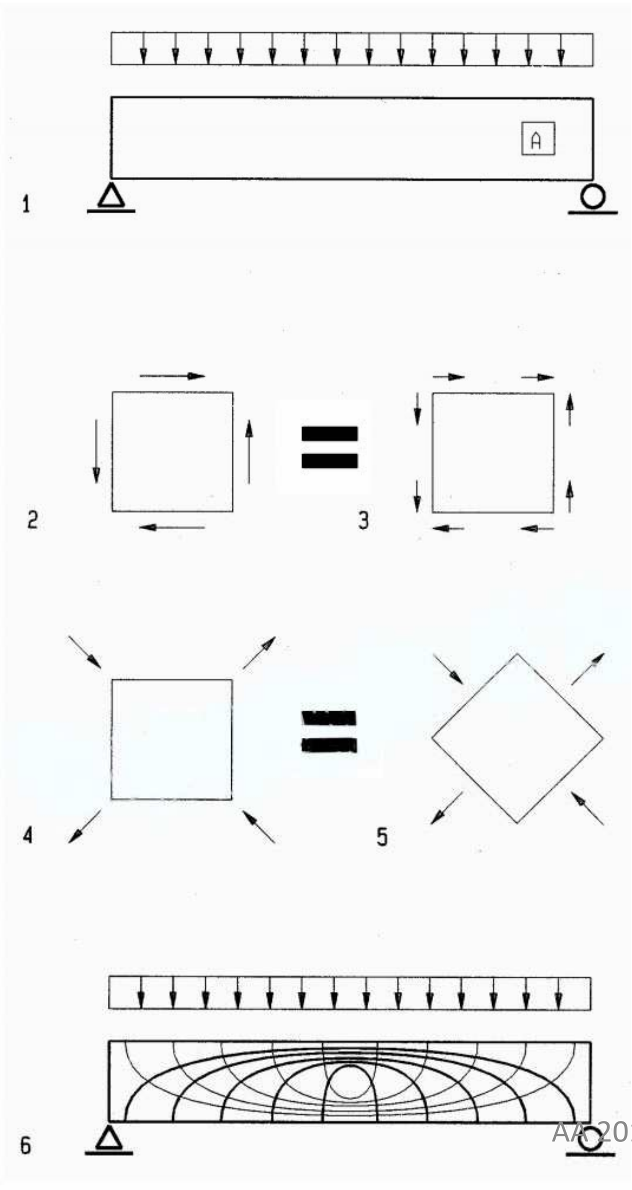
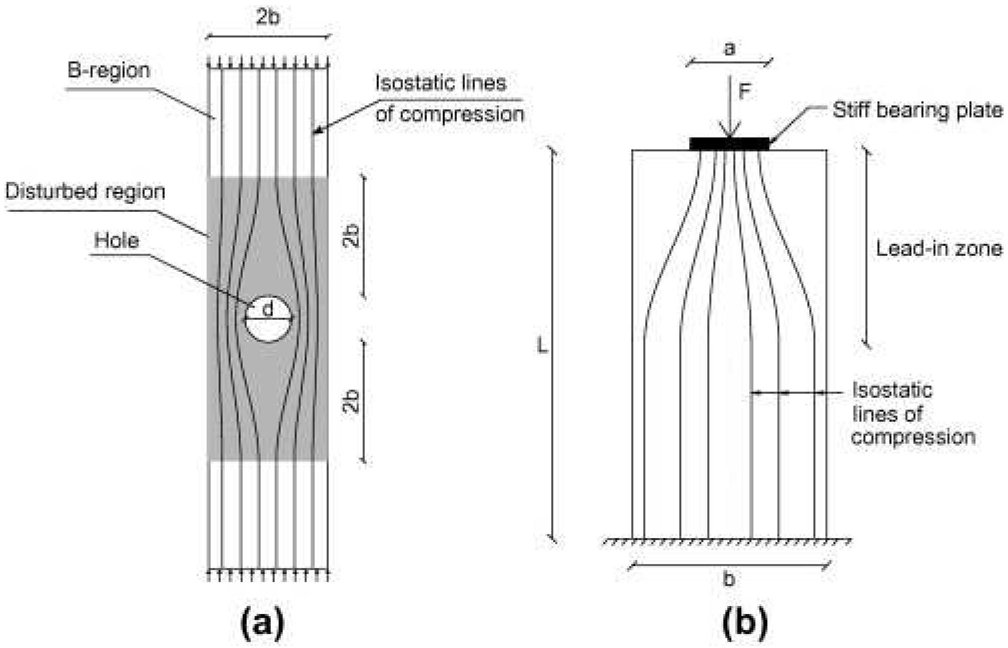
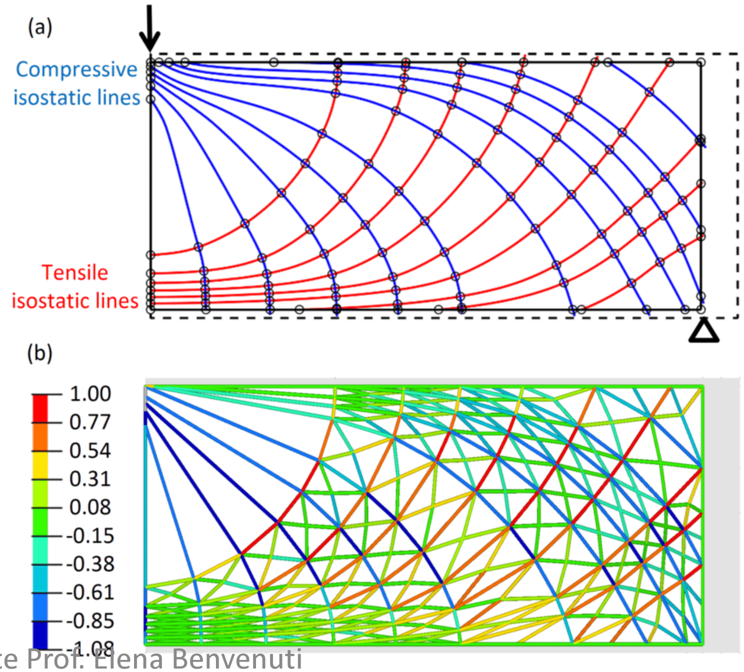
Linee isostatiche

