# Improving Quality and Efficiency in Home Health Care with Constraint Logic Programming

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# Outline

1. Home Health Care scenario
2. Modeling
   - The first prototype
   - A set variables based model
   - A 'bin-packing' based model
3. Integrating OR based TSP in CLP
   - Travelling Salesman Problem
   - TSP constraint
   - Solving TSP with CLP
   - Integrating in CLP
   - Solving TSP with Lagrangian Metaheuristic
4. Search in the set based model
   - Symmetry Breaking Labeling
   - Threshold Heuristic
   - Clique Partitioning

# Outline

# Home Health Care



- Some categories of patients can stay at home instead of being hospitalized full time
  - e.g. when only frequent blood tests are required
- Nurses visit patients at their homes and provide treatments on place
- More space at hospital for those who actually have to stay there
- Reduced expense for healthcare provider
- Increased comfort for patients

# Challenges in HHC



- Decentralization introduces new factors
- Nurses need to travel (routing)
- Nurses are scattered around (rostering complexity increases)
- Requirements change (new treatments or new patients added, old ones removed), sometimes suddenly (sick leaves of personnel)

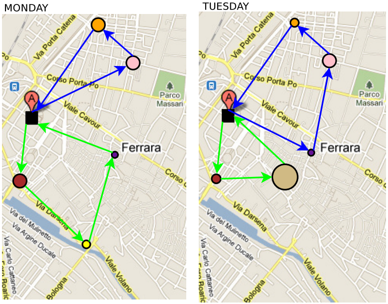
# Ferrara ASL and HHC



- 12 nurses
- Around 60 patients and 150 services per day
- Services lasting from 15 to 60 minutes
- Often more than one service per patient per day
- Nurses currently assigned 'by hand' according to zone subdivision of served area

# Goal

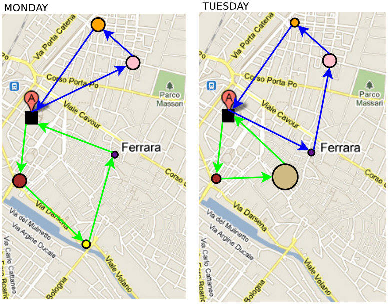
- Build a full optimized scheduling and routing plan, i.e.:
- For each day of the week:
  - Assign daily service schedule to each nurse
  - Suggest an order visiting patients which minimizes travel time (Traveling Salesman Problem)
- Some analogies with [Period Capacitated] Vehicle Routing Problem [Toth & Vigo, 2001; Christofides & Beasley, 2006]



# Requirements



- Constraints:
  - Therapeutic schedule must be followed
  - Day workload (including travel time) limited by contract
- Optimization:
  - Balanced week workloads
  - ‘Fixed’ assignment (as much as possible) of nurses to patients (‘loyalty’)



# Potential impact

*LAPS CARE, a system that was developed in 2002, uses operations research modeling to eliminate the manual planning of home care unit assignments. More than 200 units/organizations in Swedish municipalities use LAPS CARE each day to plan staff scheduling and routing for 4,000 home care workers. The system has increased operational efficiency by 10-15 percent; this corresponds to an annual savings of 20-30 million euros. In addition, the quality of home care for elderly citizens has improved.*

[Eveborn & al., 2009]

# Outline

# Input



- Services, terms whose structure is:

```
service(treatment,patient,duration,delta).
```

- For example:

```
service(28,1,52,2).
service(28,1,52,2).
...
service(30,3,22,1).
```

- Additional parameters

```
days_per_week(DaysPerWeek).
n_nurses(NNurses).
minutes_per_day(MinutesPerDay).
```

# Decision variables

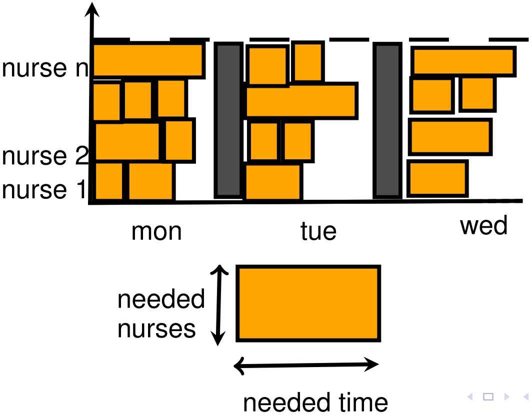
- List of assignments, terms whose structure is:

```
assignment(
  treatment,patient,duration,delta,nurse,day
).
```

