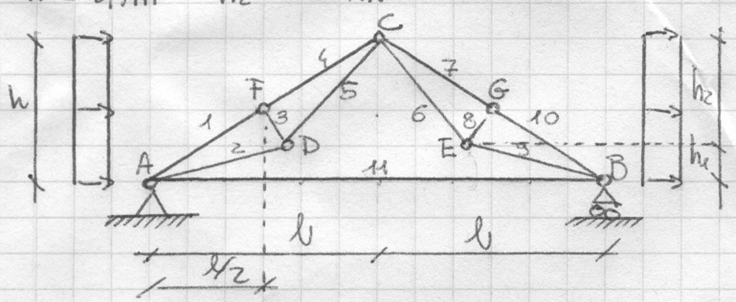
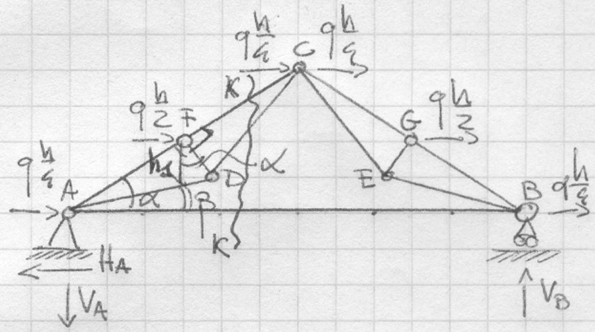
<u>ESERCIZIO 1</u>

$l = 6\ m \quad h_1 = 1\ m \quad q = 10\ kN/m$

$h = 4m \quad h_2 = 3\ m$





$\rightarrow H_A = 2qh = 80\ kN$

$A)\ V_B 2l = 8\, q\dfrac{h^2}{8} \Rightarrow V_B = q\dfrac{h^2}{2l} = 13.\bar{3}\ kN$

$V_A = 13.\bar{3}\ kN$

Definizione di alcuni parametri geometrici

$\alpha = atg\left(\dfrac{4}{6}\right) = 33.7°$ $\qquad \overline{AC} = \sqrt{l^2+h^2} = 7{,}21m$

$\overline{FD}\cos\alpha = 1 \Rightarrow \overline{FD} = \dfrac{1}{\cos\alpha} = 1.2\ m$

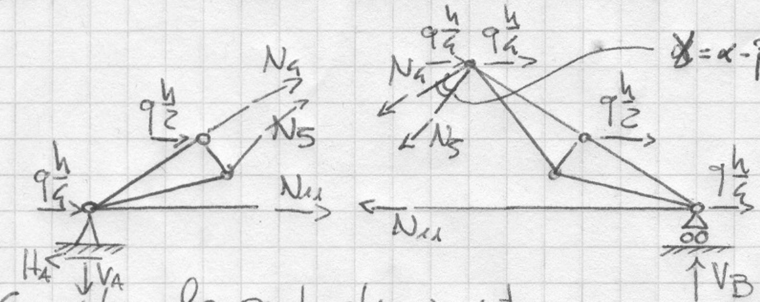
$\overline{AD}\cos\beta = \dfrac{l}{2} + \overline{FD}\sin\alpha = 3 + 1.2\sin(33.7°) = 3.66m$

$\beta = atg\left(\dfrac{h_1}{\overline{AD}\cos\beta}\right) = 15.2°$

Uso il metodo delle sezioni di Ritter

	SR^1	SR^2	SV
1	15.78	⊛	0.6*
2	17.48	0	0
3	-13.81	0	0
4	-0.85	⊛	0.6*
5	17.48	0	0
6	-17.48	0	0
7	0.85	⊛	-0.6*
8	13.81	0	0
9	-17.48	0	0
10	-15.78	⊛	-0.6*
11	40	0	0.5*

⊛ guarda i grafici nella pagina successiva



$\gamma = \alpha - \beta = 18.5°$

Considero la parte di sinistra

$A)\ -q\dfrac{h}{2}\cdot\dfrac{h}{2} + N_5\,\overline{AC}\sin\gamma = 0 \Rightarrow N_5 = q\dfrac{h^2}{4\,\overline{AC}\sin\gamma} = 17.48\ kN$

