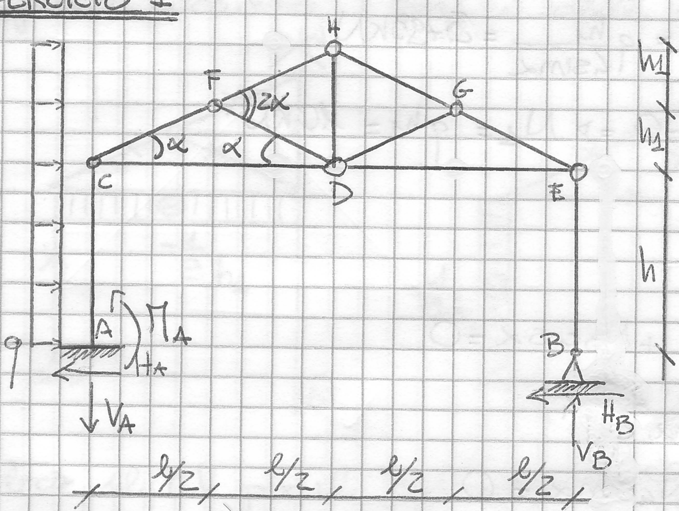
Esercizio 1



$h_1 = 1\,m$

$h = 3\,m = 3h_1$

$l = 4\,m = 4h_1$

$q = 10\,kN/m$

$\alpha = atg\left(\frac{2h_1}{l}\right) = atg(0.5) = 26.6°$

$\cos\alpha = 0.89 \qquad \sin\alpha = 0.45$

Calcolo delle reazioni vincolari

$\Sigma\rightarrow)\ H_B = 0$

$\rightarrow H_A = 5qh_1 = 50\,kN$

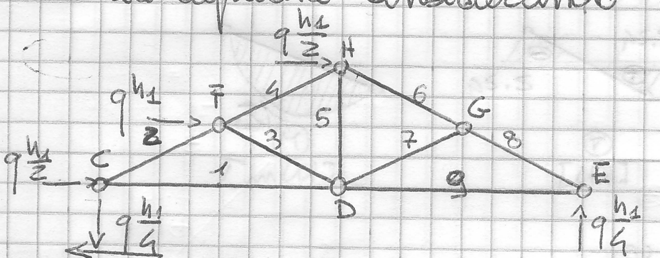
$C)\ M_A - H_A\,3h_1 + q\frac{(3h_1)^2}{2} = 0$

$M_A = 5qh_1 \cdot 3h_1 - q\frac{9h_1^2}{2} = q h_1^2\left(15 - \frac{9}{2}\right) = \frac{21}{2} q h_1^2 = 105\,kN\,m$

$A)\ V_B \cdot 2 \cdot 4h_1 + \frac{21}{2} q h_1^2 - q\frac{(5h_1)^2}{2} = 0$

$V_B = \frac{1}{8h_1}\left(q\frac{25}{2}h_1^2 - \frac{21}{2} q h_1^2\right) = \frac{1}{\cancel{8h_1}} \cdot 2 q h_1^2 = q\frac{h_1}{4} = 2.5\,kN = V_A$

Isolo la capriata considerando i carichi nodali equivalenti



	N [kN]
1	10
2	5.58
3	-5.59
4	0
5	-2.5
6	-5.58
7	0
8	-5.58
9	5

Nodo E

$\uparrow\ N_8 = -\frac{q h_1}{4\sin\alpha} = -5.58\,kN$

$\rightarrow\ N_9 = -N_8\cos\alpha = \frac{q h_1 \cos\alpha}{4\sin\alpha} = 5\,kN$

Nodo G

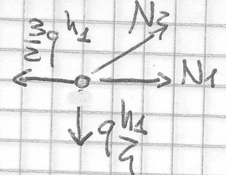
$\rightarrow\ -N_6\cancel{\cos\alpha} - N_7\cancel{\cos\alpha} - N_8\cancel{\cos\alpha} = 0$

$N_6 = -N_7 - N_8$

$\uparrow\ N_6\cancel{\sin\alpha} - N_7\cancel{\sin\alpha} + N_8\cancel{\sin\alpha} = 0 \Rightarrow N_6 - N_7 + N_8 = 0$

$-N_7 - \cancel{N_8} - N_7 + \cancel{N_8} = 0 \Rightarrow N_7 = 0 \qquad N_6 = -N_8 = -5.58\,kN$

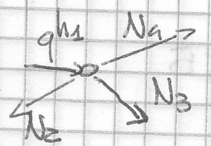
Nodo C



↑ $N_2 \sin\alpha = q\frac{h_1}{4} \Rightarrow N_2 = q\frac{h_1}{4\sin\alpha} = 5.58\,kN$

→ $-\frac{3}{2}qh_1 + q\frac{h_1}{4\,tg\alpha} + N_1 = 0 \Rightarrow N_1 = qh_1 = 10\,kN$