- Each service has to be assigned to a specific nurse (field nurse) in a specific day (field day)
- In a week: *NServices* services, *NNurses* nurses available, *DaysPerWeek* days
- List *Nurses* of length *NServices*, variables with domain 1..*NNurses*
- List *Days* of length *NServices*, variables with domain 1..*DaysPerWeek*

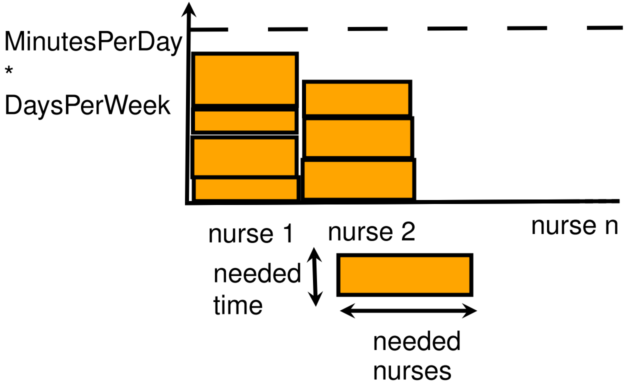
# Cumulative (1/3): all the nurses on all the weekdays

- *NNurses* on resources axis, *MinutesPerDay* $*$ *DaysPerWeek* on time one
- Dummy activities to avoid dayworkload overlap
- Not considering travel time (same applies to following kinds)



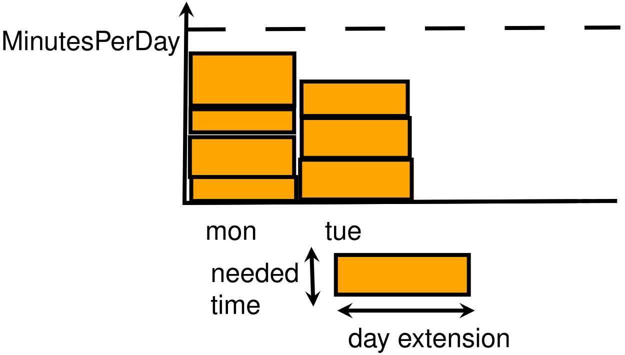
# Cumulative (2/3): all the nurses on the whole week

- Analogously:



# Cumulative (3/3): weekly schedule for each nurse

- For each nurse, *MinutesPerDay* resources every day, *DaysPerWeek* days on time axis
- We need to already know which services are assigned to the respective nurses' weekly schedules



# Spacing in therapeutic scheduling (1/2)

- Some services need to be provided more than once per week with a certain minimum of days (delta) between them

```
spacing([],[]).
spacing([Service|Services],[_|Days]):-
    Service = service(_,_,_,Delta),
    var(Delta),!,
    % this service doesn't need spacing
    spacing(Services,Days).
spacing([Service|Services],[Day|Days]):-
    spacing_loop(Service,Services,Day,Days),
    spacing(Services,Days).
```

# Spacing in therapeutic scheduling (2/2)

```
spacing_loop(_,[],_,[]).
% Same service => impose distance Delta
spacing_loop(Service,[Service|Services],
    Day1,[Day2|Days]):-!,
    Service = service(_,_,_,Delta),
    Day1+Delta #=< Day2,
    spacing_loop(Service,Services,Day1,Days).
% Otherwise, recurse
spacing_loop(Service,[_|Services],
    Day1,[_|Days]):-
    spacing_loop(Service,Services,Day1,Days).
```

