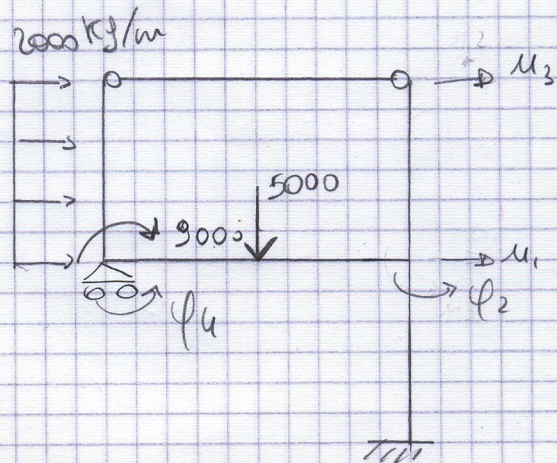
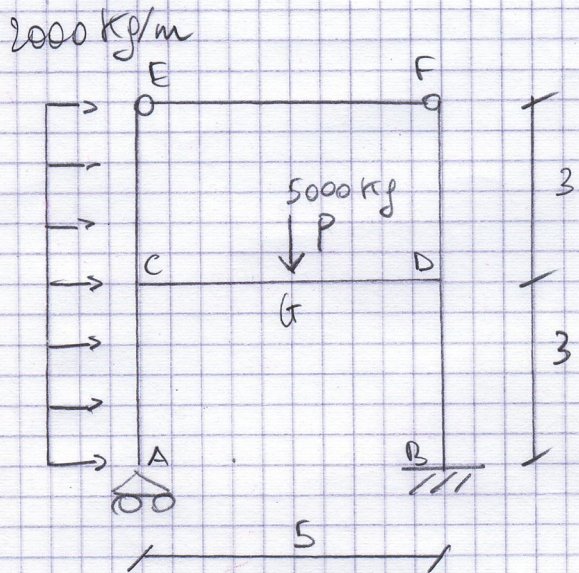


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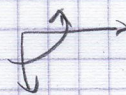
1

PROVA (A)

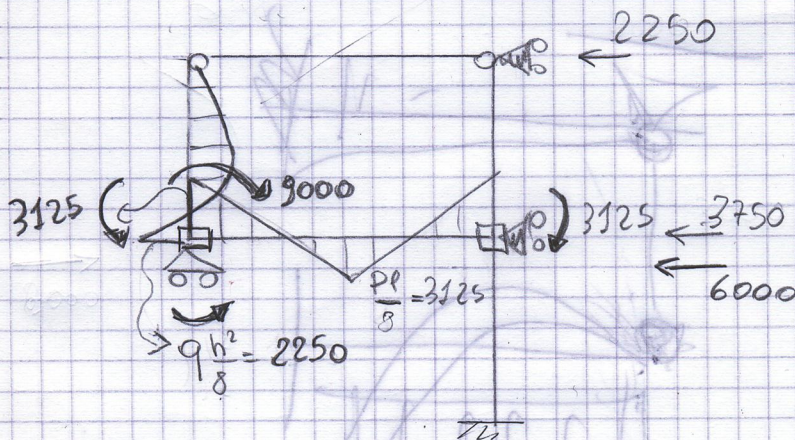


A)

Interno 0



4 spostamenti



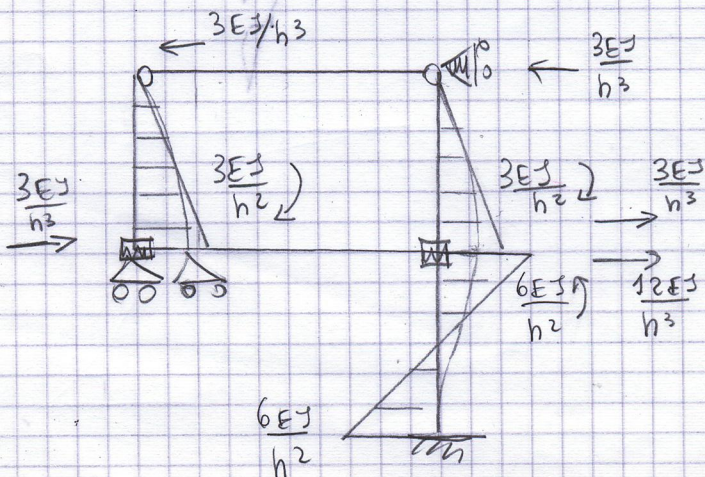
$$K_{10} = -9750$$

$$K_{20} = -3125$$

$$K_{30} = -2250$$

$$K_{40} = -3625$$

Interno 1



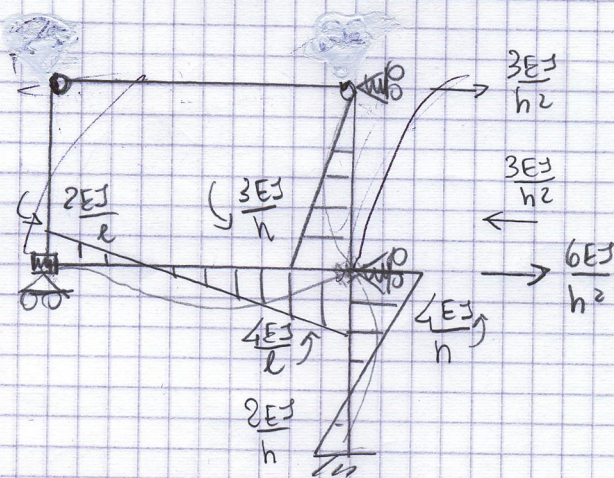
$$K_{11} = \frac{18EJ}{h^3} = \frac{18}{27} EJ = \frac{2}{3} EJ$$

