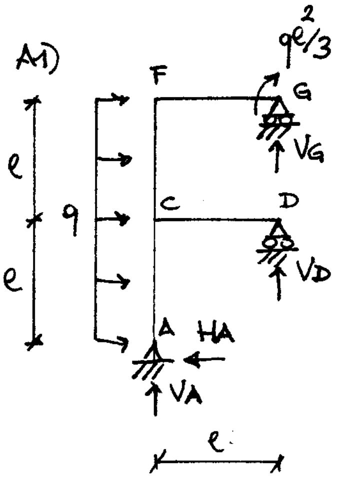
METODO DELLE FORZE: RISOLUZIONE PROVA A

A1)

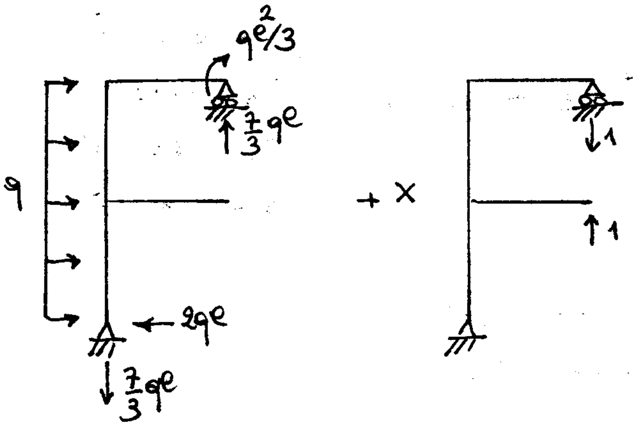


$H_A = 2ql$

$G \circlearrowleft \quad V_A l + q\frac{l^2}{3} - 2ql^2 + 4ql^2 = 0 \quad \rightarrow \quad V_A = -\frac{7}{3}ql$

$V_G + V_D = +\frac{7}{3}ql$

i) Risoluzione con $X = V_D$:



(0) (1)

Eq.ne di congruenza:

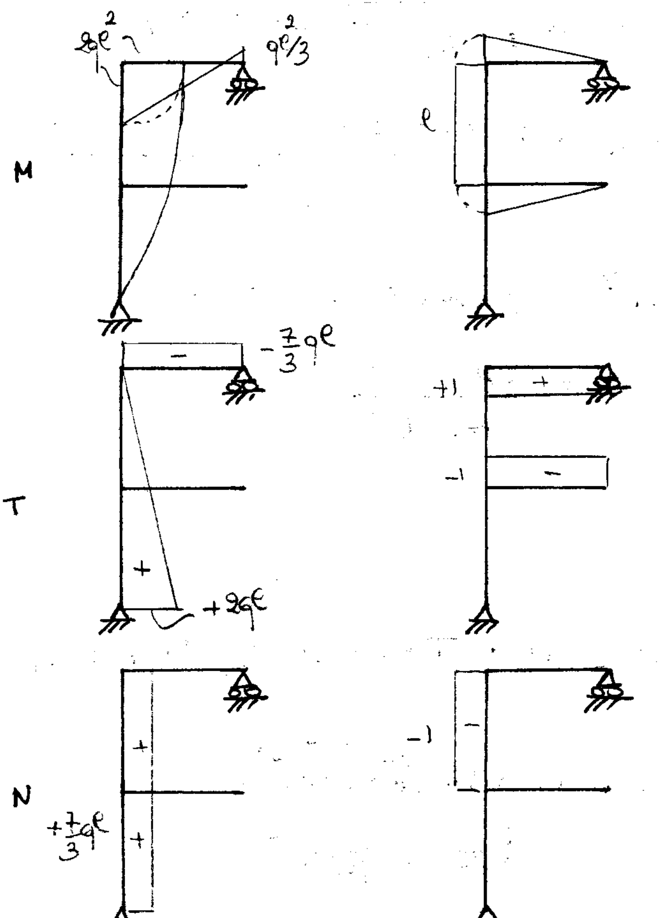
$v_D = 0$ (spost. verticale del punto D)

$= \eta_{10} + \eta_{11} X$

Calcolo dei coefficienti η_{ij}:

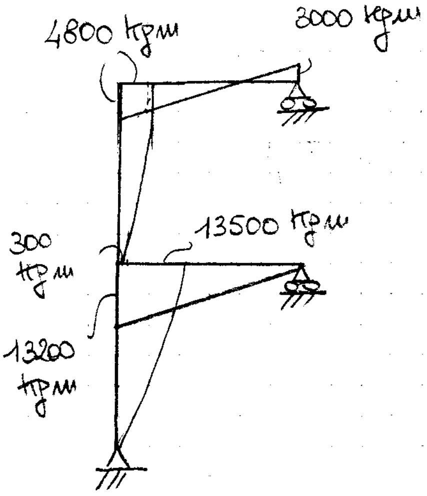
$$EJ\eta_{10} = \int_0^l (-l)\left(2ql^2 - q\frac{x^2}{2}\right)dx + \int_0^l (-x)\left(-q\frac{l^2}{3} + \frac{7}{3}ql(x)\right)dx = -\frac{22}{9}ql^4$$

$$EJ\eta_{11} = 2\int_0^l x^2 dx + l^3 = \frac{5}{3}l^3$$

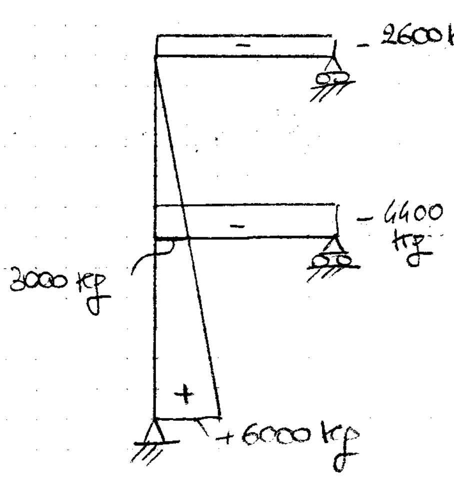


$$X = -\frac{\eta_{10}}{\eta_{11}} = \frac{22}{15}ql = 4400 \text{ kg}$$

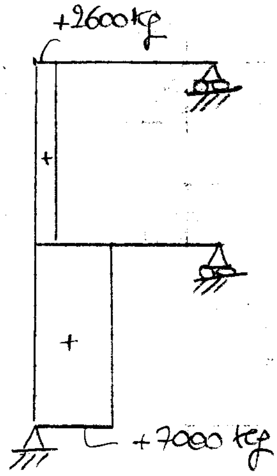
Diagrammi dell'azione interna:



(M)



(T)



(N)

Progetto: nella sezione più sollecitata a flessione $M = 13500$ kgm:

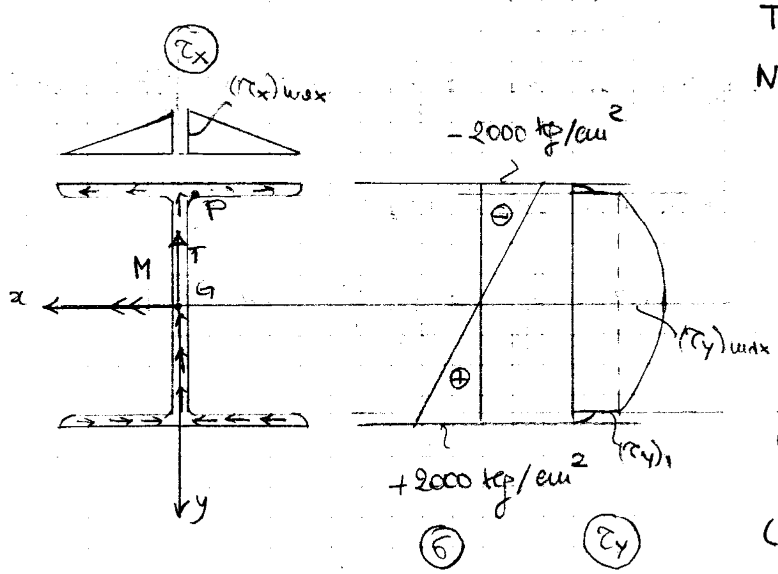
$$W_x \geq \frac{13500 \cdot 100}{2400} = 562.5 \text{ cm}^3 \Rightarrow \text{HEA } 240 \quad \begin{cases} W_x = 675 \text{ cm}^3 \\ J_x = 7763 \text{ cm}^4 \\ S_x = 372 \text{ cm}^3 \\ A = 76.8 \text{ cm}^2 \end{cases}$$

A2)

Verifica: considero ancora la sezione con $M = 13500$ kgm

$T = -4400$ kg

$N = 0$



$$\sigma_{max/min} = \pm \frac{13500 \cdot 100}{675} = \pm 2000 \text{ kg/cm}^2$$

$$(\tau_x)_{max} = \frac{4400 \cdot \frac{23}{2} \cdot \frac{1}{2} \cdot \frac{23}{4}}{7763 \cdot \frac{1}{2}} = 38 \text{ kg/cm}^2$$

$$(\tau_y)_1 = 2\,(\tau_x)_{max} \cdot \frac{12}{7.5} = 122 \text{ kg/cm}^2$$

$$(\tau_y)_{max} = \frac{4400 \cdot 372}{7763} = 281 \text{ kg/cm}^2$$

Il punto più sollecitato è indicato con P:

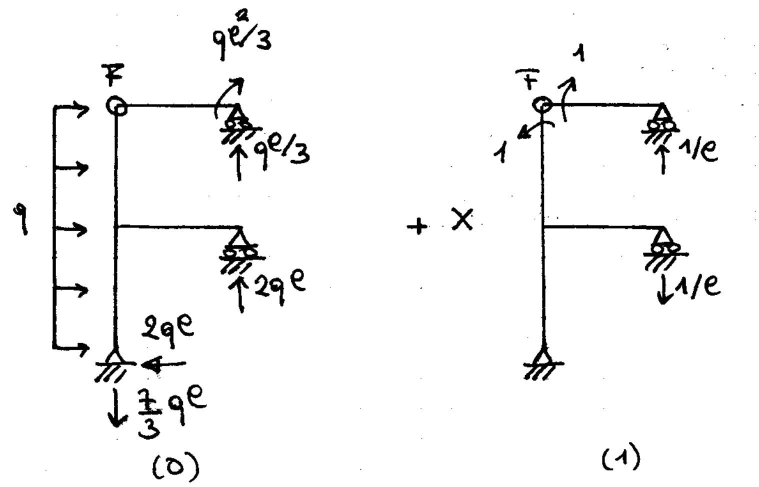
$$(\sigma_{id})_P \simeq \sqrt{2000^2 + 3 \cdot 122^2} = 2011 \text{ kg/cm}^2 < \sigma_{amm}.$$

Occorre considerare anche la sezione con $M = 13200$ kgm, $T = +3000$ kg, $N = 7000$ kg.

$$\sigma_{max/min} = \pm \frac{13200 \cdot 100}{675} + \frac{7000}{76.8} = \begin{cases} 2047 \text{ kg/cm}^2 \\ -1864 \text{ kg/cm}^2 \end{cases} \longrightarrow (\sigma_{id})_P = \sqrt{2047^2 + 3 \cdot 82^2}$$

$$(\tau_y)_1 \simeq 2 \left(\frac{12}{7.5}\right) \cdot \frac{3000 \cdot 23^2/8}{7763} = 82 \text{ kg/cm}^2 \qquad = 2052 \text{ kg/cm}^2$$

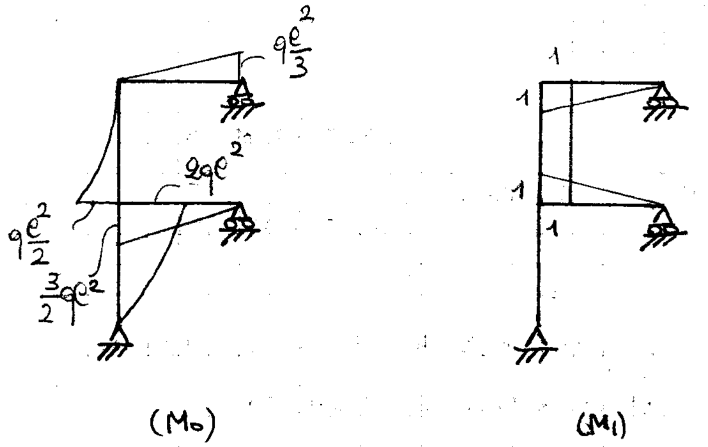
ii) Risoluzione con $X = M_F$:



Eq.ne di congruenza:

$$0 = \Delta\varphi_F = \eta_{10} + \eta_{11} X$$

$$EJ\eta_{10} = \int_0^l (2qlx)\left(-\frac{x}{l}\right)dx + \int_0^l 1\cdot\left(-q\frac{x^2}{2}\right)dx + \int_0^l \frac{x}{l}\left(q\frac{l^2}{3} - q\frac{lx}{3}\right)dx = -\frac{8}{9}ql^3$$

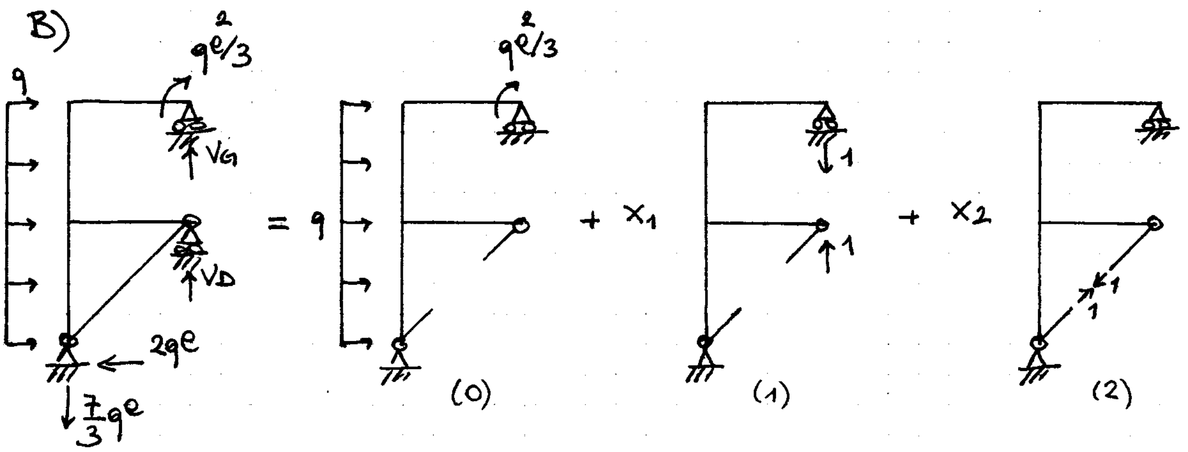


$$EJ\eta_{11} = l + \frac{2l}{3} = \frac{5}{3}l$$

$$\hookrightarrow X = -\frac{\eta_{10}}{\eta_{11}} = \frac{8}{15}ql^2 = 4800 \text{ kgm}$$

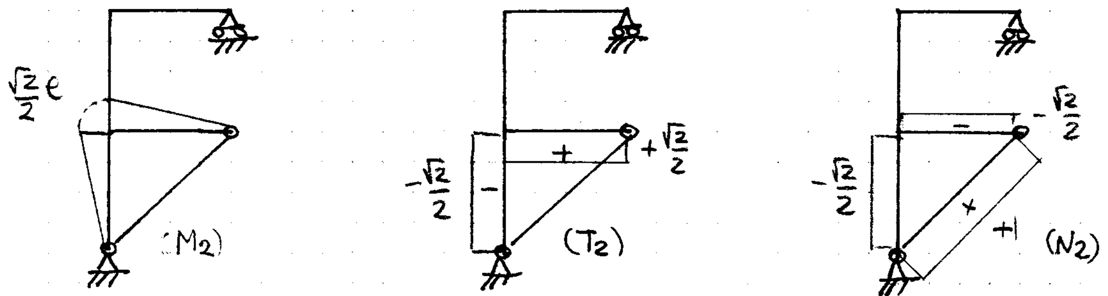
Diagrammi, progetto e verifica : vd. pag. 2.

B)



Diagrammi sistemi (0) e (1) : vedi risoluzione (i) del punto (A1)

Sistema (2):



$$\eta_{10} = -\frac{22}{9}\frac{q\ell^4}{EJ}$$ (vedi risoluz. (i) punto (A1))

$$\eta_{20} = \frac{1}{EJ}\int_0^{\ell}\left(2q\ell x - q\frac{x^2}{2}\right)\left(\frac{\sqrt{2}}{2}x\right)dx = -\frac{13\sqrt{2}}{48}\frac{q\ell^4}{EJ}$$

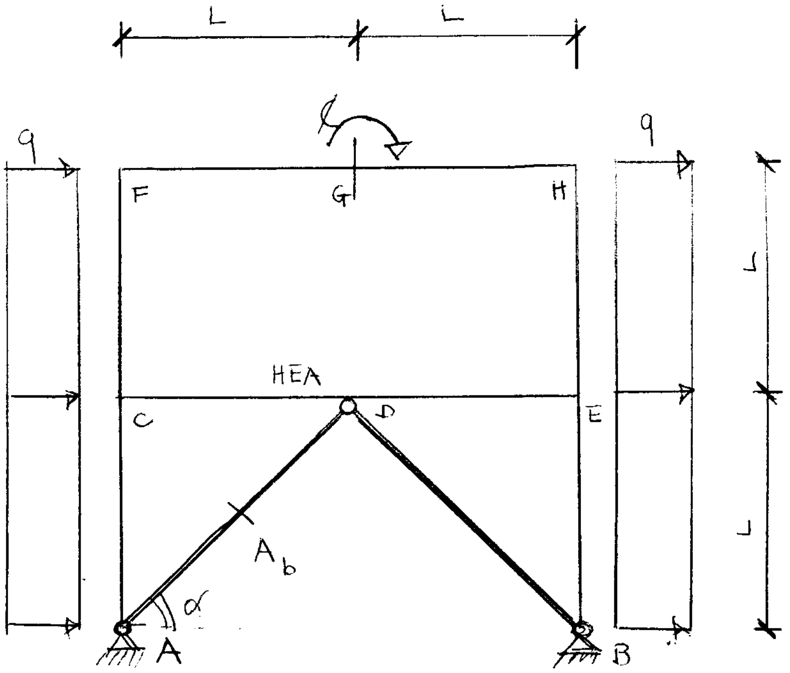
$$\eta_{11} = \frac{5\ell^3}{3EJ}$$ (vedi risoluzione (i) punto (A1))

$$\eta_{12} = -\frac{1}{EJ}\int_0^{\ell}(x)\left(\frac{\sqrt{2}}{2}x\right)dx = -\frac{\sqrt{2}}{6}\frac{\ell^3}{EJ}$$

$$\eta_{22} = \frac{2}{EJ}\int_0^{\ell}\left(\frac{\sqrt{2}}{2}x\right)^2 dx + \frac{\ell\sqrt{2}}{EA_{biella}} = \frac{\ell^3}{3EJ} + \frac{\ell\sqrt{2}}{EA_{biella}}$$

Con i dati del problema e $J = 7763\ cm^4$, si trova:
$$\begin{cases} X_1 = 5390\ kg \\ X_2 = 7002\ kg \end{cases}$$

Diagrammi delle azioni interne: vedi a pagina 8, 9, 10 della risoluzione con il metodo degli spostamenti.



DATI PROVA (A)

$L = 300 \text{ cm}$

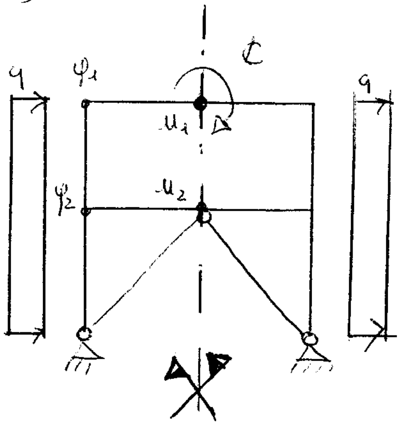
$q = 10 \ \frac{Kg}{cm} \qquad \mathcal{C} = 6 \text{ tm} = 6 \cdot 10^5 \ Kg \cdot cm$

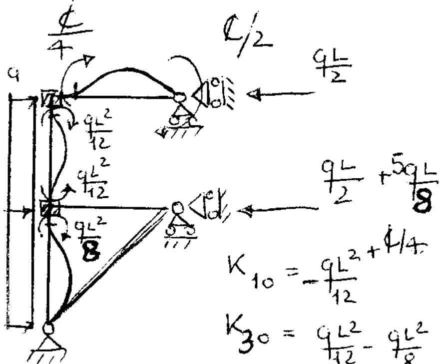
$E = 2100000 \ \frac{Kg}{cm^2}$

$\sigma_{sad} = 2400 \ \frac{Kg}{cm^2}$

$\alpha = \frac{\pi}{4}$

A1) METODO DELLE RIGIDEZZE:



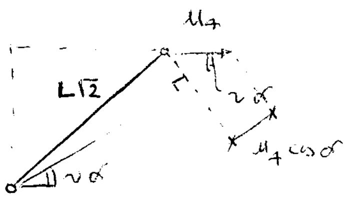


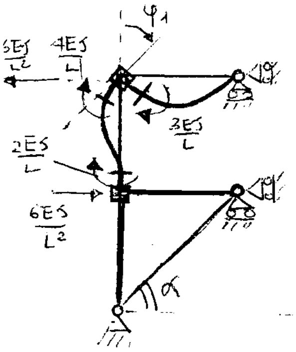
$K_{10} = -\frac{qL^2}{12} + \mathcal{C}/4 \qquad K_{40} = \frac{qL}{2} + \frac{5}{8} qL$

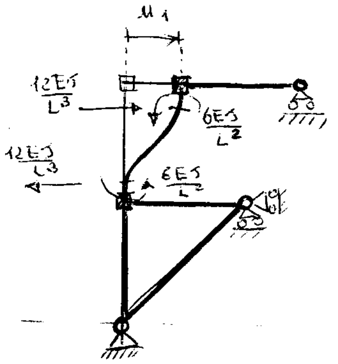
$K_{30} = \frac{qL^2}{12} - \frac{qL^2}{8}$

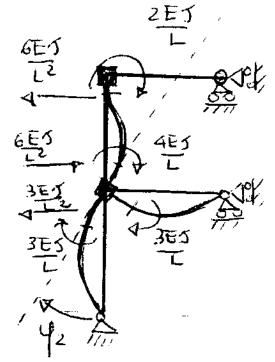
$K_{20} = \frac{qL}{2}$

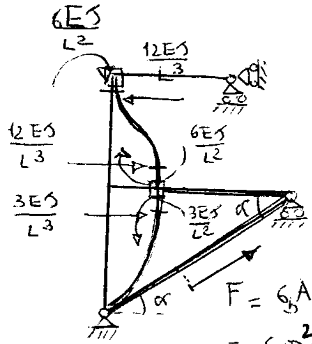
ALLUNGAMENTO BIELLA











$F = \sigma_b A_b = \varepsilon_b E A_b = \cos^2\alpha \frac{EA_b}{L}$

φ1 u1 φ2 u2

$K_{11} = \frac{7EJ}{L}$

$K_{12} = -\frac{6EJ}{L^2}$

$K_{13} = \frac{2EJ}{L}$

$K_{14} = \frac{6EJ}{L^2}$

$K_{22} = \frac{12EJ}{L^3}$

$K_{23} = -\frac{6EJ}{L^2}$

$K_{24} = -\frac{12EJ}{L^3}$

$K_{33} = \frac{10EJ}{L}$

$K_{34} = \frac{3EJ}{L^2}$

$K_{44} = \frac{15EJ}{L^3} + F\cos^3\alpha = \frac{15EJ}{L^3} + \frac{EA_b}{L}\cos^3\alpha$