# Loyalty (1/2)



- Loyalty as a penalization function
- For each patient, count the number of distinct nurses providing them a service (total function is the sum)

```
loyalty([],[],[0]).
loyalty([Service|Services], [Nurse|Nurses],
    [Count|Loyalties]) :-
    Service = service(_,P,_,_),
    same_index_del(Services,2,P,_,ServiceRest,
      Nurses,NursesPatient,NursesRest),
    count_different([Nurse|NursesPatient],Count),
    loyalty(ServiceRest,NursesRest,Loyalties).
```

# Loyalty (2/2)



- Each set has all the Nurses as upper bound
- # is cardinality constraint

```
count_different([_],1):-!.
count_different(L,Count):-
    n_nurses(NNurses),
    % we actually use intset/3
    Set :: [] ..[1,2,...,NNurses],
    all_members(L,Set),
    #(Set,Count).

all_members([],_).
all_members([H|T],Set):-
    H in Set,
    all_members(T,Set).
```

# Optimization and search (1/2)



- To obtain balanced weekly workloads, minimize the maximum
- Objective function: sum of loyalty (penalty) and biggest weekly workload

```
maxlist(WorkLoadNurses,MaxWorkLoad),
F #= LoyaltyFun+MaxWorkLoad,
minimize((
  labeling(Nurses),
  dvar_range(LoyaltyFun, LoyaltyFun, _),
  % impose the cumulative 3
  scheduling_per_nurse(NNurses,Assignment,
    MinutesPerDay),
  labeling(Days),
  workload(Services,Days,Nurses,NNurses,
    DaysPerWeek,WorkLoadMat,MinutesPerDay),
  sum_lists(WorkLoadMat,WorkLoadNurses)),
  F).
```

# Optimization and search (2/2)



- Before assigning the services to specific days we can compute a bound on the week workloads
- Lower bound on travel time imposing that all the activities are scheduled in one day (only one TSP instead of multiple ones)

```
...
scheduling_per_nurse(...),

workload(Services,Ones,Nurses,NNurses,
  DaysPerWeek,WorkLoadMatBound,_),
sum_lists(WorkLoadMatBound,WorkLoadNursesBound),
maxlist(WorkLoadNursesBound,LB),
MaxWorkLoad #>= LB,

labeling(Days)
...
```

# Problem: 'split' workload

- Assignment of services to days necessary to calculate travel time
- Week workload depends on both labelings

```
workload(Services,Days,Nurses,NNurses,
  DaysPerWeek,WorkLoadMat,MinutesPerDay):-
  %(for each Nurse,
  %for each WorkloadNurse in WorkloadMat do
  %  (for each Day,
  %    for each WorkLoad in WorkLoadNurse do
% find the services assigned to that nurse in that days
% sum their service times and put the sum in ServiceTime
% put the visited patients (without duplicates) in ListP
      solve_tsp(ListP, TravelTime),
      Workload #= TravelTime + ServiceTime,
      Workload #=< MinutesPerDay
  %  )
  %).
```

# Problem: symmetries



- To our purpose, nurses are indistinguishable, swapping the entire weekly schedule of two nurses we obtain an equivalent solution (permutations: many)

| Nurse | Mon | Tue | ... |
|---|---|---|---|
| 1 | 1, 4, 9 | 26, 45 | ... |
| 2 | 6, 7 | 52, 75, 90 | ... |
| ... | ... | ... | ... |

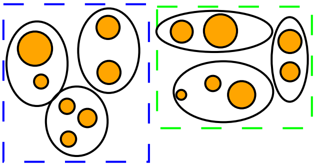
| Nurse | Mon | Tue | ... |
|---|---|---|---|
| 1 | 6, 7 | 52, 75, 90 | ... |
| 2 | 1, 4, 9 | 26, 45 | ... |
| ... | ... | ... | ... |

- Under some conditions (e.g. respecting spacing) the same applies to day swapping
- The assignment of services to nurses and days shouldn't be tightly coupled to the order of the services *in* each day: the order of patients is decided by the TSP, and it's irrelevant in which order services to the same patient in the same day are provided