Nodo F



→ $qh_1 + N_4\cos\alpha - N_2\cos\alpha + N_3\cos\alpha = 0$

$N_3 = -q\frac{h_1}{\cos\alpha} + N_2 - N_4$

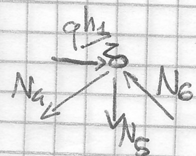
↑ $-N_2\sin\alpha - N_3\sin\alpha + N_4\sin\alpha = 0$

$N_4 = N_3 + N_2$

$N_3 = -q\frac{h_1}{\cos\alpha} + \cancel{N_2} - N_3 - \cancel{N_2} \Rightarrow N_3 = -q\frac{h_1}{2\cos\alpha} = -5.58\,kN$

$N_4 = 0$

Nodo H

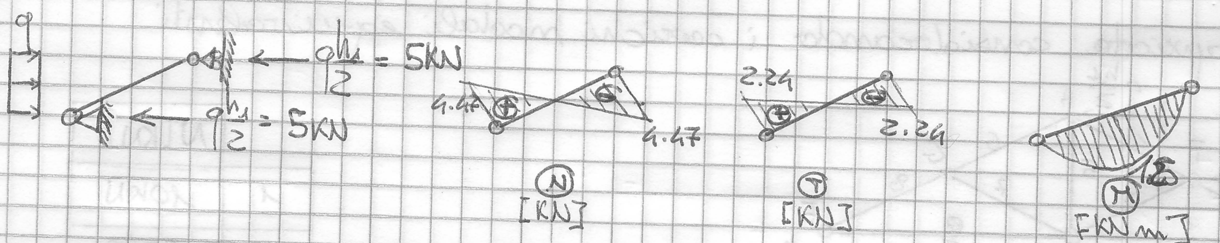


↑ $-N_4\sin\alpha - N_5 + N_6\sin\alpha = 0$

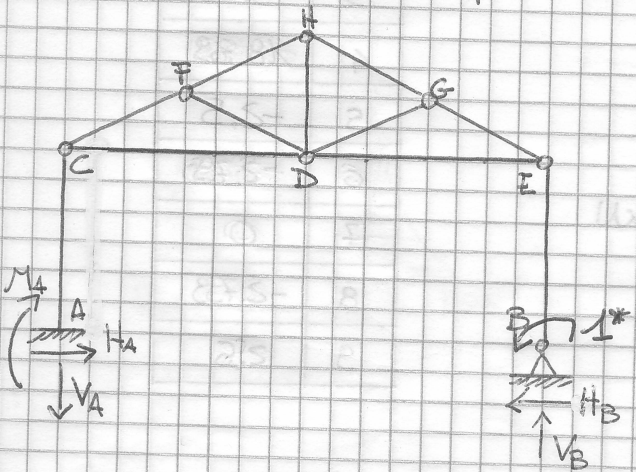
$N_5 = (N_6 - N_4)\sin\alpha = -2.5\,kN$

→ $-N_4\cos\alpha - N_6\cos\alpha + q\frac{h_1}{2} = 0 \Rightarrow -5 + 5 = 0$ ok!!!

Calcolo delle sollecitazioni secondarie nei traversi CF e FH



Sistema ausiliario per il calcolo di φ_B.

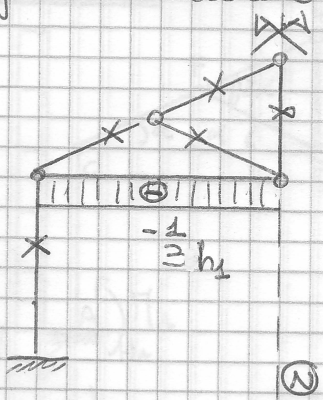


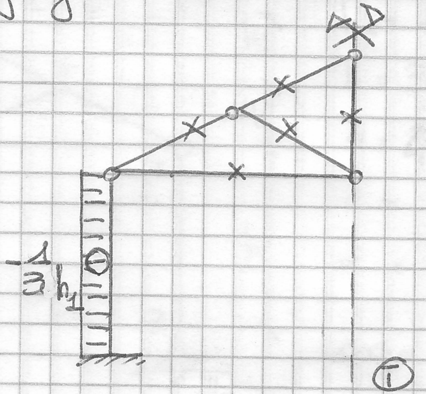
$\overset{\curvearrowright}{E})$ $H_B = \frac{1^*}{3h_1} = H_A$

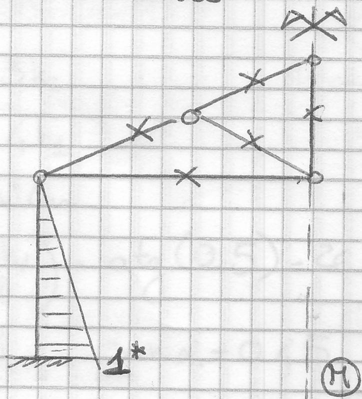
$\overset{\curvearrowright}{C})$ $\cancel{-\frac{1}{3h_1}\cdot 3h_1 + 1} + V_B\,8h_1 = 0 \Rightarrow V_B = 0 = V_A$

