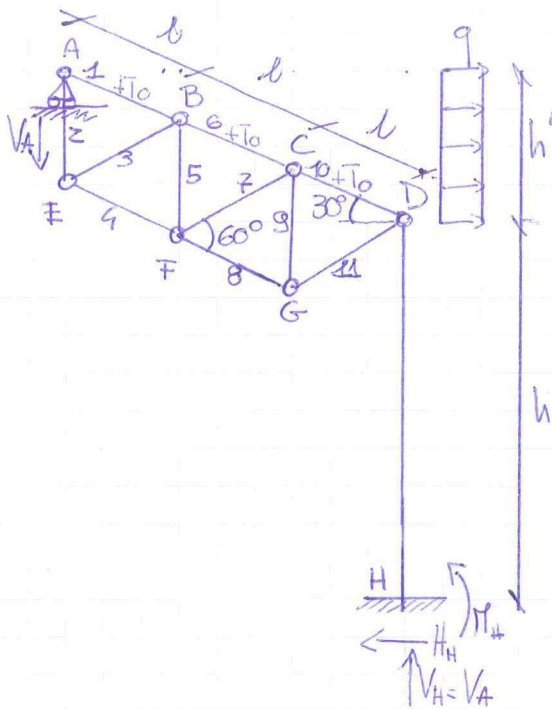


Esercizio 1



$$h = 5 \text{ m}$$

$$l = 1,5 \text{ m}$$

$$q = 20 \text{ kN}$$

$$h' = \frac{3}{2} l = 2,25 \text{ m}$$

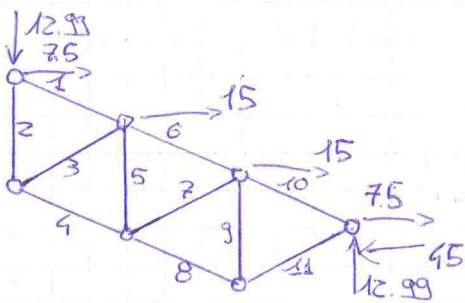
Calcolo delle reazioni vincolari

$$a) V_A = \frac{1}{3l \cos 30^\circ} \cdot q \frac{h'^2}{2} = 12,99 \text{ kN} = V_H$$

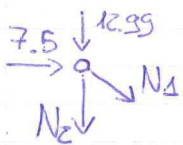
$$\rightarrow H_H = q h' = 45 \text{ kN}$$

$$b) M_H = H_H \cdot h = 225 \text{ kN}$$

Isolo la reticolare e si studiano le sollecitazioni primarie tramite applicazione dei carichi nodali equivalenti



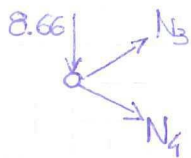
Nodo A



$$N_1 = -\frac{7,5}{\cos 30^\circ} = -8,66 \text{ kN}$$

$$N_2 = -12,99 + N_1 \sin 30^\circ = -8,66 \text{ kN}$$

Nodo E

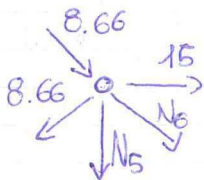


$$N_3 \cos 30 + N_4 \cos 30 = 0 \Rightarrow N_3 = -N_4$$

$$N_3 \sin 30 - N_4 \sin 30 - 8,66 = 0$$

$$N_3 = \frac{8,66}{2 \sin 30} = 8,66 \text{ kN}$$

Nodo B



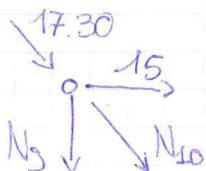
$$N_6 = \frac{-15}{\cos 30^\circ} = -17,30 \text{ kN}$$

$$-N_5 + 17,30 \sin 30 - 2 \cdot 8,66 \sin 30 = 0$$

$$N_5 = 0$$

Si capisce che $N_7 = 0$ e $N_8 = N_4$.

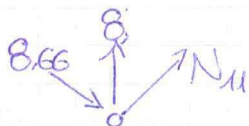
Nodo C



$$N_{10} = \frac{-15 - 17,30 \cos 30^\circ}{\cos 30^\circ} = -34,62 \text{ kN}$$

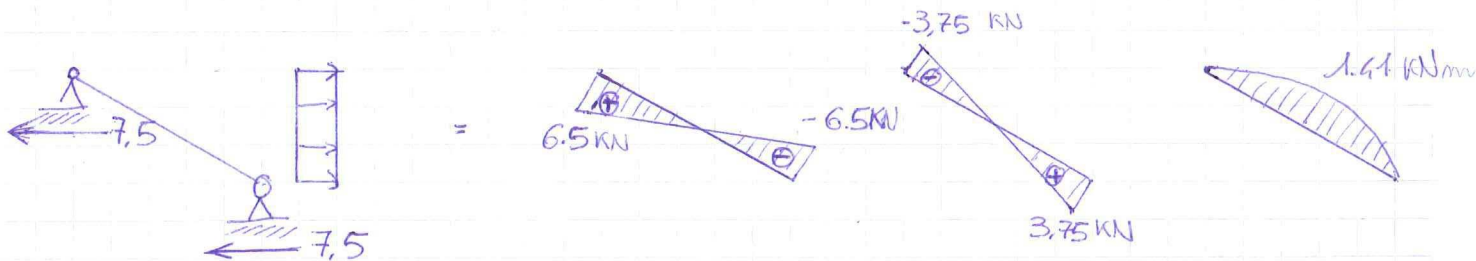
$$N_9 = -17,30 \sin 30 + 34,62 \sin 30 = 8,66 \text{ kN}$$

Nodo G



$$N_{11} = -8,66 \text{ kN}$$

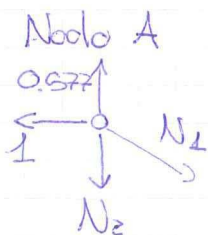
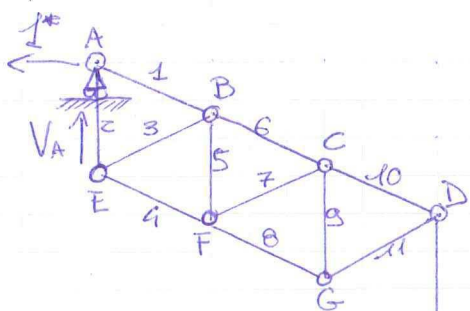
Calcolo delle sollecitazioni secondarie



Asta	SR	SV
1	-8,66	1,15
2	-8,66	0
3	8,66	0
4	-8,66	0
5	0	0
6	-17,30	1,15
7	0	0
8	-8,66	0
9	8,66	0
10	-34,62	1,15
11	-8,66	0

Per il calcolo dello spostamento dell'una orizzontale del nodo A si risolve il seguente sistema virtuale

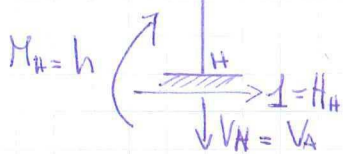
$$\vec{D}) V_A = \frac{1 \cdot 30 \sin 30^\circ}{30 \cos 30^\circ} = \tan 30^\circ = 0,577$$



$$N_1 = \frac{1}{\cos 30^\circ} = 1,15$$

$$N_2 = 0$$

Se si tracciasse una sezione di Ritz passante per le travi 1, 3 e 4 si ricaverebbe facilmente che $N_4 = 0$. Di conseguenza, si ricava che gli unici elementi non scarichi alla tralicatare sono le aste 1, 6 e 10 che sono



sollecitate dalla stessa sollecitazione uguale a 1,15.

A questo punto è possibile calcolare u_A attraverso il PLV:

$$\begin{aligned} 1^* u_A &= \int (N^*(z) E(z) + M^*(z) \chi(z)) dz = \int (N^*(z) \left(\frac{N(z)}{EA} + \alpha T(z) \right) + M^*(z) \frac{M(z)}{EI}) dz \\ &= \sum_{i=1}^{n_{aste}} \frac{N_i^*}{EA} \left(N_i^I + \alpha T_i + \int_0^{l_i} N_i^{II}(z) dz \right) = \end{aligned}$$

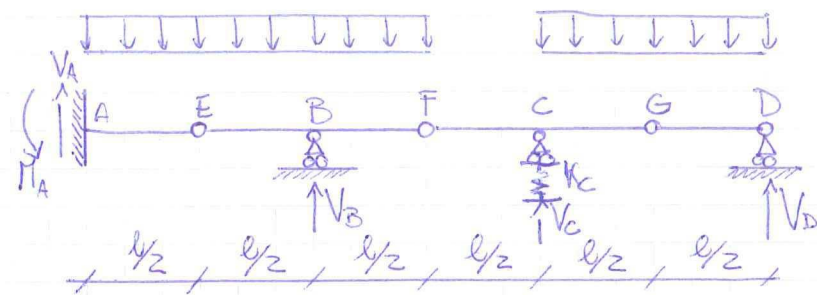
= ...

Esercizio 2

$$l = 10 \text{ m}$$

$$q = 30 \text{ kN/m}$$

$$k_c = 10 \text{ kN/m}$$



Calcolo delle reazioni vincolari

$$G) V_D = \frac{2^4}{8} \frac{q l^2}{8^4} = \frac{q l}{4} = 75 \text{ kN}$$

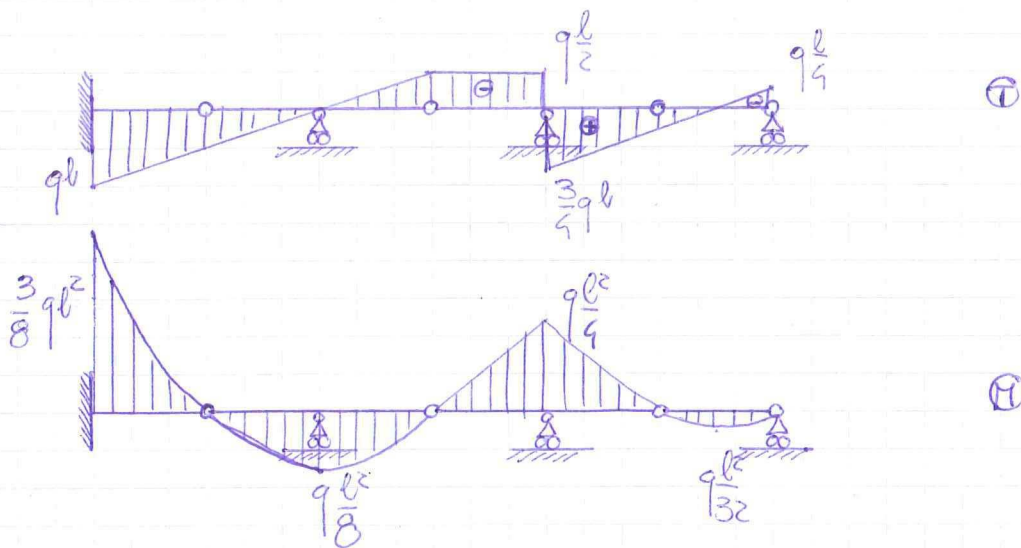
$$F) V_C = \frac{2}{8} \left(q l^2 - q \frac{l}{4} \cdot \frac{3l}{2} \right) = \frac{5}{4} q l = 375 \text{ kN}$$

$$E) V_B = \frac{2}{8} \left(q \frac{l^2}{2} + 2q l^2 - q \frac{l}{4} \cdot \frac{5}{2} l - \frac{5}{4} q l \cdot \frac{3}{2} l \right) = 0 \text{ kN}$$

$$I) V_A = \frac{5}{2} q l - q \frac{l}{4} - \frac{5}{4} q l = q l = 300 \text{ kN}$$