# Set variables based model



- Partition the set of services into $NNurses * DaysPerWeek$ subsets
- Making sure each subset is feasible (with respect to *MinutesPerDay*), taking *DaysPerWeek* ones for each nurse we surely obtain a feasible (wrt to *MinutesPerWeek*) weekly schedule
- Some symmetries are eliminated (order of services into subsets and, at this level, of subsets into weekly schedules, are not considered)
- $NNurses = 2$, $DaysPerWeek = 3$:



# Subsets and constraints on workload

```
%(for each ElementOfSOS in SubsetsOfServices do
  ElementOfSOS subset SetOfServices,
  weight(ElementOfSOS, DurationArray, ServiceTime),
  TravellingTime #>= 0,
  traveltime(ElementOfSOS, PatientArray, TravellingTime),
  TotalTime #= ServiceTime  + TravellingTime
%)
all_disjoint(SubsetsOfServicesList),
all_union(SubsetsOfServicesList, SetOfServices),
```

# Constraints on therapeutic scheduling (1/2)

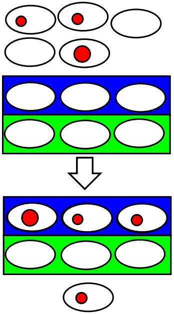


- Services to the same patient which require spacing can't be in the same subset
- Associate each service (which can have spacing issues) with a variable which represents the identifier of the subset in which the service is
- For each group of services which can't be in the same subset, impose that their subset identifiers are different
- Further: impose order (to eliminate some symmetries)

# Constraints on therapeutic scheduling (2/2)



- We don't want a patient to be visited more than once (i.e. by different nurses) in the same day
- Necessary condition is that each patient is contained in *DaysPerWeek* or less subsets
- Example with $NNurses = 2$, $DaysPerWeek = 3$ and same patient in 4 subsets:



# Symmetry breaking constraint

- Swapping two subsets, still same partitioning

```
[[1, 2], [3, 4] ...]
[[3, 4], [1, 2] ...]
```

- Lexicographic order constraint
- Order key is sum of service identifiers

```
[[1, 2], [3, 4]...] -> [3, 7, ...] (ok)
[[3, 4], [1, 2]...] -> [7, 3, ...] (no)

%(for each ElementOfSOS in SubsetsOfServicesList,
% for each ElementOfSum in SumOfServices do
    weight(ElementOfSOS, SetOfServicesArray,ElementOfSum)
%)
ordered(>=,SumOfServices),
```

# Problem: sets not as developed as fd



- Example: propagation in global constraints

```
A::[]..[1,2,3], weight(A, w(1,1,100), 1).

A = A{([] .. [1, 2, 3])
...
Yes

A::[]..[1,2,3], bpweight(A, w(1,1,100), 1).

A = A{([] .. [1, 2])
...
Yes
```

- In ECLiPSe no support for symmetry breaking during search (sbds)
- No equivalent of *search*/6 generic predicate

# A 'bin-packing' based model



- Each subset could be seen as a bin
- Objective, however, is not minimizing number of bins
- *bin_packing* constraint [Shaw, 2004] available in recent versions of ECLiPSe

# Cumulative for bin packing (1/2)

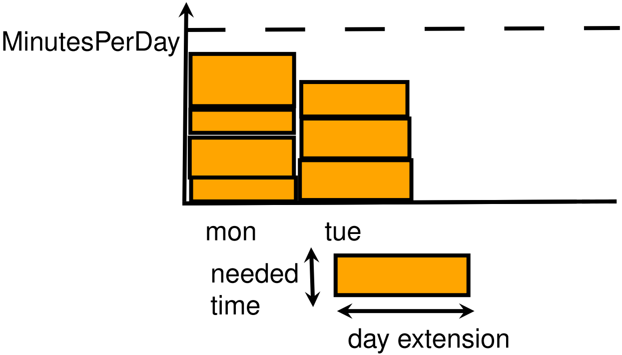


## Cumulative

$cumulative(+StartTimes, +Durations, +Resources, ++ResourceLimit)$
StartTimes: List of start times for tasks (integer variables or integers)
Durations: List of duration for tasks (integer variables or integers)
Resources: List of resource usages by tasks (integer variables or integers) ResourceLimit: Maximum amount of resource available (integer)