CASO A1 $(EA_b = 0)$

$$EJ \begin{bmatrix} \frac{7}{L} & -\frac{6}{L^2} & \frac{2}{L} & \frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} \\ \frac{2}{L} & -\frac{6}{L^2} & \frac{10}{L} & \frac{3}{L^2} \\ \frac{6}{L^2} & -\frac{12}{L^3} & \frac{3}{L^2} & \frac{15}{L^3} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ u_1 \\ \varphi_2 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\frac{qL^2}{12} - \frac{C}{4} \\ \frac{qL}{2} \\ \frac{qL^2}{12} - \frac{qL^2}{8} \\ \frac{qL}{2} + \frac{5}{8} qL \end{bmatrix} = F$$

$$\Rightarrow \begin{cases} EJ\varphi_1 = 3{,}3 \cdot 10^7 \ Kg \cdot cm^2 \\ EJ u_1 = 1{,}116 \cdot 10^{11} \ Kg \cdot cm^3 \\ EJ\varphi_2 = 1{,}32 \cdot 10^8 \ Kg \cdot cm^2 \\ EJ u_2 = 8{,}3475 \cdot 10^{10} \ Kg \cdot cm^3 \end{cases}$$

MOMENTI:

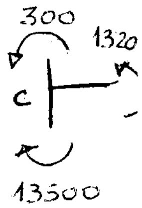
$$\circlearrowleft M_{FG} = \frac{3EJ}{L}\varphi_1 + \frac{C}{4} = 480\,000 \ Kg\,cm = 4800 \ Kg\,m$$

$$\circlearrowleft M_{FC} = \frac{4EJ}{L}\varphi_1 - \frac{6EJ}{L^2}u_1 + \frac{2EJ}{L}\varphi_2 + \frac{6EJ}{L^2}u_2 + \frac{qL^2}{12} = -480\,000 \ Kg\,cm = -4800 \ Kg$$

$$\circlearrowleft M_{CF} = \frac{2EJ}{L}\varphi_1 - \frac{6EJ}{L^2}u_1 + \frac{4EJ}{L}\varphi_2 + \frac{6EJ}{L^2}u_2 - \frac{qL^2}{12} = +30\,000 \ Kg \cdot cm = 300 \ Kg\,m$$

$$\circlearrowleft M_{CD} = \frac{3EJ}{L}\varphi_2 = 1\,320\,000 \ Kg \cdot cm = 13200 \ Kg\,m$$

$$\circlearrowleft M_{CA} = \frac{3EJ}{L}\varphi_2 - \frac{3EJ}{L^2}u_2 + \frac{qL^2}{8} = -1\,350\,000 \ Kg\,cm = -13\,500 \ Kg\,m$$



TAGLI:

$$\downarrow T_{FG} = \frac{3EJ}{L^2}\varphi_1 + \frac{3}{4}\frac{C}{L} = 2600 \ Kg$$

$$\leftarrow T_{FC} = \frac{6EJ}{L^2}\varphi_1 - \frac{12EJ}{L^3}u_1 + \frac{6EJ}{L^2}\varphi_2 + \frac{12EJ}{L^3}u_2 + \frac{qL}{2} = 0 \ Kg$$

(PROVA DELLA CORRETTEZZA DEL CALCOLO $\Rightarrow$ L'ASTA FG NON PUÒ AVERE SF. NOR

$$\leftarrow T_{CF} = -\frac{6EJ}{L^2}\varphi_1 + \frac{12EJ}{L^3}u_1 - \frac{6EJ}{L^2}\varphi_2 - \frac{12EJ}{L^3}u_2 + \frac{qL}{2} = 3000 \ Kg$$

$$\downarrow T_{CD} = \frac{3EJ}{L^2}\varphi_2 = 4400 \ Kg$$

$$\leftarrow T_{CA} = \frac{3EJ}{L^2}\varphi_2 - \frac{3EJ}{L^3}u_2 + \frac{5}{8}qL = -3000 \ Kg$$

$$\leftarrow T_{AC} = -\frac{3EJ}{L^2}\varphi_2 + \frac{3EJ}{L^3}u_2 + \frac{3}{8}qL = 6000 \ Kg$$

SFORZI NORMALI:

$$N_{GF} = 0$$

$$N_{FC} = \frac{3}{4}\frac{\mathcal{C}}{E} + \frac{3EJ\varphi_1}{L^2} = 2600\ Kg$$

$$N_{CA} = N_{FC} + \frac{3EJ}{L^2}\varphi_2 = 7000\ Kg$$

PROGETTO:

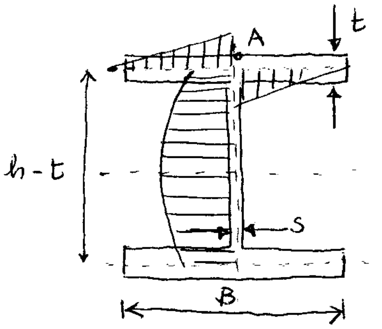
$$M_{max} = M_{CA} = 13500\ Kg\,m$$

$$W_{min} = \frac{M_{max}}{\sigma_{sadm}} = 562{,}5\ cm^3 \Rightarrow HEA\,240$$

h	B	t	s	W	A	J
23	24	1,2	0,75	675	76,8	7763
cm	cm	cm	cm	cm^3	cm^2	cm^4

A2) VERIFICA NELLA SEZIONE MAGG. SOLLECITATA A FLESSIONE

$T = 3000\ Kg \qquad M = 1\,350\,000\ Kg\,cm \qquad N = 7000\ Kg$



VERIFICA IN A

$$\sigma_M = \frac{M}{W} = 2000\ \frac{Kg}{cm^2}$$

$$\tau_A = \frac{ST}{bJ} = \frac{T\left(\cancel{t}\,\frac{B}{2}\cdot\frac{h-t}{2}\right)}{\cancel{t}\,J} = \frac{T\,B\,(h-t)}{4J} = 50{,}55\ \frac{Kg}{cm^2}$$

$$\sigma_N = \frac{N}{A} = 91{,}1\ \frac{Kg}{cm^2}$$

$$\sigma_{id} = \sqrt{(\sigma_M+\sigma_N)^2 + 3\tau^2} = 2093 < \sigma_{sadm} \Rightarrow \text{VERIFICATO}$$

B)

$$EJ\begin{bmatrix} \frac{7}{L} & -\frac{6}{L^2} & \frac{2}{L} & \frac{6}{L^2} \\ & \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} \\ & & \frac{10}{L} & \frac{3}{L^2} \\ & & & \frac{15}{L^3} + \frac{\cos^3\alpha}{L}\,\frac{EA_b}{EJ} \end{bmatrix}\begin{bmatrix} \varphi_1 \\ u_1 \\ \varphi_2 \\ u_2 \end{bmatrix} = \mathbf{F}$$

$$\Rightarrow \begin{cases} ES\varphi_1 = 3{,}29133 \cdot 10^6 \quad Kg\,cm^2 \\ ESu_1 = 9{,}10507 \cdot 10^9 \quad Kg\,cm^3 \\ ES\varphi_2 = 1{,}31653 \cdot 10^7 \quad Kg\cdot cm^2 \\ ESu_2 = 3{,}26158 \cdot 10^9 \quad Kg\cdot cm^3 \end{cases}$$

MOMENTI:

$M_{FG} = 182913\ Kg\cdot cm$

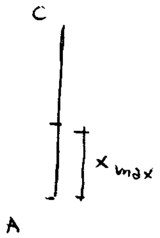
$M_{FC} = -182913\ Kg\cdot cm$

$M_{CF} = -267086\ Kg\cdot cm$

$M_{CD} = 131653\ Kg\cdot cm$

$M_{CA} = 135434\ Kg\cdot cm$

MASSIMO MOMENTO IN CAMPATA



$$x_{max} = \frac{T_{AC}}{T_{AC}+T_{CA}} \cdot L = 104{,}8\,cm$$

$$M_{max} = T_{AC}\cdot x_{max} - q\frac{x_{max}^2}{2} = 54978\ Kg\,cm$$

TAGLI;

$T_{FG} = 1609{,}7\ Kg$

$T_{FC} = -0{,}02\ Kg$

$T_{CF} = -3000\ Kg$

$T_{CD} = 438{,}8\ Kg$

$T_{CA} = 1951\ Kg$

$T_{AC} = 1048{,}6\,Kg$

$T_{DC} = T_{CD}$

SFORZI NORMALI:

$N_{GF} = 0$

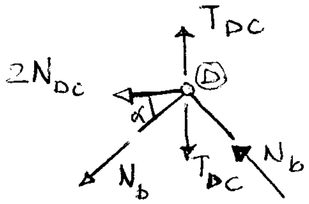
$N_{FC} = T_{FG} = 1609{,}7\ Kg$

$N_{CA} = N_{FC} + T_{CD} = 2048{,}5\ Kg$

$N_b = 7002\ Kg$

$N_{DC} = -4951\ Kg$

$T_{CF} - T_{CA}$

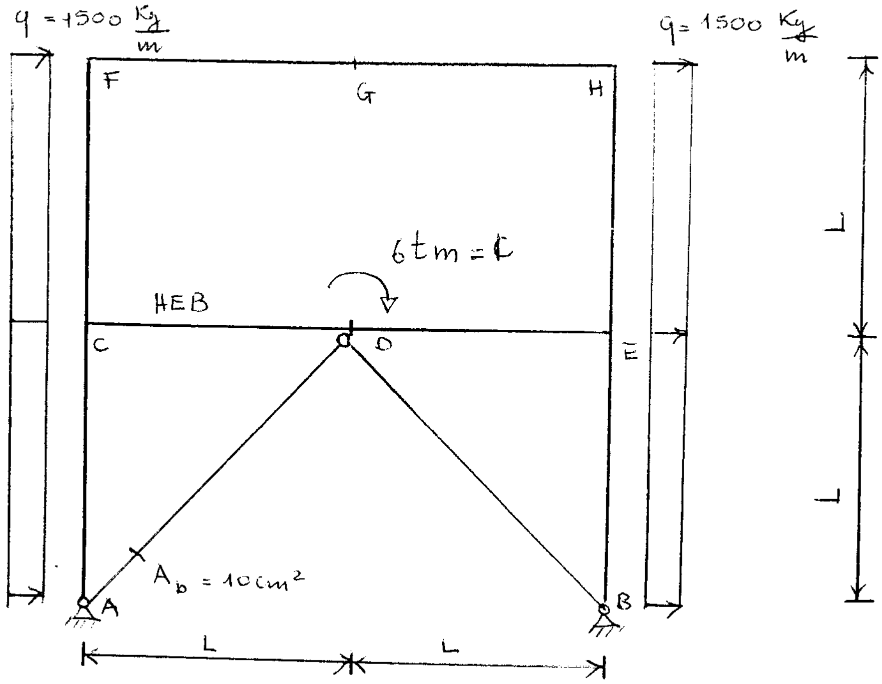


$N_b \cos\alpha = -N_{D}$

c) I VALORI DEGLI SPOSTAMENTI DI F E C SONO

$$\underbrace{\begin{matrix} w_F = 6{,}84\ cm \\ w_C = 5{,}12\ cm \end{matrix}}_{\text{Pto A1}} \qquad \underbrace{\begin{matrix} w_F = 0{,}558\ cm \\ w_C = 0{,}200\ cm \end{matrix}}_{\text{Pto B}}$$

PROVA B



$\sigma_{s\,adm} = 2400 \frac{Kg}{cm^2}$

$E = 2\,100\,000 \frac{Kg}{cm^2}$

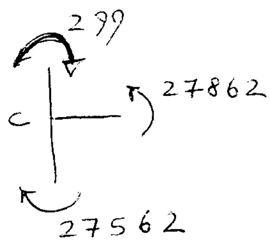
$L = 350$ cm

$q = 15 \frac{Kg}{cm}$

A1) LA MATRICE DI RIGIDEZZA NON CAMBIA RISPETTO ALLA PROVA A (CON L = 350 cm)
PER IL Pto A1) $A_b = 0$
CAMBIA IL VETTORE DELLE FORZE NODALI.