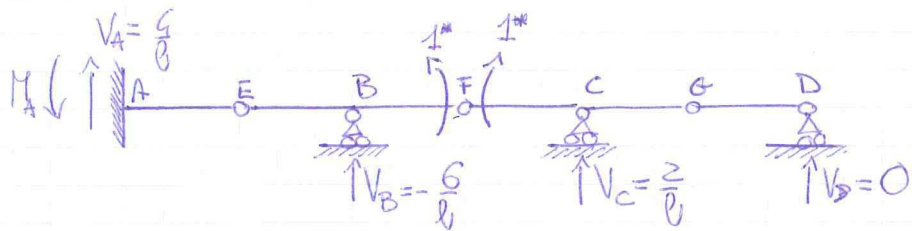
$$E) M_A = -q \frac{l^2}{8} + q \frac{l^2}{2} = \frac{3}{8} q l^2 = 1125 \text{ kNm}$$

Grafici delle sollecitazioni



Il grafico di N non è riportato in quanto nullo lungo tutta la trave.

Per il calcolo della rotazione relativa $\Delta\varphi_F$ in F utilizzo il seguente sistema virtuale



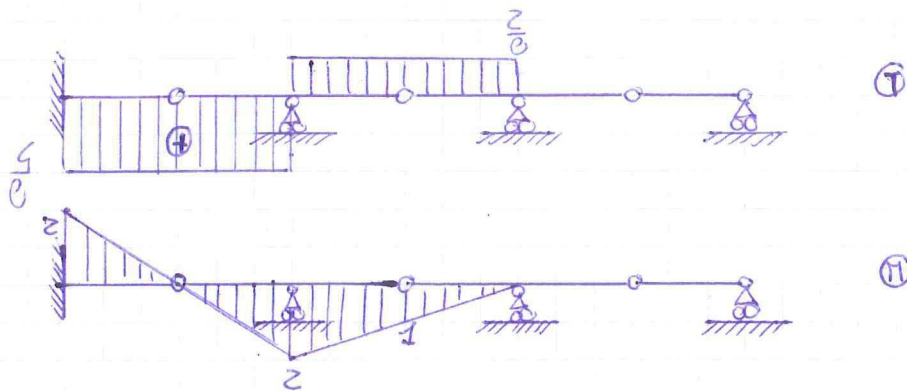
$$f) V_C = \frac{2}{l} \cdot 1^* = \frac{2}{l}$$

$$E) \sum \frac{3l}{l} + V_B \frac{l}{2} = 0 \Rightarrow V_B = -\frac{6}{l}$$

$$\uparrow V_A = \frac{6}{l} - \frac{2}{l} = \frac{4}{l}$$

$$E) M_A = \frac{1}{l} \cdot \frac{l^2}{2} = \frac{l}{2}$$

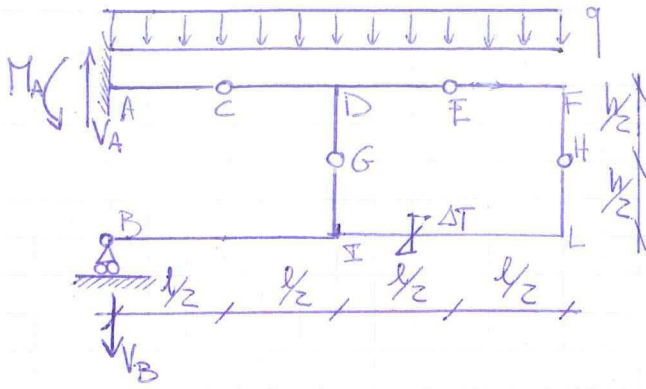
I grafici delle sollecitazioni nel sistema virtuale sono



Si applica ora il PLV per il calcolo di $\Delta\varphi_F$:

$$1^* \Delta\varphi_F = \int_0^l \frac{M^*(z) M(z)}{EJ} dz + V_C^* \cdot \frac{V_C}{k_C}$$

ESERCIZIO 3



$$l = 3 \text{ m}$$

$$h = 2 \text{ m}$$

$$q = 10 \text{ kN/m}$$

$$\alpha = 1.2 \cdot 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

$$\Delta T = 20^\circ\text{C}$$

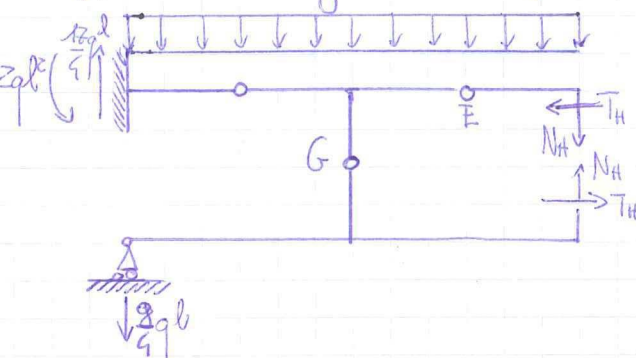
Calcolo delle reazioni vincolari

$$\uparrow V_B = \frac{2}{l} \cdot q \left(\frac{3/2 l}{2}\right)^2 = \frac{9}{4} q l = 67.5 \text{ kN}$$

$$\uparrow V_A = q l \left(\frac{9}{4} + 2\right) = \frac{17}{4} q l = 127.5 \text{ kN}$$

$$\curvearrowright M_A = \left(\frac{17}{8} - \frac{1}{8}\right) q l^2 = 2 q l^2 = 180 \text{ kN}$$

Aperto la maglia in H per ricavare le sollecitazioni interne

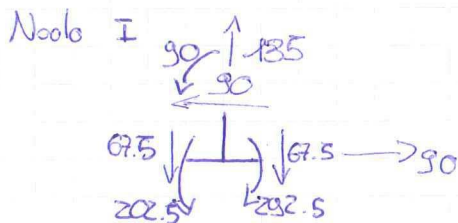
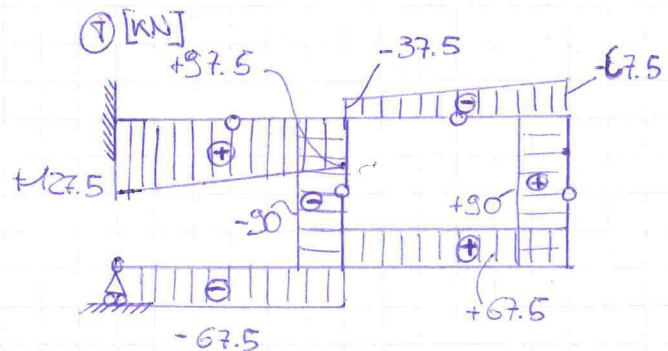
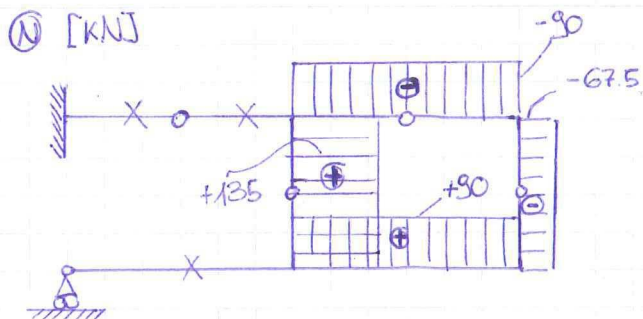


$$\curvearrowright N_H = -\frac{1}{l} \frac{9}{4} q l^2 = -\frac{9}{4} q l = -67.5 \text{ kN}$$

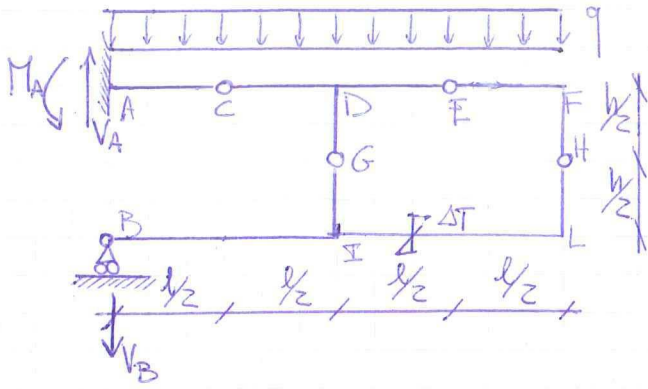
$$\uparrow \sum F = 0 \Rightarrow +\frac{9}{8} q l^2 - q \frac{l^2}{8} - T_H \frac{h}{2} = 0$$

$$T_H = 2 q l^2 = 90 \text{ kN}$$

Grafici delle sollecitazioni



ESERCIZIO 3



$$l = 3 \text{ m}$$

$$h = 2 \text{ m}$$

$$q = 10 \text{ kN/m}$$

$$\alpha = 1.2 \cdot 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

$$\Delta T = 20^\circ\text{C}$$

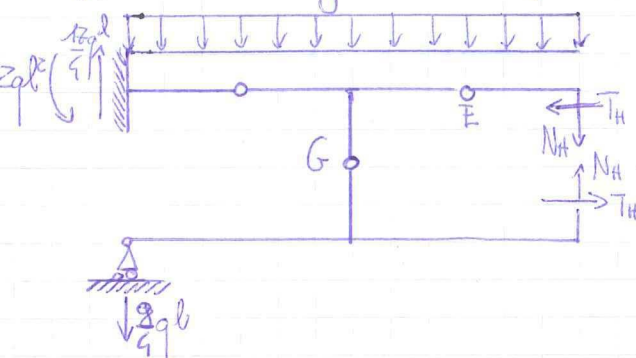
Calcolo delle reazioni vincolari

$$\uparrow V_B = \frac{2}{l} \cdot q \left(\frac{3}{2}l\right)^2 = \frac{9}{4} q l = 67.5 \text{ kN}$$

$$\uparrow V_A = q l \left(\frac{9}{4} + 2\right) = \frac{17}{4} q l = 127.5 \text{ kN}$$

$$\curvearrowright M_A = \left(\frac{17}{8} - \frac{1}{8}\right) q l^2 = 2 q l^2 = 180 \text{ kN}$$