# Cumulative for bin packing (2/2)






- No information on each column (bin) resource usage (occupied space)
- Variable item size is allowed

# Bin packing constraint



## Bin packing

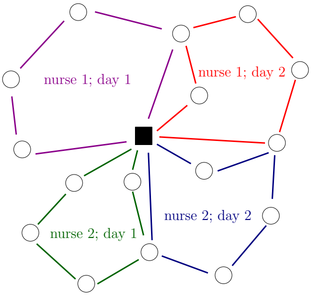
$bin\_packing(+Items, ++ItemSizes, +BinLoads)$
packing M items into N bins, each bin having a load
Items: A collection of M variables or integers (domain/value between 0 and N)
ItemSizes: A collection of M non-negative integers
BinLoads: A collection of N variables or non-negative integers

- Information on load of each bin
- Items' sizes are fixed (can't add travel time as activity of variable duration)
- Constraint still under development

# Outline

# Travelling Salesman Problem (TSP)

- All patients must be visited once
- Route must start and finish at the hospital
- Minimizing travel time



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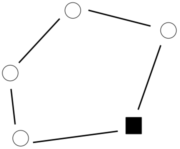

# TSP constraint



```
suspend(+Goal, +Prio, +CondList)
```

Suspend the Goal (a callable term) and wake it with priority Prio as soon as one of the conditions in CondList occurs.

```
:- dynamic cached_tsp/2.
cache_tsp(Nodes, Workload) :-
    cached_tsp(Nodes, Workload), !.
cache_tsp(Nodes, Workload) :-
    solve_tsp(Nodes, Workload),
    assertz(cached_tsp(Nodes, Workload)).
```

Read or add specified clause at the end of the dynamic procedure to the database.

# TSP constraint

```
constraint_on_tsp(Set, PatientSet, TravelTime) :-
    cache_tsp(PatientSet, TravelTimeLB),
    (ground(Set) ->
        TravelTime #= TravelTimeLB
    ;
        TravelTime #>= TravelTimeLB,
        suspend(
            constraint_on_tsp(Set, PatientSet, TravelTime),
            10, Set->add)
    ).
```

# Solving TSP with CLP

```
solve_tsp( PatientList, TravelTime):-
    length(PatientList,N),
    length(List,N),
    List #:: PatientList,
    alldifferent(List),
    append([0|List], [0], ListHospital),
    compute_travel_time(ListHospital, TravelTime),
    minimize(labeling(List), TravelTime).
```

# Solving TSP with CLP



```
solve_tsp( PatientList, TravelTime):-
    length(PatientList,N),
    length(List,N),
    List #:: PatientList,
    alldifferent(List),
    append([0|List], [0], ListHospital),
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    minimize(labeling(List), TravelTime).
```
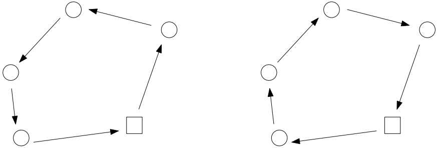

Symmetry breaking: from the hospital you can only go to the first half of patients



# Integrating in CLP



```
remote_connect(?Address, ?Peer, ?InitGoal)
remote_disconnect(+Peer)
```

Initiates a remote attachment at address Host/Port, the remote side must return control to the ECLiPSe side. Peer is the name of the control connection.

```
write_exdr(eclipse_to_java, Elements),
flush(eclipse_to_java),
read_exdr(java_to_eclipse, Result),
```

The term Elements is written onto the output stream eclipse_to_java. Buffered output may need to be flushed. Finally, a term Result is read from the input stream java_to_eclipse.