$\overset{\curvearrowright}{C})$ $M_A = 1^*$

È facile ricavare i grafici delle sollecitazioni interne







Applico il PLV per il calcolo di φ_B

$$1^* \varphi_B = \int_\Omega \left(N^* \frac{N}{EA} + M^* \frac{M}{EJ} \right) d\Omega = -\frac{1}{3h_1} \frac{10}{EA} 4h_1 - \frac{1}{3h_1} \frac{5}{EA} 4h_1 + \int_0^{3h_1} \left(1 - \frac{z}{3h_1}\right)\left(-105 + \right.$$

$$\left. - \frac{10z^2}{2} + 50z \right) \frac{dz}{EJ} =$$

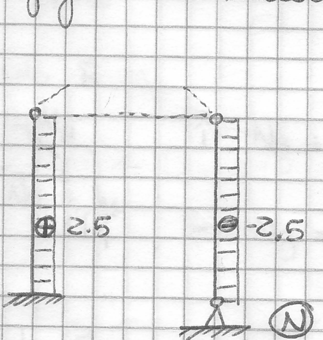
$$= - \frac{\;}{EA} + \int_0^{3h_1} \left(-105 - \frac{10z^2}{2} + 50z + \frac{105}{3h_1} z + \frac{5}{3h_1} z^3 - \frac{50}{3h_1} z^2 \right) \frac{dz}{EJ} =$$

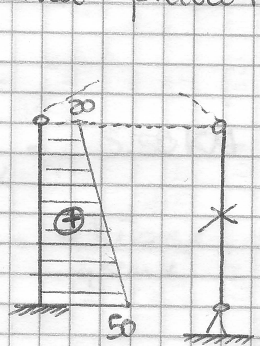
$$= - \frac{\;}{EA} + \int_0^{3h_1} \left(-105 - 21.7z^2 + 8.5z + 1.7z^3 \right) \frac{dz}{EJ} =$$

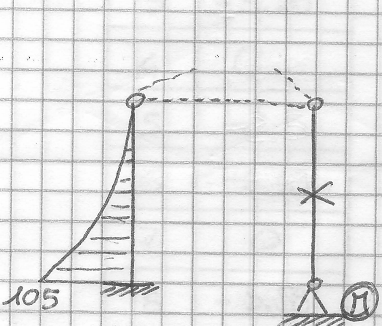
$$= - \frac{\;}{EA} + \left[-105z - 21.7\frac{z^3}{3} + 8.5\frac{z^2}{2} + 1.7\frac{z^4}{4} \right]_0^{3h_1} \cdot \frac{1}{EJ} =$$

$$= - \frac{\;}{EA} - \frac{93.4}{EJ}$$

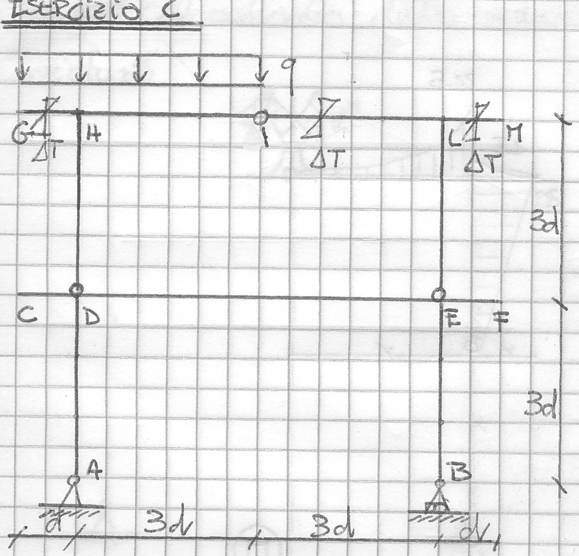
PS: I grafici delle sollecitazioni nei piedritti del sistema reale sono







Esercizio 2



$d = 1\,m$

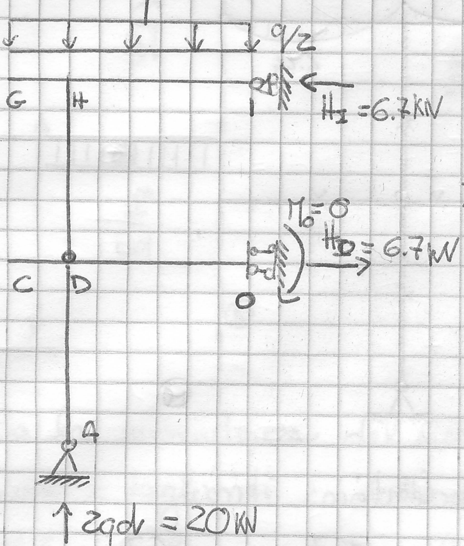
$q = 10\,kN/m$

$\alpha = 1,2\cdot 10^{-5}\,°C$

$\Delta T = 25°C$

Considero il sistema come la somma di due sistemi di cui uno è simmetrico, mentre il secondo è antisimmetrico.

