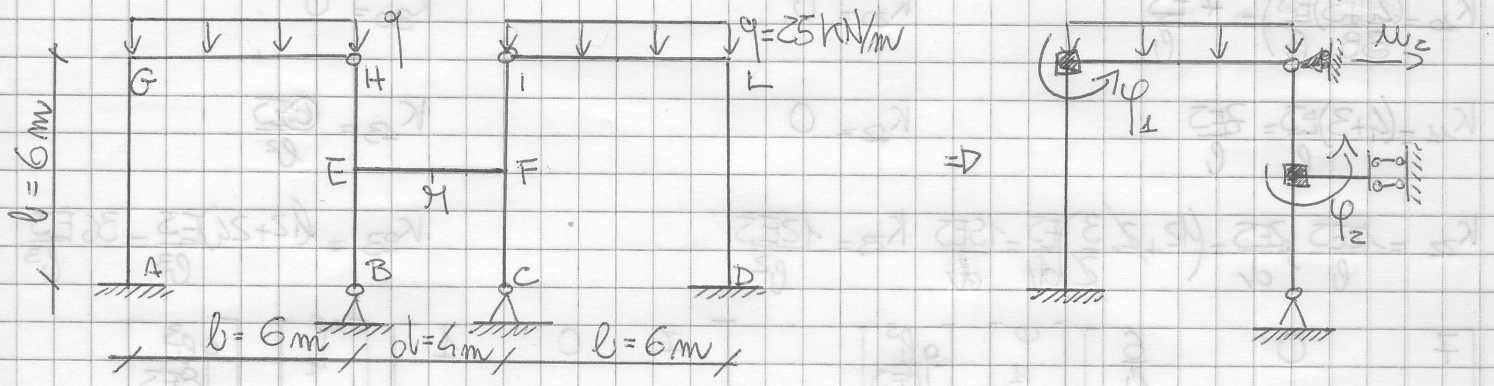
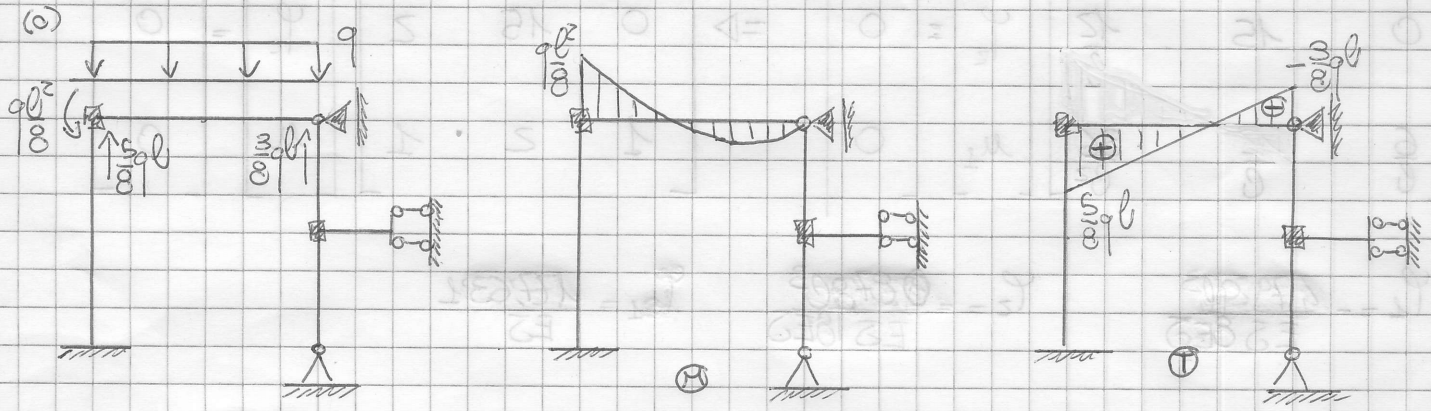
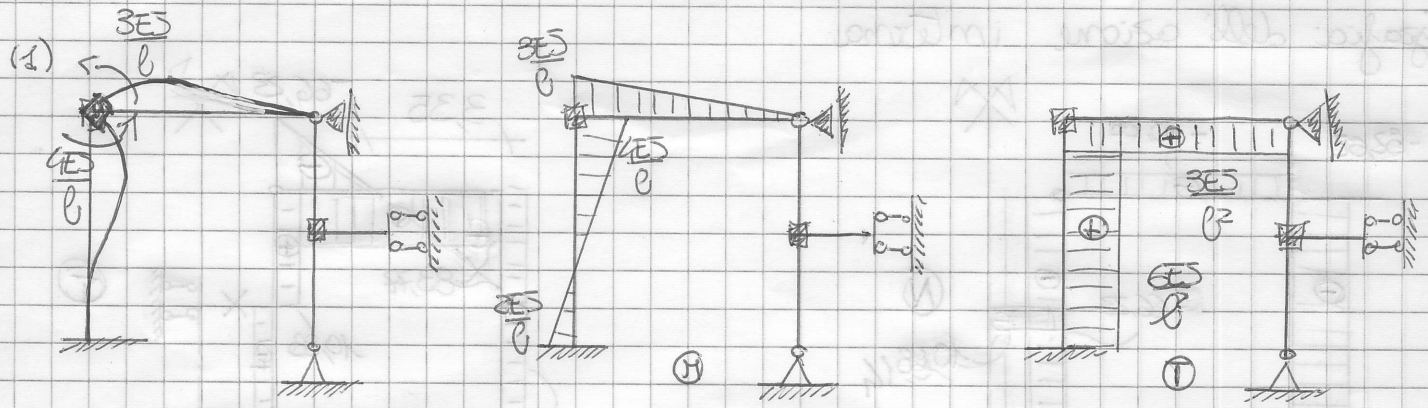
FILA A


