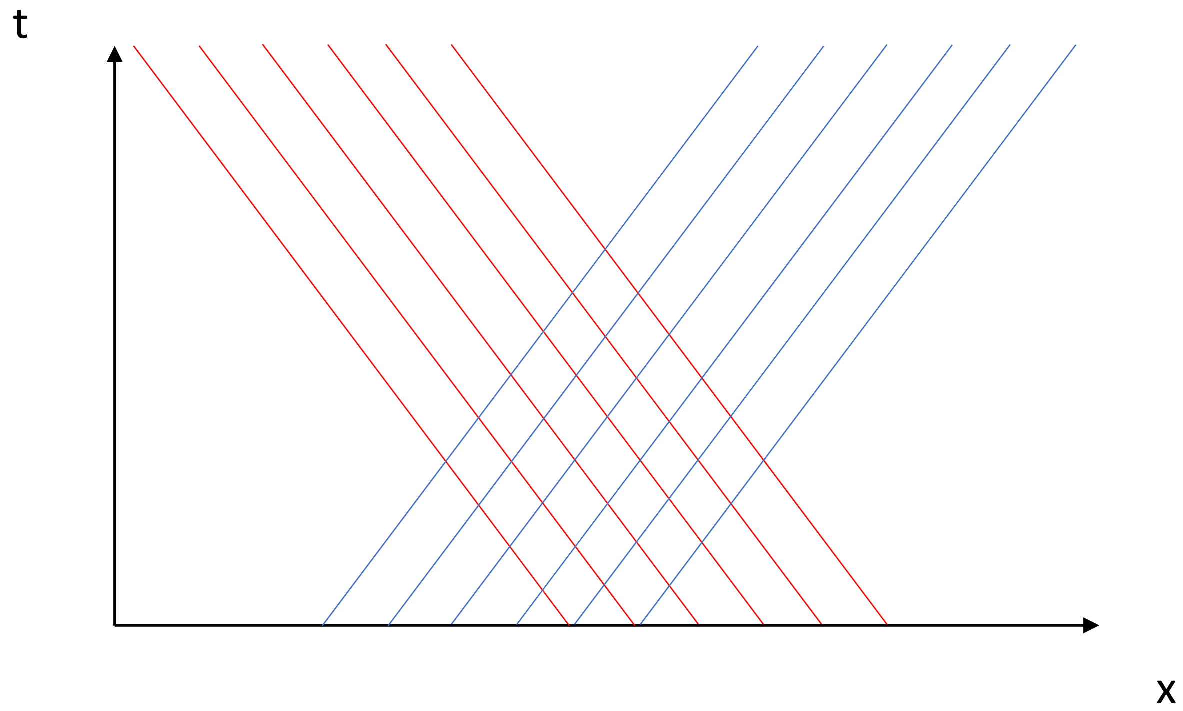
- E analogamente, per $\alpha = -c \, , l_1 = 1 \, , l_2 = +c$

$$\frac{\partial u}{\partial t} + c \frac{\partial v}{\partial t} = 0 \;\; su \;\; \frac{\partial x}{\partial t} = -c$$