Considero la parte destra

$C)\ q\dfrac{h}{2}\dfrac{h}{2} + q\dfrac{h}{4}h + q\dfrac{h^2}{2l}\, l - N_{11} h = 0$

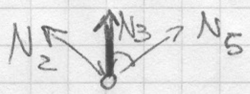
$N_{11} = \cancel{2}\cdot q\frac{h}{\cancel{4}\,2} + q\frac{h}{2} = qh = 40\ kN$

$\rightarrow\ q\frac{h}{2} + 3q\frac{h}{4} - N_4\cos\alpha - N_5\cos(\alpha+\gamma) - N_{11} = 0$

$N_4 = \frac{1}{\cos\alpha}\left[\frac{5}{4}qh - N_5\cos(\alpha+\gamma) - N_{11}\right] = -0.85\ kN$

A questo punto si procede con l'equilibrio dei nodi

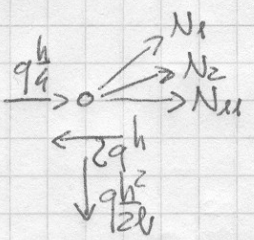
Nodo D



$N_2 = N_5 = 17.48$

$N_3 = -N_5\cos\left[\pi/2 - (\alpha+\gamma)\right] = -13.81\ kN$

Nodo A

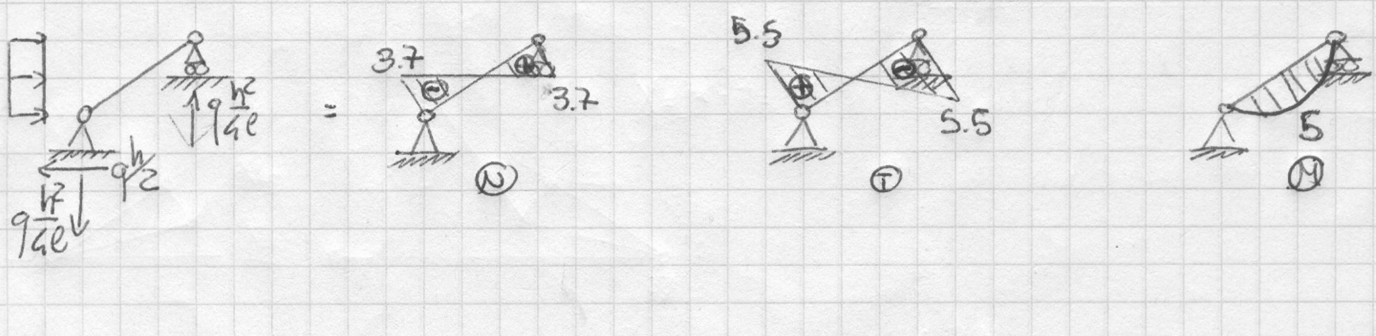


$\rightarrow\ N_1\cos\alpha + N_2\cos\beta + N_{11} + q\frac{h}{4} - 2qh = 0$

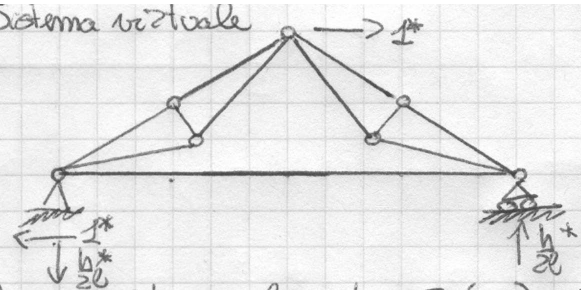
$N_1 = \frac{1}{\cos\alpha}\left[\frac{7}{4}qh - N_{11} - N_2\cos\beta\right] = 15.78\ kN$

$\uparrow\ N_1\sin\alpha + N_2\sin\alpha = q\frac{h^2}{2l} \Rightarrow 13.\bar{3} = 13.\bar{3}$ ok!!!