q
q=25 kN/m
G
H
I
L
E
F
M
A
B
C
D
l = 6m
l = 6m
dl = 4m
l = 6m
=D
M_C
φ1
φ2

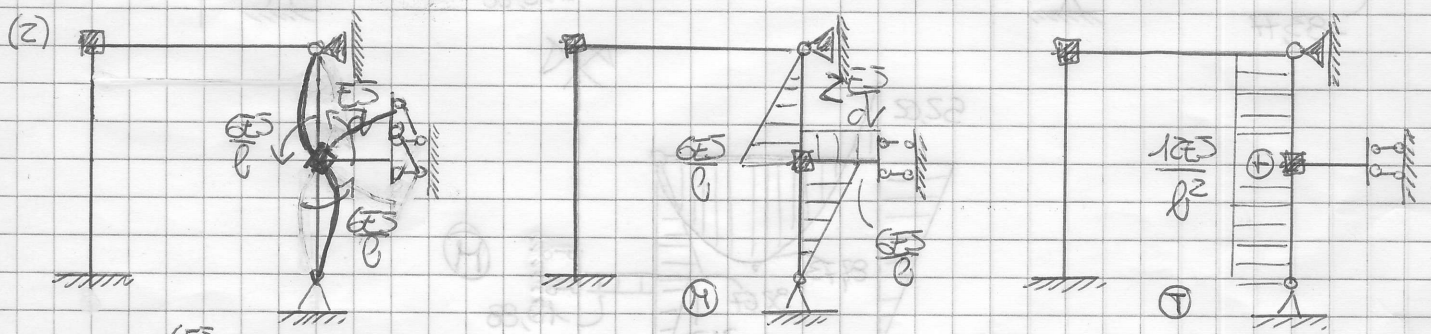
a) (0)