$$K_{12} = \frac{3EJ}{h^2} = \frac{EJ}{3}$$

$$K_{13} = -\frac{6EJ}{h^3} = -\frac{6}{27} EJ = -\frac{2}{9} EJ$$

$$K_{14} = -\frac{3EJ}{h^2} = -\frac{EJ}{3}$$

Systeme 2



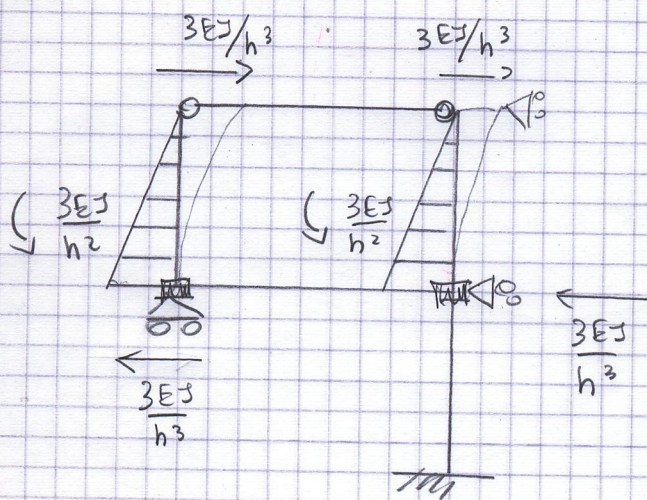
$$K_{21} = \frac{3EJ}{h^2} = \frac{3}{83} EJ = \frac{EJ}{3}$$

$$K_{22} = \frac{4EJ}{2} + \frac{3EJ}{3} + \frac{4EJ}{5} = \frac{47EJ}{15}$$

$$K_{23} = \frac{3EJ}{9} = \frac{EJ}{3}$$

$$K_{24} = \frac{2EJ}{5}$$

Systeme 3



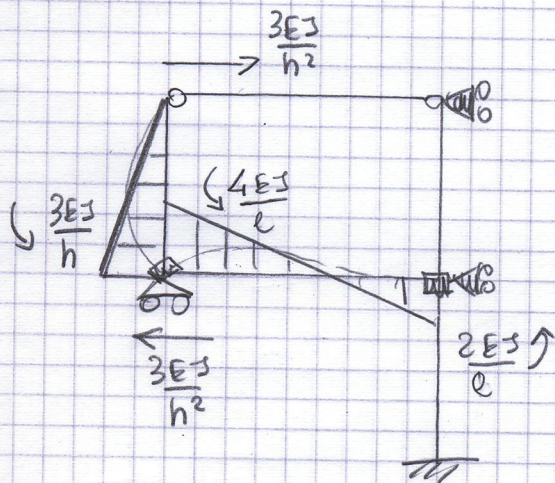
$$K_{31} = -\frac{6EJ}{h^3} = -\frac{2}{9} EJ$$

$$K_{32} = \frac{3EJ}{9} = \frac{EJ}{3}$$

$$K_{33} = \frac{6EJ}{h^3} = \frac{2}{9} EJ$$

$$K_{34} = \frac{3EJ}{h^2} = \frac{EJ}{3}$$

Systeme 4



$$K_{41} = -\frac{3EJ}{h^2} = -\frac{EJ}{3}$$

$$K_{42} = \frac{2EJ}{l} = \frac{2}{5} EJ$$

$$K_{43} = \frac{3EJ}{h^2} = \frac{EJ}{3}$$

$$K_{44} = \left(\frac{3EJ}{h} + \frac{4EJ}{l} \right) = \frac{9}{5} EJ$$

(3)

$$EJ \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & -\frac{2}{9} & -\frac{1}{3} \\ \frac{1}{3} & \frac{47}{15} & \frac{1}{3} & \frac{2}{5} \\ -\frac{2}{9} & \frac{1}{3} & \frac{2}{9} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{5} & \frac{1}{3} & \frac{9}{5} \end{pmatrix} \begin{pmatrix} u_1 \\ \varphi_2 \\ u_3 \\ \varphi_4 \end{pmatrix} = \begin{pmatrix} 9750 \\ 3125 \\ 30250 \\ 3625 \end{pmatrix}$$

$$\begin{pmatrix} u_1 \\ \varphi_2 \\ u_3 \\ \varphi_4 \end{pmatrix} = \begin{pmatrix} 43821,8 \\ -11214,5 \\ 71774,1 \\ -670,3 \end{pmatrix} \cdot \frac{1}{EJ}$$

$$M_{BD} = \frac{2EJ}{3} \left(\frac{43821,8}{EJ} \right) + \frac{2EJ}{3} \left(\frac{-11214,5}{EJ} \right) = 21738,2$$

$$M_{CE} = 2250 - \frac{EJ}{3} \left(\frac{43821,8}{EJ} \right) + \frac{EJ}{3} \left(\frac{71774,1}{EJ} \right) + EJ \left(\frac{-670,3}{EJ} \right) = 10897,1$$

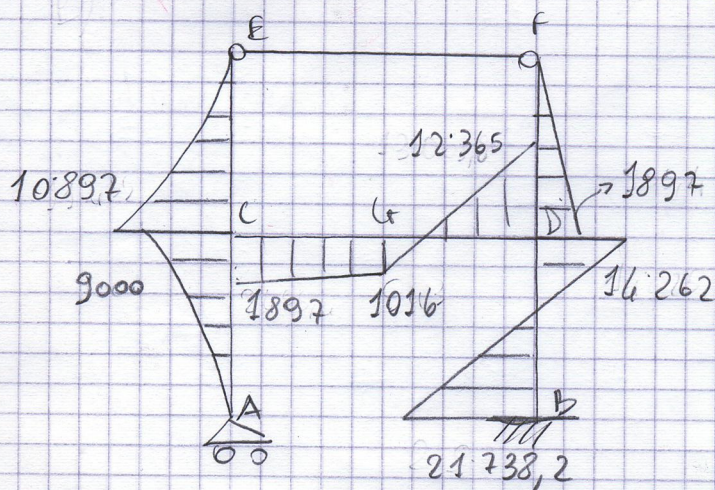
$$M_{CD} = 3125 + \frac{2EJ}{5} \left(\frac{-11214,5}{EJ} \right) + \frac{6EJ}{5} \left(\frac{-670,3}{EJ} \right) = -1897,04$$

$$M_{CC} = 3125 + \frac{EJ}{5} \left(\frac{-11214,5}{EJ} \right) - \frac{EJ}{5} \left(\frac{-670,3}{EJ} \right) = 1046,816$$