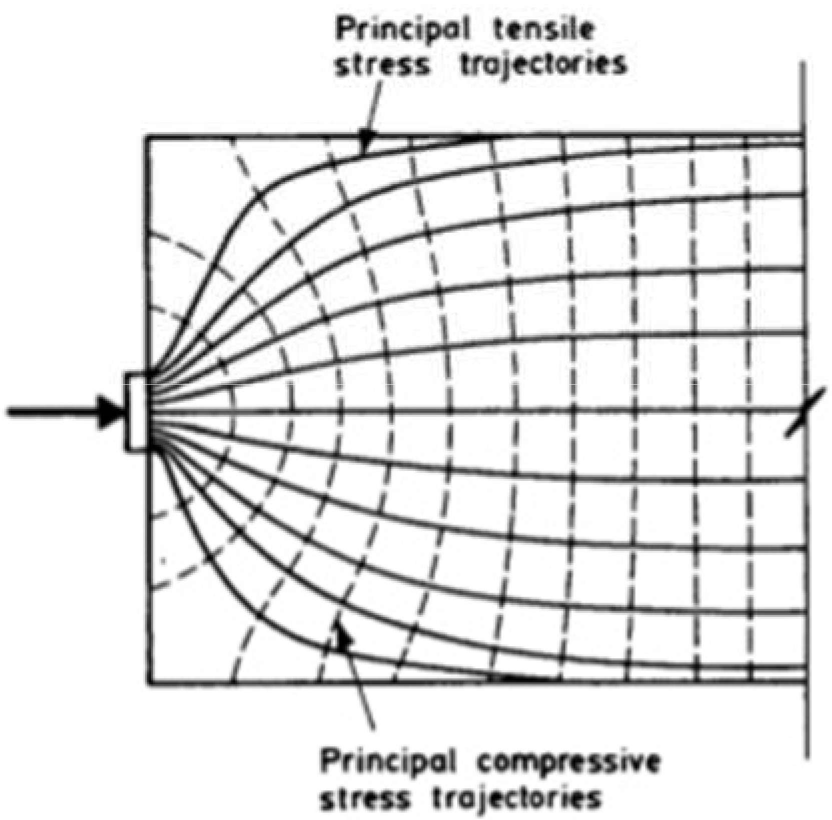




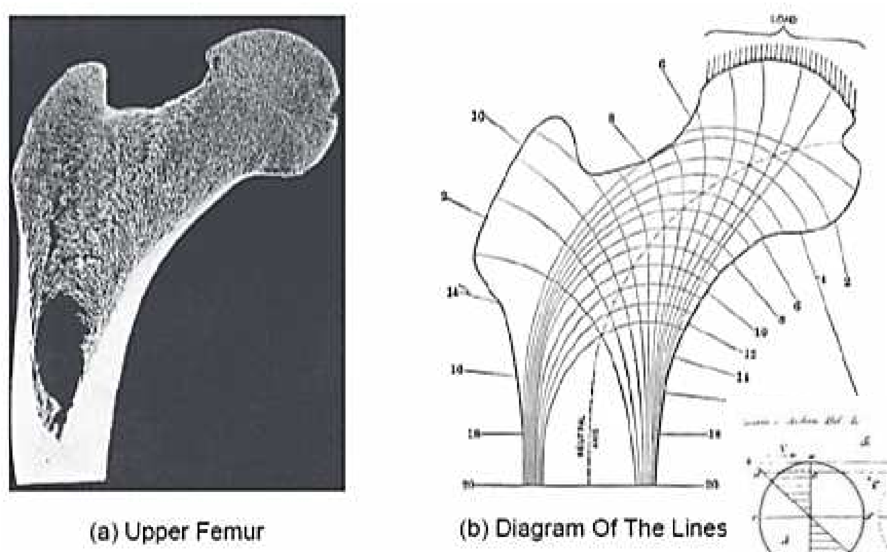


Linee isostatiche in una trave prefabbricata

The end-block of a concentrically-loaded post-tensioned member of rectangular cross-section and the distributions of principal tensile and compressive stresses within the end block is shown in the diagram below.