Aperto la maglia in H per ricavare le sollecitazioni interne

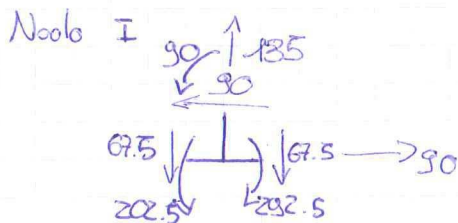
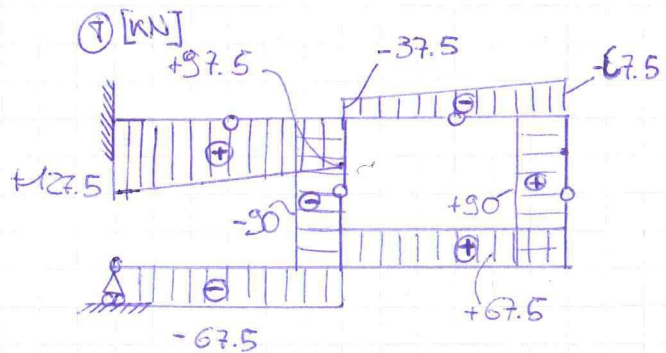
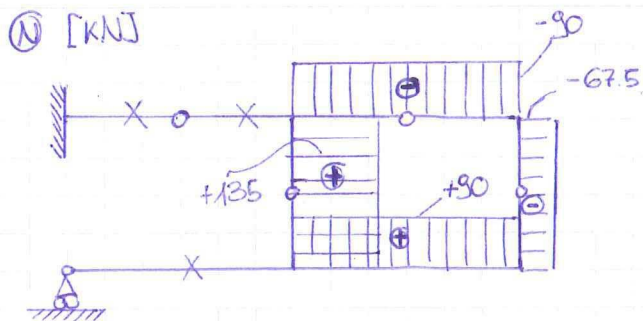


$$\curvearrowright N_H = -\frac{1}{l} \frac{9}{4} q l^2 = -\frac{9}{4} q l = -67.5 \text{ kN}$$

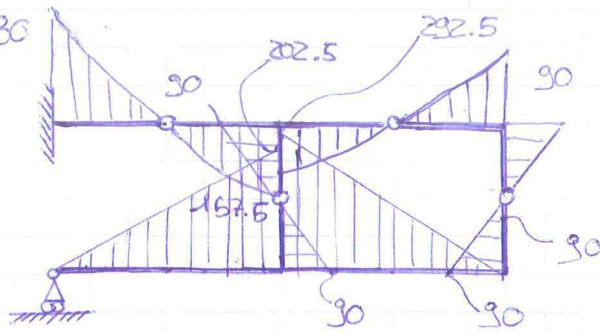
$$\curvearrowright +\frac{9}{8} q l^2 - q \frac{l^2}{8} - T_H \frac{h}{2} = 0$$

$$T_H = \frac{2 q l^2}{h} = 90 \text{ kN}$$

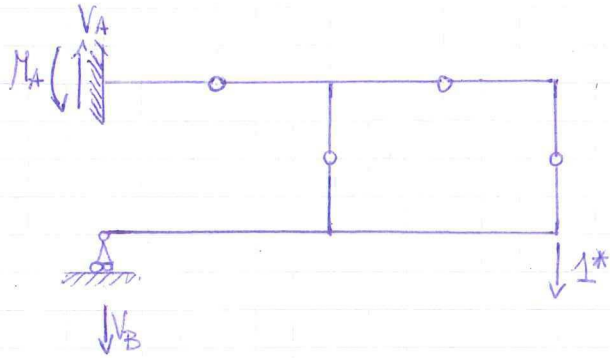
Grafici delle sollecitazioni



⑫ [kNm] -180



Per il calcolo dello spostamento v_B si considera il seguente sistema virtuale

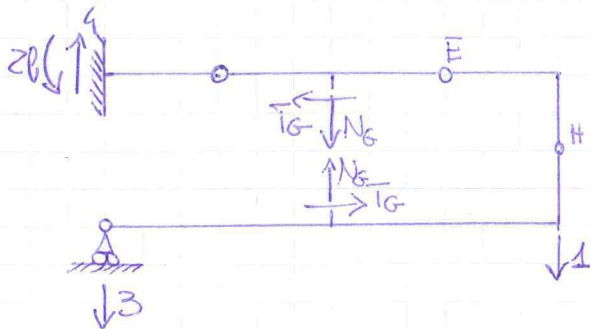


$$E) V_B = \frac{2}{l} \cdot 1 \cdot \frac{3}{2} l = 3$$

$$\uparrow V_A = 1 + 3 = 4$$

$$E) M_A = \frac{1}{2} \cdot l \cdot \frac{3}{2} l = \frac{3}{2} l$$

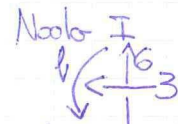
Aperto in G per il calcolo delle azioni interne



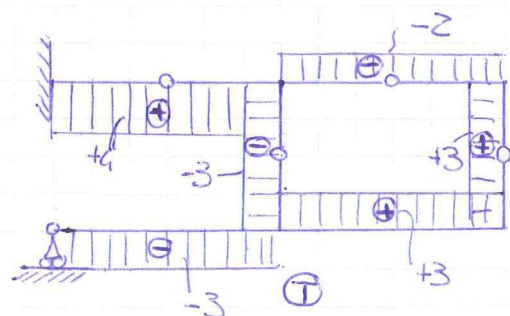
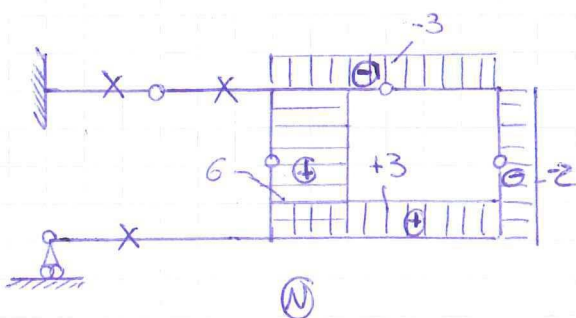
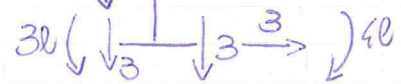
$$H) N_G = 3 \cdot 2l \cdot \frac{1}{l} = 6$$

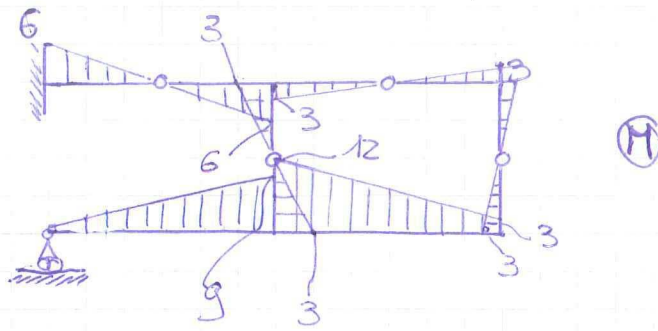
$$E) 2l - \frac{2}{l} \cdot \frac{3}{2} l + 6 \cdot \frac{l}{2} - T_G \frac{h}{2} = 0$$

$$T_G = -\frac{2l}{h} = -3$$



Grafici delle sollecitazioni del sistema virtuale





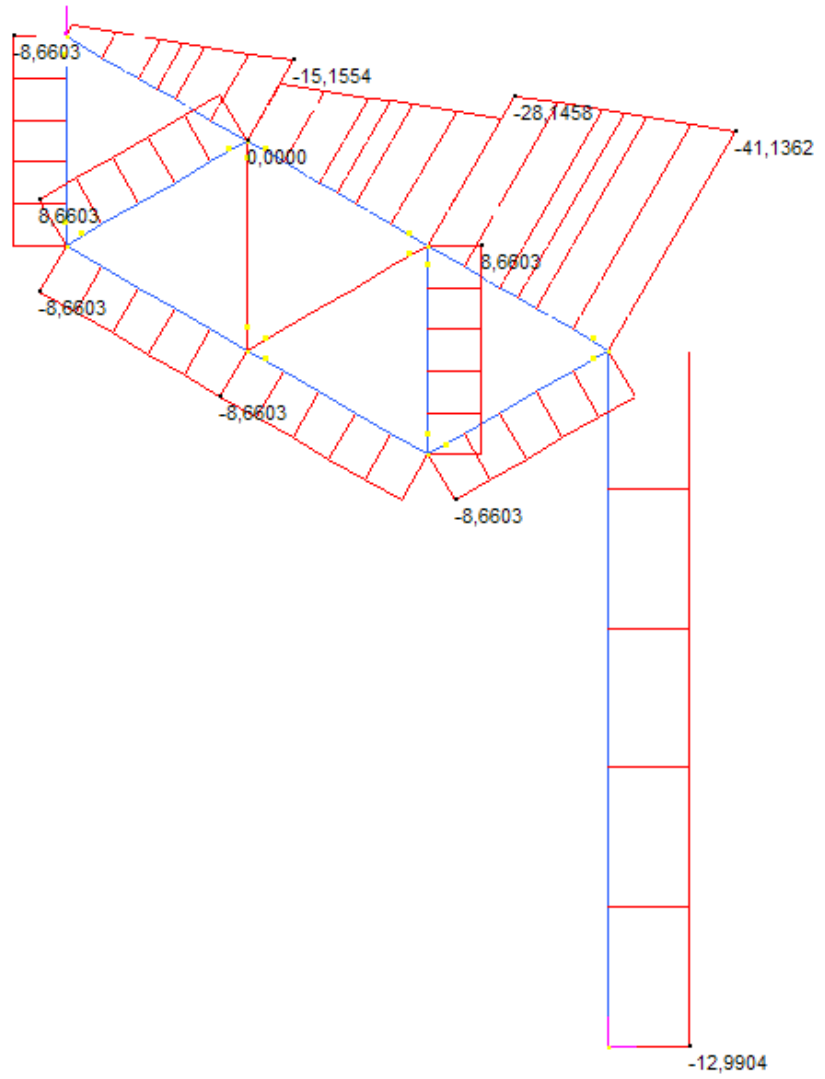
Attraverso il PLV si calcola v_L

$$1 \cdot v_L = \int \left[N^*(z) \frac{N(z)}{EA} + M^*(z) \left(\frac{M(z)}{EI} - \frac{2\alpha \Delta T}{h} \right) \right] dz = \dots$$