La struttura è geometricamente simmetrica con carichi antisimmetrici. I vincoli non son simmetrici, d'altra parte controllando le risultanti ai nodi risulta simmetrica, quindi le sollecitazioni in 6, 7, 8, 9 e 10 si ricavano da 1, 2, 3, 4, 5.
Si calcolano le sollecitazioni secondarie:



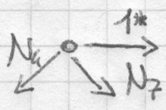
Sistema virtuale



Per equilibrio al nodo F (G), l'asta 3 (8) è scarica, quindi lo saranno anche le travi 2 e 5 (6 e 9).

Eq. ai nodi:

Nodo C

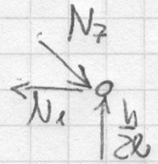


$\uparrow\ N_4 = -N_7$

$\rightarrow\ 1^* - N_4 \cos\alpha + N_7 \cos\alpha = 0$

$1^* + 2 N_7 \cos\alpha = 0 \Rightarrow N_7 = -\dfrac{1^*}{2\cos\alpha} = -0.6^*$

Nodo B



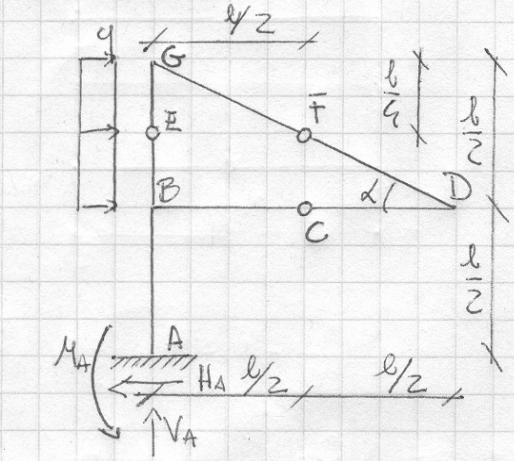
$\rightarrow\ N_7 \cos\alpha = N_1 \Rightarrow N_1 = 0.5^*$

$\uparrow\ -N_7 \sin\alpha + h/2l = 0 \Rightarrow 0 = 0$ ok!!!

Calcolo dello spostamento u_C:

$$1^* u_C = \sum_i N_i^* \frac{N^{I}}{EA} l_i + \sum_i \int_{\Omega_i} N_i^* \frac{N^{II}(z)}{EA}\, dz$$

ESERCIZIO 2



$l = 4\ m$

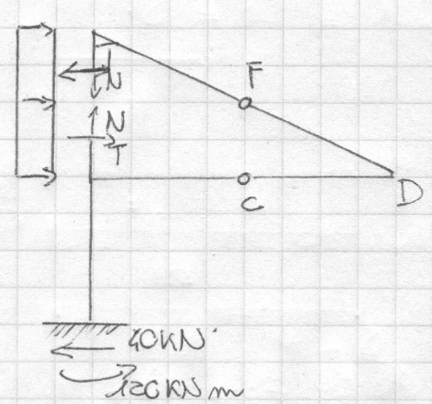
$q = 20\ kN/m$

$\alpha = atg(1/2) = 26.6°$

$H_A = q\frac{l}{2} = 40\ kN$ $\qquad M_A = q\frac{l}{2}\cdot\frac{3}{4}l = \frac{3}{8}ql^2 = 12$

$V_A = 0$

Calcolo le sollecitazioni interne in E



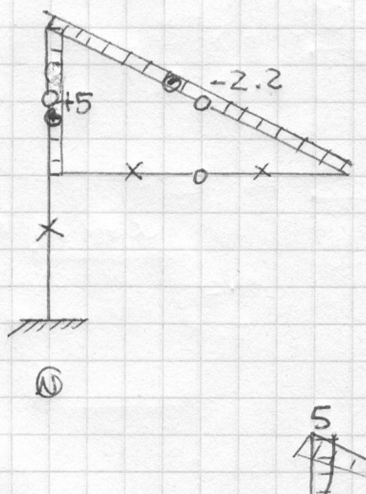
$\overset{\curvearrowright}{F})\ N\frac{\cancel{l}}{\cancel{2}} = q\frac{\cancel{l}}{4}\cdot\frac{l}{8} \Rightarrow N = q\frac{l}{16} = 5\,kN$