$$\underline{F} = \left[-\frac{qL^2}{12} \quad \frac{qL}{2} \quad \frac{qL^2}{12} - \frac{qL^2}{8} - \frac{C}{4} \quad \frac{qL}{2} + \frac{5}{8}qL \right]^T$$

$$\Rightarrow \begin{cases} EJ\varphi_1 = 1{,}03688 \cdot 10^8 \ Kg\ cm^2 \\ EJ u_1 = 3{,}1092 \cdot 10^{11} \ Kg \cdot cm^3 \\ EJ\varphi_2 = 3{,}07563 \cdot 10^8 \ Kg\ cm^2 \\ EJ u_2 = 2{,}29573 \cdot 10^{11} \ Kg\ cm^3 \end{cases}$$



MOMENTI:

$M_{FG} = \frac{3EJ}{L}\varphi_1 = 888750$ Kg cm

$M_{FC} = \frac{4EJ}{L}\varphi_1 - \frac{6EJ}{L^2}u_1 + \frac{2EJ}{L}\varphi_2 + \frac{6EJ}{L^2}u_2 + \frac{qL^2}{12} = -888750$ Kg cm

$M_{CF} = \frac{2EJ}{L}\varphi_1 - \frac{6EJ}{L^2}u_1 + \frac{4EJ}{L}\varphi_2 + \frac{6EJ}{L^2}u_2 - \frac{qL^2}{12} = -30000$ Kg cm

$M_{CD} = \frac{3EJ}{L}\varphi_2 + \frac{C}{4} = 2786250$ Kg cm

$M_{CA} = \frac{3EJ}{L}\varphi_2 - \frac{3EJ}{L^2}u_2 + \frac{qL^2}{8} = -2756250$ Kg cm

TAGLI:

$T_{FG} = \frac{3EJ}{L^2}\varphi_1 = 2539$ Kg

$T_{FC} = \frac{6EJ}{L^2}\varphi_1 - \frac{12EJ}{L^3}u_1 + \frac{6EJ}{L^2}\varphi_2 + \frac{12EJ}{L^3}u_2 + \frac{qL}{2} = 0$ Kg

$T_{CF} = -\frac{6EJ}{L^2}\varphi_1 + \frac{12EJ}{L^3}u_1 - \frac{6EJ}{L^2}\varphi_2 - \frac{12EJ}{L^3}u_2 + \frac{qL}{2} = 5250$ Kg

$$T_{CD} = \frac{3EJ}{L^2}\varphi_2 + \frac{3}{4}\frac{\phi}{L} = 8818\ Kg$$

$$T_{CA} = \frac{3EJ}{L^2}\varphi_2 - \frac{3EJ}{L^3}u_2 + \frac{5}{8}qL = 5250\ Kg$$

$$T_{AC} = -\frac{3EJ}{L^2}\varphi_2 + \frac{3EJ}{L^3}u_2 + \frac{3}{8}qL = 10500\ Kg$$

SFORZI NORMALI:

$$N_{FC} = T_{GF} = 2539\ Kg$$

$$N_{AC} = T_{CD} + N_{FC} = 11357\ Kg$$

Progetto; $M_{max} = M_{CD} = 2786250\ Kg \cdot cm$

$$W_{min} = \frac{M_{max}}{\sigma_{s\,adm}} = 1161\ cm^3 \Rightarrow \text{HEB } 280$$

h	B	t	s	W	A	J
28	28	1,8	1,05	1380	131,4	1927
cm	cm	cm	cm	cm^3	cm^2	cm^4

A2) VERIFICA NELLA SEZIONE MAGG. SOLLECITATA A FLESSIONE

$M = 2786250\ Kg \cdot cm$ $\quad T = 8818\ Kg$ $\quad N = 0$

$$\sigma_M = \frac{M}{W} = 2019\ \frac{Kg}{cm^2}$$

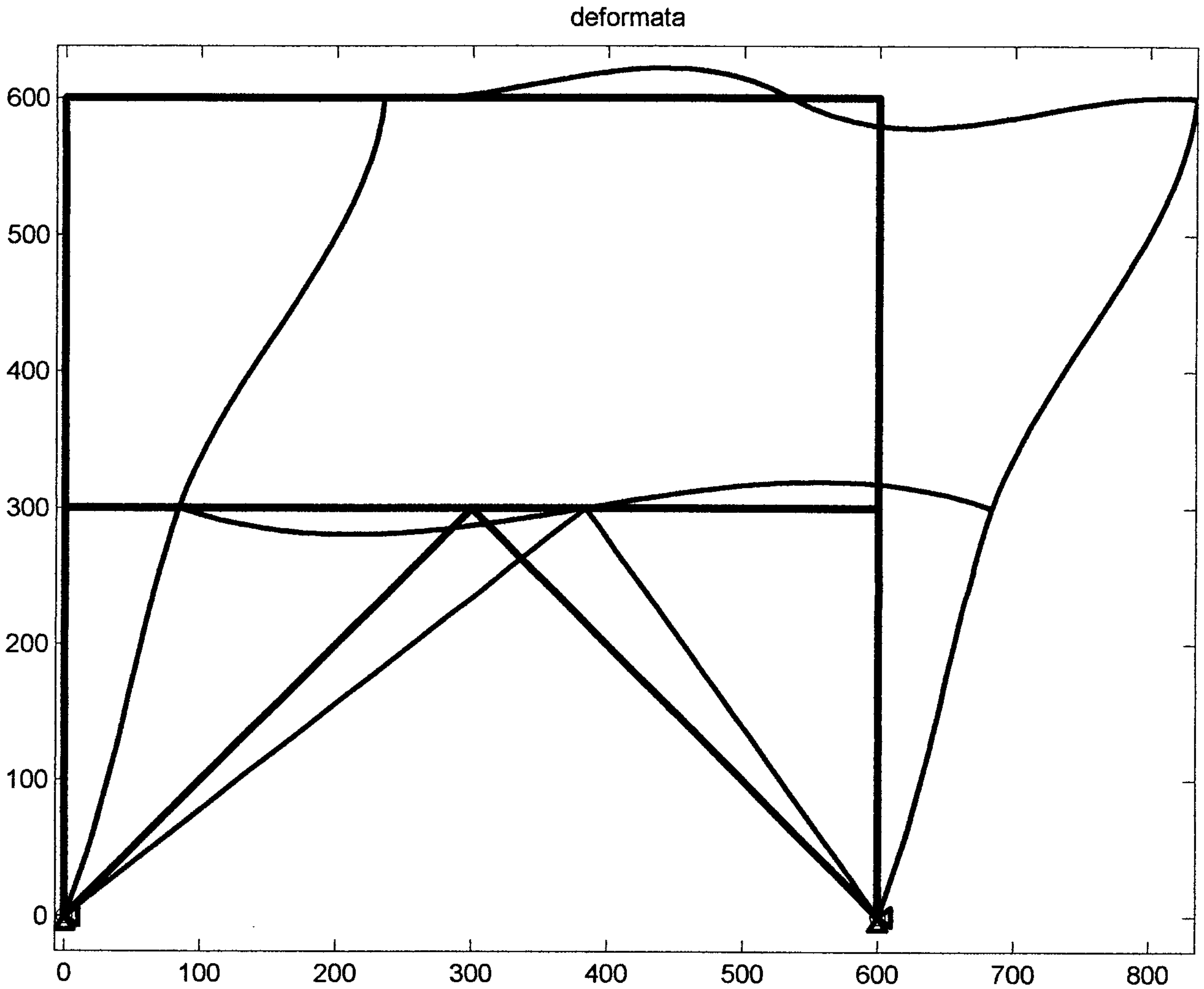
$$\sigma_N = 0$$

$$\tau = \frac{TS}{bJ} = \frac{T\left(\frac{B}{2}\cancel{t}\ \frac{h-t}{2}\right)}{\cancel{t}J} = 84\ Kg/cm^2$$

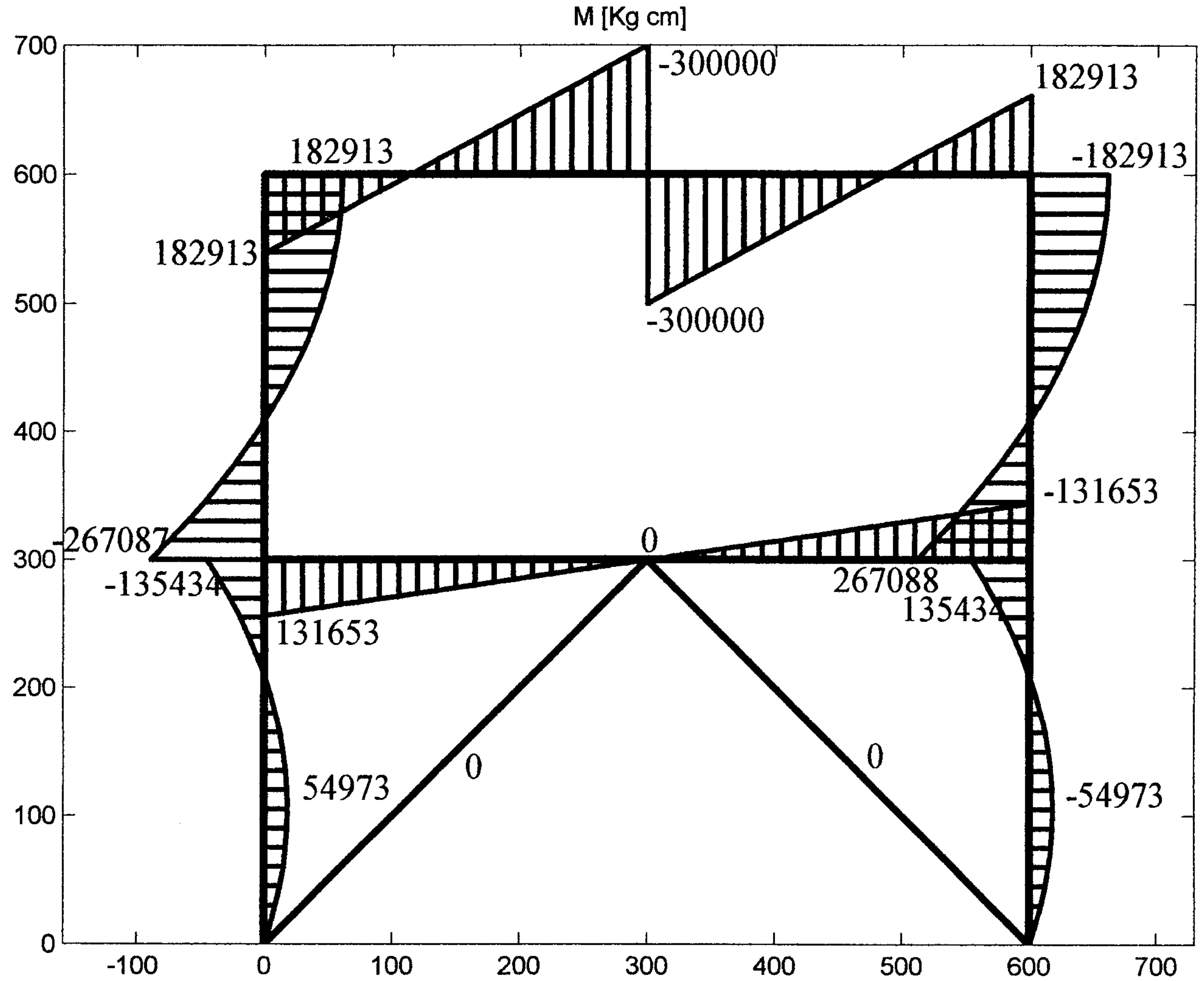
$$\sigma_{id} = \sqrt{(\sigma_M+\sigma_N)^2 + 3\tau^2} = 2024\ \frac{Kg}{cm^2} < \sigma_{s\,adm} \Rightarrow \text{VERIFICATO}$$