Ossa e linee isostatiche



Beginning in 1869, the similarities between these stress trajectories and the arched patterns of trabecular struts in various human bones profoundly influenced the work of Julius Wolff. This led to Wolff's formulation of the trajectorial hypothesis of trabecular bone architecture, which eventually evolved into common parlance as "Wolff's Law of the Functional Adaptation of Bone" or "Wolff's Law"

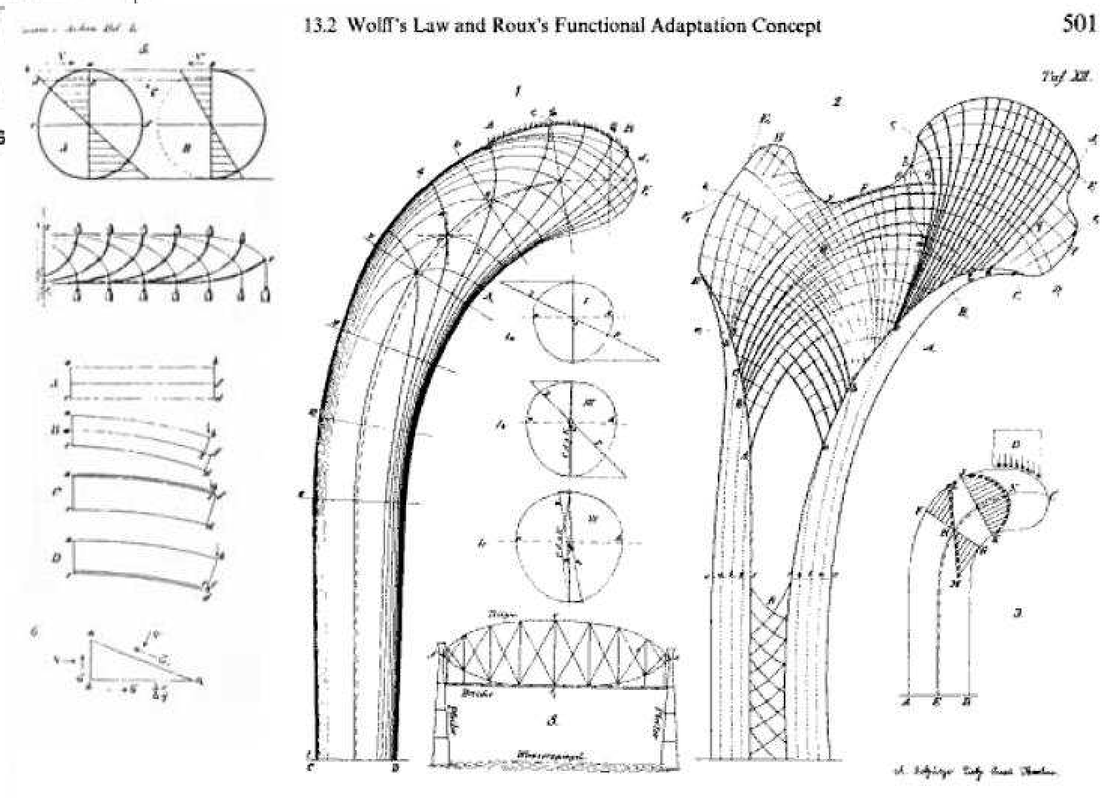
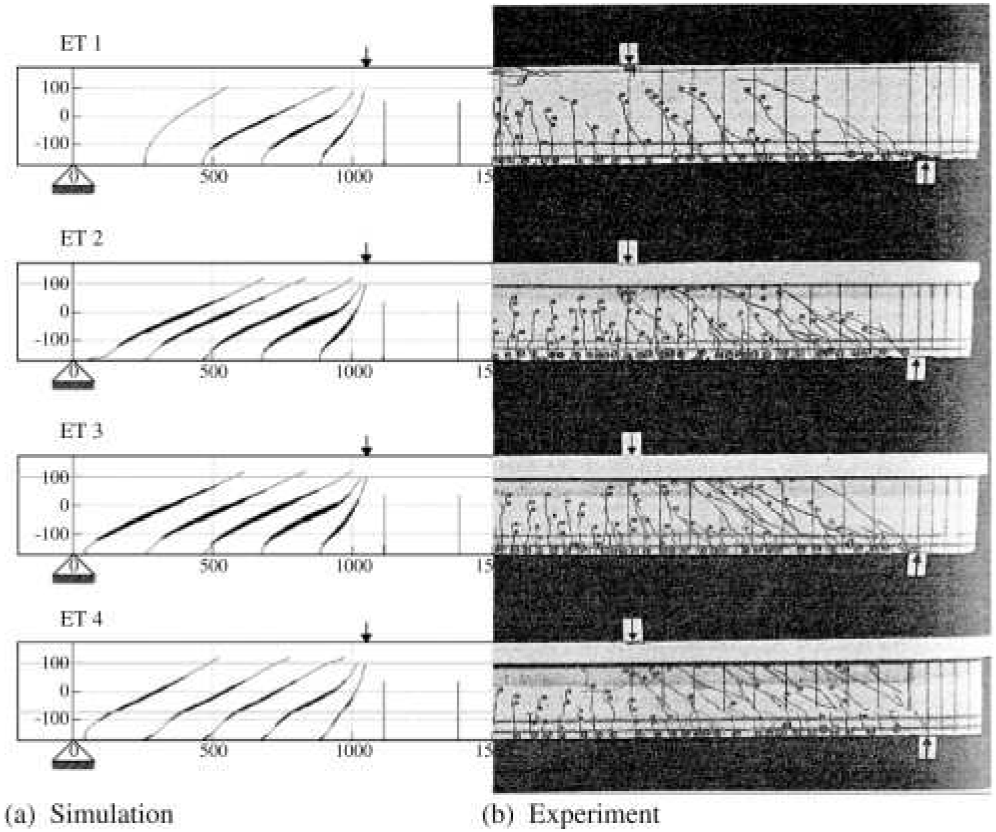