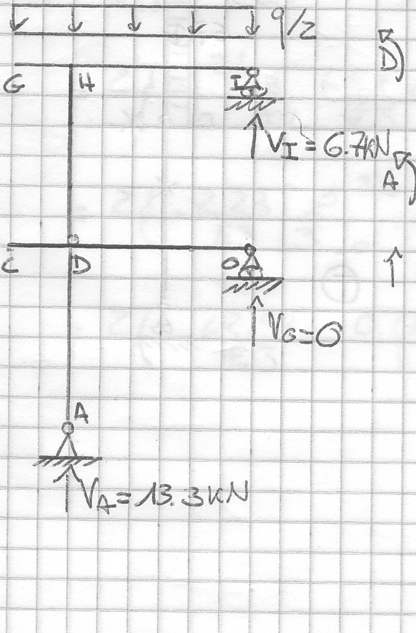
Considero quello simmetrico:



$\overset{\frown}{I})\ H_I\,\cancel{3d} = \frac{q}{\cancel{2}_1}\,\cancel{4}^{2}d\,\cancel{d} \Rightarrow H_I = \frac{2}{3}qd = H_O = 6.7\,kN$

$\overset{\frown}{D})\ M_O = 0$

e il sistema antisimmetrico



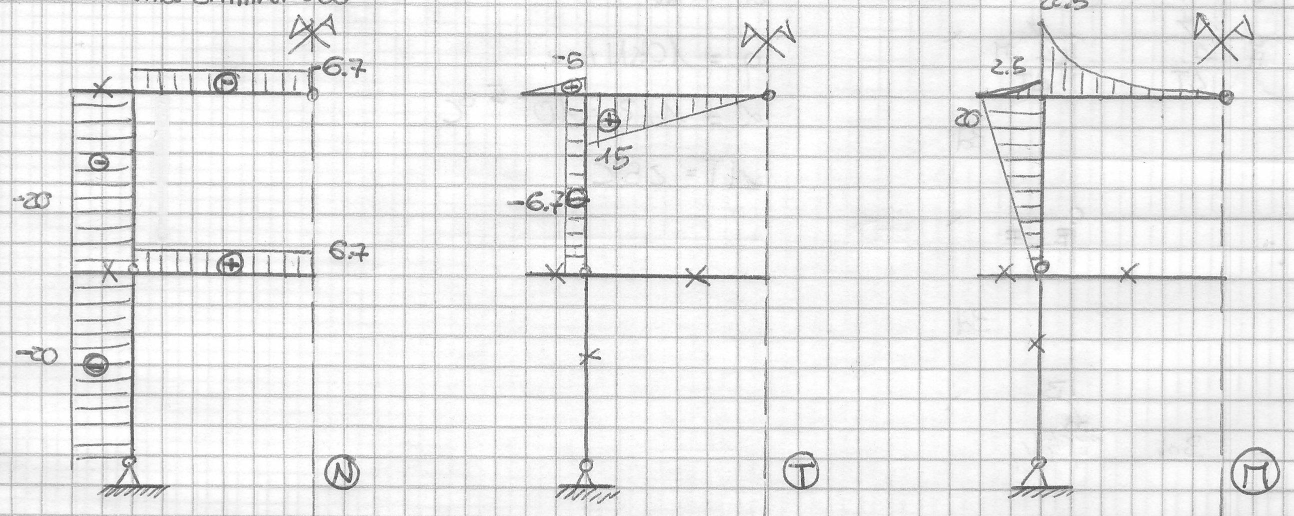
$\overset{\frown}{D})\ V_I\,\cancel{3d} = \frac{q}{\cancel{2}_1}\,\cancel{4}^{2}d\,\cancel{d} \Rightarrow V_I = \frac{2}{3}qd = 6.7\,kN$

$\overset{\frown}{A})\ \cancel{V_I\,3d} + V_O\,3d = \cancel{\frac{q}{2}\,4d\,d} \Rightarrow V_O = 0$