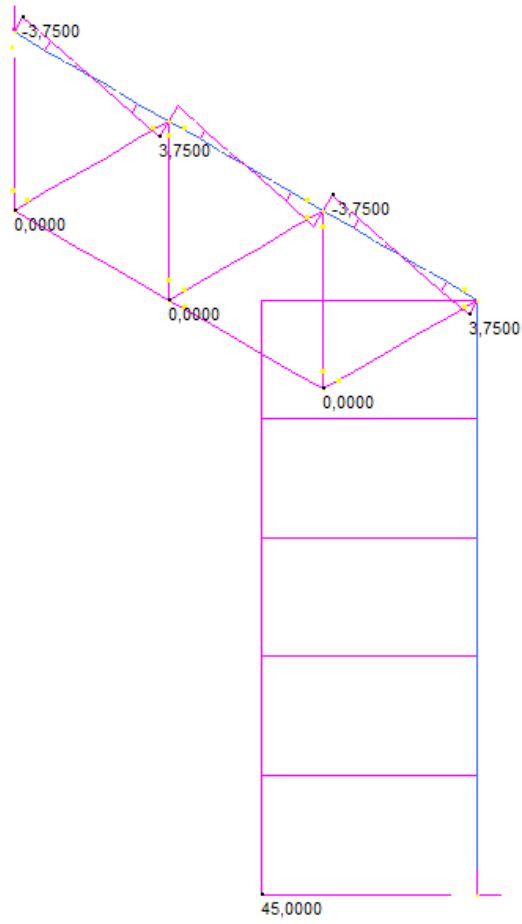
FILA A

ES1

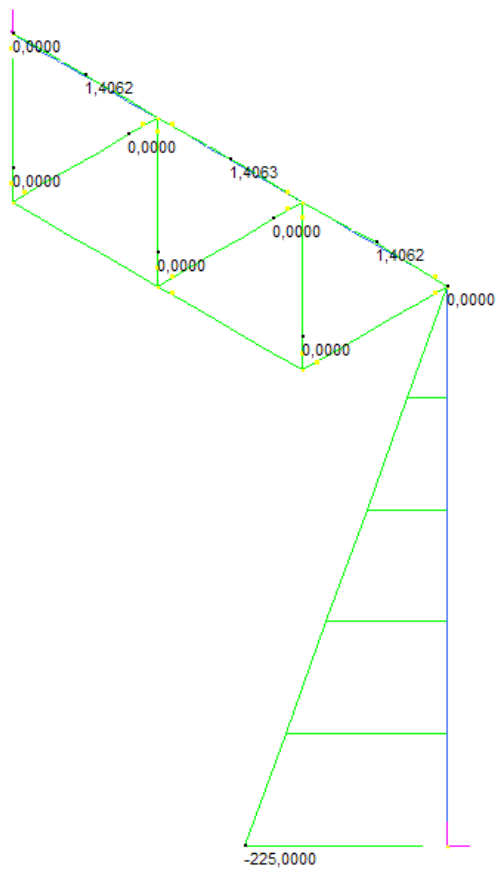
	MIN	MAX
Force(kN)	-41,1362	8,6603
	[Bm:3]	[Bm:9]



	MIN	MAX
SF2(kN)	-3,7500	45,0000
	[Bm:3]	[Bm:1]

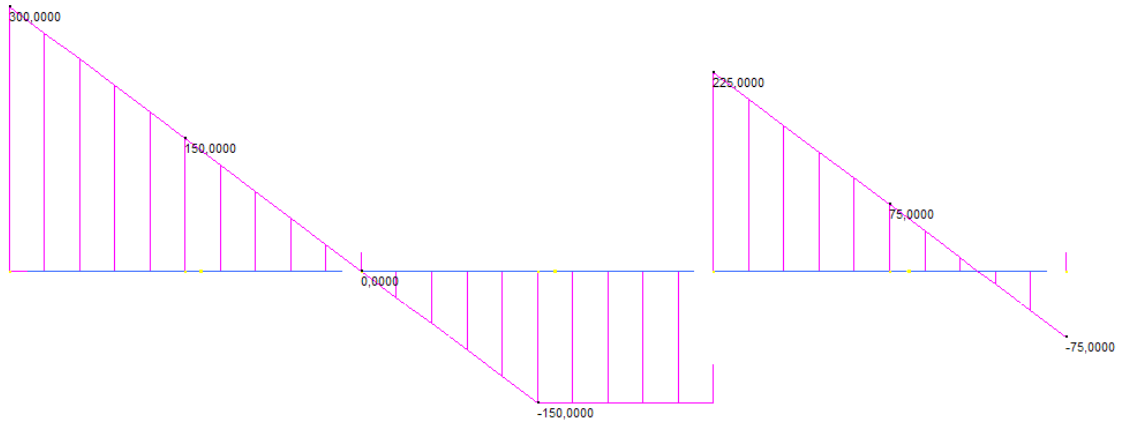


	MIN	MAX
M2(kN.m)	-225,0000	1,4063
	[Bm:1]	[Bm:4]

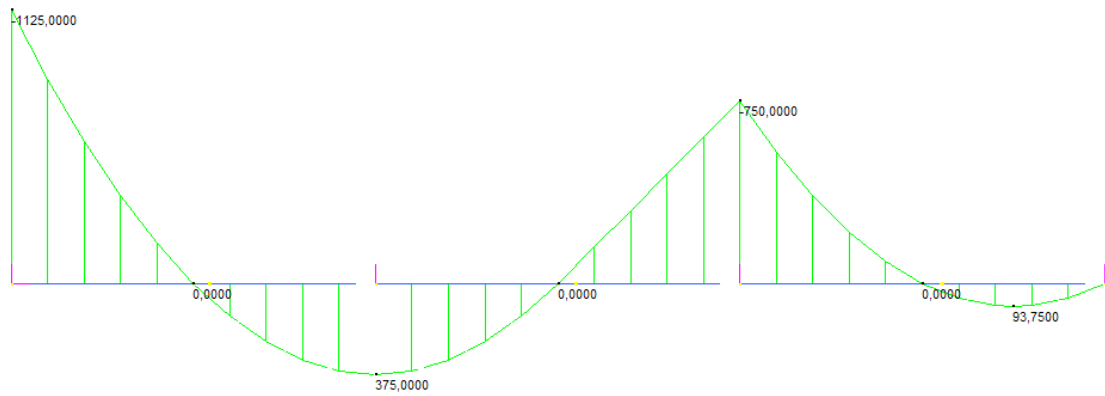


ES2

	MIN	MAX
SF2(kN)	-150,0000	300,0000
	[Bm:3]	[Bm:1]

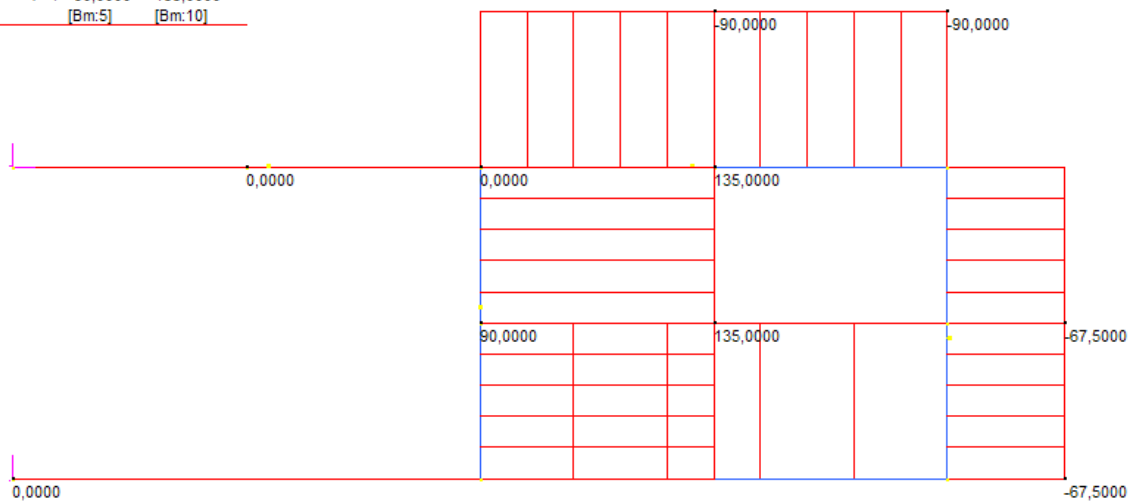


	MIN	MAX
BM2(kN.m)	-1125,0000	375,0000
	[Bm:1]	[Bm:3]

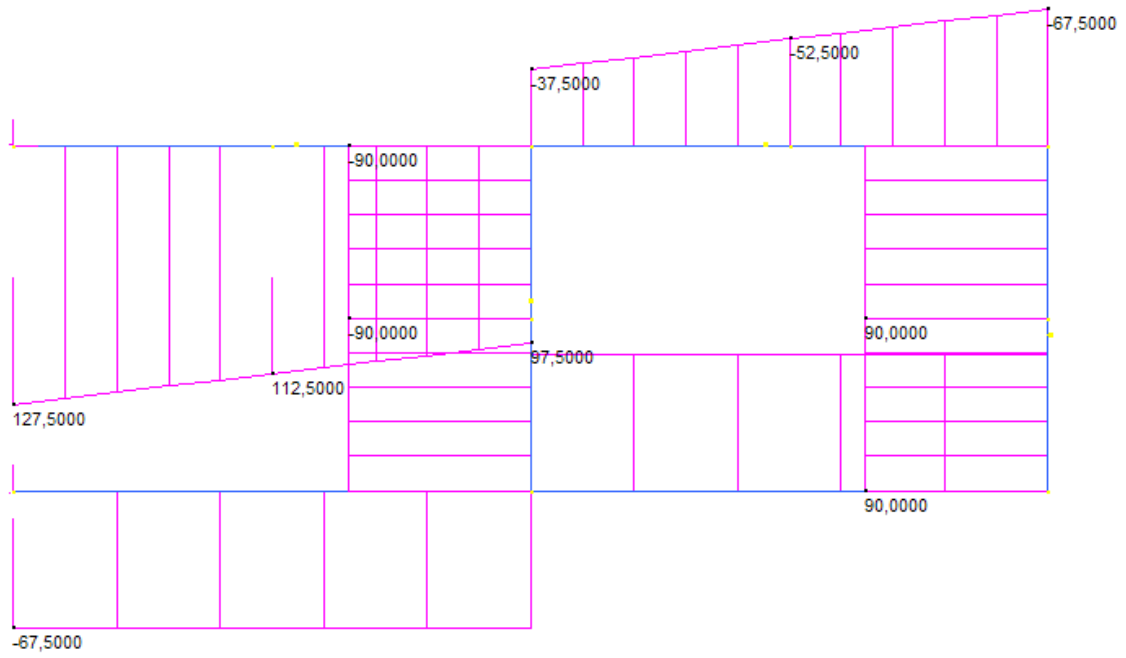


ES 3

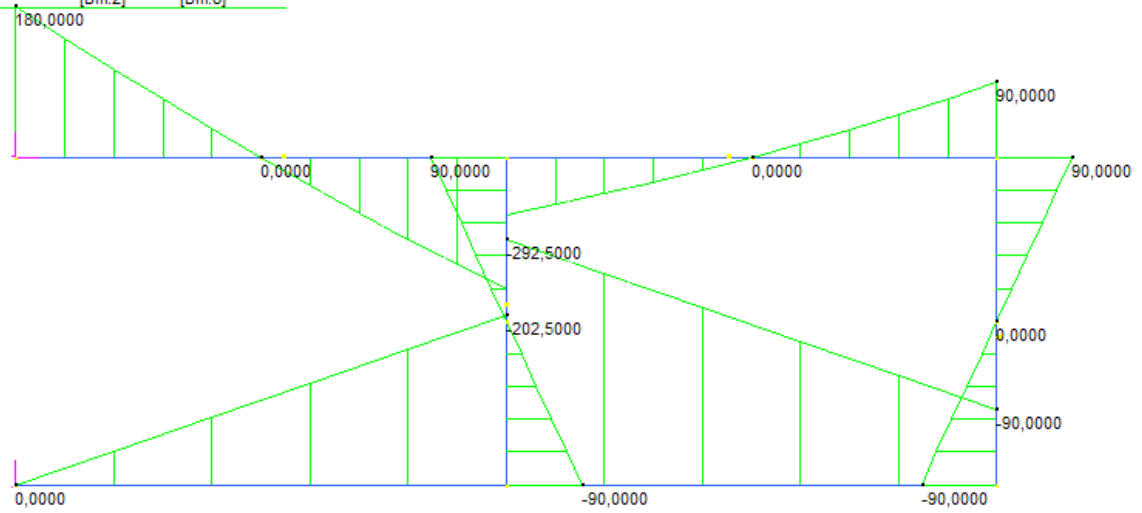
	MIN	MAX
Force(kN)	-90,0000	135,0000
	[Bm:5]	[Bm:10]