13.2 Wolff's Law and Roux's Functional Adaptation Concept 501

FIGURE 13.2:1 Culmann's crane presented by Wolff in his 1870 paper in Virchow's Archiv.

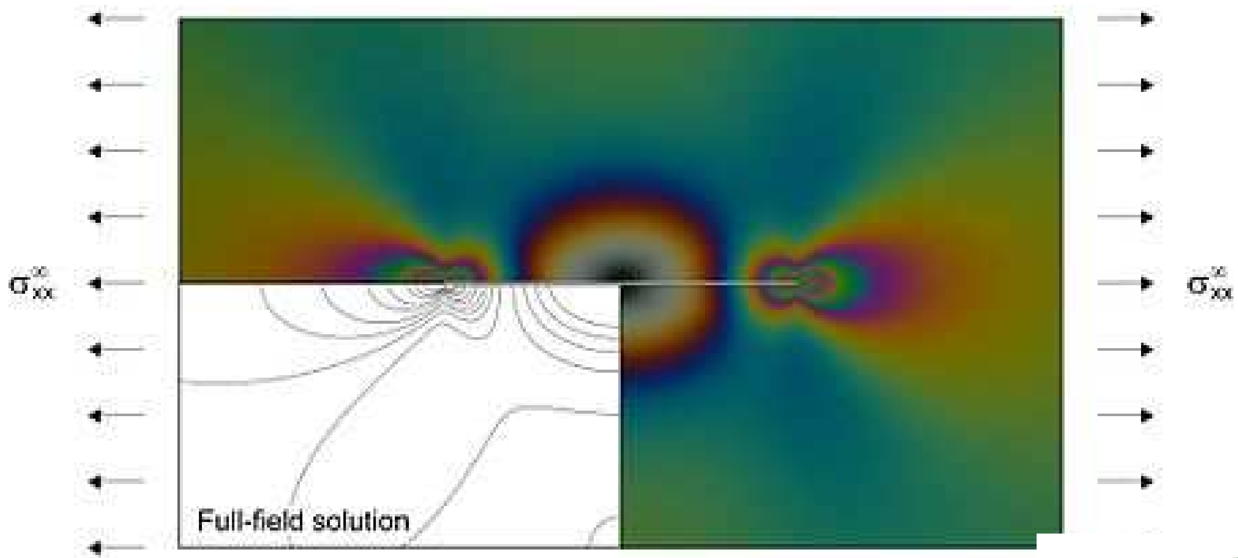
Trave inflessa (4pt bending test)

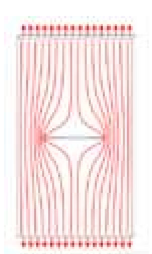


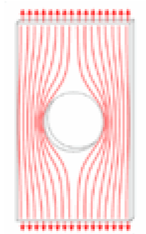
Fotoelasticità

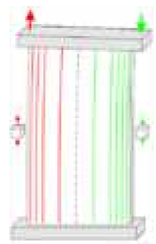
- Describes changes in the optical properties of a material under mechanical deformation
- one of the oldest experimental methods for determining stress distribution in a material
- used for determining stress points in asymmetrical geometries. This detailed stress-analysis method is based on an optomechanical property called birefringence (transparent polymers and glass)
- A very useful tool for engineers to see areas where a structure might break due to high concentrations of stress
- Birefringent plastics will reveal areas of strain within a structure in the form of colorful light fringes when placed under polarized light