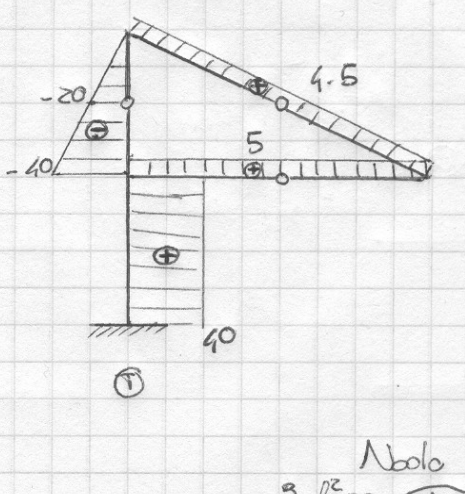
$\overset{\curvearrowright}{C})\ N\frac{\cancel{l}}{\cancel{2}} + T\frac{\cancel{l}}{\cancel{4}} - q\frac{\cancel{l}}{\cancel{4}}\frac{3}{8}l = 0$

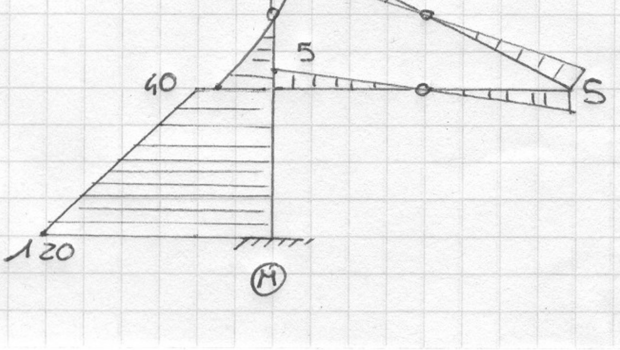
$N + \frac{T}{2} = \frac{3}{16}ql$

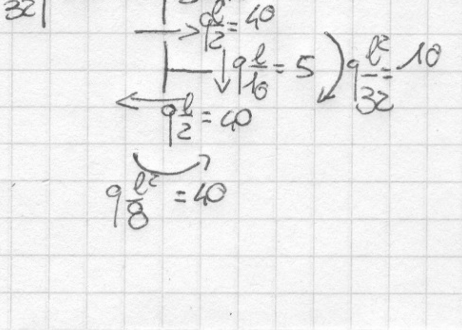
$T = 2\left(\frac{3}{16}ql - N\right) = \left(\frac{3}{8} - \frac{1}{8}\right)ql = \frac{1}{4}ql = 20\,kN$