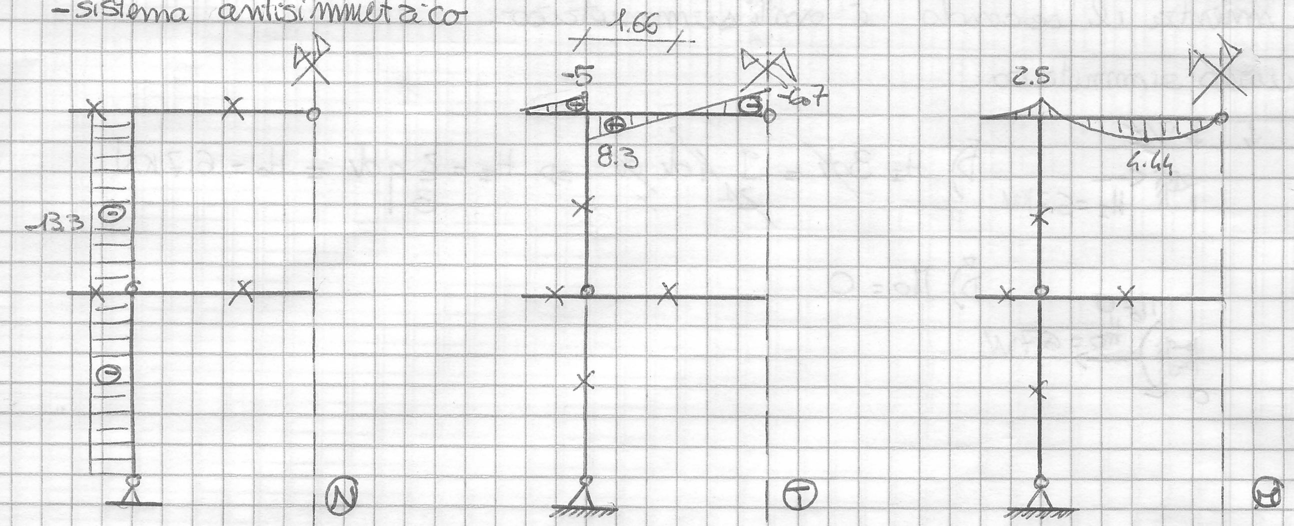
$\uparrow\ V_A + V_I = \frac{q}{\cancel{2}_1}\,\cancel{4}^{2}d \Rightarrow V_A = 2qd - V_I = \left(2 - \frac{2}{3}\right)qd = \frac{4}{3}qd$

Grafici delle azioni interne:

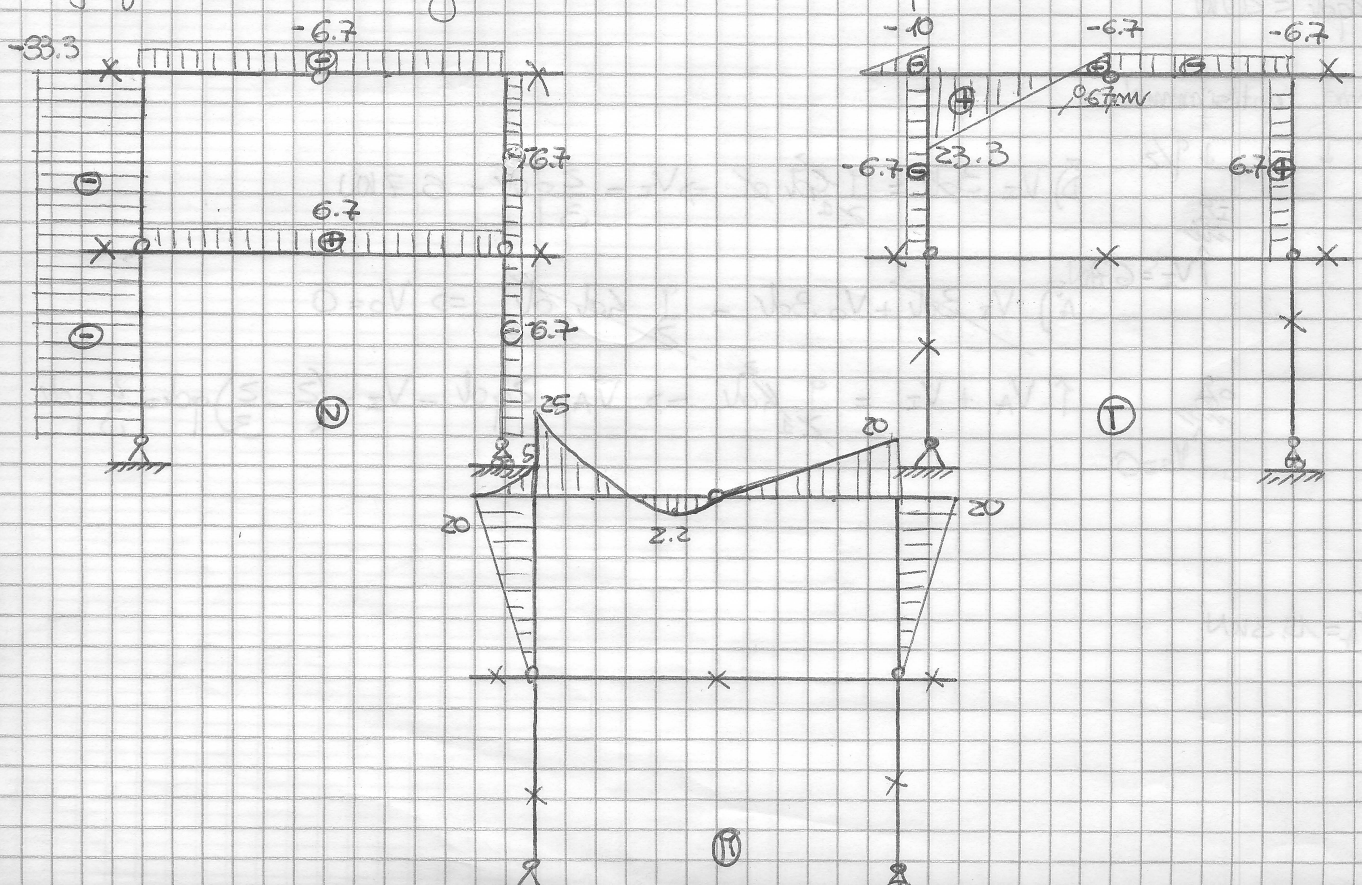
- sistema simmetrico



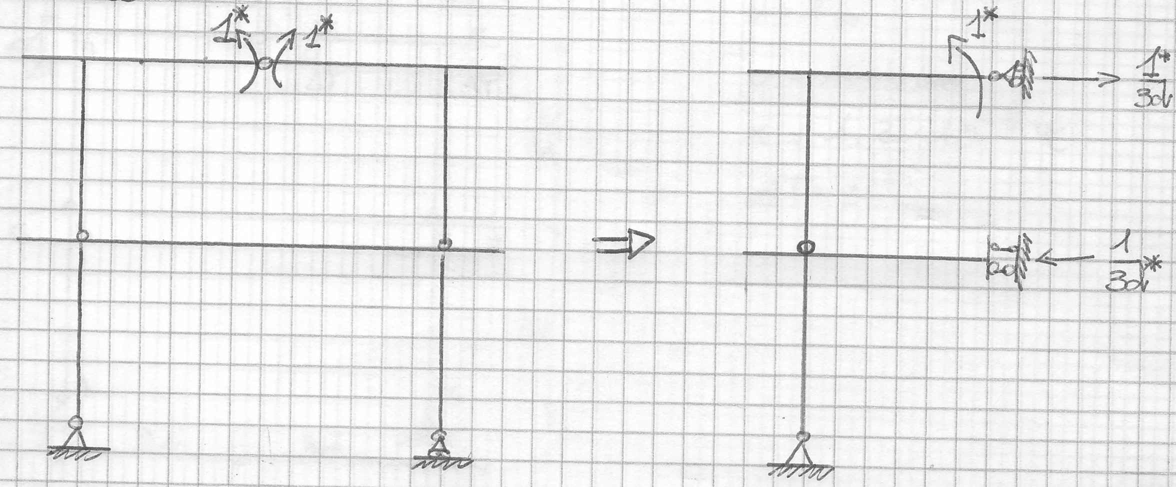
- sistema antisimmetrico



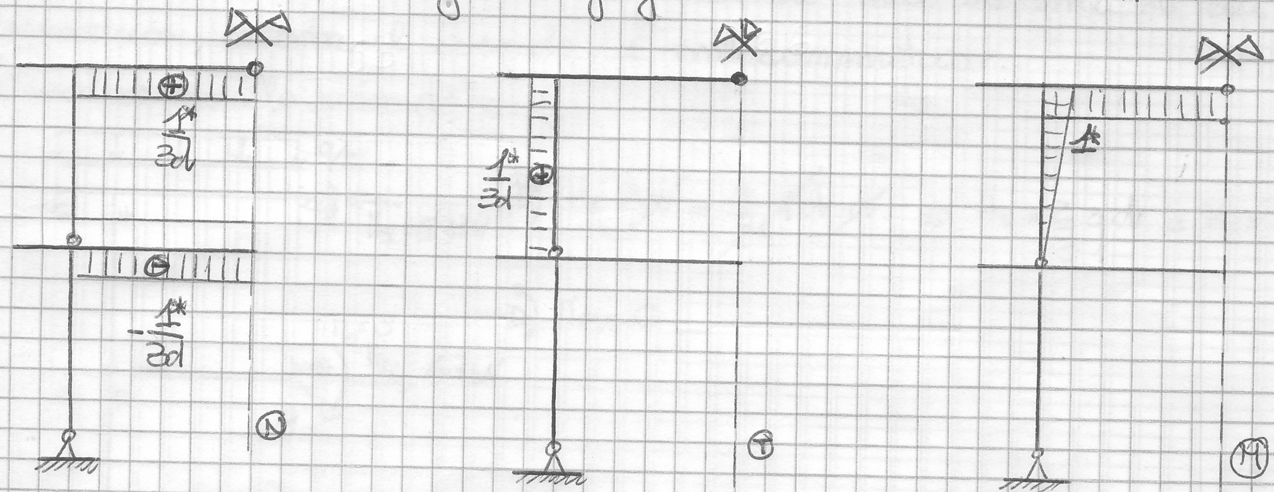
I grafici totali si ottengono dalla somma dei precedenti



Per il calcolo della rotazione relativa in I considero il seguente sistema virtuale



da cui si ricavano i seguenti grafici delle sollecitazioni interne



Data la simmetria del sistema virtuale, quindi trascurando le deformazioni assiali e taglianti indotte dai carichi, il valore di $\Delta\varphi_I$ si ottiene attraverso il PLV con la seguente relazione