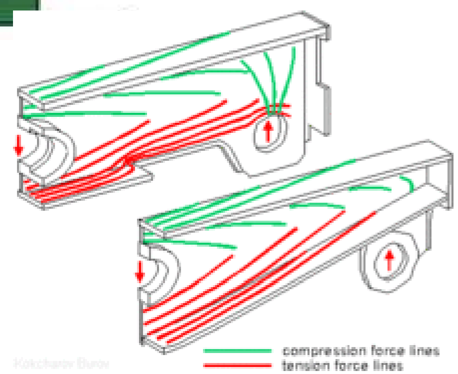
Fotoelasticità, utile soprattutto per misurare concentrazioni locali di stress











PL Nervi, il ferrocemento, e le isostatiche

Pier Luigi Nervi (Sondrio, 21 giugno 1891 – Roma, 9 gennaio 1979) è stato un ingegnere italiano, specializzato nell'edilizia civile.

Fu socio dell‘ Accademia nazionale delle scienze e autore di alcune grandi opere.

Collaborò con architetti di fama internazionale, tra cui Le Corbusier e Louis Kahn

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Il ferrocemento fu ideato e brevettato, attorno agli anni quaranta, da Pier Luigi Nervi[1]; durante il periodo fascista, infatti, in Italia l'impiego del calcestruzzo armato fu proibito perché "non italico": sia l'acciaio che il legno per le casseforme venivano importati dall'estero.

Il brevetto di Nervi si basò sul *ferciment*, prodotto inventato dal francese Joseph-Louis Lambot a metà dell'Ottocento e utilizzato prettamente per la costruzione di barche.

La differenza tra ferrocemento e il calcestruzzo armato è principalmente l'armatura: nel primo questa è costituita prevalentemente da una serie di strati di reti metalliche piccolo diametro (0,5 - 1,5 mm) tenute insieme da un numero limitato di barre di diametro più grande con la funzione irrigidente, il tutto viene poi annegato in una malta cementizia plastica molto ricca di cemento (fino a 800 chili ogni metro cubo di sabbia)

Con tale combinazione si riescono a realizzare delle lastre molto sottili (pochi centimetri) che risultano molto elastiche, flessibili, resistenti alla fessurazione, duttili, leggere e straordinariamente sagomabili in forme qualsiasi.

PL Nervi, il ferrocemento, e le isostatiche

Fig. 1. Tobacco Factory Floor System [13]

Nervi first tested this moveable formwork procedure (entitled the "Nervi System") with the construction of the Bologna Tobacco Factory in 1949 (Figure 1). While the formwork produced a pattern imitative of the traditional beam and joist systems, the successful implementation of this construction procedure inspired Nervi's departure from orthogonality to the curved forms of the isostatic ribs. Although the isostatic rib patterns are considered integral to the "Nervi System," the concept of aligning floor ribs along the isostatics of the principal bending moments is attributed to Aldo Arcangeli, a practicing engineer who worked in the office of Soc. Ing. Nervi & Bartoli [12]. Arcangeli is noted as the author of the invention listed in the corresponding 1949 patent (No. 455,678), filed by Soc. Ing. Nervi & Bartoli, which included the improvement in

2. NERVI'S METHOD FOR EVALUATING ISOSTATICS

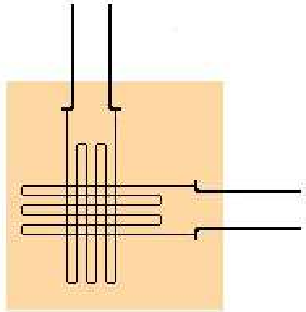
While the structural behavior on which the isostatic rib patterns are based has been highlighted by Nervi, the manner in which these patterns have been generated has seen limited literary discussion. This section explores the experimental and theoretical analysis methods emphasized in Nervi's writings and the application of these analysis techniques to generating isostatics.

2.1. Strain Gauge Methods

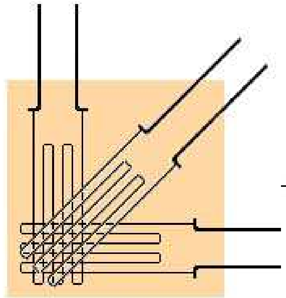
In *Structures*, Nervi identifies two domains of experimental stress analysis: strain gauge methods and photoelasticity. Strain gauge methods rely on devices capable of measuring strain via mechanical, optical, electrical, acoustical, and pneumatic methods, to determine the displacements and stresses at points on a small-scale model [15]. Nervi first used mechanical strain gauges to determine the magnitude of the stresses in the ribs of a scale model for the 1935 Orvieto hangars [13]. In the late 1930s, Simmons and Ruge independently developed bonded-wire electrical-resistance strain gauges [15], which permitted the calculation of principal stress directions.