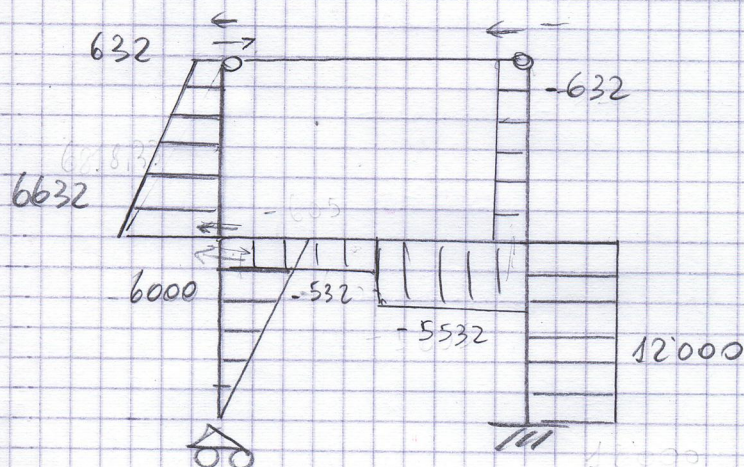
$$M_{DE} = -\frac{EJ}{3} \left(\frac{43821,8}{EJ} \right) + EJ \left(\frac{-11214,5}{EJ} \right) + \frac{EJ}{3} \left(\frac{71774,1}{EJ} \right) = -1897,1$$

$$M_{DC} = -3125 + \frac{6EJ}{5} \left(\frac{-11214,5}{EJ} \right) + \frac{2EJ}{5} \left(\frac{-670,3}{EJ} \right) = -12365$$

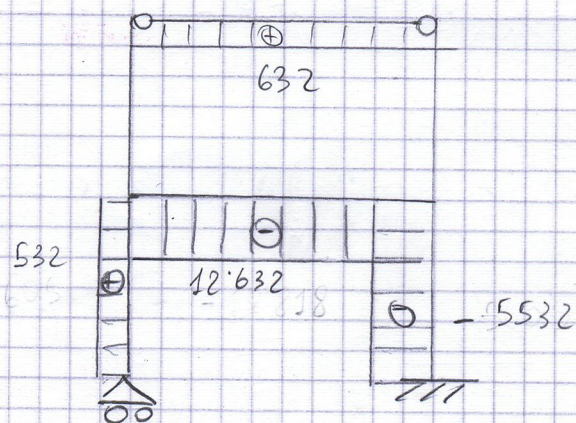
$$M_{DS} = \frac{2EJ}{3} \left(\frac{43821,8}{EJ} \right) + \frac{6EJ}{3} \left(\frac{-11214,5}{EJ} \right) = 14262$$



(M) [kg.m]



(T) [kg]



(N) [kg]

B) Proyecto del Celar. (HEA)

$$M_{max} = 21738,2 \text{ kg.m}$$

$$\Rightarrow W_{min} = \frac{21738,2 \cdot 100}{2600} = 836,1 \text{ cm}^3 \Rightarrow \text{HEA 280}$$

$$\Rightarrow W_x = 1010 \text{ cm}^3; \quad J_x = 13673 \text{ cm}^4, \quad A = 97,3 \text{ cm}^2$$

c) Verificare dell'ammontamento delle sollecitazioni

$$M_{max} = 21738,2 \text{ kg.m}$$

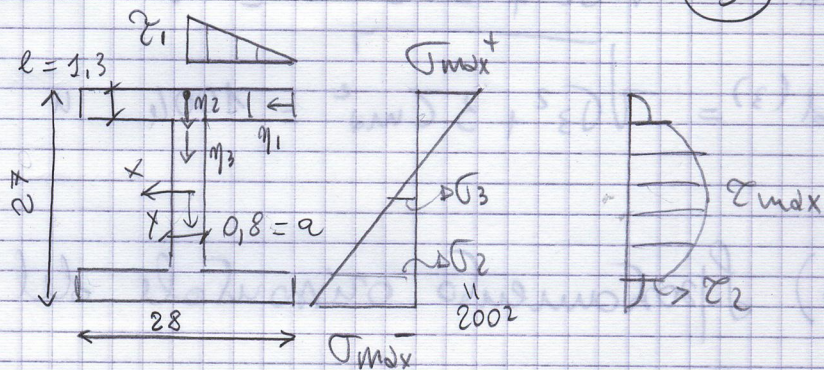
$$T = 12000 \text{ kg}$$

$$N = -5532 \text{ kg}$$

$$A = 97,3 \text{ cm}^2$$

$$W_x = 1010 \text{ cm}^3$$

$$J_x = 13673 \text{ cm}^4$$



$$\sigma = \frac{N}{A} + \frac{M}{W} = \frac{-5532}{97,3} + \frac{21738,2 \cdot 100}{1010} = \begin{cases} \sigma_{max}^+ = 2095,4 \text{ kg/cm}^2 \\ \sigma_{max}^- = -2209,15 \text{ "} \end{cases}$$

$< 2600 \text{ kg/cm}^2 = OK$

$$\sigma_3 = \frac{N}{A} = -56,85 \text{ kg/cm}^2$$

$$S_1 = e \cdot \eta_1 \cdot \left(\frac{h-e}{2} \right) \rightarrow S_{1max} \left(\eta_1 = \left(\frac{b}{2} - \frac{a}{2} \right) \right) = -227,18 \text{ cm}^3$$

$$\tau_1 = - \frac{T \cdot S_{1max}}{e \cdot J_x} = - \frac{12000 \cdot -227,18}{1,3 \cdot 13673} = 153,4 \frac{\text{kg}}{\text{cm}^2}$$

$$S_2 = -b \cdot \eta_2 \cdot \left(\frac{h}{2} - \frac{\eta_2}{2} \right) \rightarrow S_{2max} (\eta_2 = e) = -467,74 \text{ cm}^3$$

$$\tau_{ALA} = - \frac{T \cdot S_{2max}}{b \cdot J_x} = - \frac{12000 \cdot -467,74}{28 \cdot 13673} = 14,6 \text{ kg/cm}^2$$