Grafici dell'azione interna

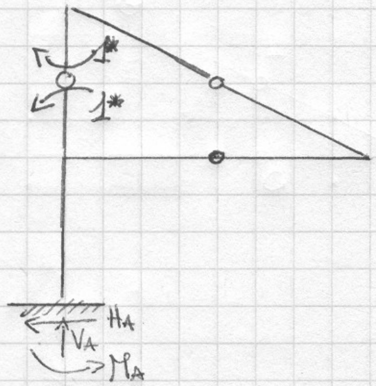




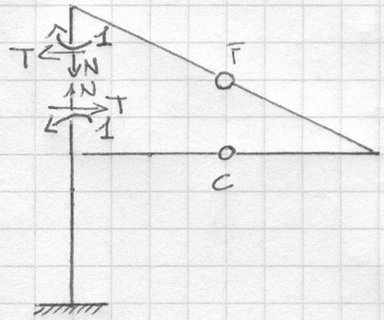




Sistema virtuale per il calcolo di $\Delta\varphi_E$

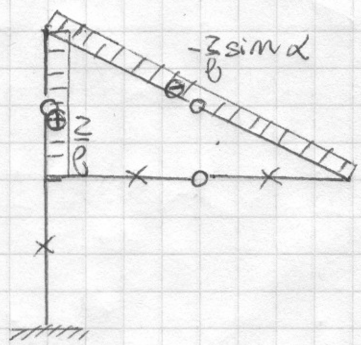


$H_A = V_A = M_A = 0$

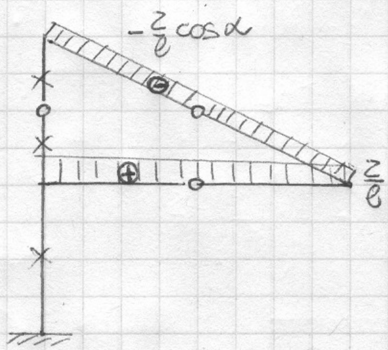


$\overset{\curvearrowleft}{F})\; N\frac{l}{2} = 1 \Rightarrow N = \frac{2}{l}$

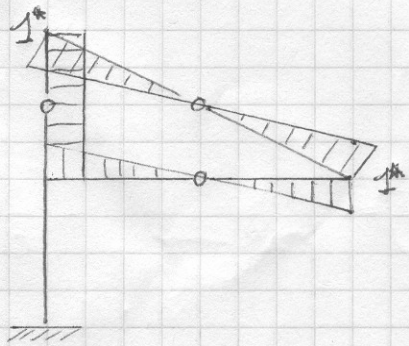
$\overset{\curvearrowleft}{C})\; -T\frac{l}{4} - N\frac{l}{2} + 1 = 0 \Rightarrow -T\frac{l}{4} - \cancel{\frac{2}{l}}\,\cancel{\frac{l}{2}}^{\,1} + 1 = 0 \Rightarrow T = 0$



N



T

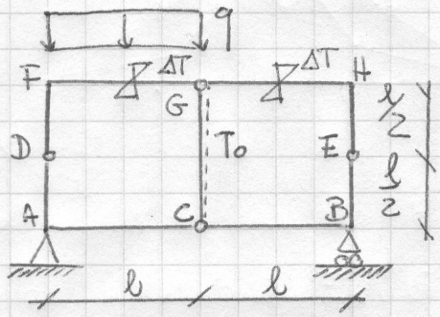


M

Applico il PLV:

$$1 \cdot \Delta\varphi_E = \int_\Omega M^* \frac{M}{EJ}\, dz$$

Esercizio 3



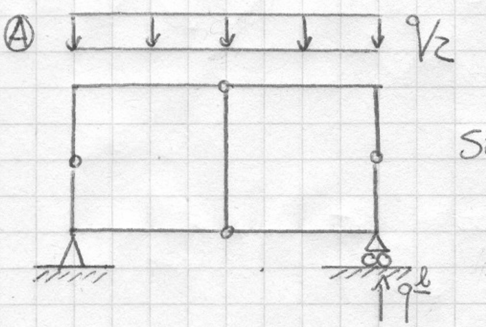
$\Delta T = 50°C$

$T° = 30°C$

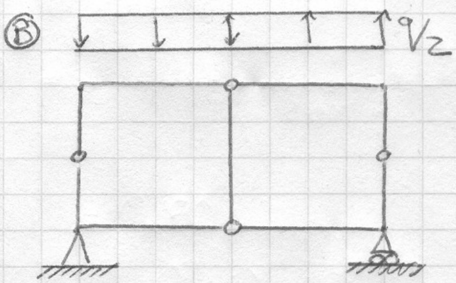
$l = 4m$

$q = 10kN/m$

Considero la struttura soggetta ad un carico simmetrico ed uno anti-simmetrico tali da ottenere la situazione iniziale.

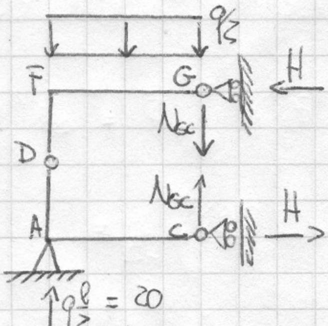


Sim.