To find the stress field on the surface of a flat slab, it is necessary to use three-element strain gauge rosettes. These rosettes include three strain-gauges, each oriented at a different angle relative to the two in-plane axes (x and y), which provide three strain measurements corresponding to the three orientation angles. Using strain-transformation equations, the three Cartesian components of strain (ε_{xx}, ε_{yy}, and γ_{xy}) can be calculated and can be used to find the principal strain direction at the measurement location [15]. While calculating the principal stress directions from the rosette readings is simple, the experimental preparations and procedure are costly and time-consuming. Several rosettes would be needed to obtain enough data to clearly represent the full field of principal stress trajectories. As this method does not provide a more accessible way to obtain isostatics than theoretical calculations, strain gauge methods could not have been Nervi's initial means of finding isostatics.

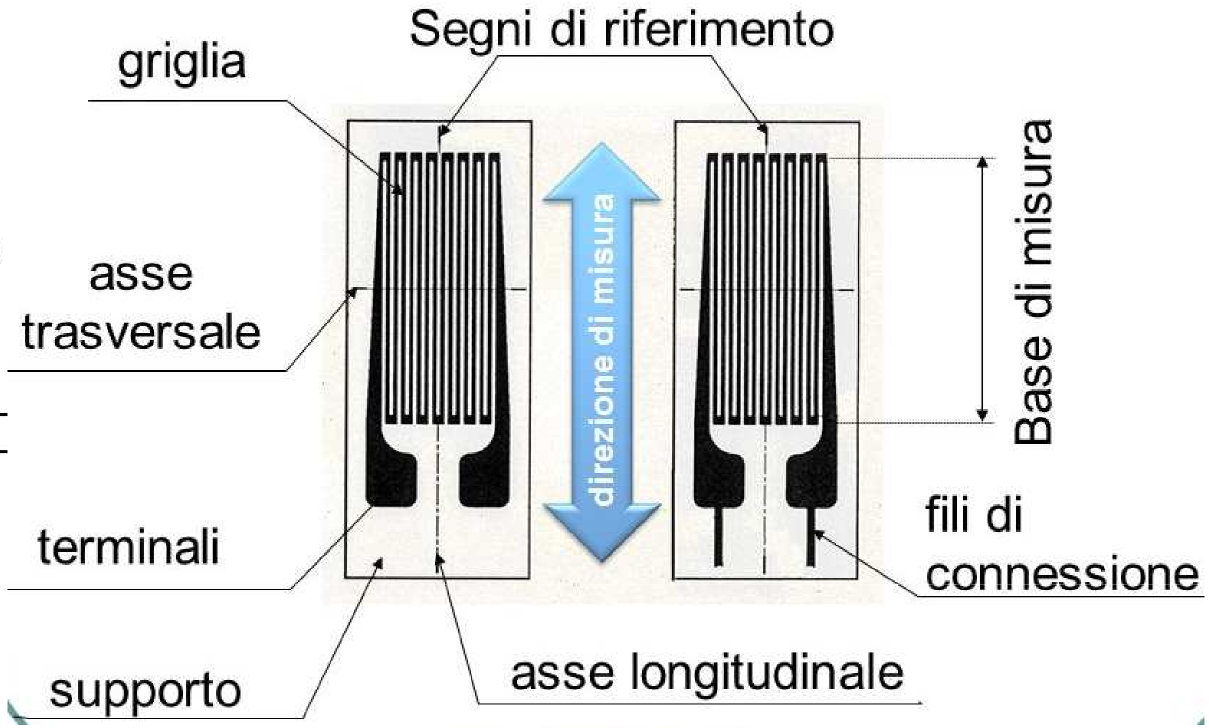
Rosette estensimetriche



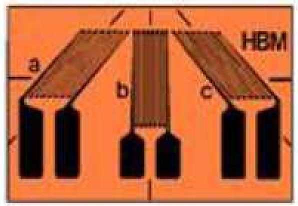
Estensimetro biassiale con griglie a 90°



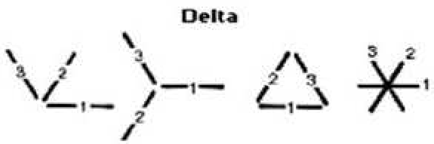
Estensimetro triassiale con griglia intermedia a 45°

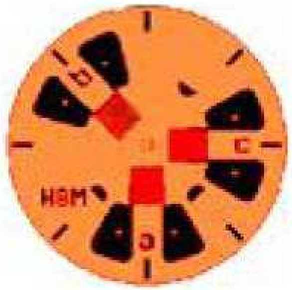


rosette rettangolari sono quelle in cui il secondo e terzo asse di misura sono messi a 45° e a 90° rispetto al primo asse



rosette a delta sono quelle in cui gli assi sono disposti rispettivamente a 60° e 120° rispetto al primo asse.