$$\tau_2 = - \frac{T \cdot S_{2max}}{a \cdot J_x} = - \frac{12000 \cdot -467,74}{0,8 \cdot 13673} = 513,13$$

$$S_3 = -a \cdot \eta_3 \cdot \left(\frac{h}{2} - e - \frac{\eta_3}{2} \right) \rightarrow S_{3max} \left(\eta_3 = \frac{h}{2} - e \right) = -59,56 \text{ cm}^3$$

$$\tau_{max} = \tau_2 + - \frac{T \cdot S_{3max}}{a \cdot J_x} = 578,44 \text{ kg/cm}^2$$

$$\begin{aligned} \sigma_{id}^{(1)} &= \sqrt{\sigma_{max}^2 + 3\tau_1^2} = 2225 \text{ kg/cm}^2 \\ \sigma_{id}^{(2)} &= \sqrt{\sigma_2^2 + 3\tau_2^2} = 2190 \text{ " } \\ \sigma_{id}^{(3)} &= \sqrt{\sigma_3^2 + 3\tau_{md}^2} = 1004 \text{ " } \end{aligned} \quad \left. \vphantom{\begin{aligned} \sigma_{id}^{(1)} \\ \sigma_{id}^{(2)} \\ \sigma_{id}^{(3)} \end{aligned}} \right\} < 2600 \text{ kg/cm}^2 \quad (6)$$

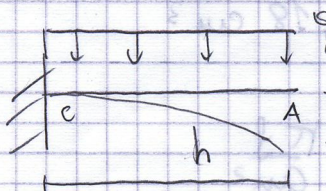
D) Spostamento orizzontale del punto A

Lo spostamento del nodo c era noto precedentemente calcolato e risultava pari a:

$$\varphi_u = \varphi_c = -\frac{6793}{EI} = \frac{-670,3}{2,1 \cdot 10^{10} \cdot 13,673 \cdot 10^{-8}} = 0,000233$$

(rotazione in senso orario $\curvearrowright$)

L'abbondamento di una mensola con carico distribuito è pari a:



$$\mu_A = \frac{qh^4}{8EI} = \frac{2000 \cdot 3^4}{8 \cdot 2,1 \cdot 13,673 \cdot 10^2} = 0,00705$$

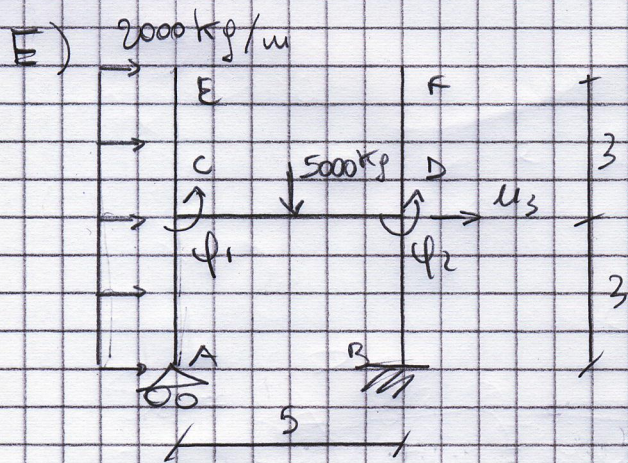
Potanto lo spostamento di A dovuto alle sole rotazioni

$$\mu_{A_n} = -0,000233 \cdot 3 \text{ m} + 0,00705 = 0,006 \text{ m} = 6 \text{ mm}$$

A questo si aggiunge lo spostamento orizzontale del nodo c pari a:

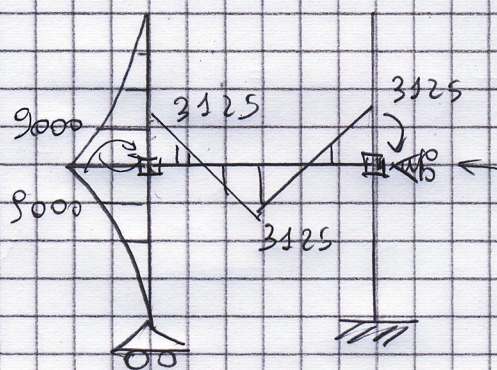
$$\mu_c = \mu_1 = \frac{63821,8}{EI} = 0,01526 \text{ m}$$

$$\Rightarrow \mu_A = \mu_{A_n} + \mu_c = 0,006 + 0,01526 = 0,0213 \text{ m} \approx 2,1 \text{ cm}$$