q
q l²/8
5/8 l
3/8 l
q l/8
-3/8 l
5/8 l
M
T

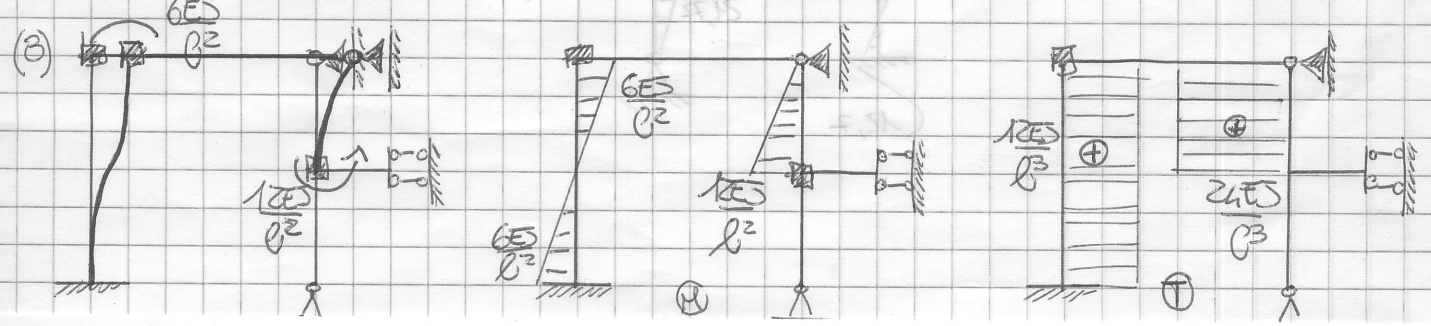
(1)


3EJ/l
4EJ/l
3EJ/l
4EJ/l
4EJ/l
3EJ/l²
6EJ/l²
M
T

(2)


EJ
6EJ/l
6EJ/l
2EJ/dl
6EJ/l
6EJ/l
12EJ/l²
M
T

(3)


6EJ/l²
12EJ/l²
6EJ/l²
6EJ/l²
12EJ/l²
12EJ/l³
24EJ/l³
M
T

Calcolo dei coefficienti K_{ij}:

$K_{10} = q\frac{\ell^2}{8}$ $\quad K_{20} = 0$ $\quad K_{30} = 0$

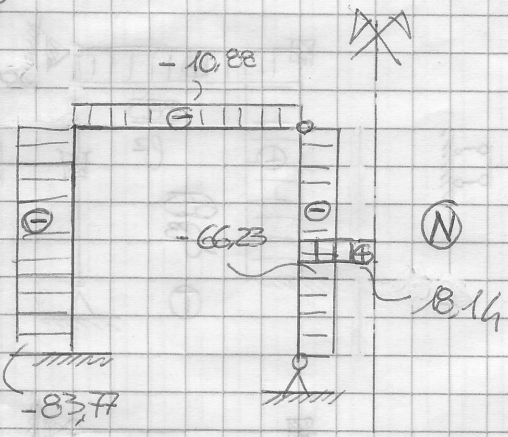
$K_{11} = (4+3)\frac{EJ}{\ell} = \frac{7EJ}{\ell}$ $\quad K_{12} = 0$ $\quad K_{13} = \frac{6EJ}{\ell^2}$

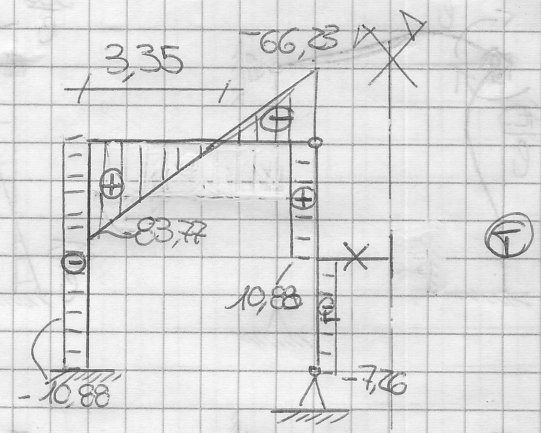
$K_{22} = \frac{12EJ}{\ell} + \frac{3EJ}{\ell} = (12+3)\frac{EJ}{\ell} = \frac{15EJ}{\ell}$ $\quad K_{23} = \frac{12EJ}{\ell^2}$ $\quad K_{33} = (12+24)\frac{EJ}{\ell^3} = 36\frac{EJ}{\ell^3}$

