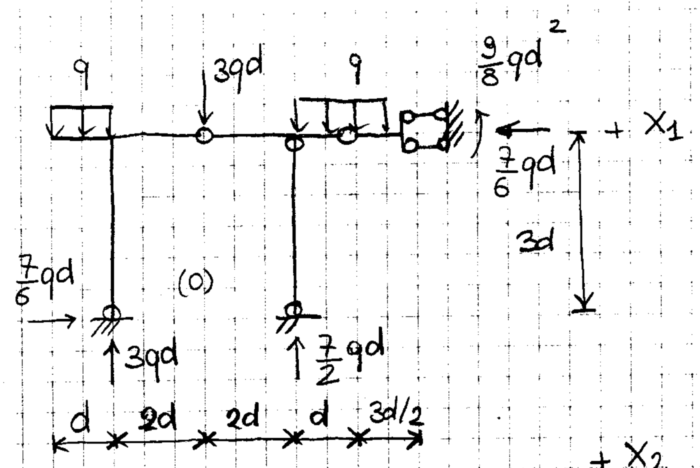
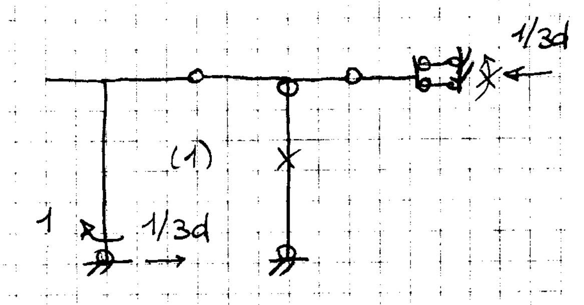
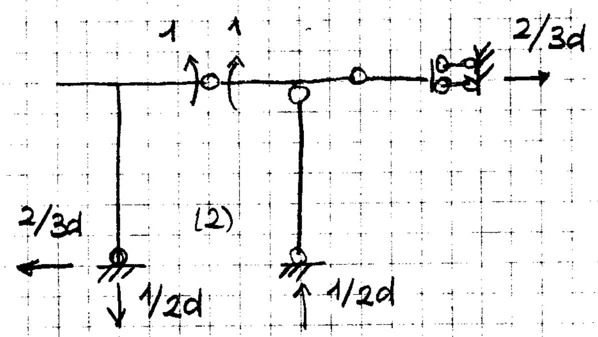
RISOLUZIONE CON IL METODO DELLE FORZE