# Solving TSP with Lagrangian Metaheuristic



The proposed mathematical formulation for the symmetric TSP is as follows:

$$\min \sum_{e \in E} c_e x_e$$

subject to

$$\sum_{e \in \delta(i)} x_e = 2, \qquad \forall i \in I$$

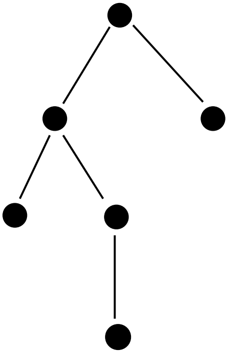
$$\sum_{e \in E(S)} x_e \leq | S | -1, \quad \forall\, S \subset I\,,\ | S | \leq \frac{1}{2} \mid I \mid$$

binary variable

$$x_e = \begin{cases} 1 & \text{if patient } j \text{ is visited immediately after } i \\ 0 & \text{otherwise} \end{cases} \quad , \ \forall e = (i,j) \in E$$

Lagrangian Relaxation [Held & Karp, 1971]

- All subcycle can be avoided constructing the solution $x$ as a 1-tree.
- A 1-tree is a tree on the graph induced by patients $\{i_1, \ldots, i_n\}$ plus two incident edges to the Hospital $i_0$.



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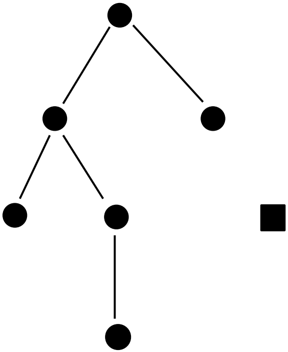

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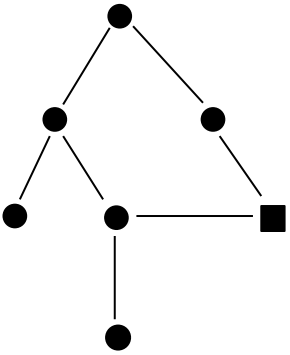


# Solving TSP with Lagrangian Metaheuristic



The Lagrangean Dual problem obtained from TSP is:

$$\max_{u \in \mathbb{R}^n} L(u)$$

where

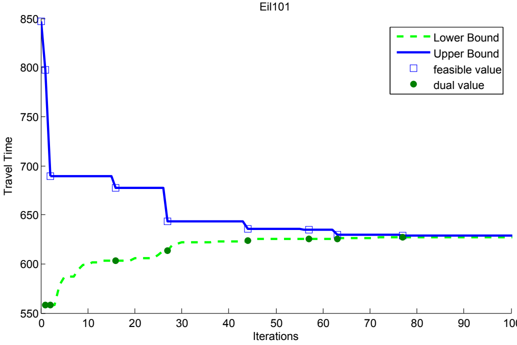
$$L(u) \quad = \quad \min_{x\ 1-tree} \sum_{e \in E} c_e x_e + \sum_{i \in I} u_i (2 - \sum_{e \in \delta(i)} x_e)$$

Its subgradient direction is $\gamma = \left(2 - \sum_{e \in \delta(i)} x_e\right)_i$

- Subgradient Optimization converges to the optimal iteratively.
- Finding a optimal solution of the dual problem is $O(n^2)$.

## Lagrangian Metaheuristic

- A heuristic obtains a feasible solution from the dual variable.
- Improving the upper bound ensures algorithm's convergence.



# Outline

# Symmetry Breaking Labeling

Symmetry Breaking on Sets:

- $Set_1 = [s_1, s_2], Set_2 = [s_3, s_4], Set_3 = [s_5]$.
- $Set_1 = [s_1, s_2], Set_2 = [s_5], Set_3 = [s_3, s_4]$.

How to avoid it within the search labeling?

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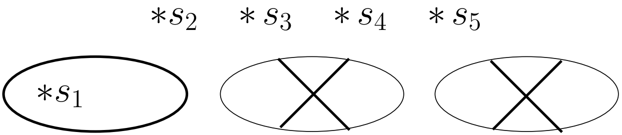
How to avoid it within the search labeling?
[Meseguer & Torras, 1999]

- Assign the first service to the first set.
- Assign the next services to the opened sets or open a new one.