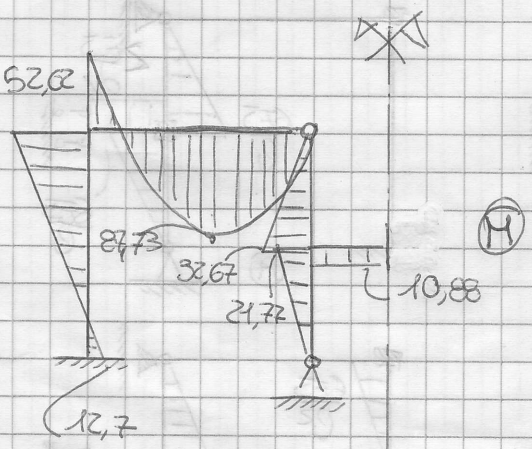
$$\begin{bmatrix} 7 & 0 & \frac{6}{\ell} \\ 0 & 15 & \frac{12}{\ell} \\ \frac{6}{\ell} & \frac{12}{\ell} & \frac{36}{\ell^2} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} -q\frac{\ell^3}{8EJ} \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 7 & 0 & 1 \\ 0 & 15 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} -q\frac{\ell^3}{8EJ} \\ 0 \\ 0 \end{bmatrix}$$

$\varphi_1 = -\frac{119,76}{EJ}$ $\quad \varphi_2 = -\frac{21,77}{EJ}$ $\quad u_1 = \frac{163,31}{EJ}$

b) Grafici delle azione interna







c) Progetto della sezione

$$W_{min} \geq \frac{M_{max}}{\sigma_{amm}} = \frac{\cdot \cdot 10^3}{260 \cdot 10^6} \cdot 10^6 = 202{,}38\ cm^3 \Rightarrow HEA = \dots$$

$h = 152\ mm$ $\quad b = 160\ mm$ $\quad a = 6\ mm$ $\quad e = 9\ mm$

$A = 38{,}8\ cm^2$ $\quad W = 220\ cm^3$ $\quad J = 616\ cm^4$

d) Calcolo dell'abbassamento nel punto M

Ricavo lo spostamento dalla linea elastica della trave, con le seguenti condizioni al contorno

$u(0) = u(L) = 0$ $\qquad \varphi(0) = -\varphi_2$ $\qquad \varphi(L) = \varphi_2$

dove $L = 4d$. Si ha

$u(x) = C_3x^3 + C_2x^2 + C_1x + C_0$

$u(0) = 0 \Rightarrow C_0 = 0$

$u(L) = 0 \Rightarrow C_3L^3 + C_2L^2 + C_1L = 0$

$u'(0) = -\varphi_2 \Rightarrow C_1 = -\varphi_2 \Rightarrow C_2L^2 = -C_1L - C_3L^3 = \varphi_2 L - C_3L^3 \Rightarrow C_2 = \frac{\varphi_2}{L} - C_3L$

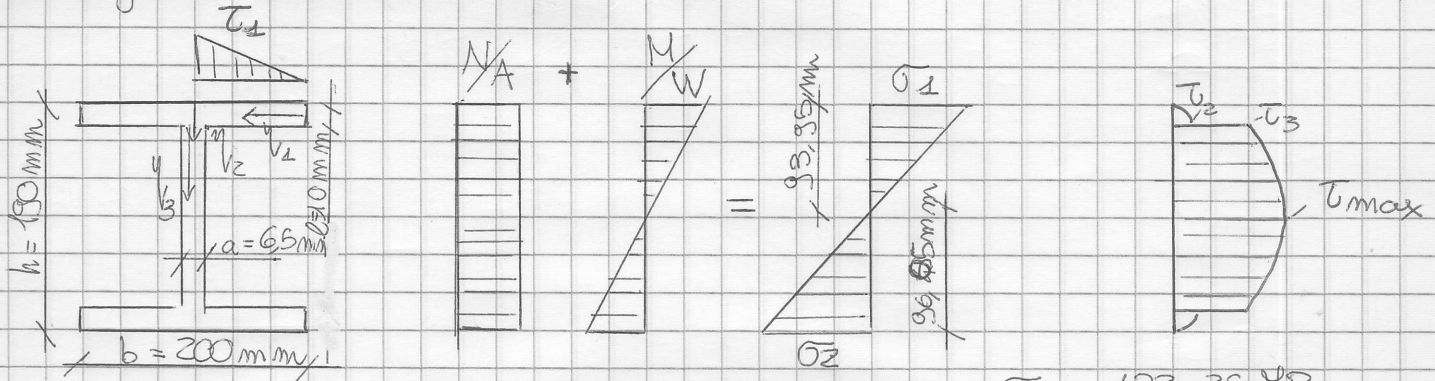
$u'(L) = \varphi_2 \Rightarrow 3C_3L^2 + 2C_2L + C_1 = 0$