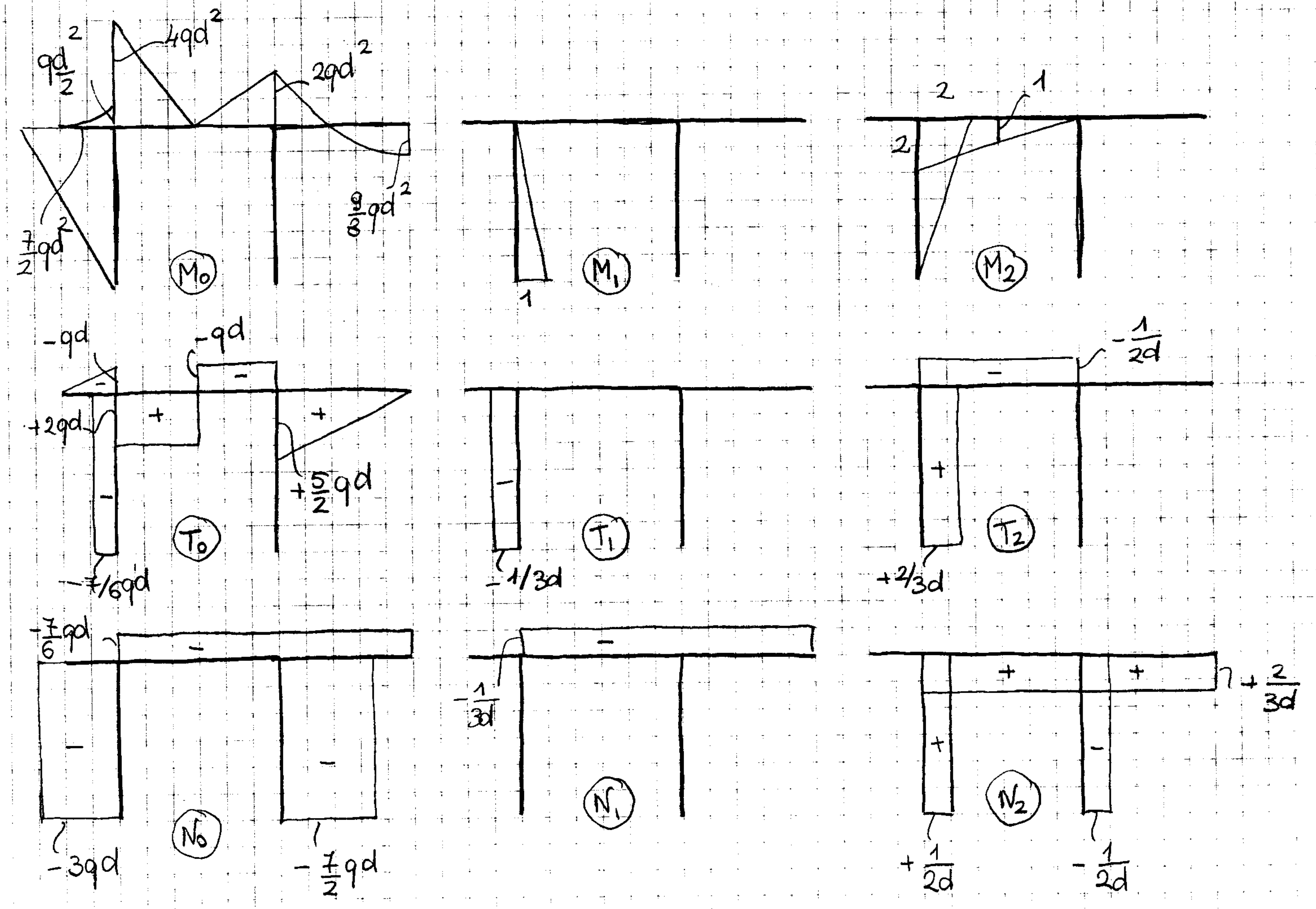
+ X_1



+ X_2



Diagrammi parziali:



Coefficienti:

$$\eta_{10} = \frac{1}{EJ}\int_0^{3d}\left(1-\frac{z}{3d}\right)\left(-\frac{7}{6}qzd\right)\,dz = -\frac{7qd^3}{4EJ}$$

$$\eta_{20} = \frac{1}{EJ}\left[\int_0^{3d}\left(-\frac{7}{6}qdz\right)\left(\frac{2z}{3d}\right)dz + \int_0^{2d}(-qdz)\left(1-\frac{z}{2d}\right)dz + \int_0^{2d}(-2qdz)\left(1+\frac{z}{2d}\right)dz\right] = -\frac{43}{3}\frac{qd^3}{EJ}$$

$$\eta_{11} = \frac{1}{EJ}\int_0^{3d}\left(\frac{z}{3d}\right)^2 dz = \frac{d}{EJ}$$