	MIN	MAX
SF2(kN)	-90,0000	127,5000
	[Bm:10]	[Bm:8]



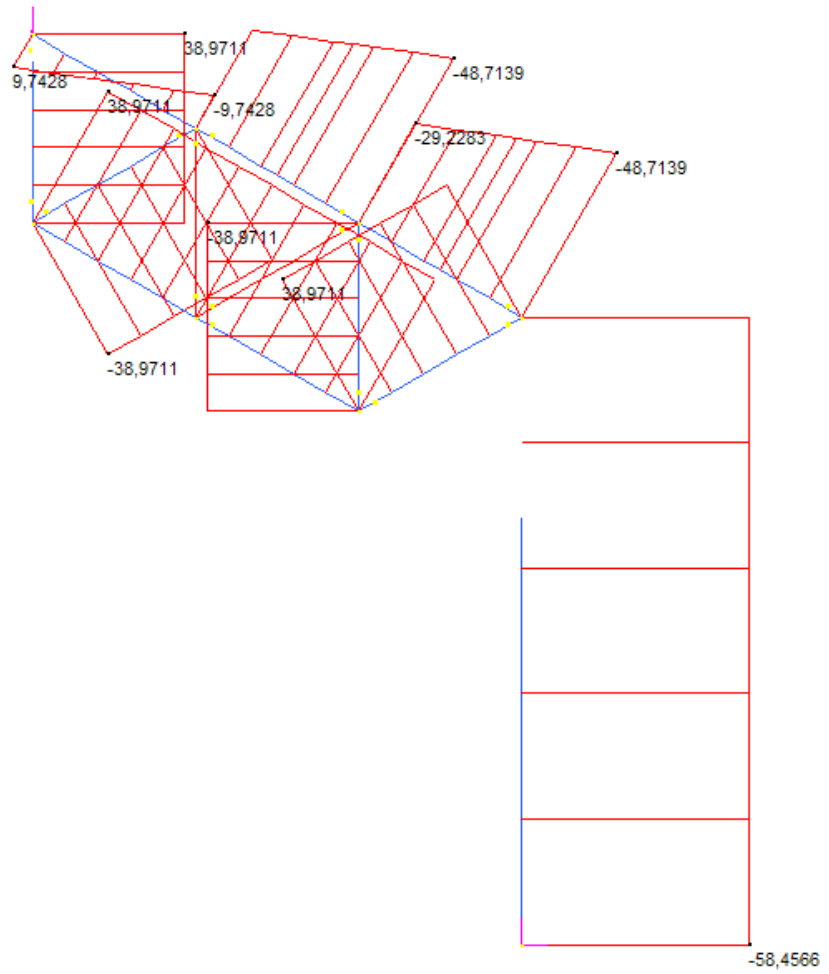
	MIN	MAX
BM2(kN.m)	-292,5000	180,0000
	[Bm:2]	[Bm:8]



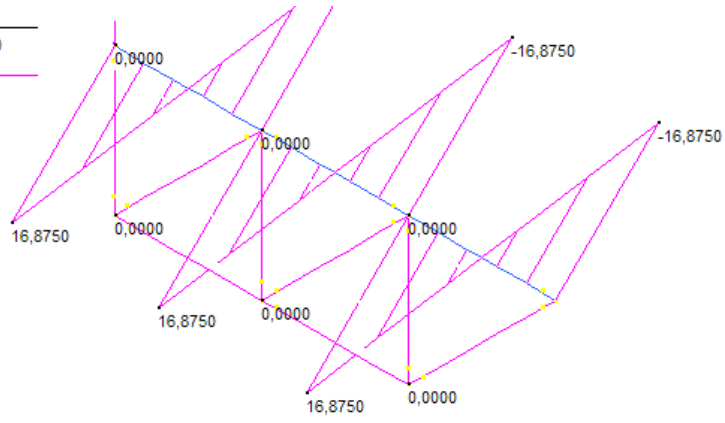
FILA B

ES 1

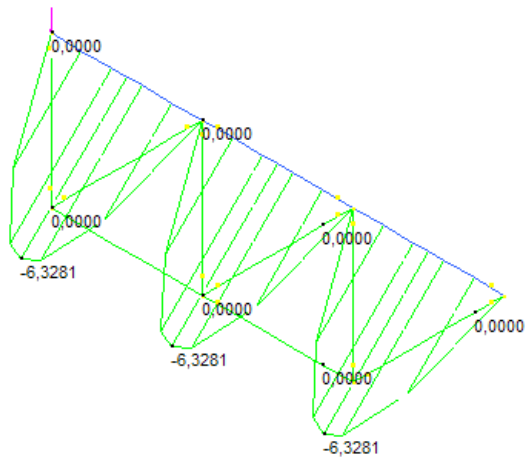
	MIN	MAX
Force(kN)	-58,4566	38,9711
	[Bm:1]	[Bm:5]



	MIN	MAX
SF2(kN)	-16,8750	16,8750
	[Bm:3]	[Bm:2]

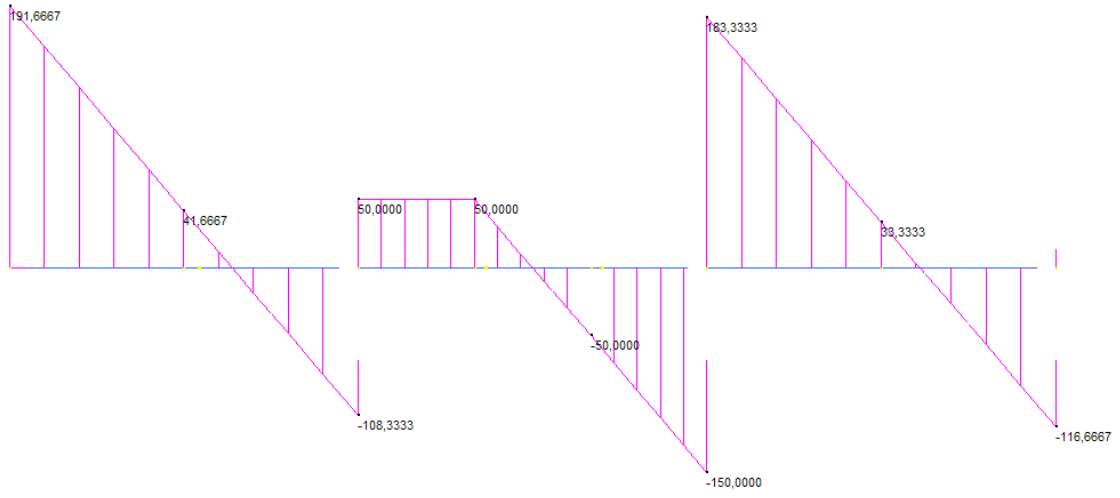


	MIN	MAX
BM2(kN.m)	-6,3281	0,0000
	[Bm:3]	[Bm:11]

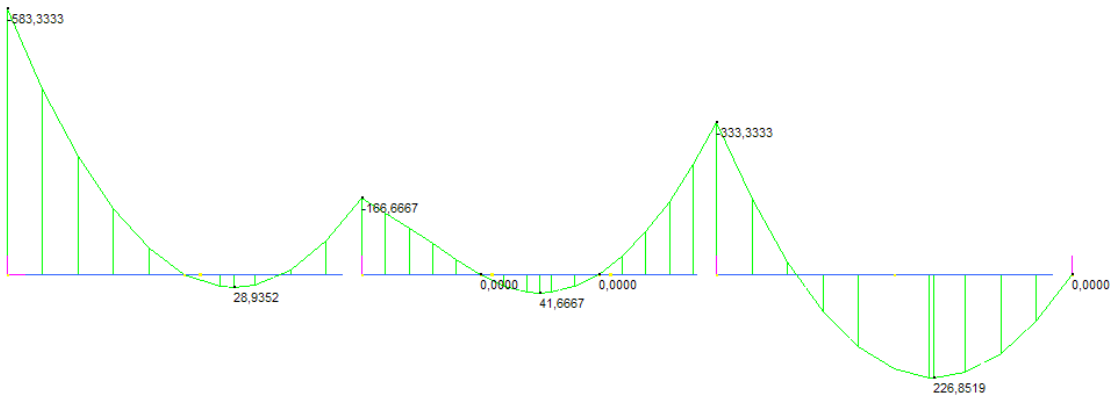


ES 2

	MIN	MAX
SF2(kN)	-150,0000	191,6667
	[Bm:5]	[Bm:1]