$$3C_3L^2 + 2\left(\frac{\varphi_2}{L} - C_3L\right)L - \varphi_2 = 0$$

$$C_3L^2 + \varphi_2 = 0 \Rightarrow C_3 = -\frac{\varphi_2}{L^2}$$

Noti C_i, con $i = 0, \dots 3$, è possibile calcolare $w_M = u(x = L/2)$, assunto $L = 4d$.

② Verifica della sezione G: $M = 52{,}62\ kN\,m$ $\quad T = 83{,}77\ kN$ $\quad N = -10{,}88\ kN$



$$\sigma = \frac{N}{A} + \frac{M}{W} = -\frac{10{,}88 \cdot 10^{-3}}{53{,}8 \cdot 10^{-4}} \pm \frac{52{,}62 \cdot 10^{-3}}{389 \cdot 10^{-6}} = \begin{cases} \sigma_1 = 133{,}25\ MPa \\ \sigma_2 = -137{,}29\ MPa \end{cases}$$

Calcolo delle tensioni tangenziali

$$\tau_1 = \frac{T S_1}{e J} \simeq \frac{83{,}77 \cdot 10^{-3}}{10 \cdot 10^{-3} \cdot 3692 \cdot 10^{-8}} \cdot \frac{190 \cdot 10^{-3}}{2} \cdot \left(\frac{200}{2} \cdot 10 \cdot 10^{-6}\right) = 21{,}5\ MPa$$

$$\tau_2 = \frac{T S_2}{b J} \simeq \frac{83{,}77 \cdot 10^{-3}}{200 \cdot 10^{-3} \cdot 3692 \cdot 10^{-8}} \cdot \frac{190 \cdot 10^{-3}}{2} \cdot \left(200 \cdot 10 \cdot 10^{-6}\right) = 2{,}16\ MPa$$

$$\tau_3 = \frac{T S_3}{a J} \simeq \frac{83{,}77 \cdot 10^{-3}}{6{,}5 \cdot 10^{-3} \cdot 3692 \cdot 10^{-8}} \cdot \frac{190 \cdot 10^{-3}}{2} \cdot \left(200 \cdot 10 \cdot 10^{-6}\right) = 66{,}32\ MPa$$

$$\tau_{max} = \frac{T S_{max}}{a J} \simeq \tau_3 + \frac{83{,}77 \cdot 10^{-3}}{6{,}5 \cdot 10^{-3} \cdot 3692 \cdot 10^{-8}} \cdot \frac{190 \cdot 10^{-3}}{4} \cdot \left(\frac{190}{2} \cdot 6{,}5 \cdot 10^{-6}\right) = 76{,}56\ MPa$$