3 Stufenansatz

Linien 0

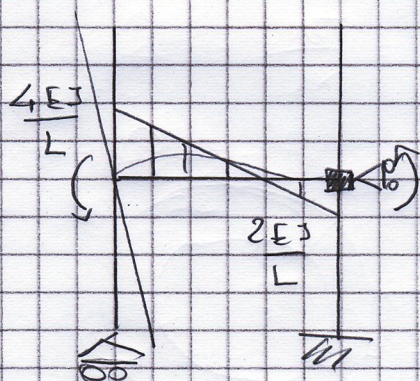


$$K_{10} = 3125$$

$$K_{20} = -3125$$

$$K_{30} = -12000$$

Linien 1

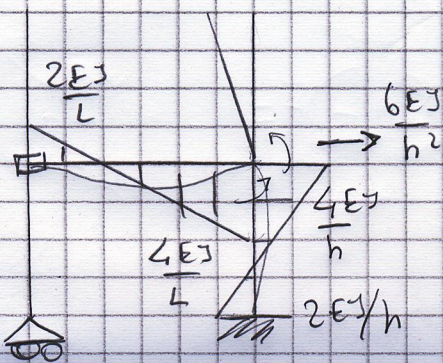


$$K_{11} = \frac{4}{5} EI$$

$$K_{12} = \frac{2}{5} EI$$

$$K_{13} = 0$$

Linien 2



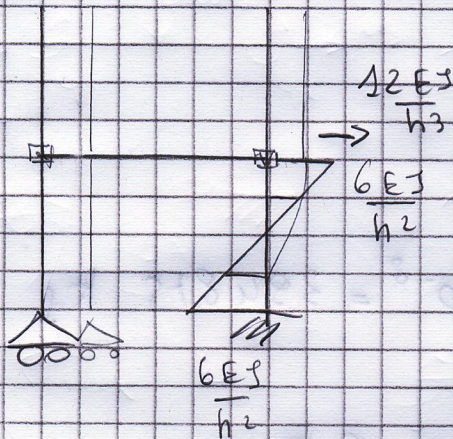
$$K_{21} = \frac{2}{5} EI$$

$$K_{22} = \frac{4}{5} EI + \frac{4}{3} EI = \frac{32}{15} EI$$

$$K_{23} = \frac{26}{9} EI = \frac{2}{3} EI$$

Beispiel 3

8



$$K_{31} = 0$$

$$K_{32} = \frac{2}{3} EJ$$

$$K_{33} = \frac{4 \cdot 12}{27} EJ = \frac{4 EJ}{9}$$

$$EJ \begin{pmatrix} 4/5 & 2/5 & 0 \\ 2/5 & 32/15 & 2/3 \\ 0 & 2/3 & 4/9 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} -3125 \\ 3125 \\ 12000 \end{pmatrix}$$

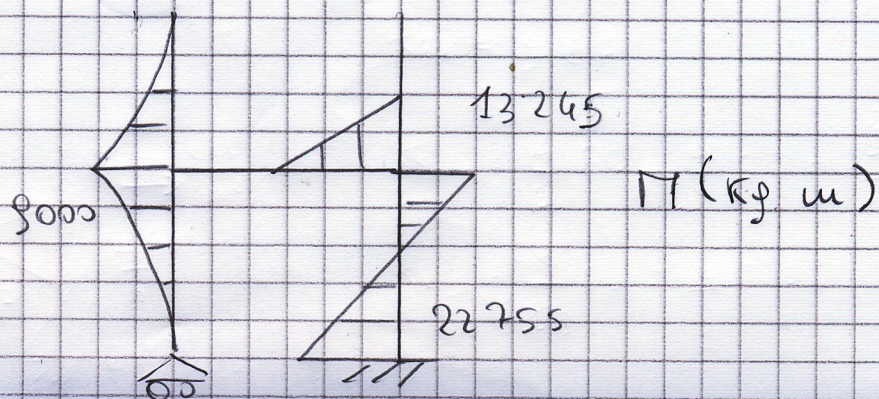
$$\Rightarrow \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ u_3 \end{pmatrix} = \frac{1}{EJ} \begin{pmatrix} 3225,45 \\ -16263,4 \\ 48395,1 \end{pmatrix}$$

$$M_{AG} = -3125 + \frac{2}{5} EJ \left(\frac{3225,45}{EJ} \right) + \frac{4}{5} EJ \left(\frac{-16263,4}{EJ} \right) = -13245$$

$$M_{DB} = \frac{4}{3} EJ \left(\frac{-16263,4}{EJ} \right) + \frac{2}{3} EJ \left(\frac{48395,1}{EJ} \right) = 13245$$

$$M_B = \frac{2}{3} EJ \left(\frac{-16263,4}{EJ} \right) + \frac{2}{3} EJ \left(\frac{48395,1}{EJ} \right) = 22755$$

$$M_{CD} = 0$$



(F)

$$P_{crit} = \frac{\pi^2 E J_{min}}{l^2}$$

$$J_{min} = 4763$$

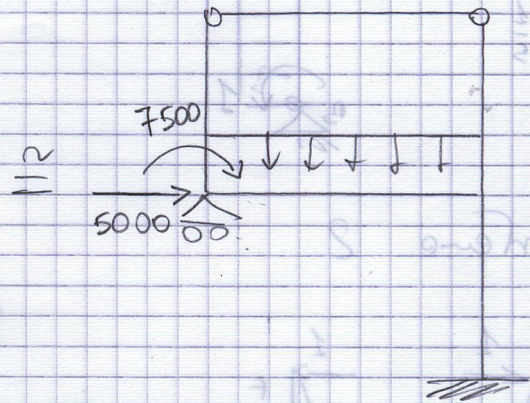
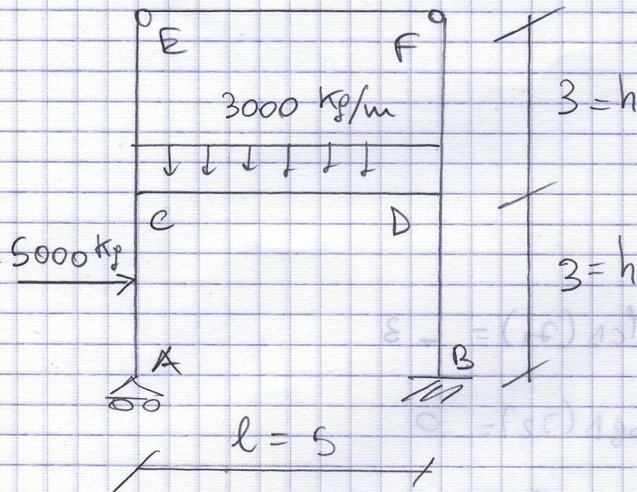
$$P_{crit} = \frac{\pi^2 \cdot 2,1 \cdot 10^{10} \cdot 4763 \cdot 10^{-8}}{25} = 394875 \text{ kg}$$

$$P_{crit} > M_{eff} = 632 \text{ kg} \rightarrow 0 \text{ m}$$

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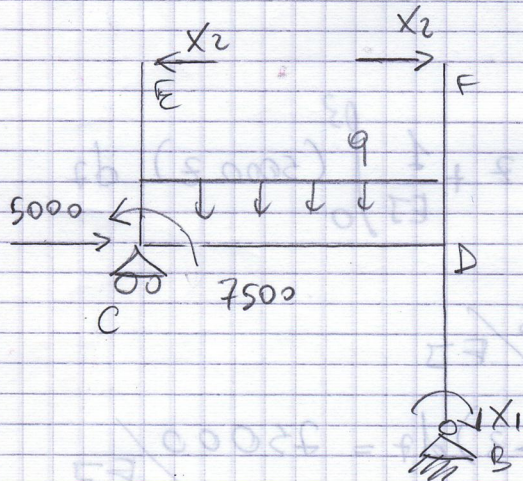
1