Anti-Sim.

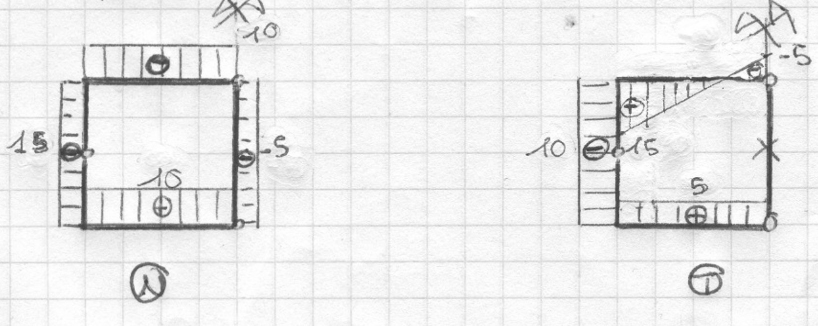
Struttura Ⓐ

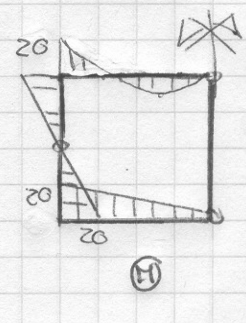


$D_{sotto})\ H\frac{\cancel{l}}{2} + N_{GC}\cancel{l} = 0 \Rightarrow H = -2N_{GC}$

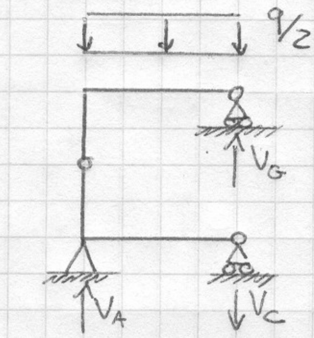
$D_{sopra})\ H\frac{\cancel{l}}{2} - N_{GC}\cancel{l} - \frac{q}{2}\frac{l^2}{2} = 0 \Rightarrow -N_{GC}\cdot 2 - q\frac{l}{4} = 0 \Rightarrow N_{GC} = -q\frac{l}{8} = -5kN$

$H = q\frac{l}{4} = 10kN$





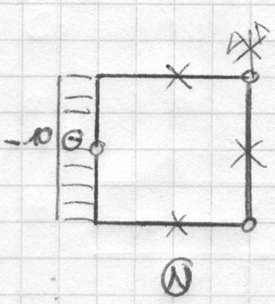
Struttura B

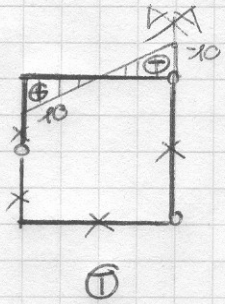


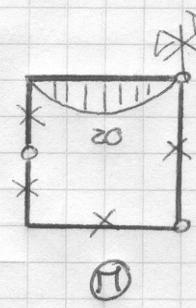
$$\overset{D}{\text{sopra}}\Big)\quad V_G \cancel{l} - \frac{q}{2}\frac{l^{\cancel{2}}}{2} = 0 \Rightarrow V_G = q\frac{l}{4}$$

$$G\Big)\quad -V_A l + \frac{q}{2}\frac{l^2}{2} = 0 \Rightarrow V_A = q\frac{l}{4}$$