Calcolo della tensione normale nel baricentro e all'ala

$\sigma_* = 1{,}5\ MPa$ $\qquad \sigma_a = 122{,}99\ MPa$

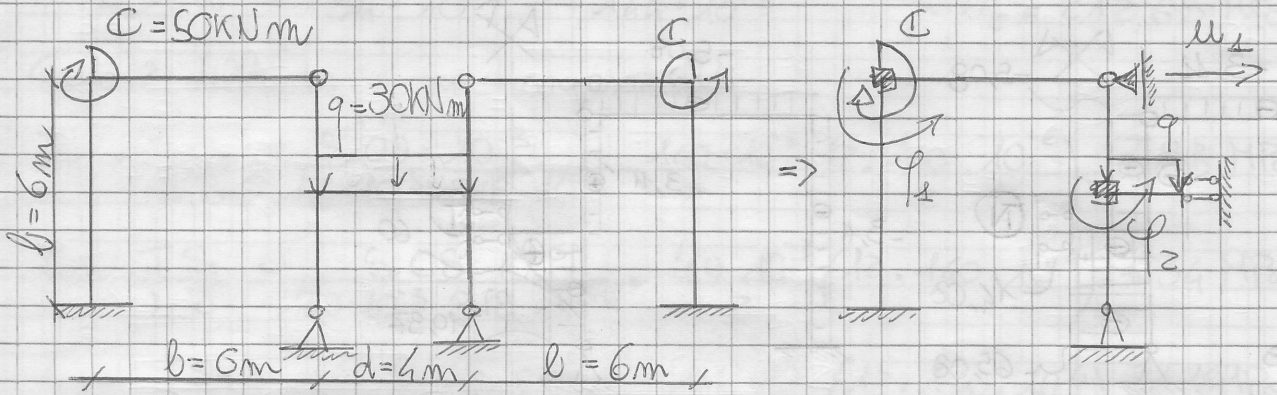
Verifica delle tensioni ideali:

$\sigma_{id}^{(1)} = \sqrt{133{,}25^2 + 3 \cdot 21{,}5^2} = 139{,}36\ MPa < \sigma_{amm}$

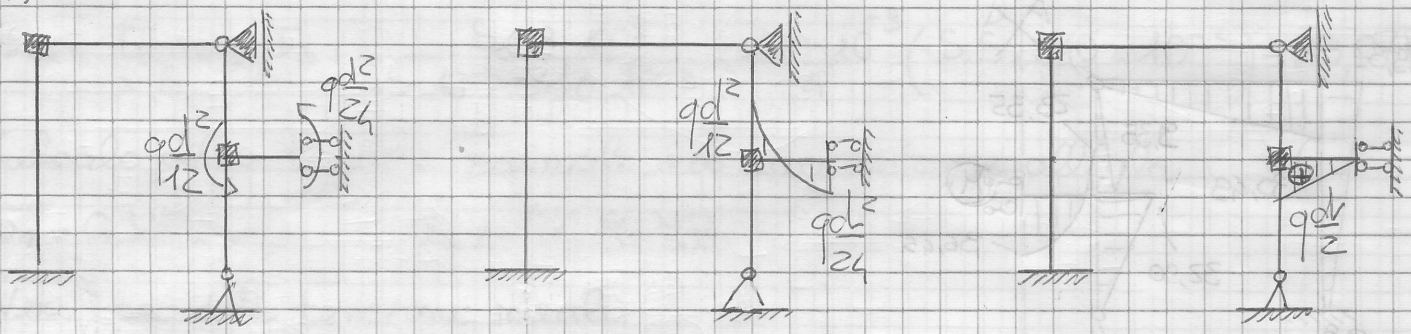
$\sigma_{id}^{(2)} = \sqrt{122{,}99^2 + 3 \cdot 2{,}16^2} = 123{,}05\ MPa < \sigma_{amm}$

$\sigma_{id}^{(*)} = \sqrt{1{,}5^2 + 3 \cdot 76{,}56} = 132{,}61\ MPa < \sigma_{amm}$

FILA B



a) (6)



I sistemi (1),(2),(3) sono uguali a quelli della FILA A.

$K_{10} = 0 + C = C$ $\quad K_{20} = q\frac{d^2}{12}$ $\quad K_{30} = 0$

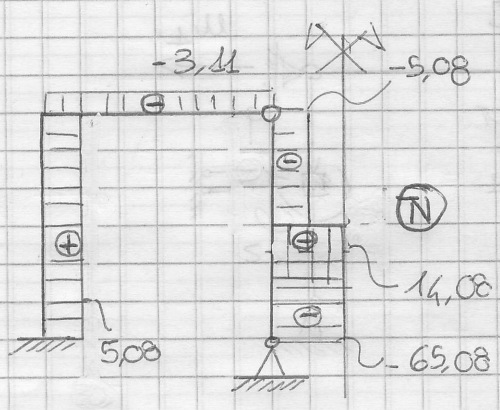
$K_{11} = \frac{7ES}{l}$ $\quad K_{12} = 0$ $\quad K_{13} = \frac{6ES}{l^2}$

