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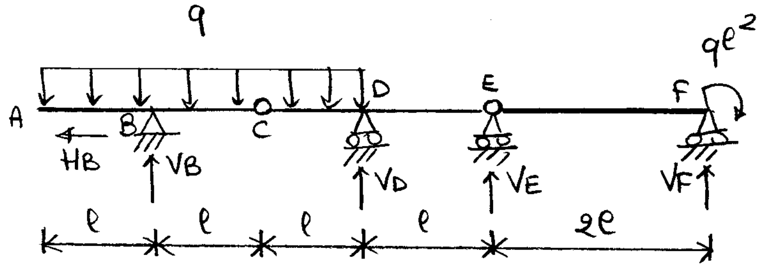
Risoluzione Es. 1



$H_B = 0$

$E \circlearrowleft : \; V_F \cdot 2l - ql^2 = 0$

$\rightarrow \boxed{V_F = q\frac{l}{2}}$

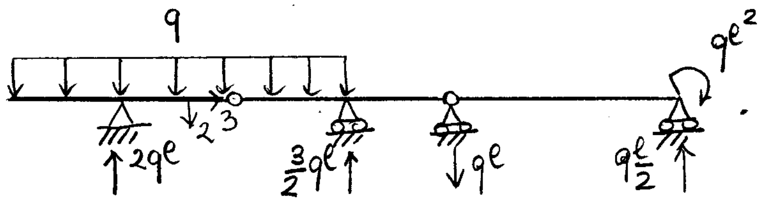
$C \circlearrowleft : \; 2ql^2 - V_B \cdot l = 0$

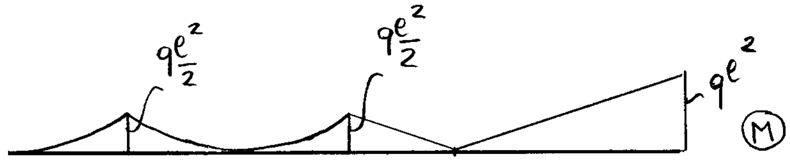
$\rightarrow \boxed{V_B = 2ql}$

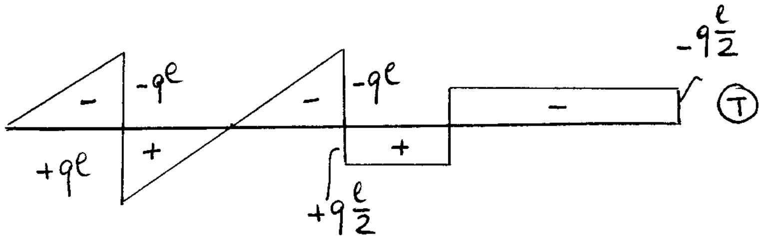
$D \circlearrowleft : \; V_E \cdot l + V_F \cdot 3l - ql^2 - V_B \cdot 2l + 3ql \cdot \frac{3}{2} l = 0$

$\rightarrow \boxed{V_E} = -\frac{3}{2} ql + ql + 4ql - \frac{9}{2} ql = \boxed{-ql}$

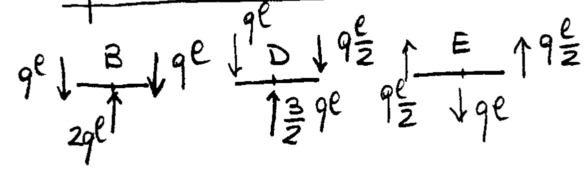
$\boxed{V_D} = -V_E - V_B - V_F + 3ql = +ql - 2ql - q\frac{l}{2} + 3ql = \boxed{\frac{3}{2} ql}$



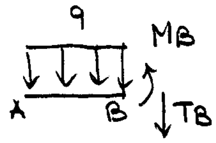




Disegno prima il diagramma di T utilizzando i seguenti equilibri ai nodi:

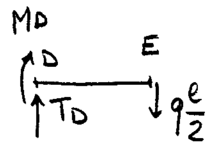


Infine determino M_B e M_D:



$B \circlearrowleft : \; M_B + q\frac{l^2}{2} = 0$

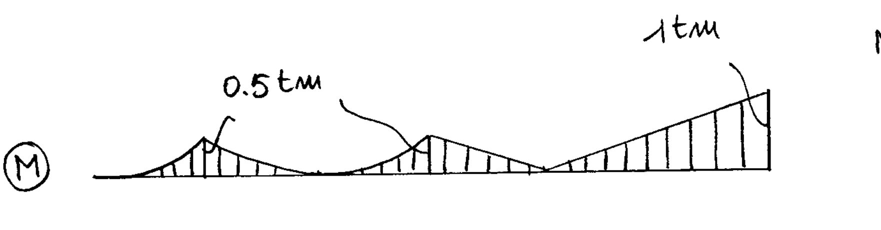
$\rightarrow M_B = -q\frac{l^2}{2}$

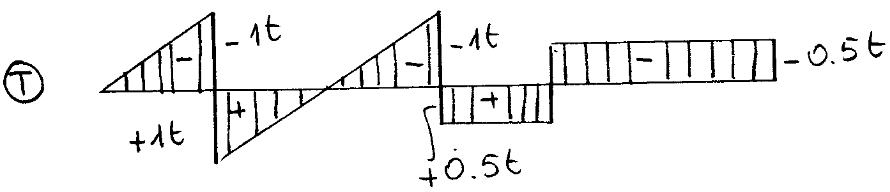


$D \circlearrowleft : \; M_D + q\frac{l}{2} \cdot l = 0$

$\rightarrow M_D = -q\frac{l^2}{2}$

Diagrammi quotati ($ql = 1t$; $ql^2 = 1tm$):