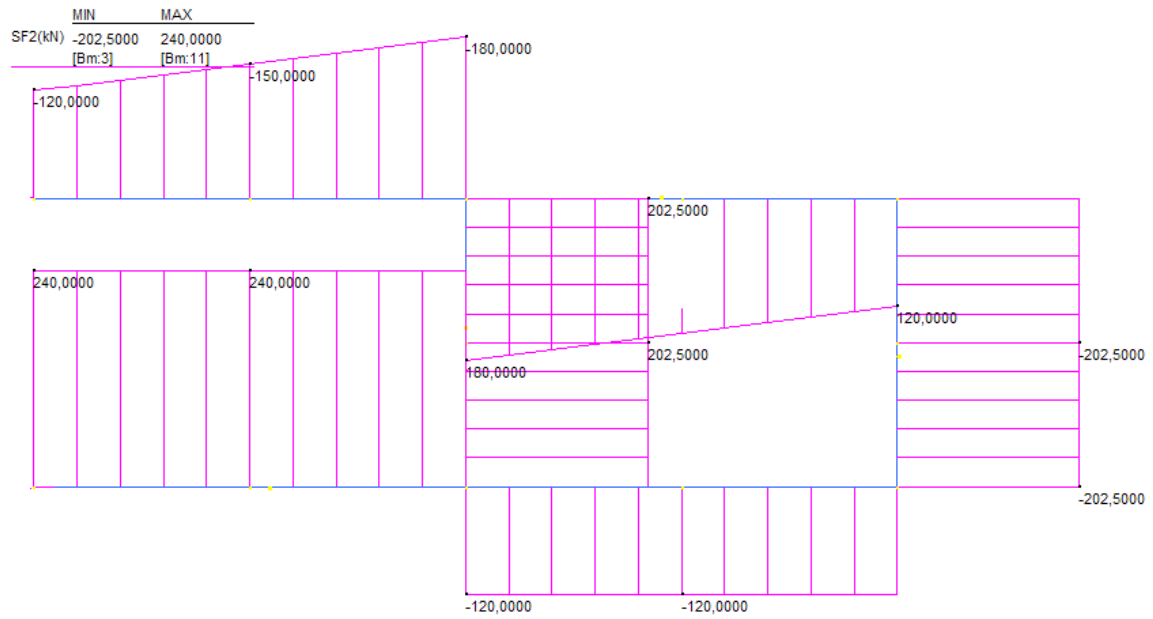
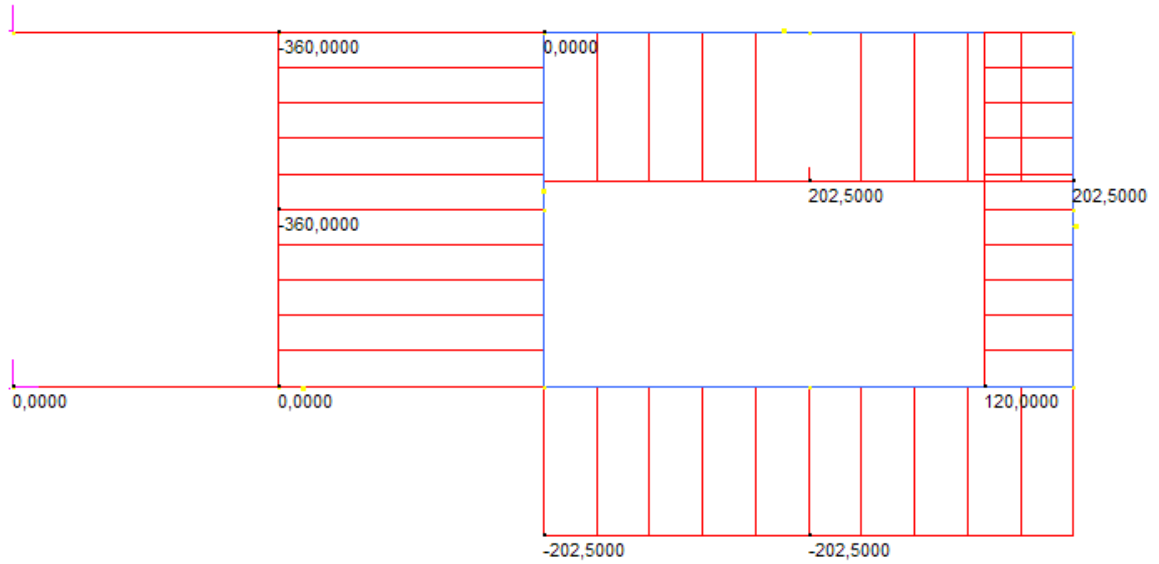


	MIN	MAX
BM2(kN.m)	-583,3333	226,8519
	[Bm:1]	[Bm:4]

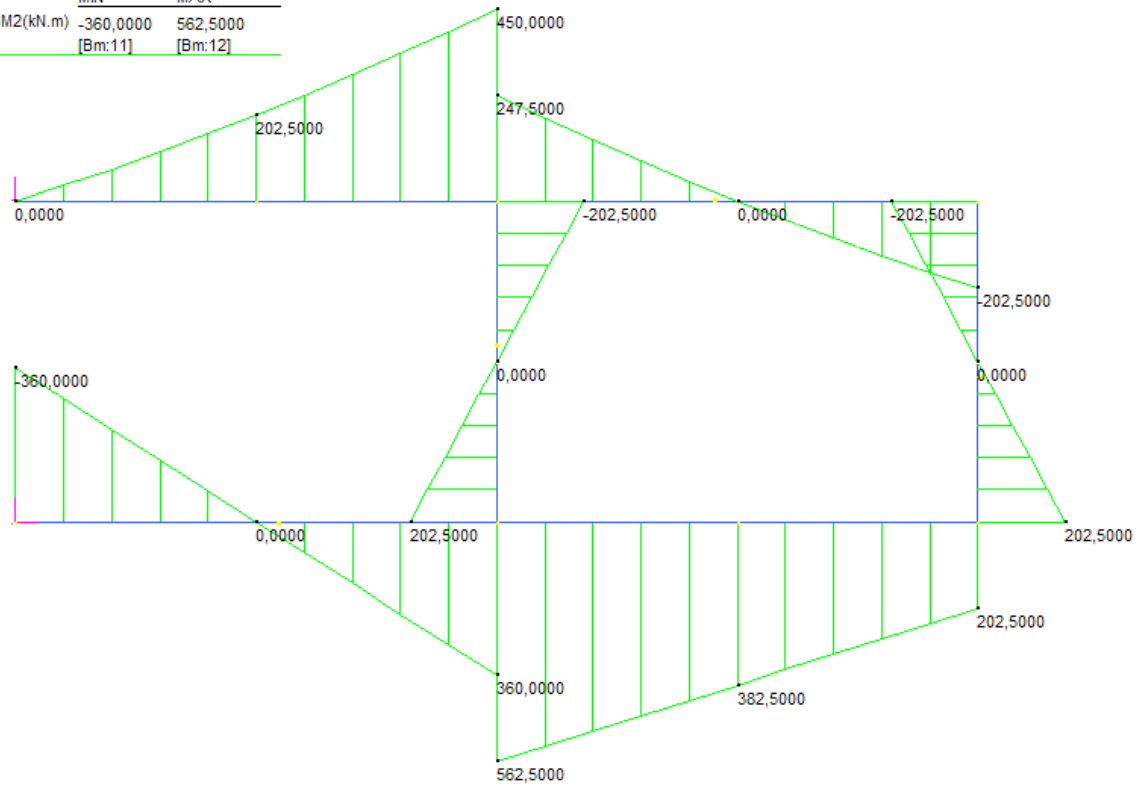


ES 3

	MIN	MAX
Force(kN)	-360,0000	202,5000
	[Bm:10]	[Bm:6]



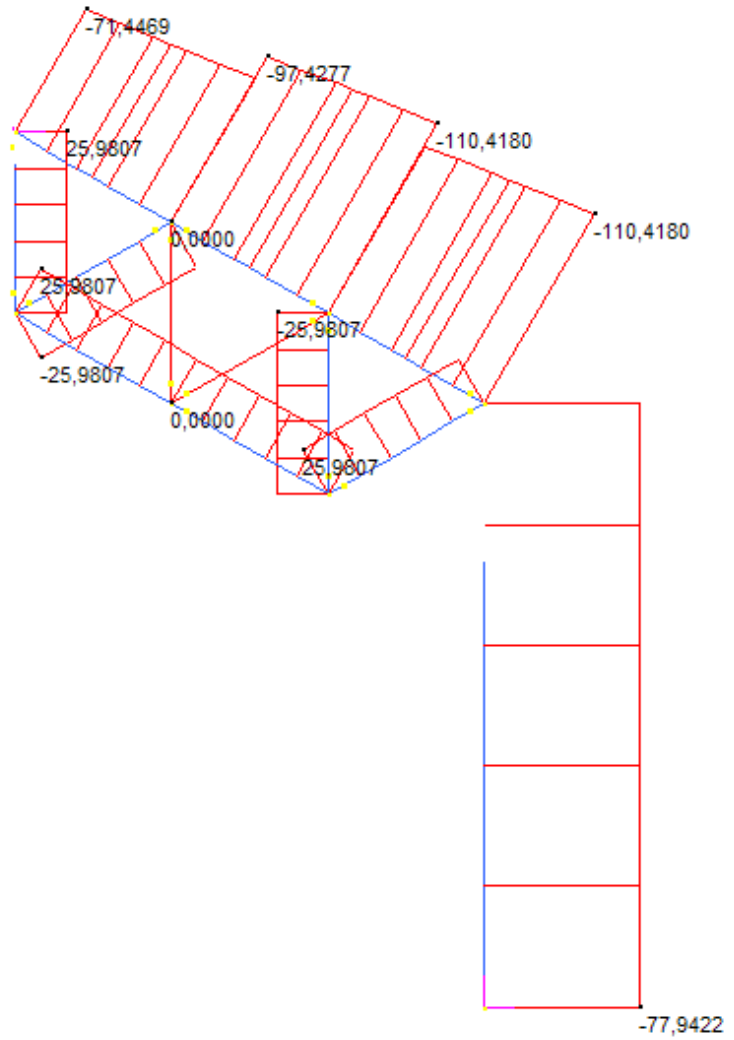
	MIN	MAX
BM2(kN.m)	-360,0000	562,5000
	[Bm:11]	[Bm:12]



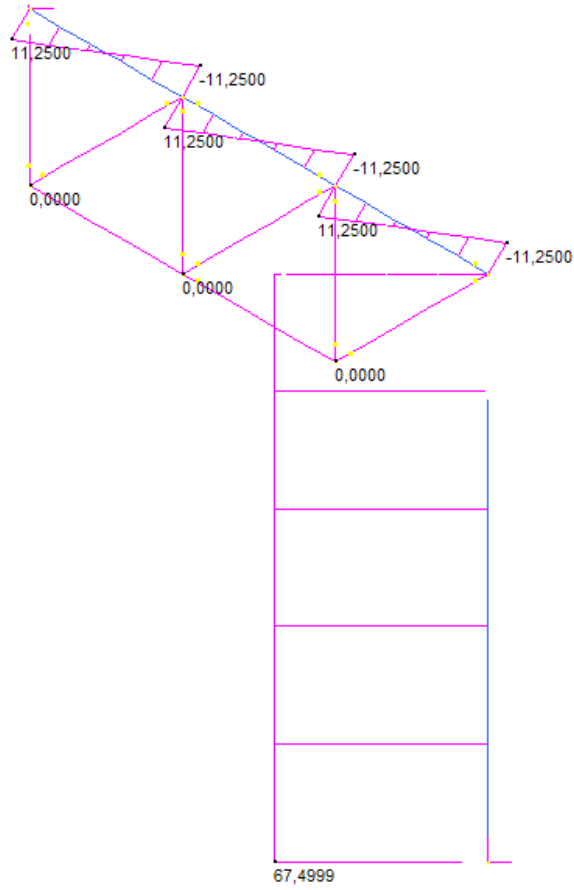
FILA C

ES 1

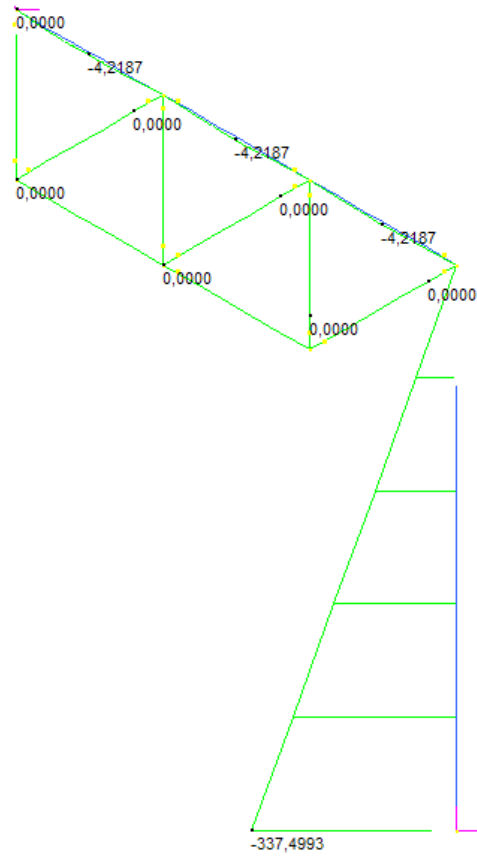
	MIN	MAX
Force(kN)	-110,4180	25,9807
	[Bm:4]	[Bm:5]