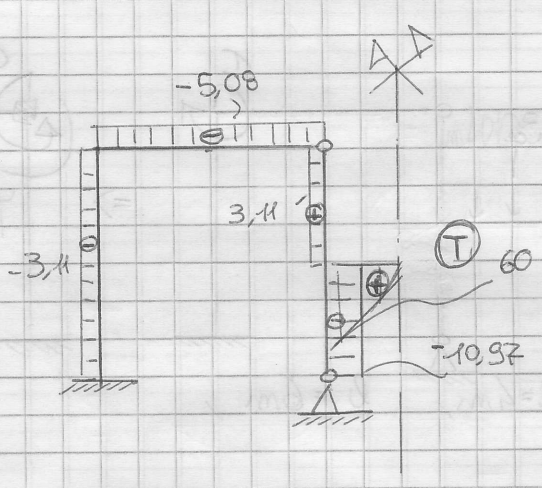
$K_{22} = \frac{15ES}{l}$ $\quad K_{23} = \frac{12ES}{l^2}$ $\quad K_{33} = \frac{36ES}{l^3}$

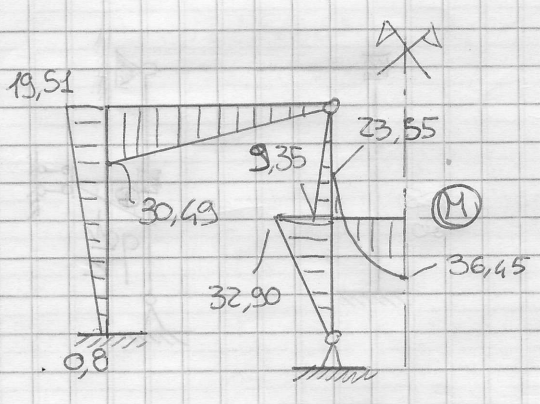
$$\begin{bmatrix} 7 & 0 & \frac{6}{l} \\ 0 & 15 & \frac{12}{l} \\ \frac{6}{l} & \frac{12}{l} & \frac{36}{l^2} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -\frac{Cl}{ES} \\ -\frac{qd^2}{12ES} \\ 0 \end{bmatrix} \cdot 240$$

$\varphi_1 = -\frac{60,97}{ES}$ $\quad \varphi_2 = -\frac{32,90}{ES}$ $\quad u_3 = \frac{126,77}{ES}$

b) Grafici delle azioni interne







c) Progetto della sezione

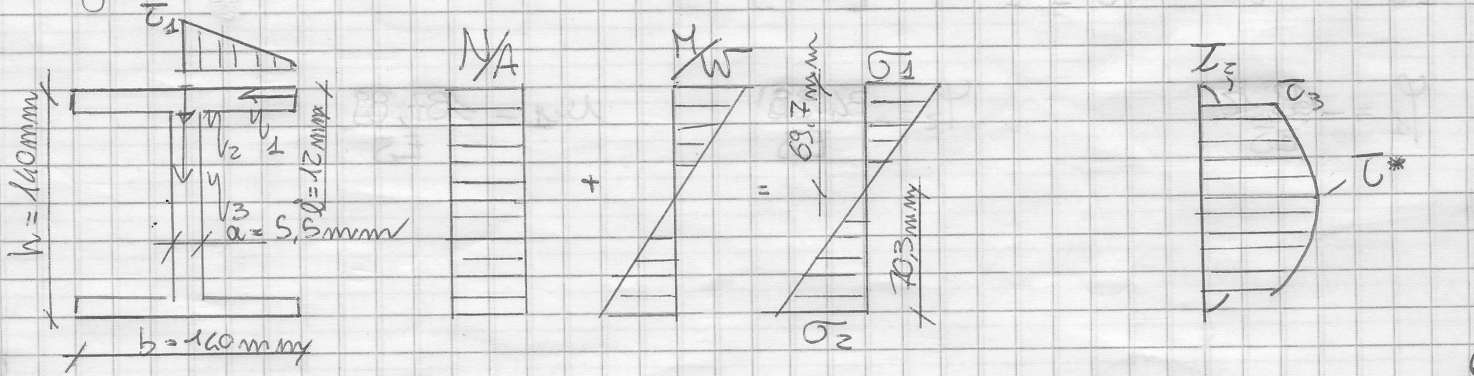
$W_{min} \geq \frac{M_{max}}{W} = \frac{36,45 \cdot 10^3}{260 \cdot 10^6} \cdot 10^6 = 140,19\ cm^3 \Rightarrow$ HEB 140