Prova B

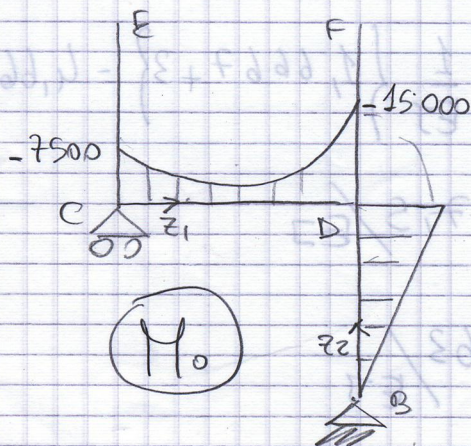
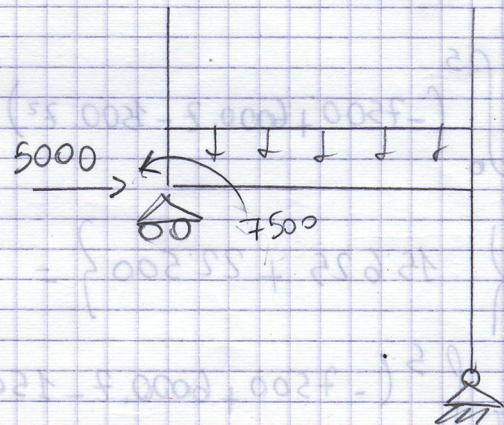


2 ip.

Isotetica equivalente:



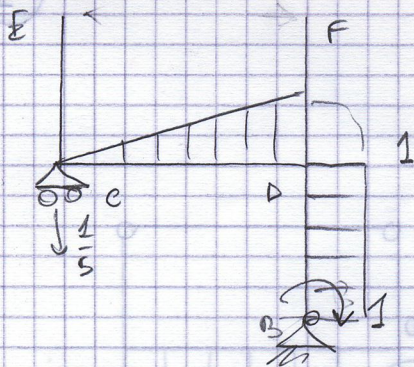
Linea



$$M_{CD}(z_1) = -7500 + 6000 \cdot z_1 - 1500 \cdot z_1^2$$

$$M_{BD}(z_2) = 5000 \cdot z_2$$

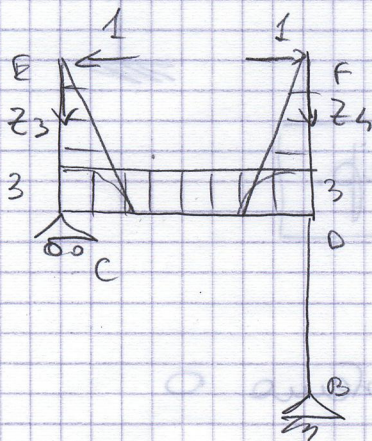
hintene 1



$$M_{CD}(z_1) = -\frac{1}{5} \cdot z_1$$

$$M_{BD}(z_2) = 1$$

hintene 2



$$M_{CD}(z_1) = -3$$

$$B_{BD}(z_2) = 0$$

$$M_{EC}(z_3) = 1 \cdot z_3$$

$$M_{ED}(z_4) = -1 \cdot z_4$$

$$\eta_{10} = \frac{1}{EI} \int_0^5 (-7500 + 6000 \cdot z - 1500 \cdot z^2) \left(-\frac{z}{5}\right) dz + \frac{1}{EI} \int_0^3 (5000 \cdot z) dz$$

$$= \frac{1}{EI} \left\{ 15625 + 22500 \right\} = 38125/EI$$

$$\eta_{20} = \frac{1}{EI} \int_0^5 (-7500 + 6000 \cdot z - 1500 \cdot z^2) (-3) dz = 75000/EI$$

$$\eta_{11} = \frac{1}{EI} \int_0^5 \left(-\frac{z}{5}\right)^2 dz + \frac{1}{EI} \int_0^3 1 dz = \frac{1}{EI} \left\{ 1,6667 + 3 \right\} = 4,666/EI$$

$$\eta_{12} = \frac{1}{EI} \int_0^5 \left(-\frac{z}{5}\right) (-3) dz = \frac{1}{EI} \cdot 7,5 = 7,5/EI$$

$$\eta_{22} = \frac{2}{EI} \int_0^3 z^2 dz + \frac{1}{EI} \int_0^5 9 dz = 63/EI$$

$$\frac{1}{EI} \begin{pmatrix} 4,666 & 7,5 \\ 7,5 & 63 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} -38125 \\ -75000 \end{pmatrix} \cdot \frac{1}{EI}$$

$$X_1 = -7738$$

$$X_2 = -269,3$$

$$M_{BD} = -7738 \text{ kg.m}$$

(3)

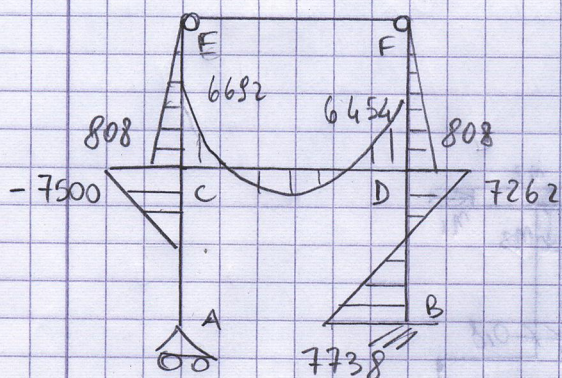
$$M_{DF} = -3 \cdot (-269,3) = 807,9 \text{ u}$$

$$M_{DE} = -15000 - 1(-7738) - 3(-269,3) = -6454 \text{ u}$$

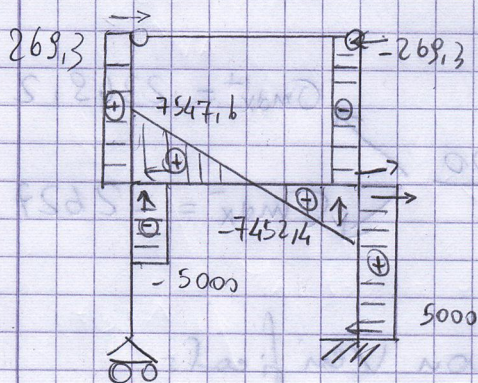
$$M_{CD} = -7500 - 3(-269,3) = -6692,1$$