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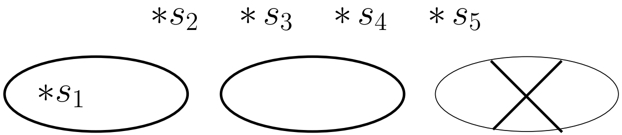

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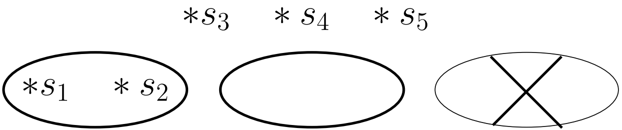

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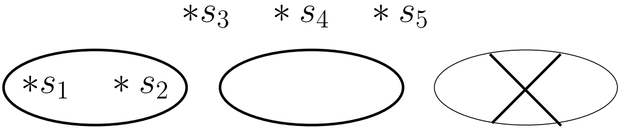



Advantage: reduce search effort!

# Threshold Heuristic

A service is inserted into the closest set, that is, if distance between their patients is lower than some threshold:

- Try to insert it into the closest sets.
- Try to open new one.
- Try to insert into the other ones.

# Threshold Heuristic

A service is inserted into the closest set, that is, if distance between their patients is lower than some threshold:

- Try to insert it into the closest sets.
- Try to open new one.
- Try to insert into the other ones.

Challenge: Parameter Threshold!

# Threshold Heuristic



Given a list of elements:

```
Key-(Set-Index)
```

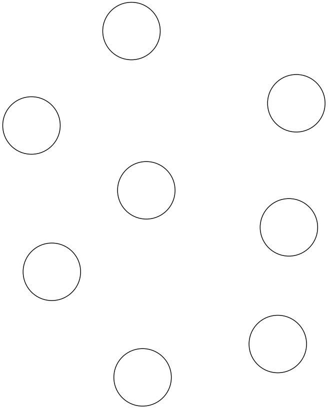
The sort is done according to the value of the Key dividing the list into two list the closest ones and the farthest ones.

```
make_sortable_dist([Set|LOpenSubs],Index,Act,
        Patient,[Key-(Set-Index)|Sortable]):-
    set_range(Set,SetLB,_),
    compute_distance_key(SetLB,Act,Patient,100000,Key),
    Index1 is Index + 1,
    make_sortable_dist(LOpenSubs,Index1,Act,
        Patient,Sortable).
```

# Clique Partitioning



Sets of services:



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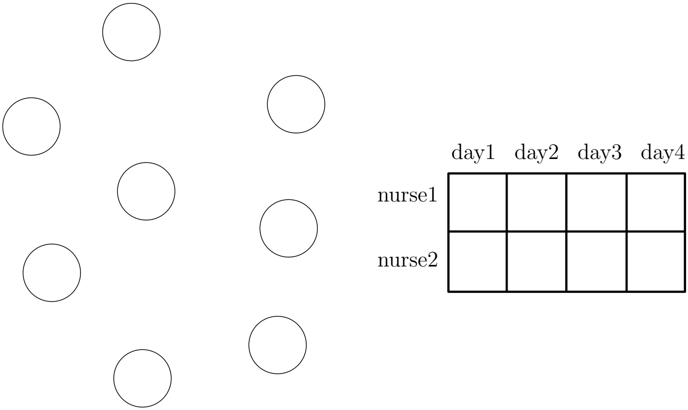

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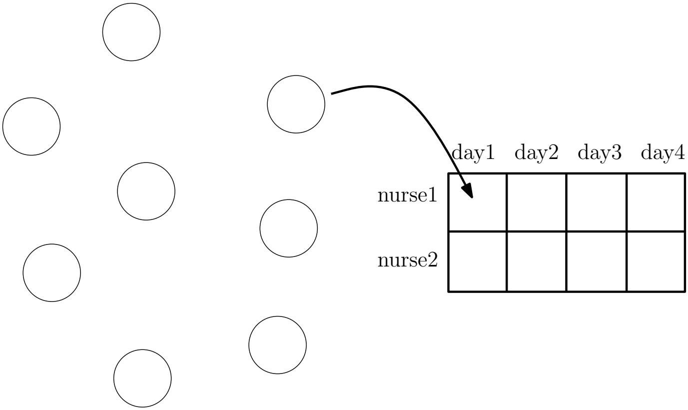
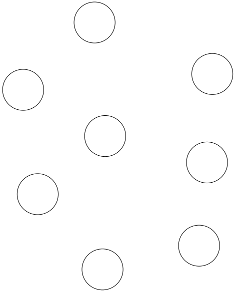


# Clique Partitioning



For each nurse, *Ndays* sets should be assigned.