$A = 34\ cm^2$ $W = 144\ cm^3$ $J = 318\ cm^4$

Andrebbe bene anche un profilo HEB 120, ma poi si rischierebbe di non soddisfare le verifiche successive

d) Analoga a quanto fatto per la Fila A

e) Verifica della sezione G: $M = 30,49\ KN\,m$ $T = 5,08\ KN$ $N = -3,11\ KN$



$$\sigma = \frac{N}{A} \pm \frac{M}{W} = -\frac{3,11 \cdot 10^{-3}}{34 \cdot 10^{-4}} \pm \frac{30,49 \cdot 10^{-3}}{144 \cdot 10^{-6}} = \begin{cases} \sigma_1 = 210,82 \text{ MPa} \\ \sigma_2 = 212,65 \text{ MPa} \end{cases}$$

Calcolo delle tensioni tangenziali

$$\tau_1 = \frac{T S_1}{e \bar{J}} \simeq \frac{5,08 \cdot 10^{-3}}{12 \cdot 10^{-3} \cdot 318 \cdot 10^{-8}} \cdot \frac{140 \cdot 10^{-3}}{2} \left(12 \cdot \frac{140}{2} \cdot 10^{-6}\right) = 7,83 \text{ MPa}$$

$$\tau_2 = \frac{T S_2}{b \bar{J}} \simeq \frac{5,08 \cdot 10^{-3}}{140 \cdot 10^{-3} \cdot 318 \cdot 10^{-8}} \cdot \frac{140 \cdot 10^{-3}}{2} \left(12 \cdot 140 \cdot 10^{-6}\right) = 1,34 \text{ MPa}$$

$$\tau_3 = \frac{T S_3}{a \bar{J}} \simeq \frac{5,08 \cdot 10^{-3}}{5,5 \cdot 10^{-3} \cdot 318 \cdot 10^{-8}} \cdot \frac{140 \cdot 10^{-3}}{2} \left(12 \cdot 140 \cdot 10^{-6}\right) = 34,16 \text{ MPa}$$

$$\tau_{max} = \frac{T S_{max}}{a \bar{J}} \simeq \tau_3 + \frac{5,08 \cdot 10^{-3}}{5,5 \cdot 10^{-3} \cdot 318 \cdot 10^{-8}} \cdot \frac{140 \cdot 10^{-3}}{4} \left(5,5 \cdot \frac{140}{2} \cdot 10^{-6}\right) = 38,07 \text{ MPa}$$

Calcolo della tensione normale nel baricentro e all'ala

$$\sigma_* = 0,9 \text{ MPa} \qquad \sigma_a = 176,35 \text{ MPa}$$

Verifica delle tensioni ideali

$$\sigma_{id}^{(1)} = \sqrt{212,65^2 + 3 \cdot 7,83^2} = 213,08 \text{ MPa} < \sigma_{amm}$$

$$\sigma_{id}^{(2)} = \sqrt{176,35^2 + 3 \cdot 34,16^2} = 186,01 \text{ MPa} < \sigma_{amm}$$

$$\sigma_{id}^{(*)} = \sqrt{0,9^2 + 3 \cdot 38,07^2} = 65,95 \text{ MPa} < \sigma_{amm}$$