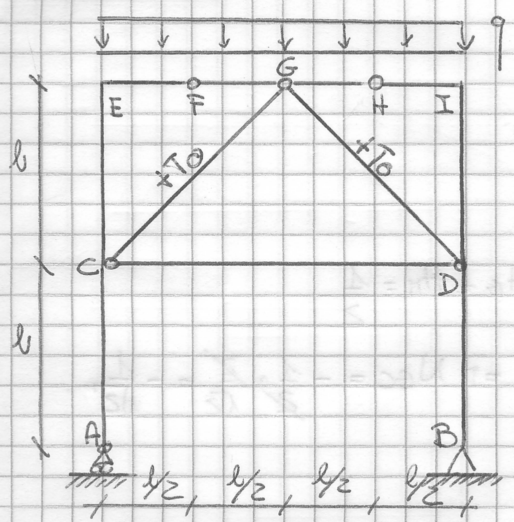
$$1^* \Delta\varphi = \int_\Omega M^* \left(\frac{M}{EJ} + \chi^T \right) d\Omega = 2 \int_{\Omega_s} M^* \left(\frac{M}{EJ} + \chi^T \right) d\Omega_s =$$

$$= 2 \left\{ \int_0^{3d} \frac{z}{3d} \frac{(-6.7 z)}{EJ} dz + \int_0^{3d} \frac{1^*}{EJ} \left(-\frac{10}{2} \frac{z^2}{2} \right) dz + \int_0^{3d} 1^* \left(-\frac{2\alpha \Delta T}{h} \right) dz \right. =$$

$$= 2 \left\{ -\frac{2.23}{EJ} \left[\frac{z^3}{3} \right]_0^{3d} - \frac{2.5}{EJ} \left[\frac{z^3}{3} \right]_0^{3d} - \frac{2\alpha \Delta T}{h} \left[z \right]_0^{3d} \right\} =$$

$$= 2 \left(-\frac{42.57}{EJ} - \frac{0.0018}{h} \right) = -\left(\frac{85.14}{EJ} + \frac{0.0036}{h} \right)$$

Esercizio 3



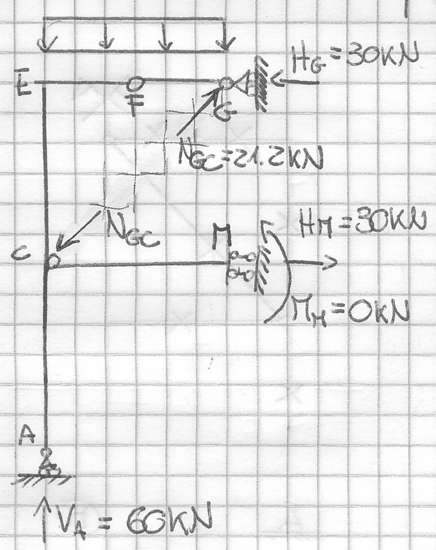
$l = 3\,m$

$q = 20\,kN/m$

$\alpha = 1.2 \cdot 10^{-5}\,°C^{-1}$

$T° = 50\,°C$

Sfrutto la simmetria del problema e studio solo metà struttura:



C) $M_M = 0$

$\rightarrow$ $H_G = H_M$

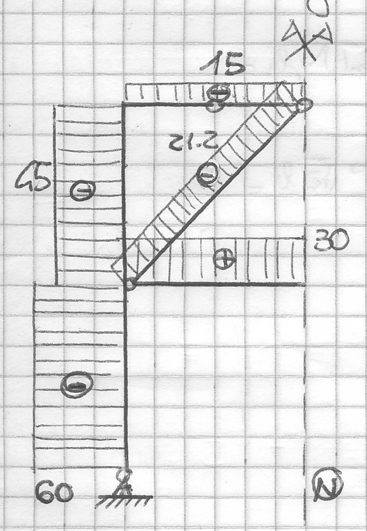
$\uparrow$ $V_A = q\,l$

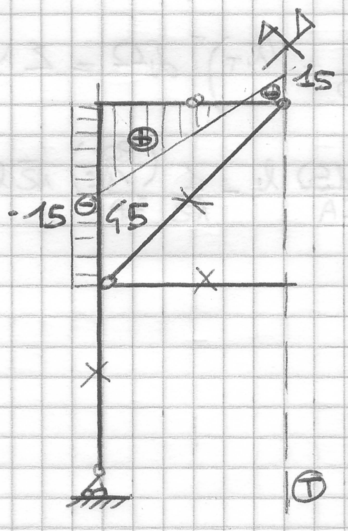
A) $H_G\, 2\cancel{l} - H_G \cancel{l} - q \frac{l^{\cancel{2}}}{2} = 0 \Rightarrow H_G = q \frac{l}{2}$

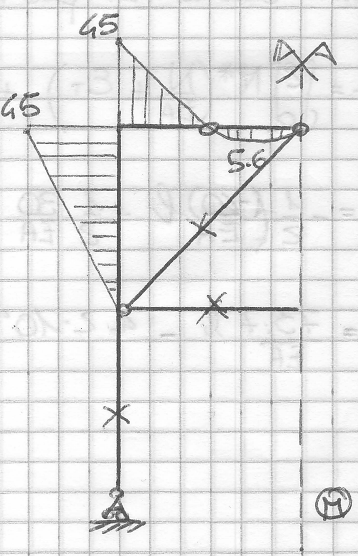
F_G) $N_{GC} \cdot \frac{l}{\cancel{2}} \sin 45° = q \frac{(l/2)^2}{8}$

$$N_{GC} = q \frac{l}{4} \cdot \frac{1}{\sin 45°} = q \frac{l}{2\sqrt{2}} = 21.2\ kN$$