	MIN	MAX
SF2(kN)	-11,2500	67,4999
	[Bm:3]	[Bm:1]

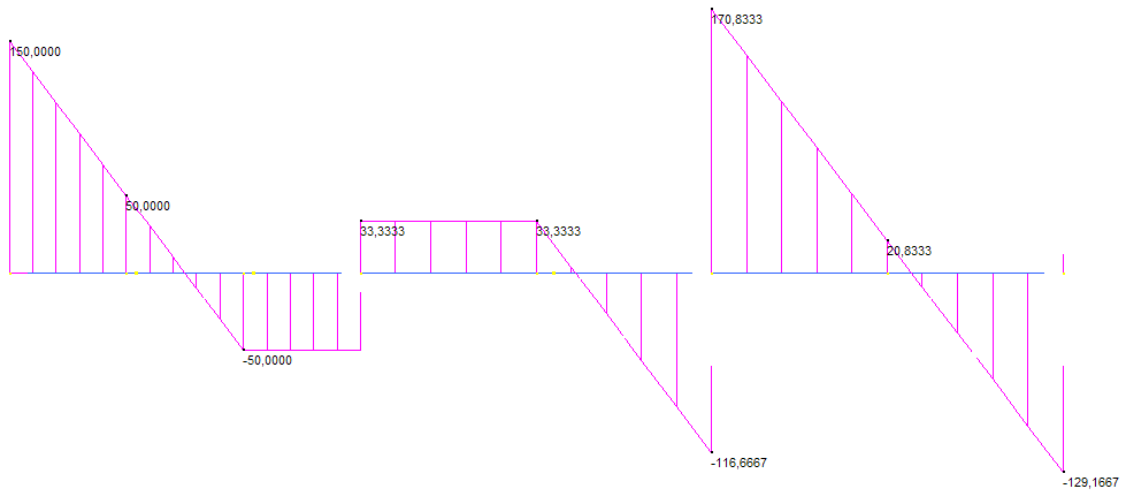


	MIN	MAX
BM2(kN.m)	-337,4993	0,0000
	[Bm:1]	[Bm:11]

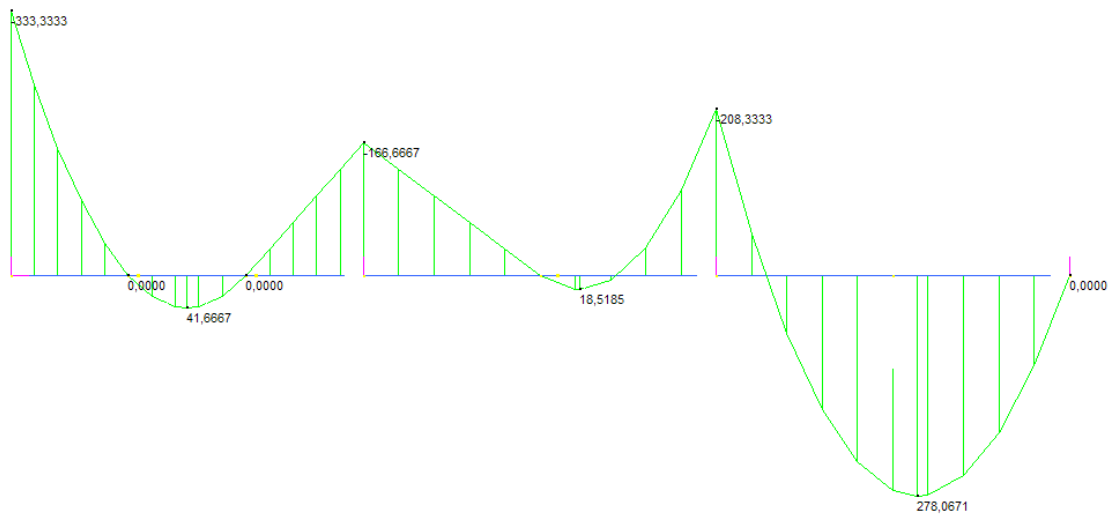


ES 2

	MIN	MAX
SF2(kN)	-129,1667	170,8333
[Bm:2]	[Bm:1]	

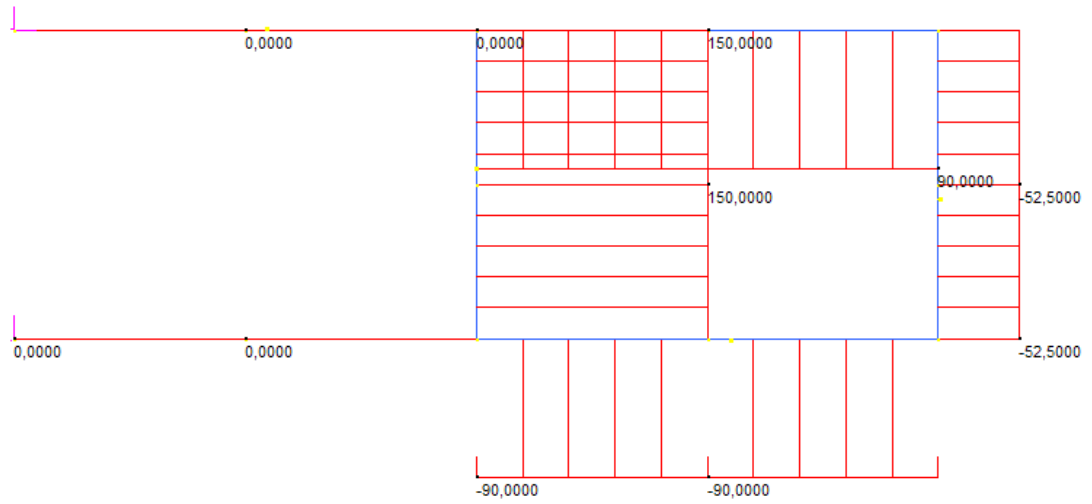


	MIN	MAX
BM2(kN.m)	-333,3333	278,0671
[Bm:5]	[Bm:2]	

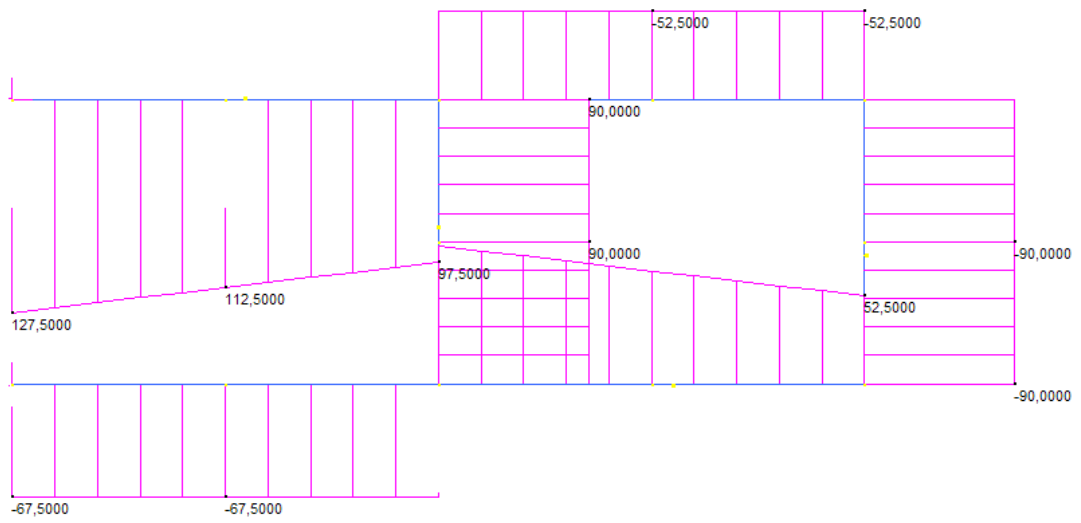


ES 3

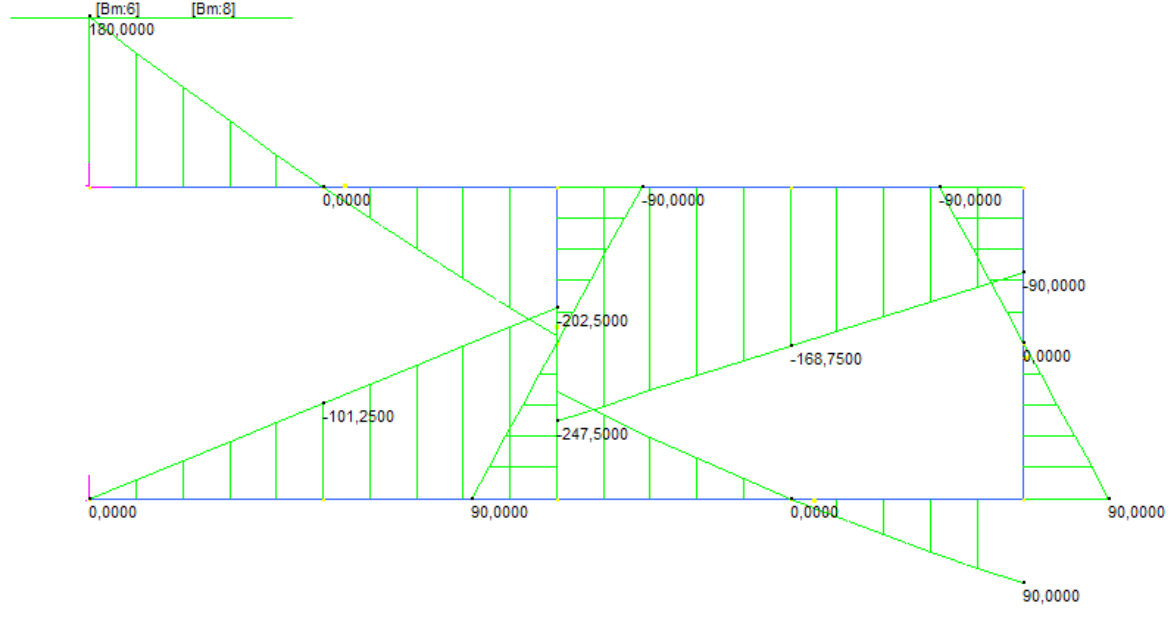
	MIN	MAX
Force(kN)	-90,0000	150,0000
	[Bm:12]	[Bm:9]