PL Nervi, il ferrocemento, e le isostatiche

2. NERVI'S METHOD FOR EVALUATING ISOSTATICS

While the structural behavior on which the isostatic rib patterns are based has been highlighted by Nervi, the manner in which these patterns have been generated has seen limited literary discussion. This section explores the experimental and theoretical analysis methods emphasized in Nervi's writings and the application of these analysis techniques to generating isostatics.

Fig. 4. Gatti Wool Factory Floor System [13]

2.2. Photoelasticity

Nervi's fascination with stress visualization in photoelasticity experiments (led by Danusso at the Polytechnic of Milan) suggests Nervi's possible use of photoelasticity to generate isostatics [12].

Photoelasticity is derived from the strain- and stress-optics laws (Neumann 1841, Maxwell 1852) on the theory of artificial double refraction (anisotropic birefringence) in a stressed isotropic, transparent solid. In 1816, Brewster coined the term photoelasticity due to the color pattern produced in clear glass when stressed and examined under polarized light. When certain transparent materials undergo stress, the material exhibits birefringence. As polarized light passes through the material, the rays refract and separate into two perpendicular components each parallel to the principal refractive indices of the material. A condition of the stress-optics laws states that these principal indices correspond to the principal stress directions [16].

2.3. Mathematical Theory

As no efficient experimental approach existed at the time, Arcangeli theoretically studied the concept of placing ribs along the isostatics of principal moments in proposing the idea to Nervi [14]. The two most commonly used plate theories are the Kirchoff-Love and Reisner-Mindlin plate theories. The Kirchoff-Love theory, applicable to thin plates, was developed by Love in 1888 using Kirchhoff's 1850 boundary condition assumptions [22]. Reissner-Mindlin plate theory, an extension of Kirchoff-Love plate theory and applicable to thick plates, takes into account shear deformations through the thickness of a plate and was proposed by Reissner in 1945, but not fully developed by Mindlin until 1951. Given this timeline, Arcangeli's theoretical calculations for the principal bending moment directions must have been based on Kirchoff-Love thin plate theory. Although thin plate theory involves high-order partial differential equations, numerous analytical (Navier, Lévy, Timoshenko [23]), approximate (Ritz), and design solutions (Westergaard and Slater) for thin plate-bending theory were already well-established and in widespread use when the patent for isostatic rib floors was filed in 1949. Additional resources were developed in Italy, including the analytical solutions of Botasso [22] and the design solutions of Santarella, who wrote and edited a plethora of practical manuals and theoretical texts on reinforced concrete produced as a result of the 1927 updates to the building regulations [3].

PL Nervi, il ferrocemento, e le isostatiche

Fig. 4. Gatti Wool Factory Floor System [13]

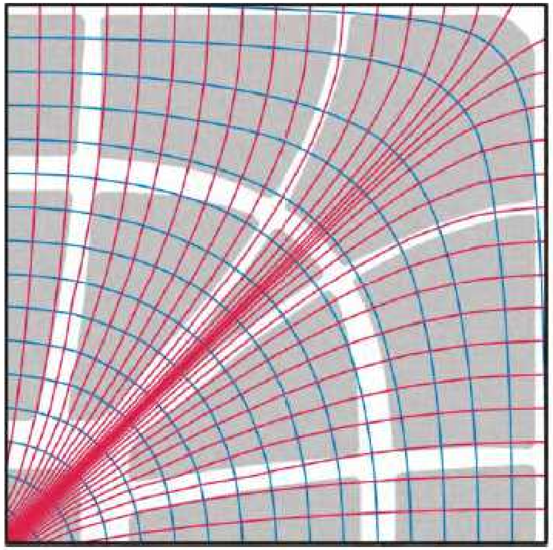


Fig. 6. Gatti Wool Factory Quarter Slab Analysis

The red lines the primary isostatics, corresponding to the maximum principal bending moments, and the blue lines represent the secondary isostatics, corresponding to the minimum principal bending moments.

The concentrations of lines indicate convergence of the principal bending moment directions, highlighted by the prescribed method of starting node generation. The grey and white outline shows an approximate plan of the Gatti Wool Factory floor to illustrate the correlation between the theoretical isostatics and the as-built rib pattern. The concentric secondary isostatics produced around the column support at the bottom left of the quarter slab reflects the reasoning for placing concentric curves of reinforcement around the column head and verifies Nervi's placement of the rib encircling the column head. The ribs emanating from the central column also show close correlation to the generated primary isostatics.