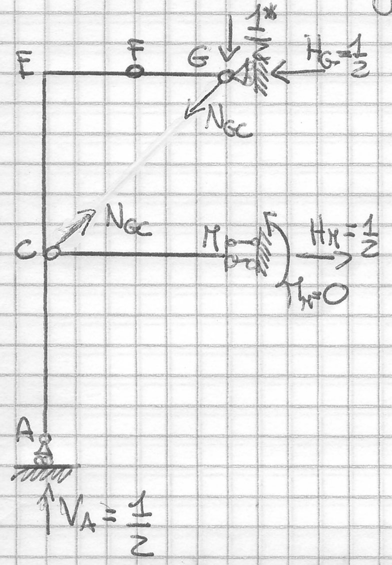
Si tracciano i grafici dell'azione interna







Sistema virtuale (tengo già conto della simmetria):



$\rightarrow$ $H_G = H_M$

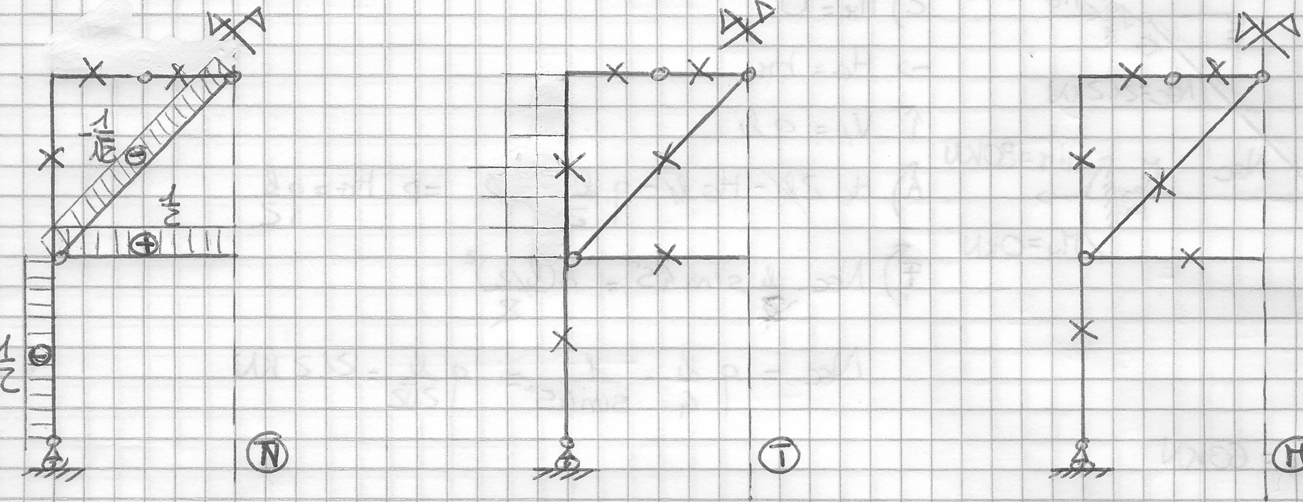
$\circlearrowleft C)$ $M_M = 0$

$\uparrow$ $V_A = \frac{1}{2}$

$\circlearrowleft A)$ $H_G\, 2\cancel{l} - H_G \cancel{l} - \frac{1}{2}\cancel{l} = 0 \Rightarrow H_G = H_M = \frac{1}{2}$

$\circlearrowleft F)$ $-N_{GC}\frac{\cancel{l}}{\cancel{2}}\sin 45° - \frac{1}{2}\frac{\cancel{l}}{\cancel{2}} = 0 \Rightarrow N_{GC} = -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$

e relativi grafici delle sollecitazioni interne



Si applica ora il PLV per il calcolo dello spostamento verticale v_G

$$1^* v_G = \int_\Omega \left[N^*\left(\frac{N}{EA} + \varepsilon_T\right) + \overset{=0}{M^*}\left(\frac{M}{EJ} + \chi_T\right)\right] d\Omega = \sum N_i^*\left(\frac{N_i}{EA} + \alpha T^i\right) l_i =$$

$$= -\frac{1}{2}\frac{(-60)}{EA} l + \frac{1}{2}\frac{30}{EA} l - \frac{1}{2}\frac{(-15)}{EA} l - \frac{1}{\sqrt{2}}\frac{\left(-\frac{30}{\sqrt{2}}\right)}{EA}\sqrt{2}\, l - \frac{1}{\sqrt{2}}\alpha T^0 \sqrt{2}\, l =$$

$$= \frac{73.7}{EA} l - 4.2 \cdot 10^{4}\, l$$