$$M_{CE} = 3 \cdot (-269,3) = -807,9$$

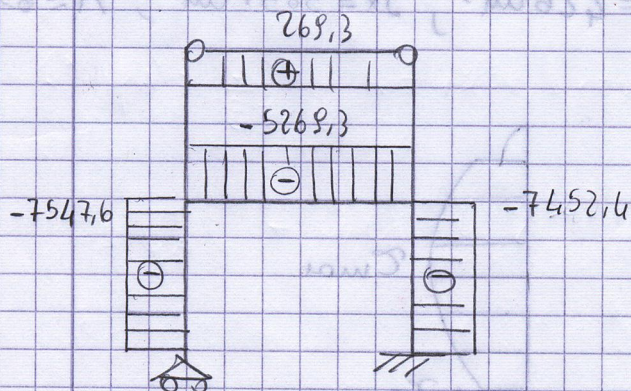
$$M_{DB} = 15000 + 1(-7738) = 7262$$



(M) [kg]



(T) [kg]



(N) [kg]

B) Progetto del Telaio (HEB)

(4)

$$M_{max} = 7738 \text{ Kg.m}$$

$$\Rightarrow W_{min} = \frac{7738 \cdot 100}{2600} = 297,62 \text{ cm}^3$$

$$\Rightarrow \text{HEB } 160 \Rightarrow W_x = 311 \text{ cm}^3$$

c)

$$M_{max} = 7738 \text{ Kg.m}$$

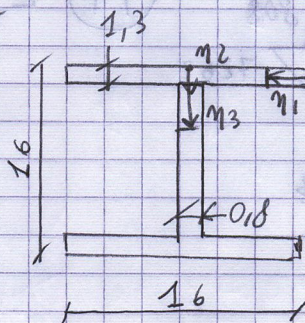
$$T = 5000 \text{ Kg}$$

$$N = 7452 \text{ Kg}$$

$$A = 54,3 \text{ cm}^2$$

$$W_x = 311 \text{ cm}^3$$

$$J_x = 2492 \text{ cm}^4$$



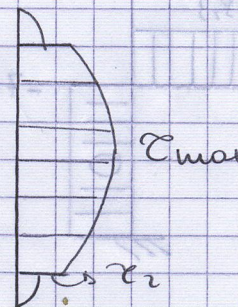
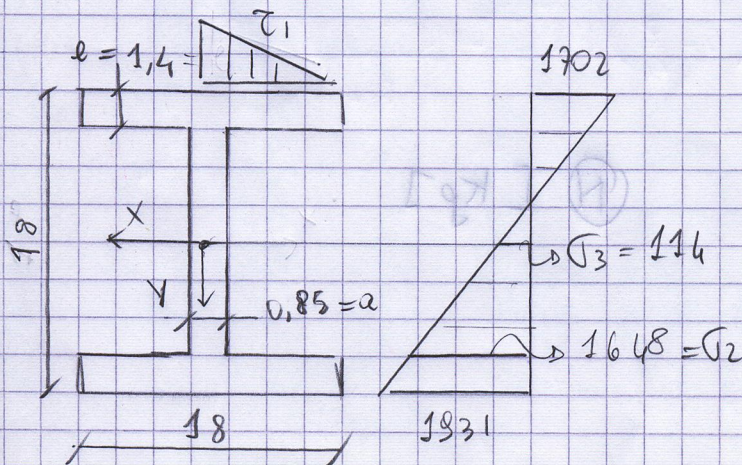
$$\sigma = \frac{N}{A} + \frac{M}{W} = \frac{-7452}{54,3} + \frac{7738 \cdot 100}{311}$$

$$\sigma_{max}^+ = 2349,2 \text{ kg/cm}^2$$

$$\sigma_{max}^- = -2627 \text{ u}$$

$\Rightarrow \sigma_{max}^- > 2600 \text{ kg/cm}^2 \rightarrow \text{non verificato}$

$\Rightarrow \text{Assumo HEB } 180 \Rightarrow W_x = 426 \text{ cm}^3; J_x = 3831 \text{ cm}^4; A = 65,3 \text{ cm}^2$



$$\sigma = \begin{cases} \sigma_{max}^+ = 1702 \text{ kg/cm}^2 \\ \sigma_{max}^- = -1931 \text{ u} \end{cases}$$

$$< 2600 \text{ kg/cm}^2 \Rightarrow \text{OK}$$

$$S_1 = -e \cdot \eta_1 \left(\frac{h-e}{2} \right) \quad S_{1\max} \left(\eta_1 = \frac{b}{2} - \frac{a}{2} \right) = -99,6 \text{ cm}^3 \quad (5)$$

$$\tau_1 = \frac{-5000 \cdot -99,6}{1,4 \cdot 3831} = 92,94 \text{ kg/cm}^2$$

$$S_2 = -b \cdot \eta_2 \left(\frac{h}{2} - \frac{\eta_2}{2} \right) \quad S_{2\max} (\eta_2 = e) = -209,2 \text{ cm}^3$$

$$\tau_{ALA} = \frac{-5000 \cdot -209,2}{18 \cdot 3831} = 15,2 \text{ kg/cm}^2$$

$$\tau_2 = \frac{-5000 \cdot -209,2}{0,85 \cdot 3831} = 321,22 \text{ "}$$

$$S_3 = -a \cdot \eta_3 \left(\frac{h}{2} - e - \frac{\eta_3}{2} \right) \quad S_{3\max} \left(\eta_3 = \frac{h}{2} - e \right) = -24,55 \text{ cm}^3$$

$$\tau_3 = 3 \cdot 21,22 + \frac{5000 \cdot 24,55}{0,85 \cdot 3831} = 358,92 \text{ kg/cm}^2$$