Significato fisico



Significato fisico



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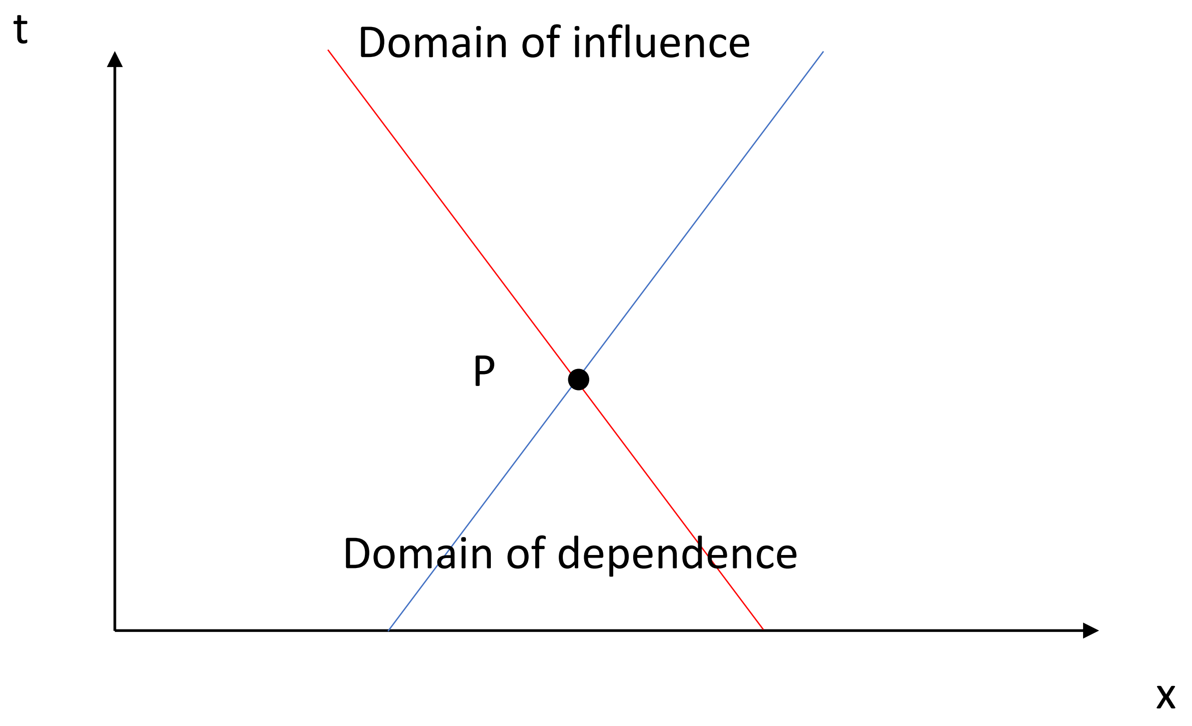
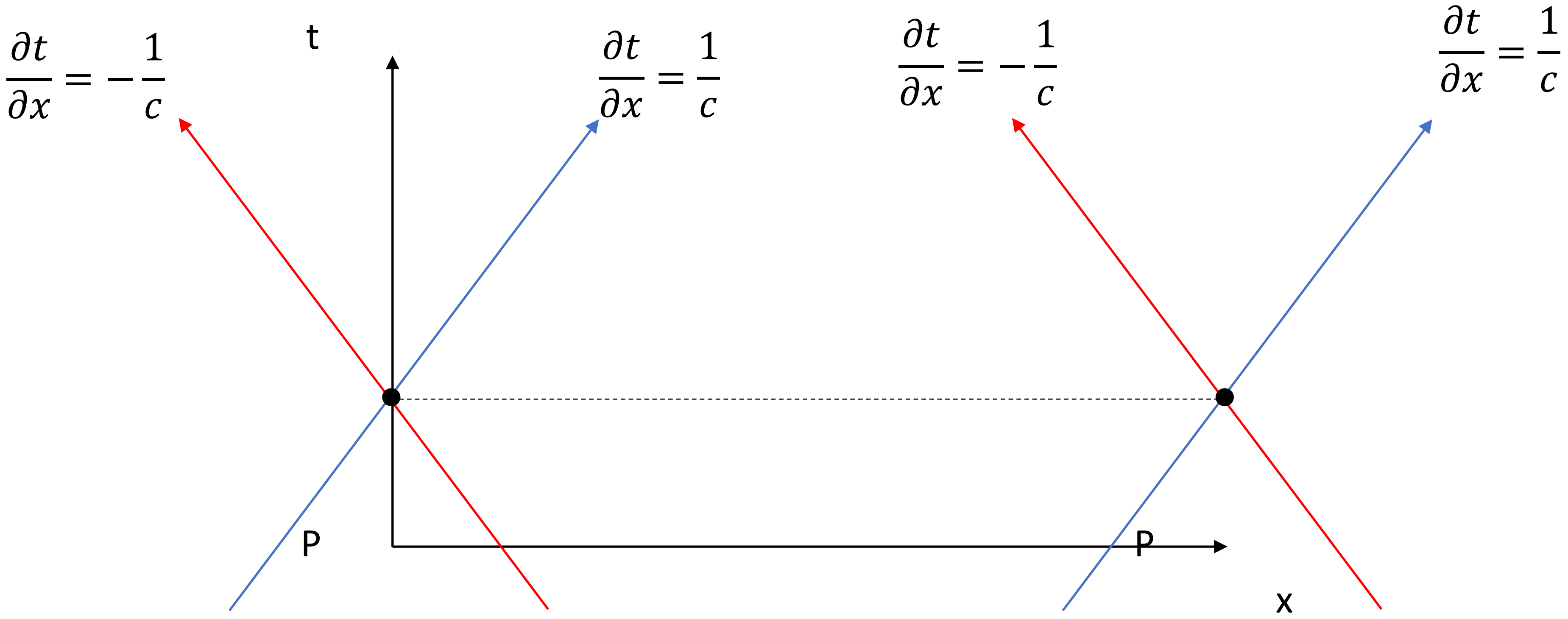
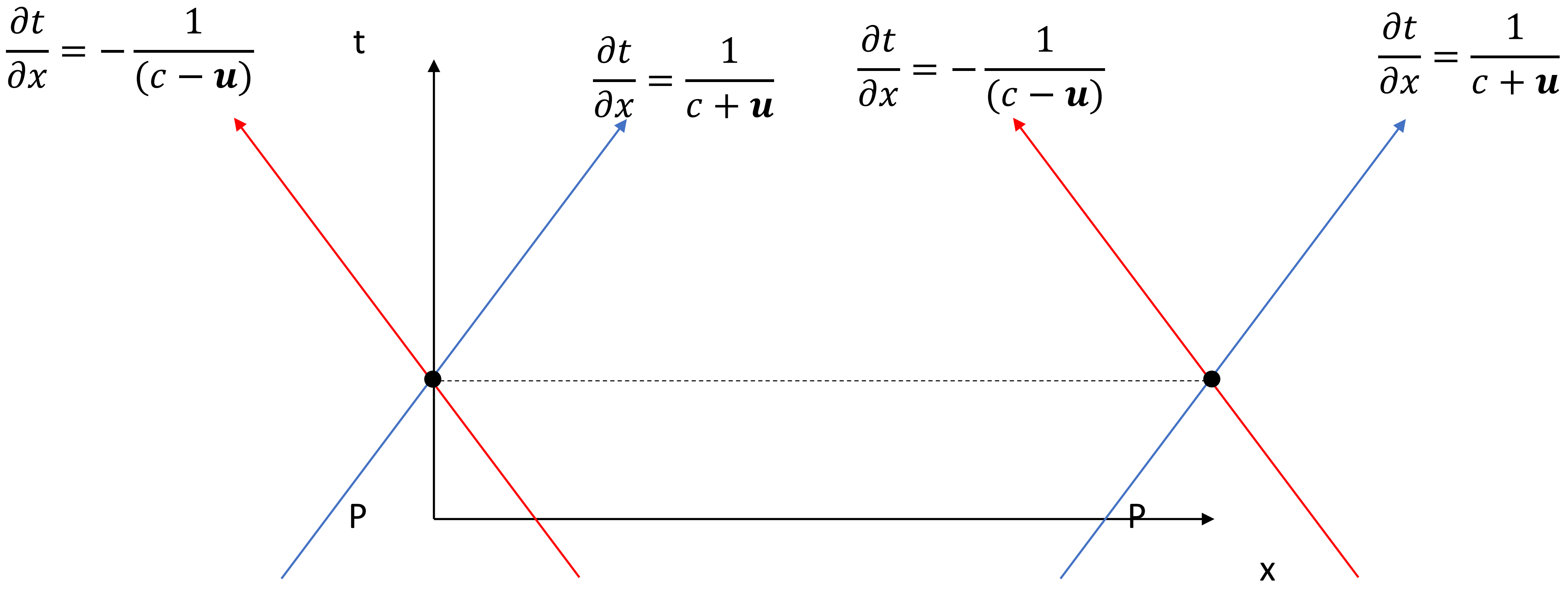