B)
$$\begin{cases} EJ\varphi_1 = 3,58386 \cdot 10^7\ Kg\ cm^2 \\ EJ u_1 = 3,78285\ 10^{10}\ Kg \cdot cm^3 \\ EJ\varphi_2 = 3,6167 \cdot 10^7\ Kg \cdot cm^2 \\ EJ u_2 = 1,58487 \cdot 10^{10}\ Kg \cdot cm^3 \end{cases}$$

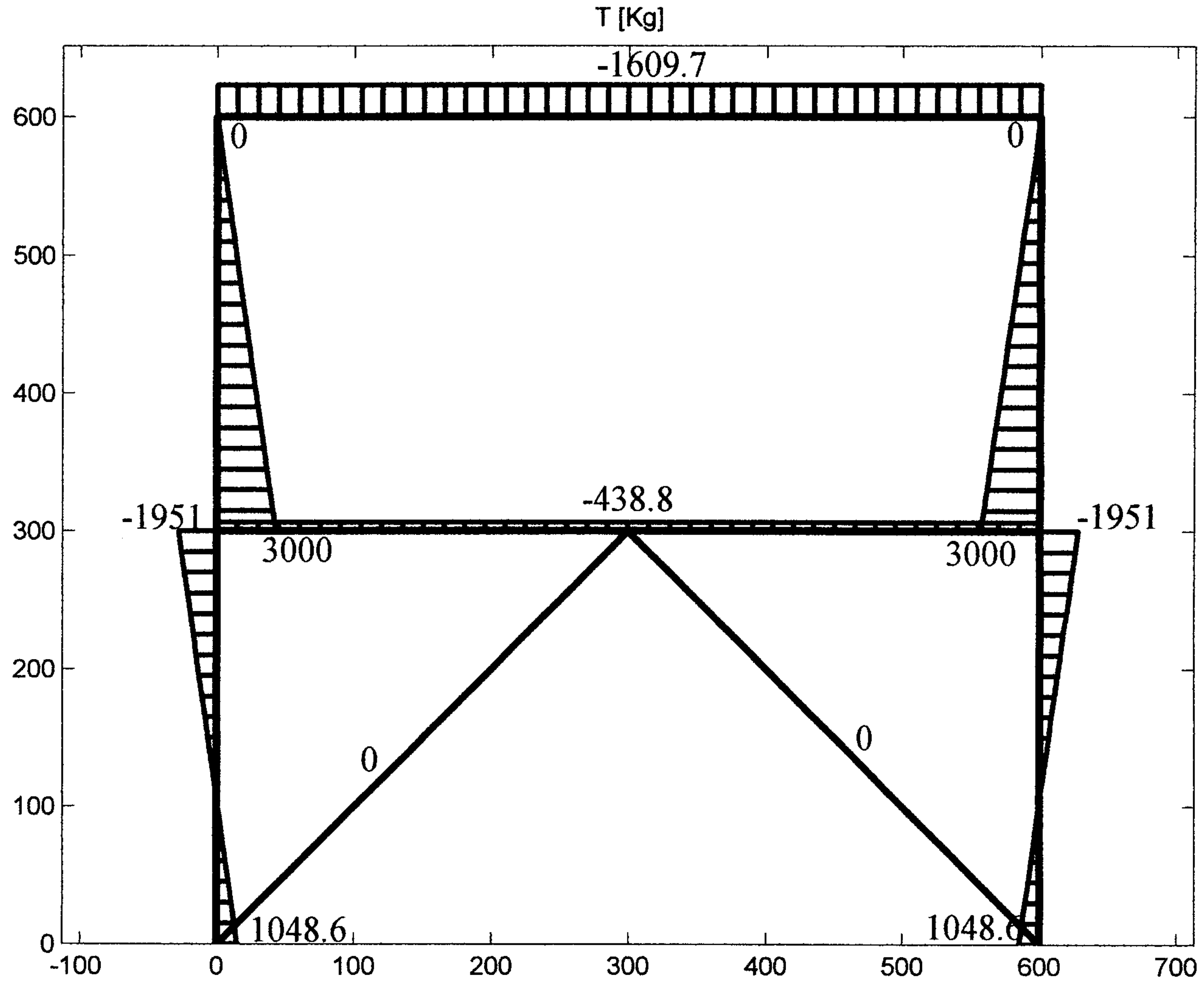
PROVA A   Pto B


deformata
600
500
400
300
200
100
0
0
100
200
300
400
500
600
700
800

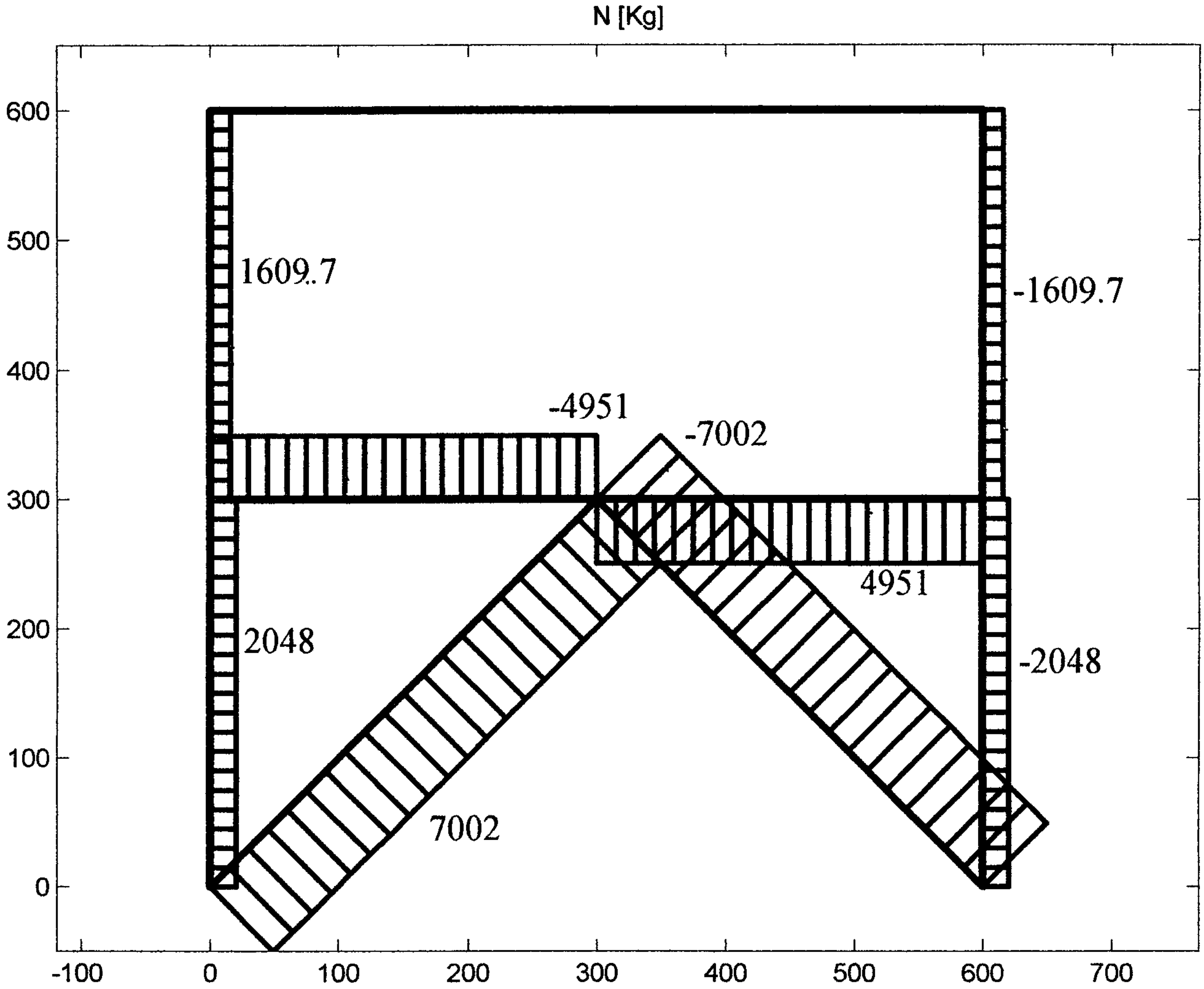
PROVA A PTO B


M [Kg cm]
-300000
182913
182913
-182913
182913
-300000
-267087
0
-131653
-135434
267088
131653
135434
0
0
54973
-54973
700
600
500
400
300
200
100
0
-100
0
100
200
300
400
500
600
700