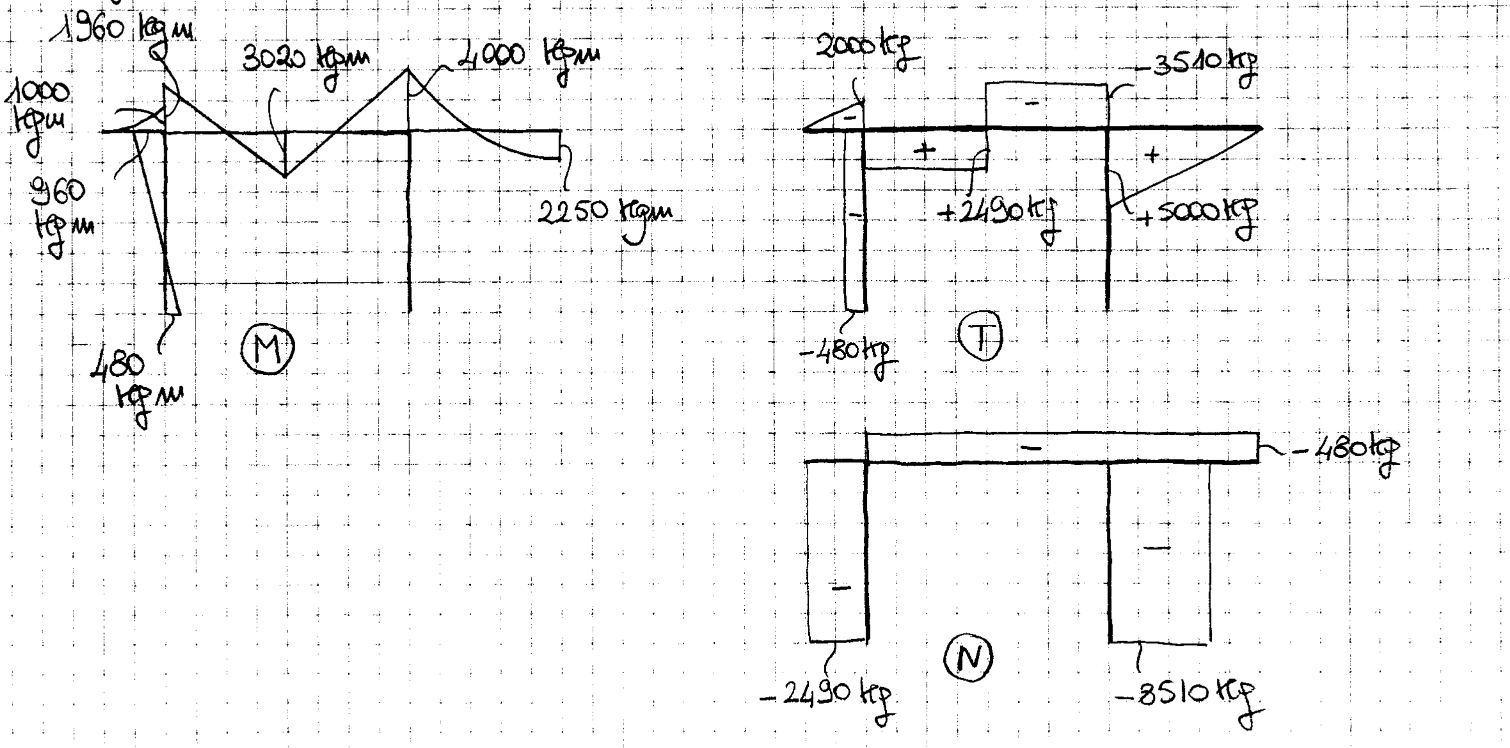
$$\eta_{12} = \frac{1}{EJ}\int_0^{3d}\left(\frac{2z}{3d}\right)\left(1-\frac{z}{3d}\right)dz = \frac{d}{EJ}$$

$$\eta_{22} = \frac{1}{EJ}\left[\int_0^{3d}\left(\frac{2z}{3d}\right)^2 dz + \int_0^{4d}\left(\frac{z}{2d}\right)^2 dz\right] = \frac{28d}{3EJ}$$

Sistema risolutivo:

$$\begin{bmatrix} d/EJ & d/EJ \\ d/EJ & 28d/3EJ \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 7qd^3/4EJ \\ 43qd^3/3EJ \end{bmatrix} \longrightarrow \begin{cases} X_1 = \dfrac{6}{25}qd^2 = 480 \text{ kgm} \\ X_2 = \dfrac{151}{100}qd^2 = 3020 \text{ kgm} \end{cases}$$

Diagrammi di M, T, N:



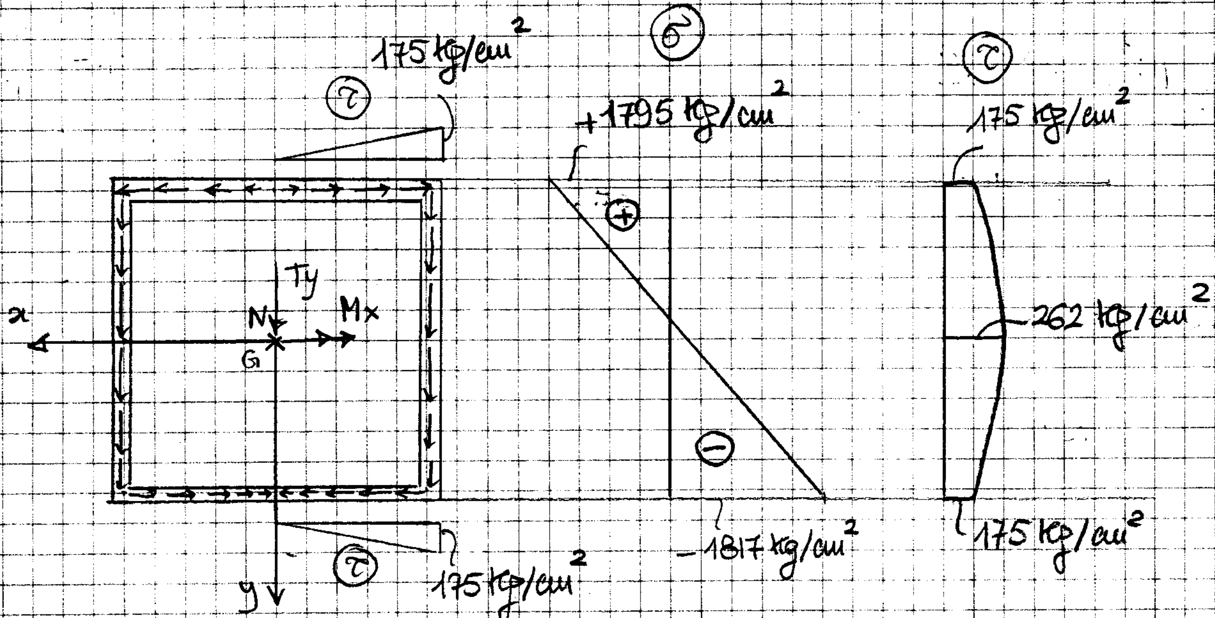
<u>Progetto</u>: la sezione più sollecitata a flessione è quella nel punto G con:

$$M_x = 4000\ \text{kgm}$$

$$\hookrightarrow \frac{M_x}{W_x} \le \sigma_{am} \longrightarrow W_x \ge \frac{M_x}{\sigma_{am}} = \frac{4000 \cdot 100}{2400} = 167\ \text{cm}^3$$

↳ TUBI QUADRI CON LATO ESTERNO 160 mm E SPESSORE s = 7,1 mm.

<u>Verifica</u> (sezione in G dalla parte di H)

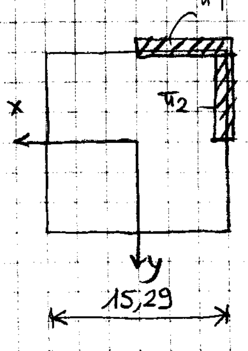


$M_x = -4000\ \text{kgm}$ $\qquad W_x = 221\ \text{cm}^3$

$N = -480\ \text{kg}$ $\qquad A = 43\ \text{cm}^2$

$T_y = 5000\ \text{kg}$ $\qquad J_x = 1693\ \text{cm}^4$

$$\sigma_{c,T} = -\frac{480}{43} \mp \frac{4000 \cdot 100}{221} = \begin{cases} -1817\ \text{kg/cm}^2 \\ +1795\ \text{kg/cm}^2 \end{cases}$$



$$S_{\pi_1} = -\left(\frac{15{,}29}{2}\right)^2 \cdot 0{,}71 = -42\ \text{cm}^3 \longrightarrow \tau = \frac{5000}{1693}\,\frac{42}{0{,}71} = 175\ \text{kg/cm}^2$$

$$S_{\pi_1 \cup \pi_2} = -42 - \left(\frac{15{,}29}{2} \cdot 0{,}71\right)\left(\frac{15{,}29}{4}\right) \longrightarrow \tau = \frac{5000}{1693}\,\frac{63}{0{,}71} = 262\ \text{kg/cm}^2$$

$$= -63\ \text{cm}^3$$

Massima tensione ideale: (secondo Von Mises) $\sigma_{id} = \sqrt{1817^2 + 3 \cdot 175^2} = 1842\ kg/cm^2 < \sigma_{am}$