	MIN	MAX
SF2(kN)	-90,0000	127,5000
	[Bm:3]	[Bm:8]

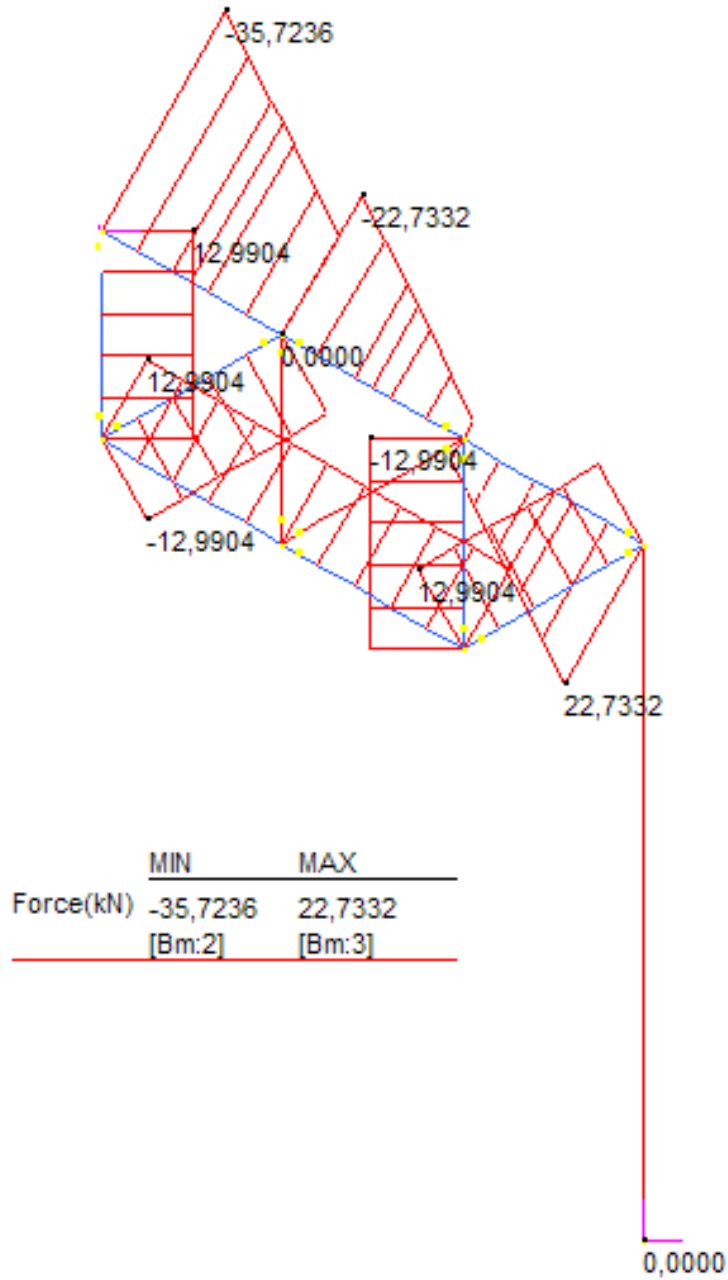


	MIN	MAX
BM2(kN.m)	-247,5000	180,0000
	[Bm:8]	[Bm:8]

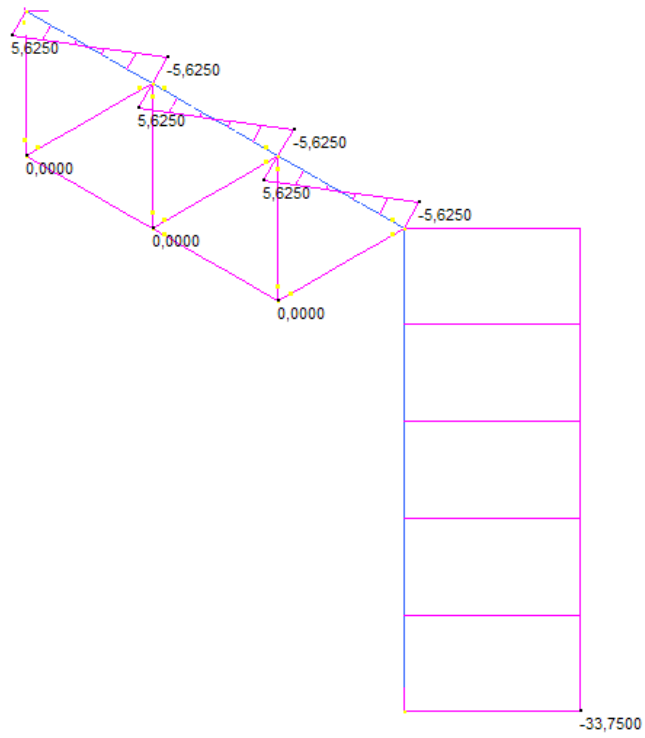


FILA D

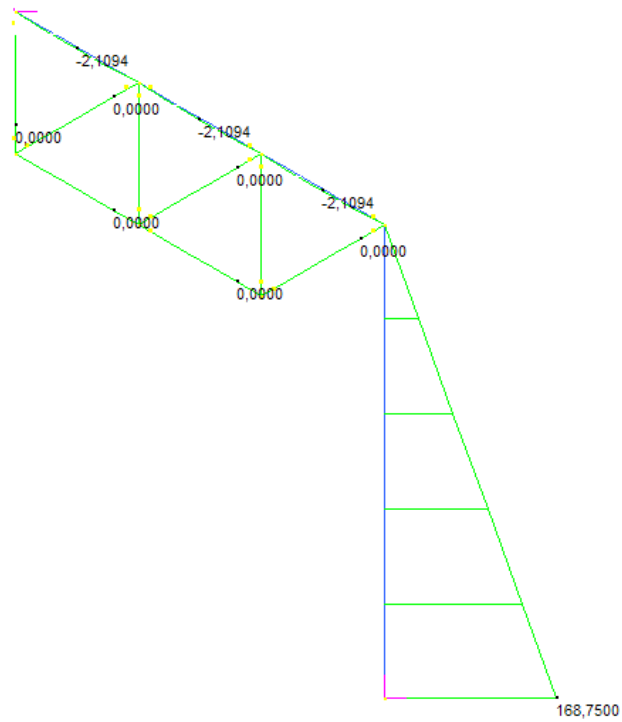
ES 1



	MIN	MAX
SF2(kN)	-33,7500	5,6250
	[Bm:1]	[Bm:3]

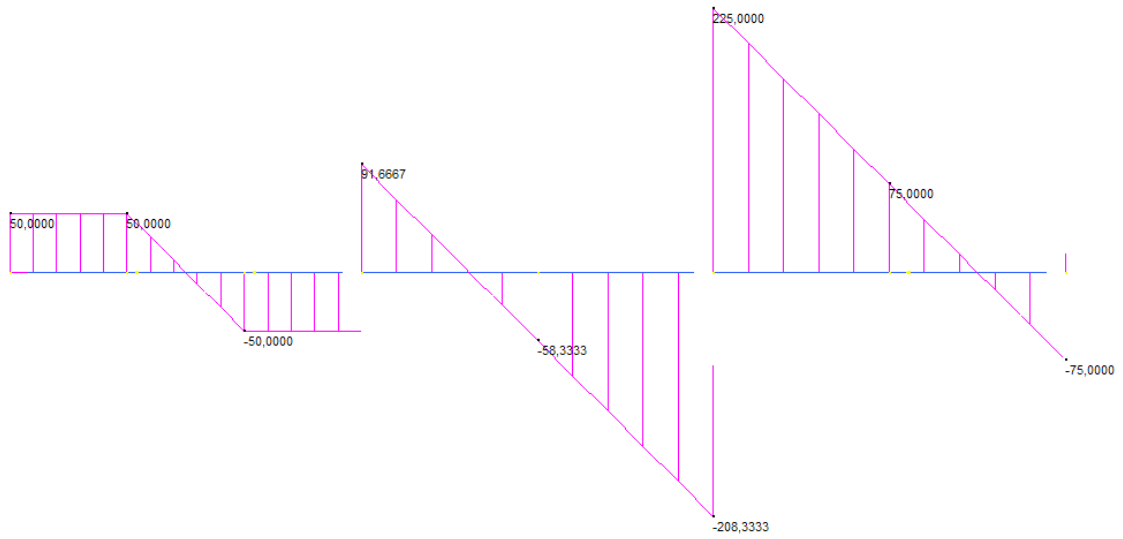


	MIN	MAX
BM2(kN.m)	-2,1094	168,7500
	[Bm:4]	[Bm:1]

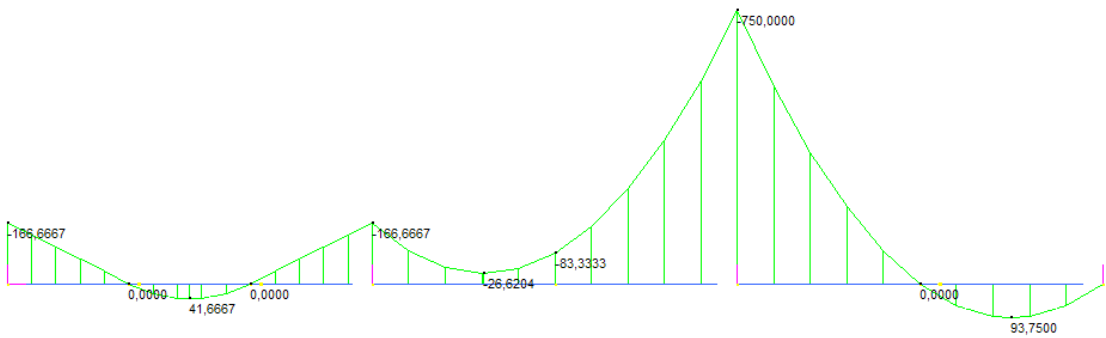


ES 2

	MIN	MAX
SF2(kN)	-208,3333	225,0000
	[Bm:3]	[Bm:1]



	MIN	MAX
BM2(kN.m)	-750,0000	93,7500
	[Bm:3]	[Bm:2]



ES 3

	MIN	MAX
Force(kN)	-165,0000	101,2500
	[Bm:10]	[Bm:2]