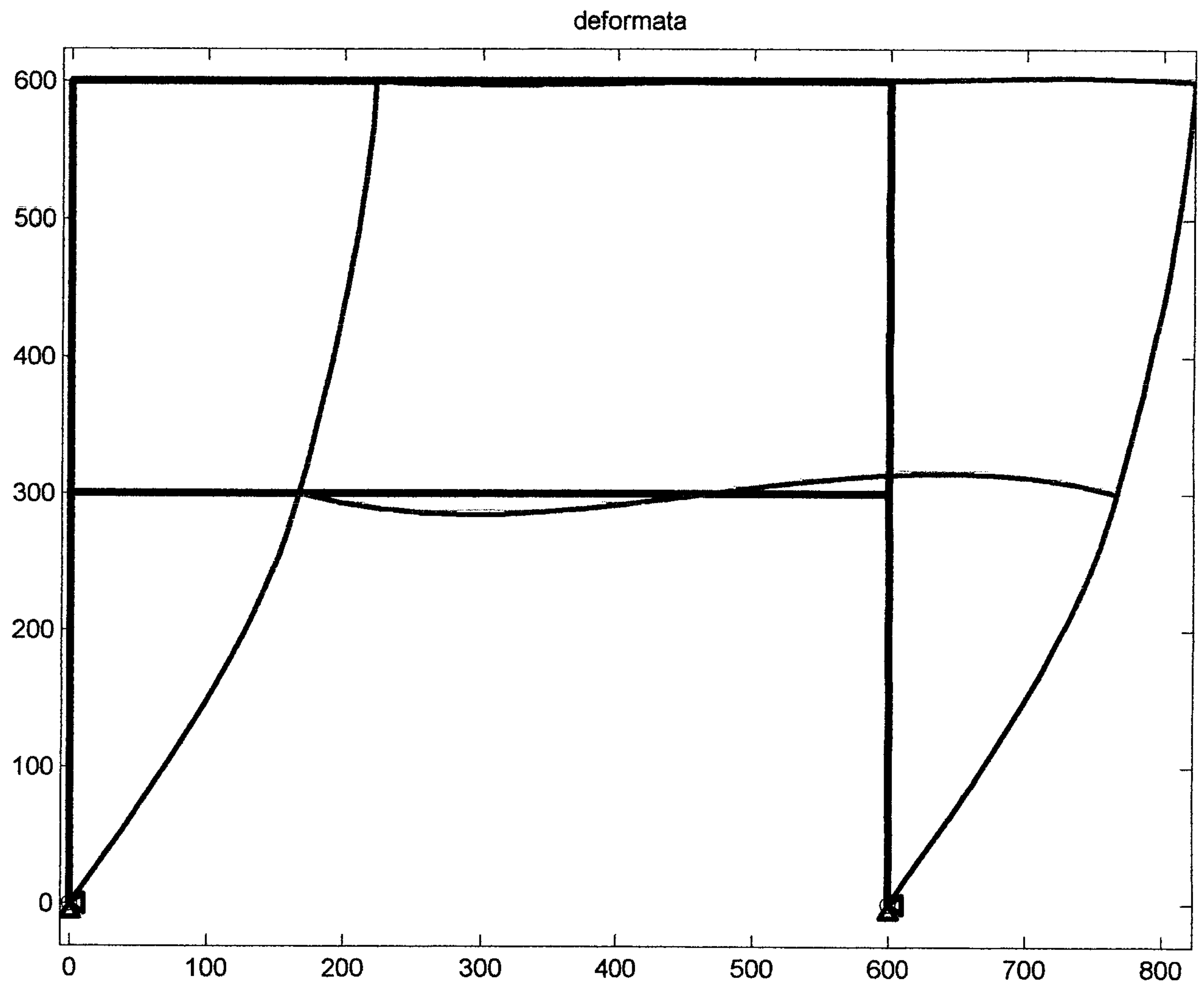
PROVA Pto B



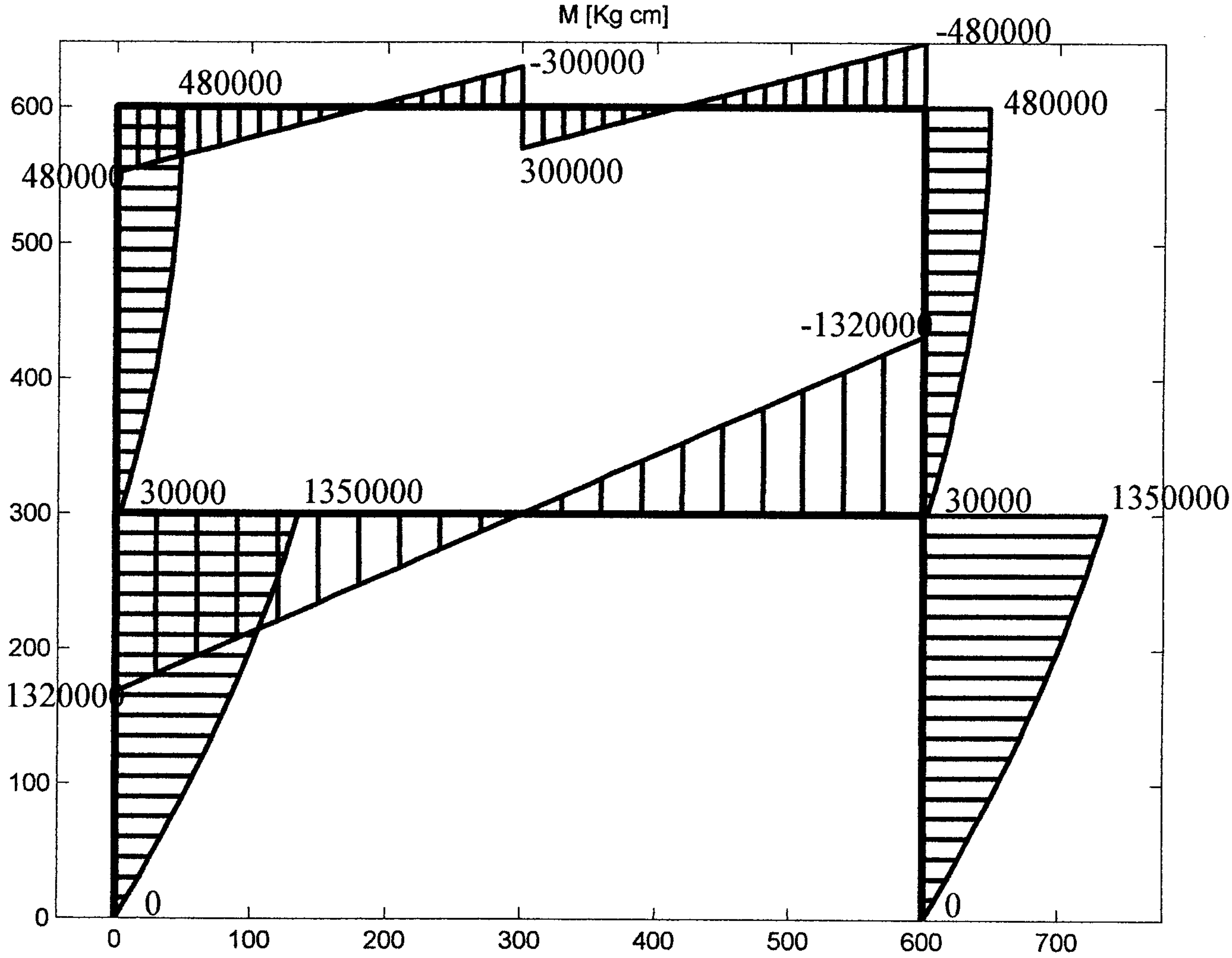
T [Kg]
-1609.7
0
0
-438.8
-1951
-1951
3000
3000
0
0
1048.6
1048.6
600
500
400
300
200
100
0
-100
0
100
200
300
400
500
600
700


N [Kg]
1609.7
-1609.7
-4951
-7002
4951
2048
-2048
7002
600
500
400
300
200
100
0
-100
0
100
200
300
400
500
600
700