Risoluzione con il metodo delle forze tenendo conto della cedevolezza del nucleo in A e delle deformazioni assiali:

$$\eta_{10} = -\frac{7qd^3}{4EJ} + \frac{14qd}{3EA}$$

$$\eta_{20} = -\frac{43qd^3}{3EJ} - \frac{133qd}{36EA}$$

$$\eta_{11} = \frac{d}{EJ} + \frac{1}{k} + \frac{13qd}{18EA}$$

$$\eta_{12} = \frac{d}{EJ} - \frac{13qd}{9EA}$$

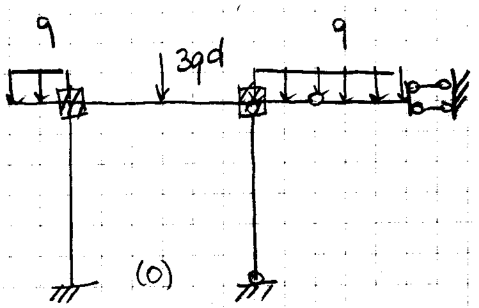
$$\eta_{22} = \frac{28d}{3EJ} + \frac{79qd}{18EA}$$

(attenzione! Nei calcoli si deve utilizzare il valore di k in tm/rad:

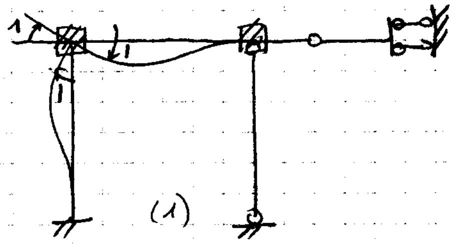
$k = 10\ tm/grado = \frac{\pi \cdot 10}{180}\ tm/rad \overset{!}{=} 174\ kgm/rad$)

$$\begin{cases} X_1 = 663\ kgm \\ X_2 = 3014\ kgm \end{cases}$$

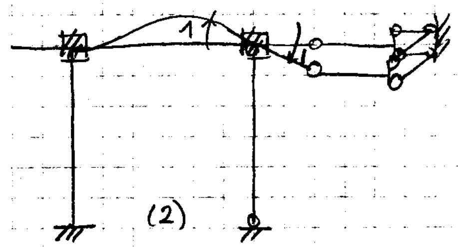
RISOLUZIONE CON IL METODO DELLE RIGIDEZZE



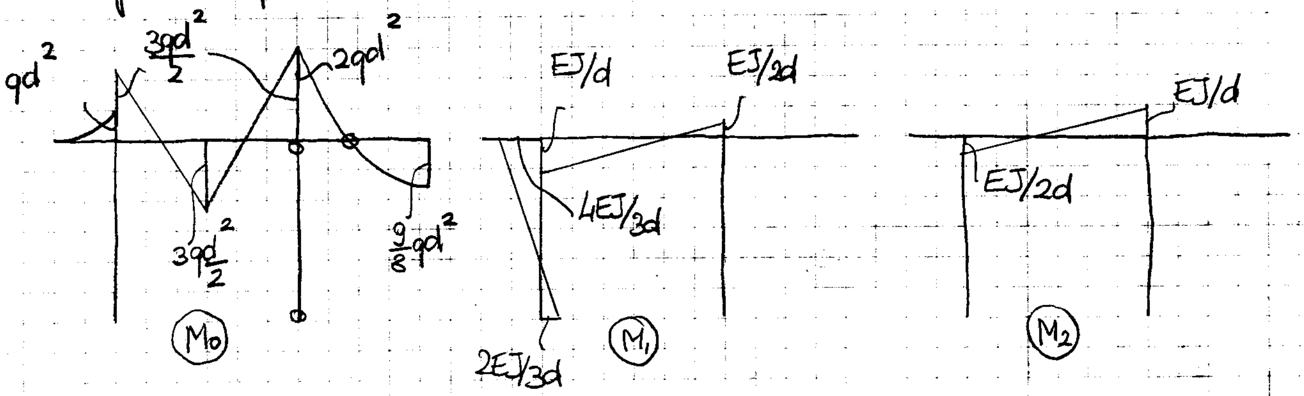
$+ Y_1$



$+ Y_2$

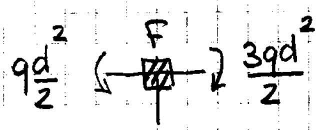


Diagrammi parziali:

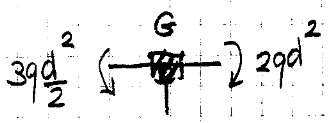


Coefficienti di rigidezza:

Sistema (0)

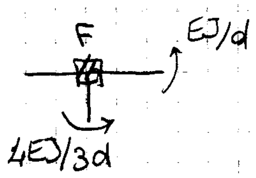


$k_{10} = -qd^2$

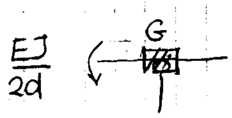


$k_{20} = -qd^2/2$

Sistema (1)

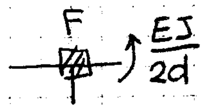


$k_{11} = \left(\frac{4}{3}+1\right)\frac{EJ}{d} = \frac{7EJ}{3d}$

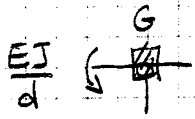


$k_{21} = +\frac{EJ}{2d}$

Sistema (2)



$$k_{12} = \frac{EJ}{2d}$$

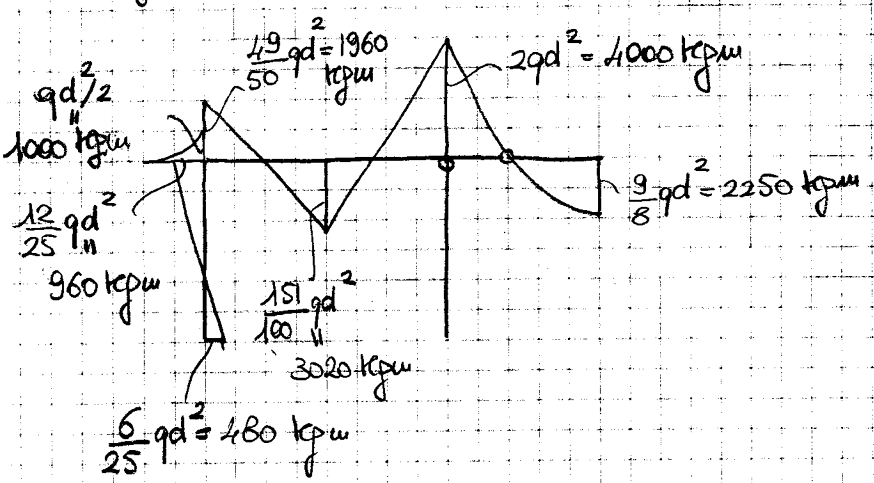


$$k_{22} = EJ/d$$

Sistema di equazioni risolutivo:

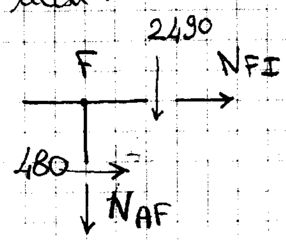
$$\begin{bmatrix} 7EJ/3d & EJ/2d \\ EJ/2d & EJ/d \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} +qd^2 \\ +qd^2/2 \end{bmatrix} \rightarrow \begin{cases} Y_1 = \dfrac{9qd^3}{25EJ} \\ Y_2 = \dfrac{8qd^3}{25EJ} \end{cases}$$

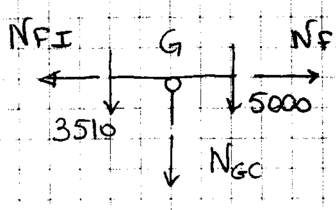
Diagramma di M (si ottiene per sovrapposizione $M = M_0 + Y_1 M_1 + Y_2 M_2$)



Il diagramma di T si ottiene per equilibrio sui tratti isostatici EF e GI e calcolando la pendenza di M negli altri tratti. Si ottiene il diagramma riportato a pagina 2.

Il diagramma dello sforzo normale N si ottiene con i seguenti equilibri ai nodi:





$N_{AF} = -2490$ kgf

$N_{FI} = -480$ kgf

$N_{GC} = -(3510 + 5000) = -8510$ kgf

e si ottiene il diagramma di N riportato a pag. 2.