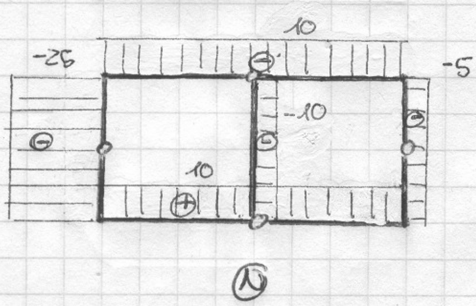
$$V_C = 0$$

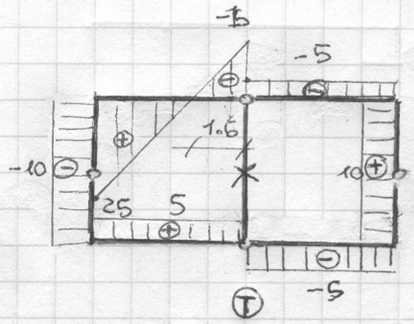


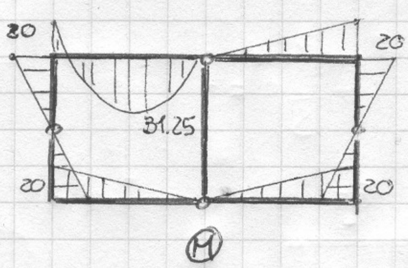




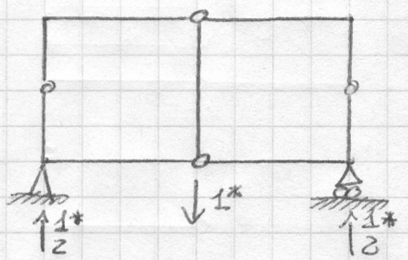
Grafici finali

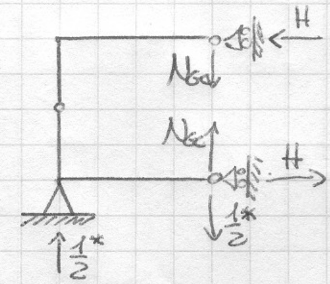


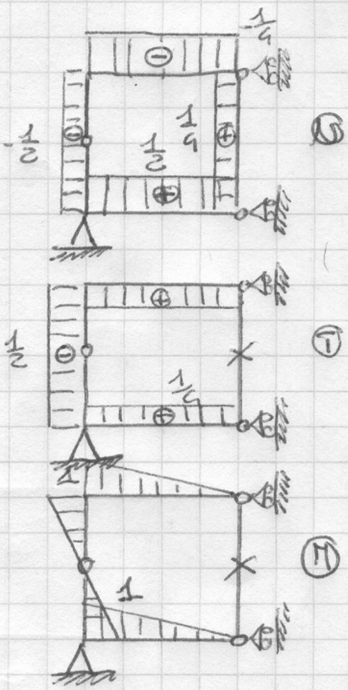




Sistema virtuale







$D^{\Gamma}_{sotto})\quad H\frac{\ell}{2} + N_{GC}\cancel{\ell} - \frac{1^*}{2}\cancel{\ell} = 0$

$D^{\Gamma}_{sopra})\quad H\frac{\cancel{\ell}}{2} - N_{GC}\cancel{\ell} = 0 \quad \Rightarrow H = 2N_{GC}$

$N_{GC}\cdot 2 = \frac{1^*}{2} \Rightarrow N^*_{GC} = \frac{1^*}{4} \qquad H = \frac{1^*}{2}$

Calcolo dello spostamento v_C tramite PLV

$$1^* v_C = \int_\Omega N^* \varepsilon_t \, dz + \int_\Omega M^*\left(\frac{M}{EJ} + \chi_t\right) dz =$$

$$= 2\frac{1}{4}\alpha T^\circ \ell + 2\int_0^\ell -z\left(\frac{2\alpha\Delta T}{h}\right) dz + 2\int_0^\ell -z\left(5z - \frac{9}{2}\frac{z^2}{2}\right)\frac{1}{EJ}\, dz =$$

$$= 15\alpha + 1600\frac{\alpha}{h} + \frac{2}{EJ}\int_0^\ell\left(9\frac{z^3}{4} - 5z^2\right) dz = 15\alpha + 1600\frac{\alpha}{h} + \frac{2}{EJ}\left[9\frac{\ell^4}{16} - \frac{5\ell^3}{3}\right] =$$

$$= 15\alpha + 1600\frac{\alpha}{h} + \frac{106,\overline{6}}{EJ}$$

Oss: siccome il grafico di M del sistema virtuale, solo la componente simmetrica del grafico di M nel sistema reale è stata considerata nel calcolo di v_C.