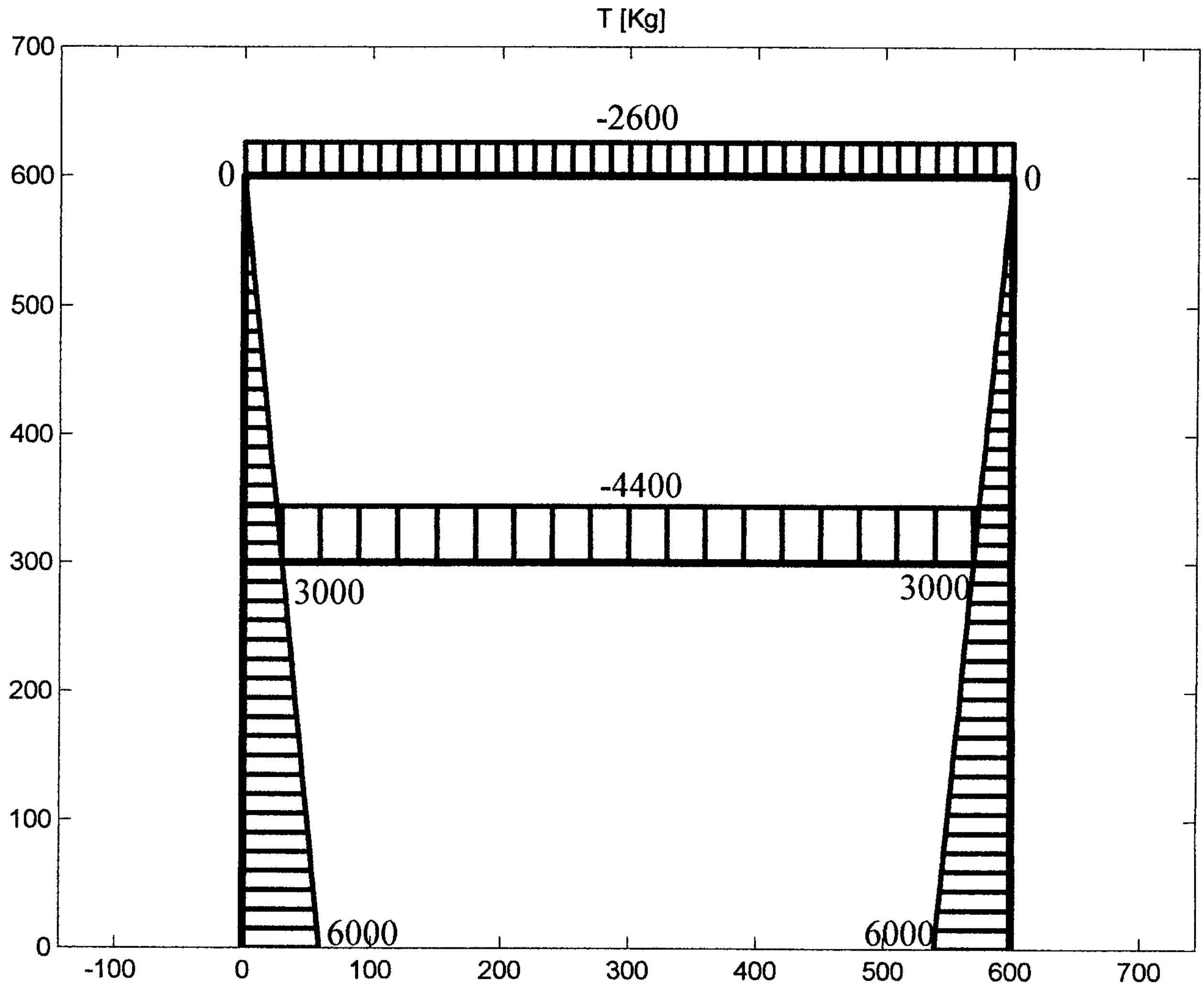
PROVA A   Pto A1



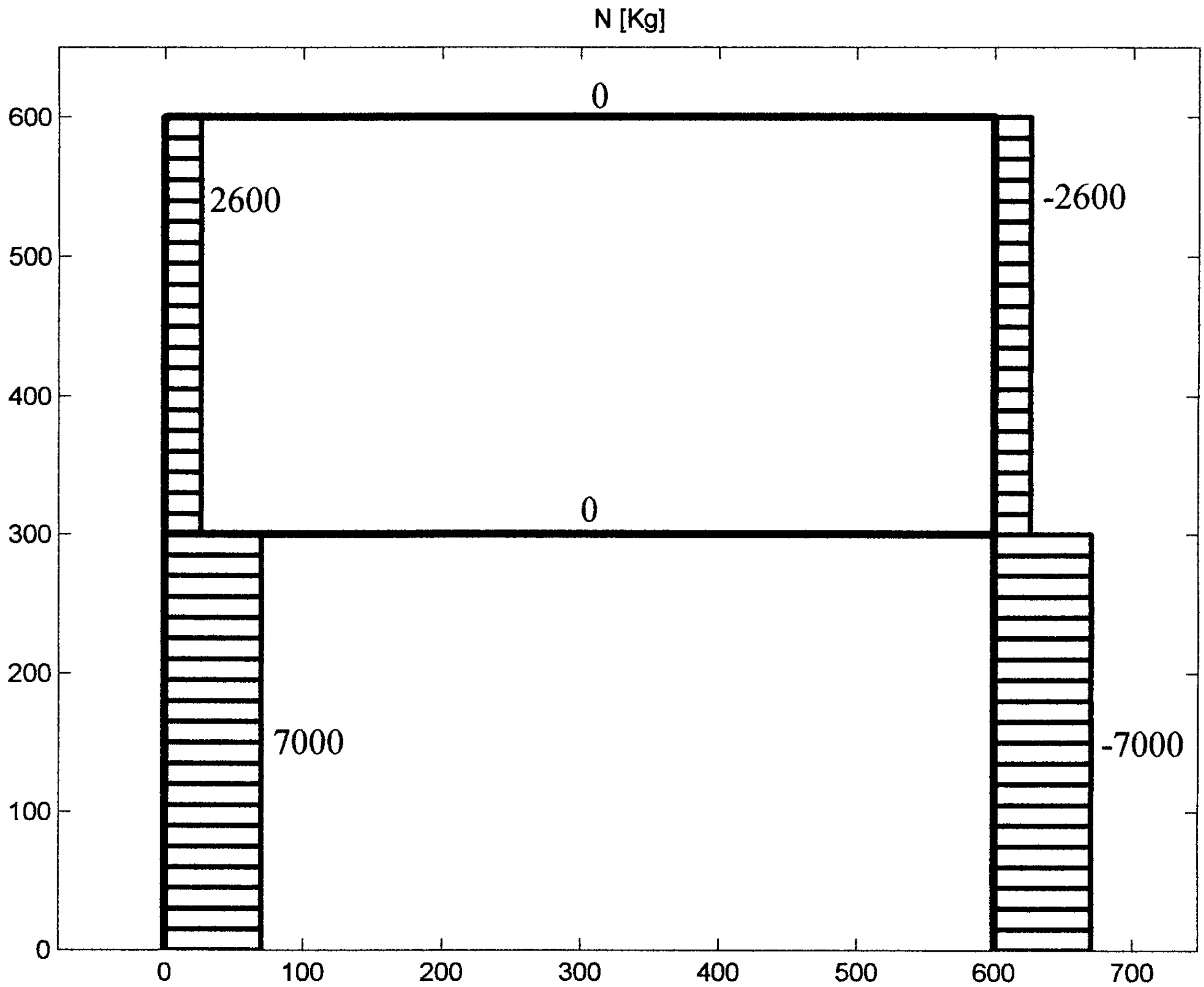
PROVA A   Pto A1



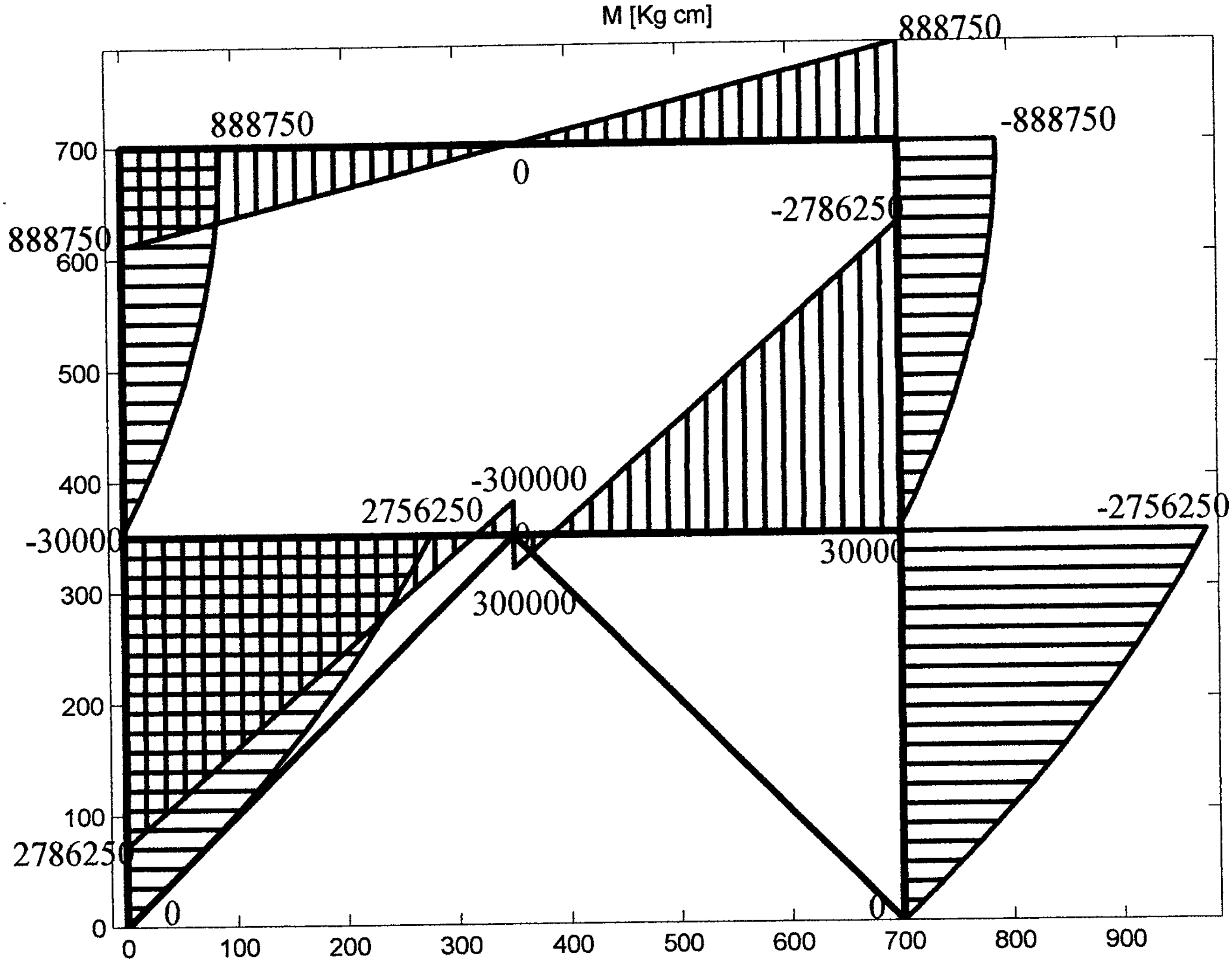
PROVA A   Pto A1



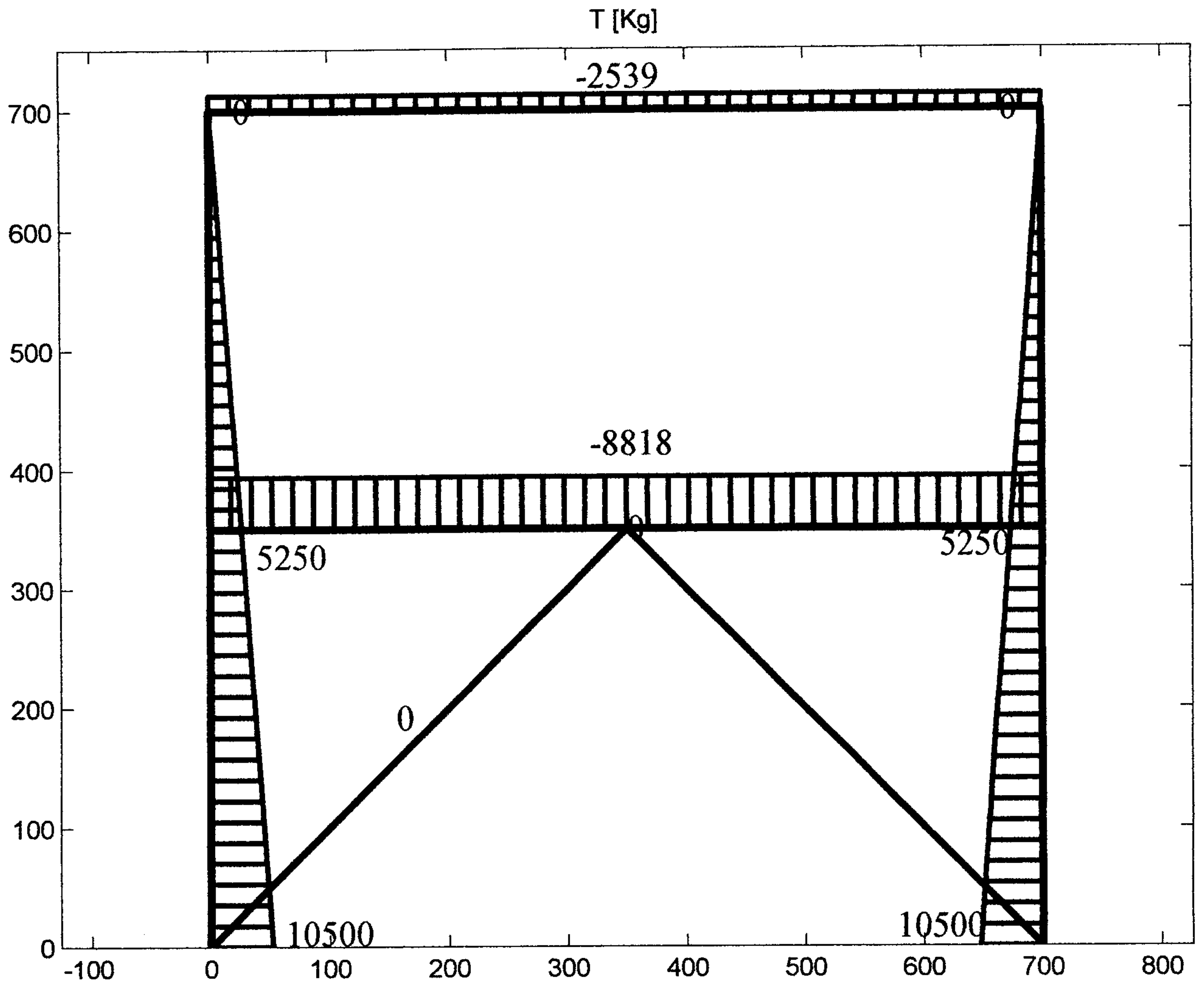
PROVA A   Pto A1



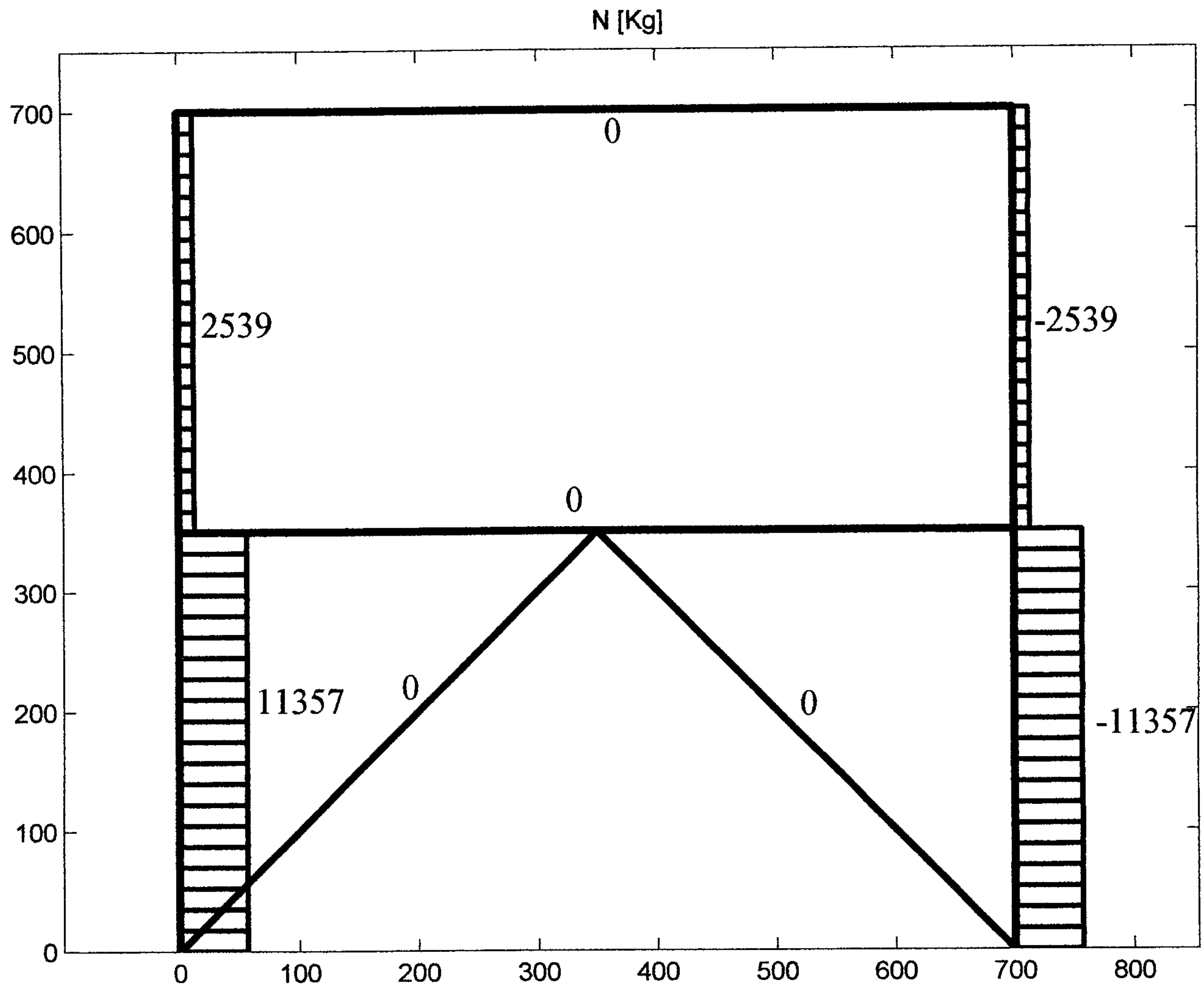
PROVA B Pto A1



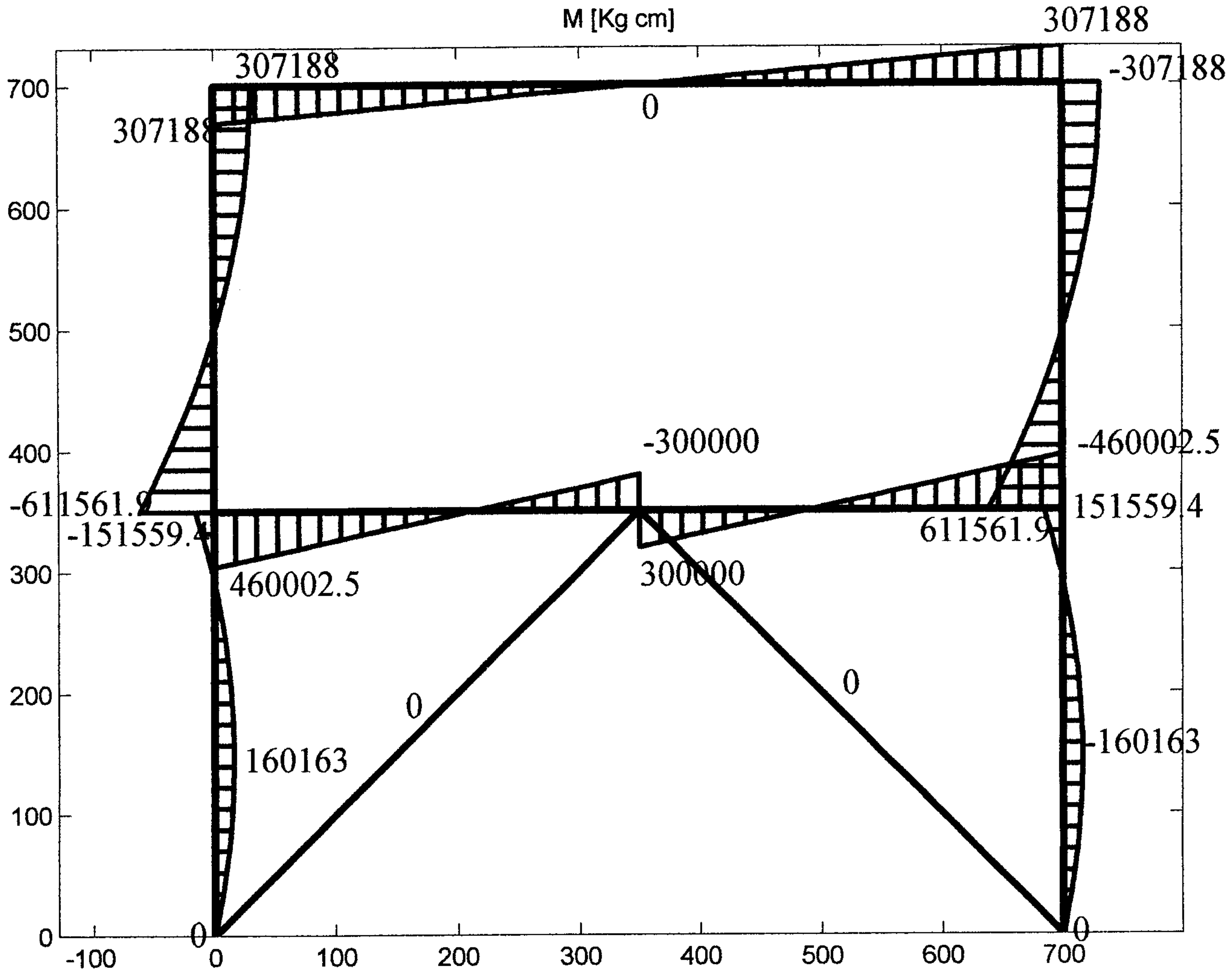
PROVA B Pto A1



PROVA B Pto A1

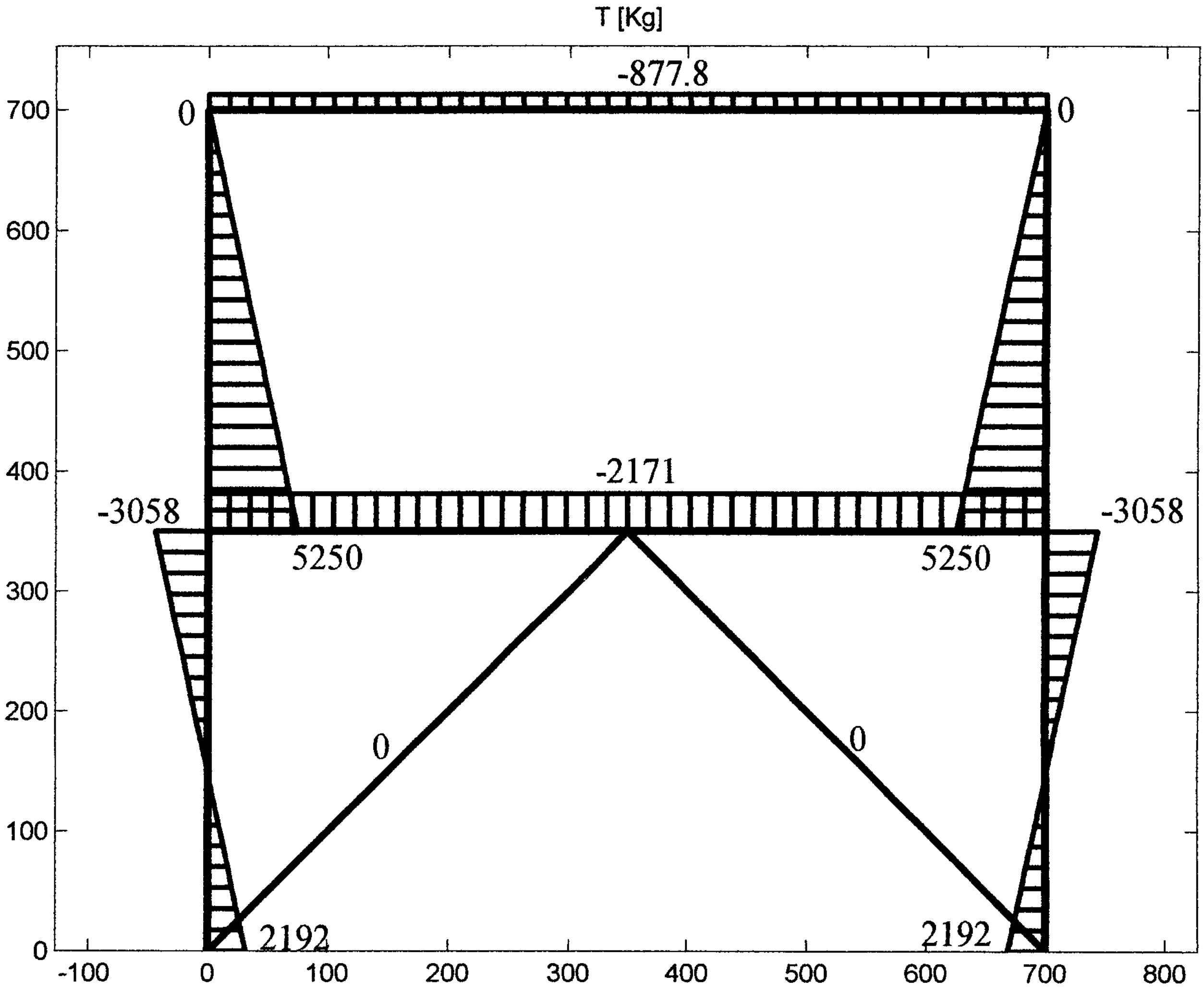


Prova B   Pto B

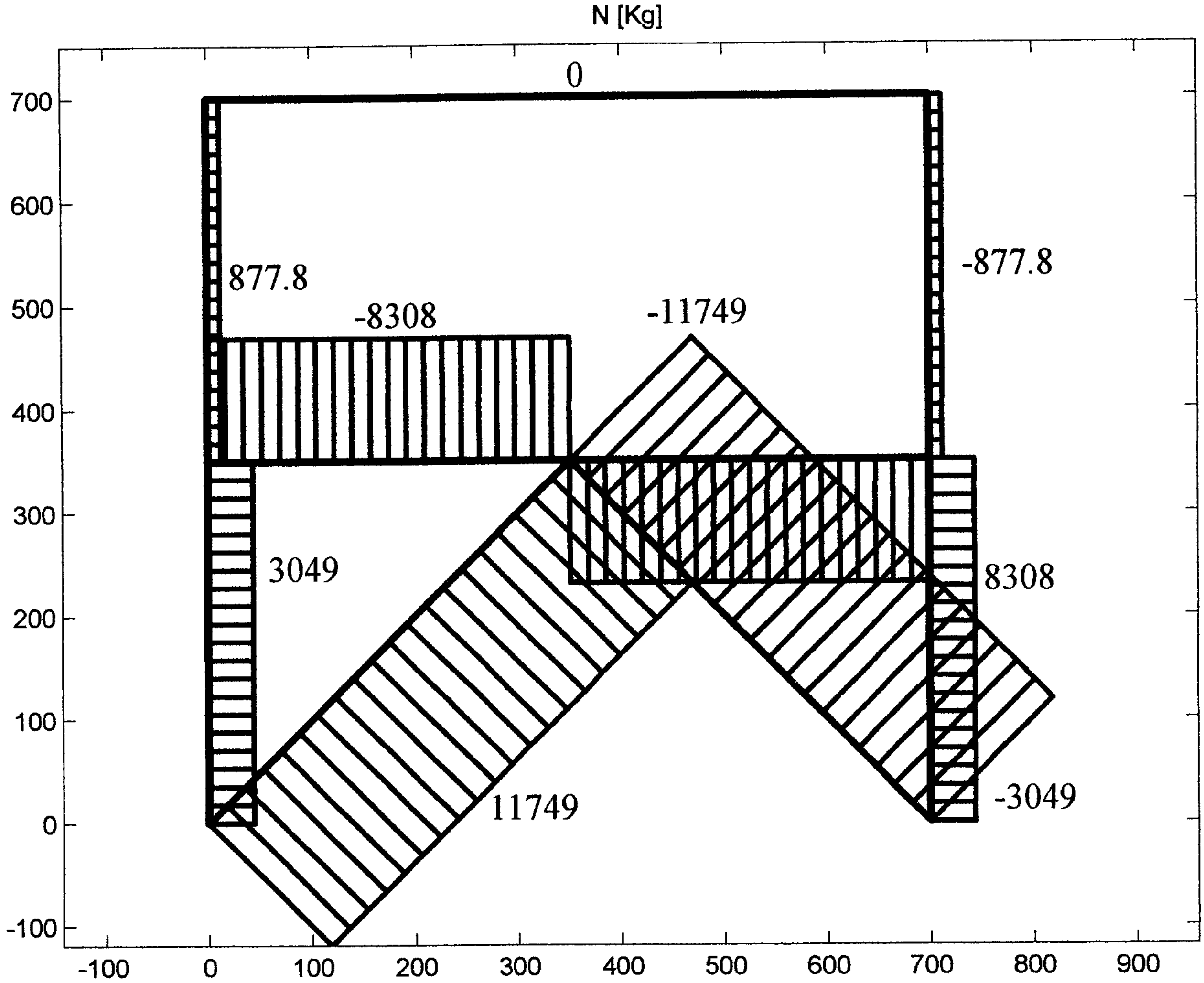


M [Kg cm]
307188
307188
-307188
307188
0
-611561.9
-151559.4
460002.5
-300000
300000
-460002.5
151559.4
611561.9
0
0
160163
-160163
0
0
700
600
500
400
300
200
100
0
-100
0
100
200
300
400
500
600
700

PROVA B Pto B


T [Kg]
-877.8
0
0
-2171
-3058
-3058
5250
5250
0
0
2192
2192
700
600
500
400
300
200
100
0
-100
0
100
200
300
400
500
600
700
800

PROVA B Pto B


N [Kg]
0
877.8
-877.8
-8308
-11749
3049
8308
11749
-3049
700
600
500
400
300
200
100
0
-100
-100
0
100
200
300
400
500
600
700
800
900