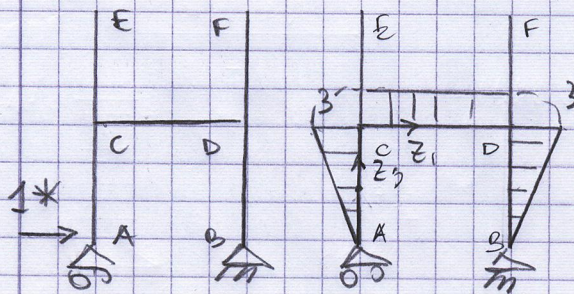
$$\sigma_{id}^{(1)} = \sqrt{\sigma_{mx}^2 + 3 \tau_1^2} = 1938 \text{ kg/cm}^2$$

$$\sigma_{id}^{(2)} = \sqrt{\sigma_2^2 + 3 \tau_2^2} = 1739 \text{ "}$$

$$\sigma_{id}^{(3)} = \sqrt{\sigma_3^2 + 3 \tau_3^2} = 632 \text{ "}$$

$$\left. \begin{array}{l} \sigma_{id}^{(1)} \\ \sigma_{id}^{(2)} \\ \sigma_{id}^{(3)} \end{array} \right\} \leq 2600 \frac{\text{kg}}{\text{cm}^2} \Rightarrow \text{OK}$$

D) Spontamento unitario in A



$$M_{AC}(z_5) = -z_5$$

$$M_{CD}(z_1) = -3$$

$$M_{BD}(z_2) = z_2$$

$$1^* u_A = \frac{1}{EI} \int_0^{1,5} (-1,5 - z) \cdot (-5000 \cdot z) dz + \frac{1}{EI} \int_0^5 (-6692 + 7547,6 \cdot z - 1500 \cdot z^2) \cdot (-3) dz + \frac{1}{EI} \int_0^3 (-7738 + 5000 \cdot z) z dz$$

$$1^* \mu_A = \frac{29086,5}{\text{EJ}} = \frac{29086,5}{2,1 \cdot 10^{10} \cdot 3831 \cdot 10^{-8}} = 0,036 \text{ m}$$

$$\mu_A \approx 3,6 \text{ cm}$$

E) Tensile test - (see p. 10)

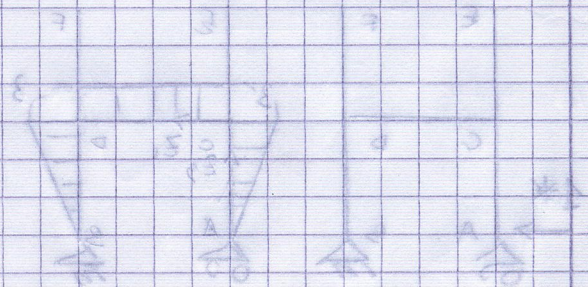
$$K = \frac{EA}{l} = \frac{210 \cdot 10^9 \cdot 6,28 \cdot 10^{-4}}{0,02} = 6,58 \cdot 10^7 \frac{N}{m}$$

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$$1 \times \frac{1}{K} = 3,64 \cdot 10^{-8}$$

$$\begin{array}{r} 63 + 0.02937 \\ \hline 63.02937 \\ \hline 1588.28,0 \end{array}$$

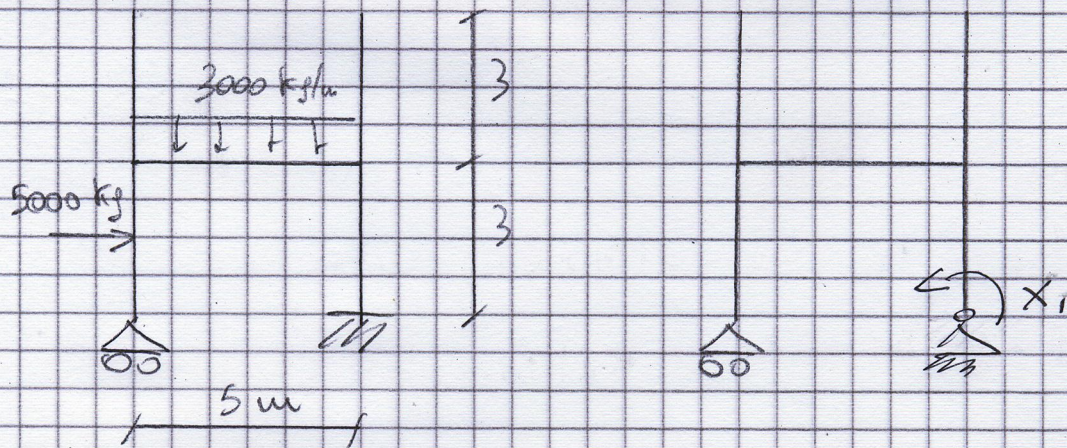
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -772,8981 \\ -269,144 \end{pmatrix}$$



$$-F_d + F_N + (P_d) = \frac{1}{0.02} + \frac{1}{0.02} (2.000 + 2.000) = 100 + 100 = 200$$

E)

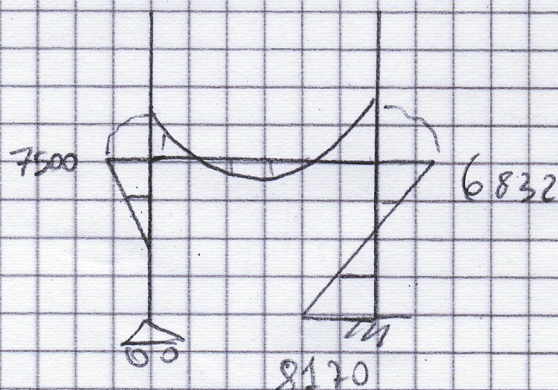
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→ M_0 ed M_1 sono gli sforzi di primo

$$\Rightarrow X_1 = - \frac{M_{10}}{\eta_{11}} = \frac{38125}{4.666} = 8170$$

→



(M) [kg·m]

F) Conico critico dell'asta EF

(8)

$$P_{crit} = \pi^2 \cdot \frac{EJ_{min}}{(l)^2}$$

$$J_{min} = 1363 \text{ cm}^4$$

$$P_{crit} = \frac{\pi^2 \cdot 2,1 \cdot 10^{10} \cdot 1363 \cdot 10^{-8}}{25} = 112999 \text{ kg}$$

$$P_{crit} > N_{EF} = 269 \text{ kg}$$