Significato fisico – condizioni al contorno



Significato fisico – condizioni al contorno
Flusso reale



Significato fisico – condizioni al contorno
Flusso reale

$$if\ \boldsymbol{u}(\boldsymbol{x}=\boldsymbol{0})<c\ \rightarrow subsonic$$

$$if\ \boldsymbol{u}(\boldsymbol{x}=\boldsymbol{L})<c\ \rightarrow subsonic$$

$$\frac{\partial t}{\partial x}=-\frac{1}{(c-\boldsymbol{u})}$$

$$\frac{\partial t}{\partial x}=\frac{1}{c+\boldsymbol{u}}$$

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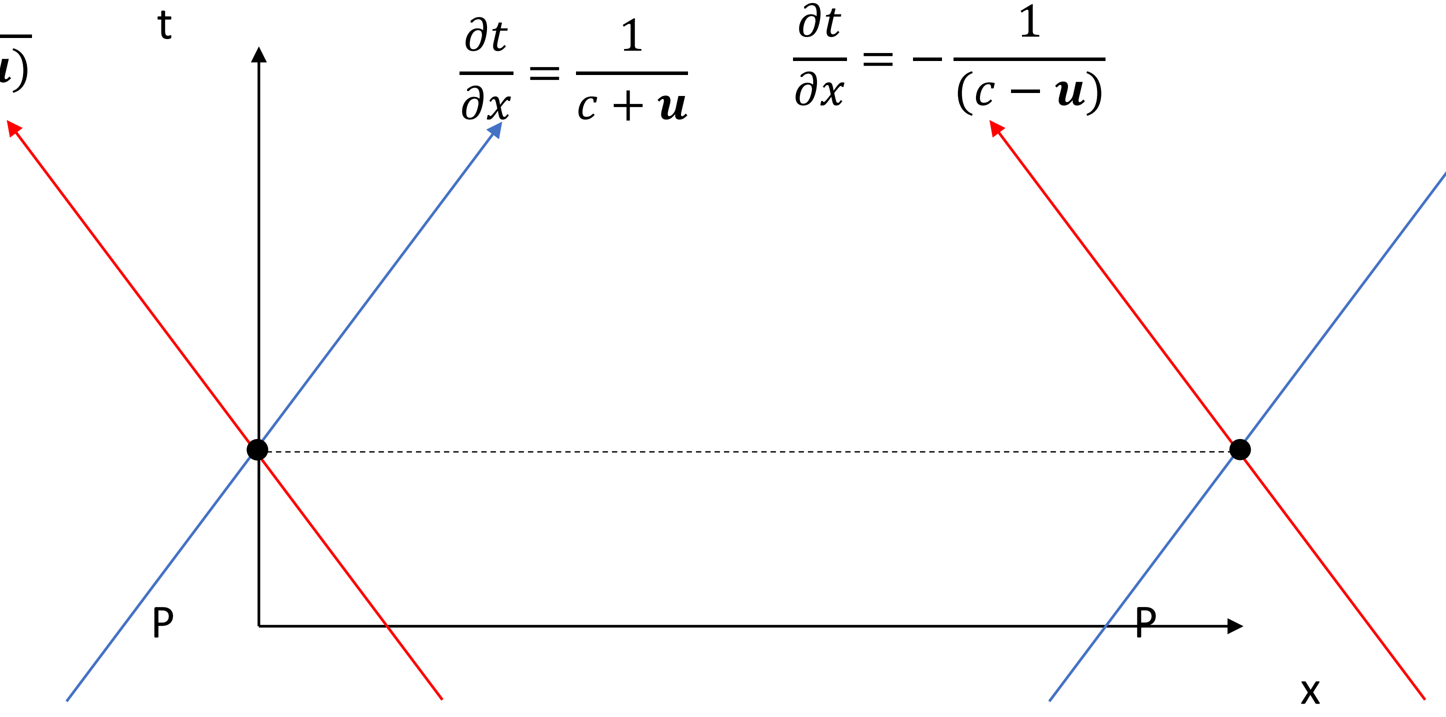

Significato fisico – condizioni al contorno Flusso reale

$if\ \boldsymbol{u}(\boldsymbol{x}=\boldsymbol{0}) > c \ \rightarrow supersonic$

$if\ \boldsymbol{u}(\boldsymbol{x}=\boldsymbol{L}) < c \ \rightarrow subsonic$