PL Nervi, il ferrocemento, e le isostatiche

Fig. 7. Palace of Labor Floor System [13]

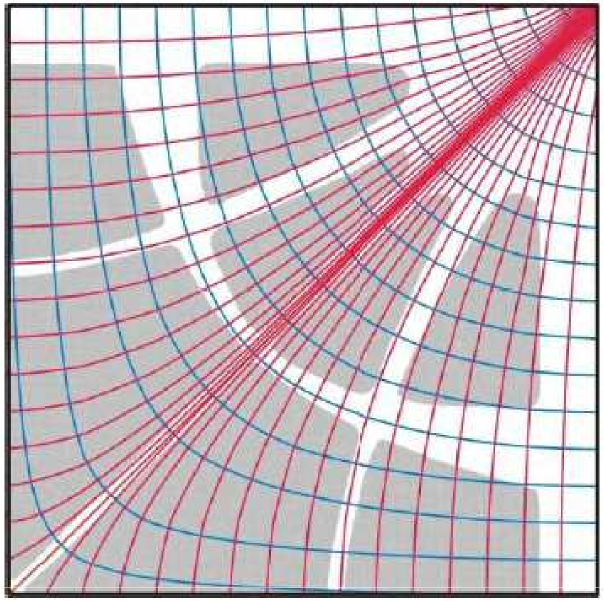
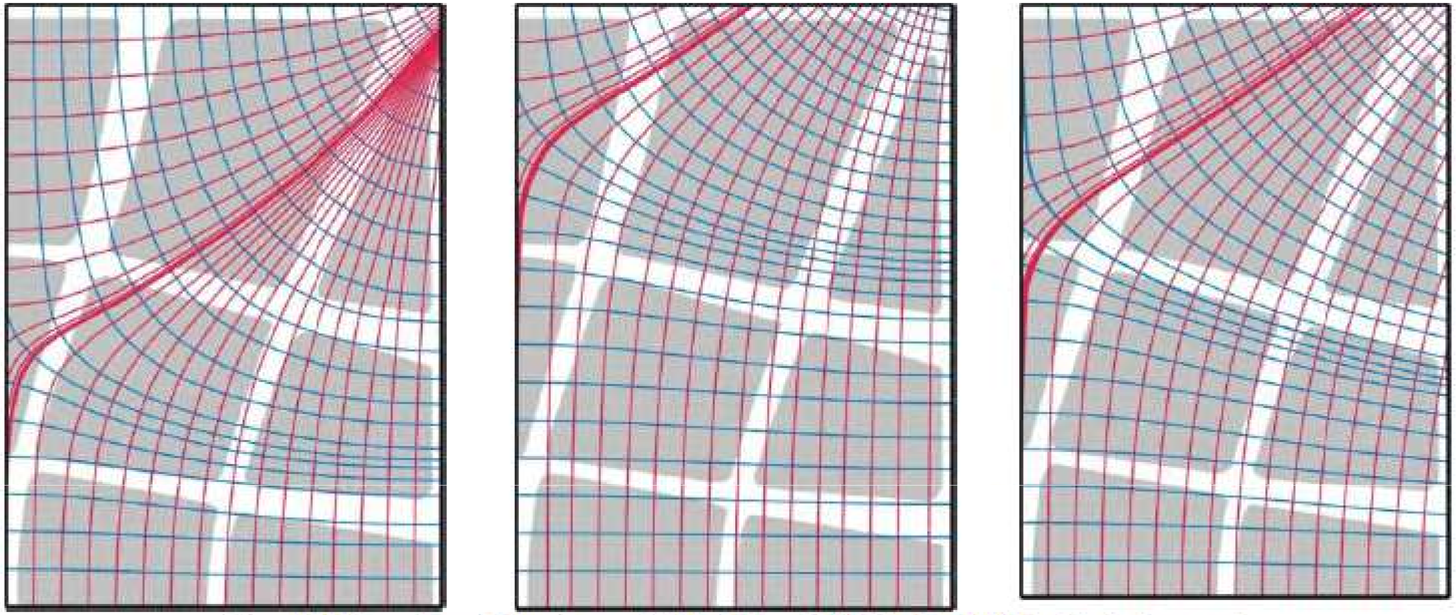


Fig. 8. Palace of Labor Quarter Slab Analysis

The red lines the primary isostatics, corresponding to the maximum principal bending moments, and the blue lines represent the secondary isostatics, corresponding to the minimum principal bending moments.

The concentrations of lines indicate convergence of the principal bending moment directions, highlighted by the prescribed method of starting node generation. The grey and white outline shows an approximate plan of the Gatti Wool Factory floor to illustrate the correlation between the theoretical isostatics and the as-built rib pattern. The concentric secondary isostatics produced around the column support at the bottom left of the quarter slab reflects the reasoning for placing concentric curves of reinforcement around the column head and verifies Nervi's placement of the rib encircling the column head. The ribs emanating from the central column also show close correlation to the generated primary isostatics.

PL Nervi, il ferrocemento, e le isostatiche



Figs. 10-12. Large Sports Palace Quarter Slab Analysis – Full Fixity, Partial Fixity, Simply-Supported

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The Ribbed Floor Slab Systems of Pier Luigi Nervi

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