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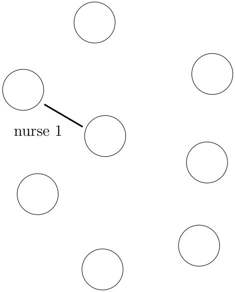

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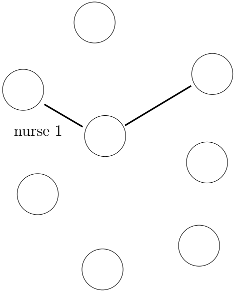

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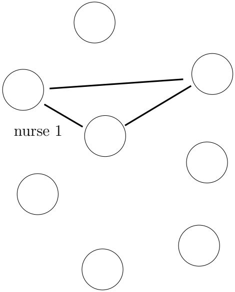

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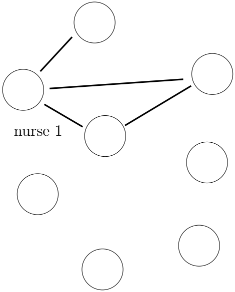

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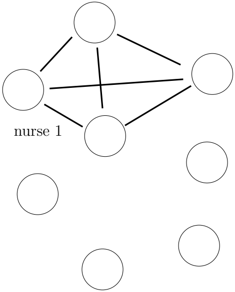

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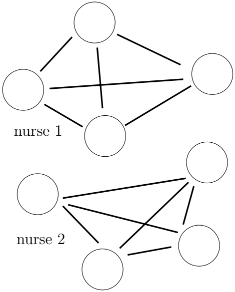
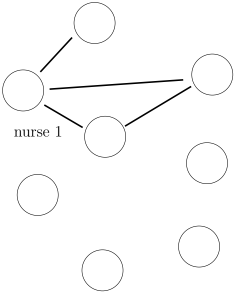



This assignment can be seen as a variation of Clique Partitioning, [Grötschel & Wakabayashi, 1990].

# Clique Partitioning

Variable: *Arc* is a boolean symmetric matrix having zero on the diagonal.
Arc[i,j]=1 arguments that the sets *i* and *j* are assigned to the same nurse.




```
ArcIJ + ArcIH - ArcJH #=< 1,
```

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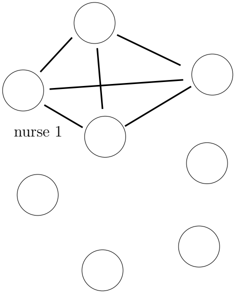


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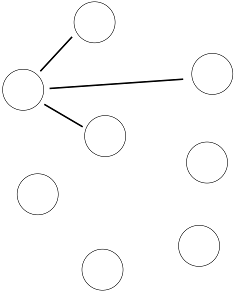

```
occurrences(1, Arc[i, 1..Nsets],  Ndays-1),
```

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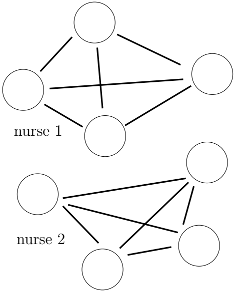


```
occurrences(1, Arc, (Ndays-1)*Nsets),
```

# Clique Partitioning

```
suspend(
    loyalty_cell(ArcIJ, PatSetI, PatSetJ, LoyaltyIJ) ,
    5, ArcIJ->inst ),


loyalty_cell(ArcIJ, PatSetI, PatSetJ, LoyaltyIJ) :-
    (ArcIJ==0 ->
        #(PatSetI /\ PatSetJ, LoyaltyIJ)
    ;
        LoyaltyIJ #= 0
    ).
```

# Improving Quality and Efficiency in Home Health Care with Constraint Logic Programming

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