N = 0

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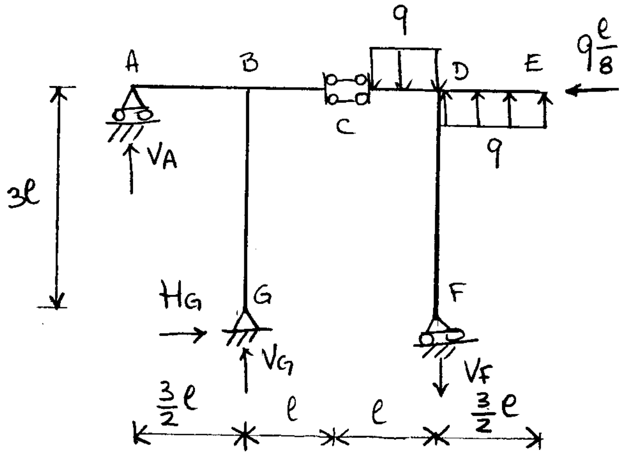
Risoluzione Es. 2



$$\boxed{H_G = q\frac{\ell}{8}}$$

$$V_F + q\ell - \frac{3}{2}q\ell = 0 \qquad (T_c = 0)$$

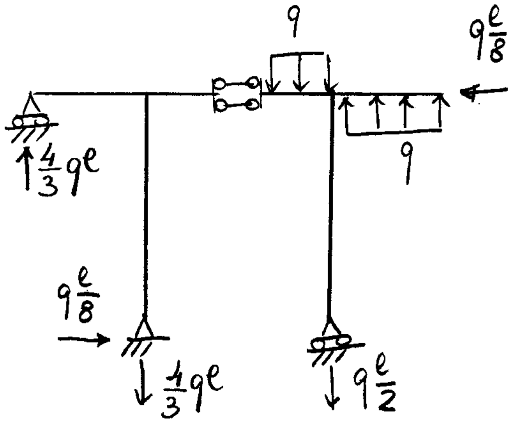
$$\hookrightarrow \boxed{V_F = q\frac{\ell}{2}}$$

$$G \circlearrowright : \; V_A \cdot \frac{3}{2}\cancel{\ell} + V_F \cdot 2\cancel{\ell} + q\ell\,\frac{3}{2}\cancel{\ell} - \frac{3}{2}q\cancel{\ell}\cdot\frac{11}{4}$$

$$- q\frac{\ell}{8}\cdot 3\cancel{\ell} = 0$$

$$\hookrightarrow \frac{3}{2}V_A = -q\ell - \frac{3}{2}q\ell + \frac{33}{8}q\ell + \frac{3}{8}q\ell = 2q\ell$$

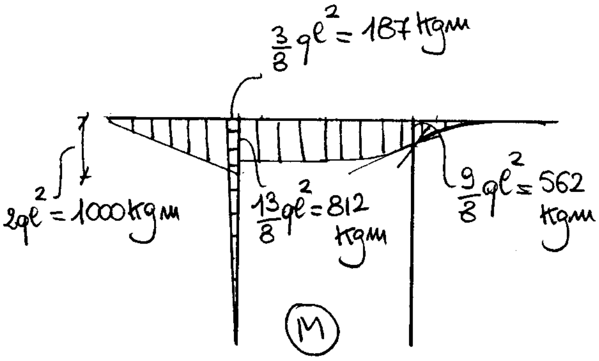
$$\hookrightarrow \boxed{V_A = \frac{4}{3}q\ell}$$

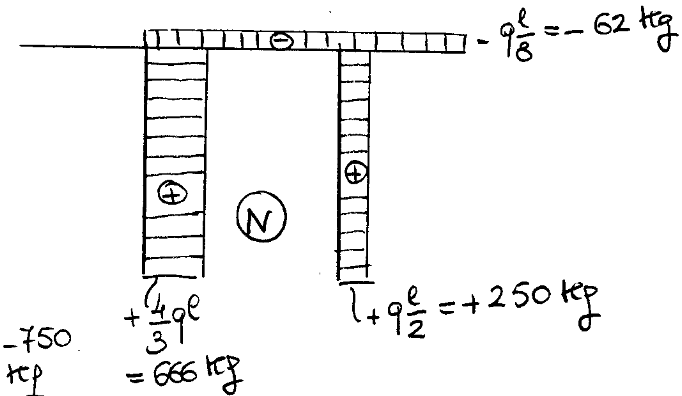


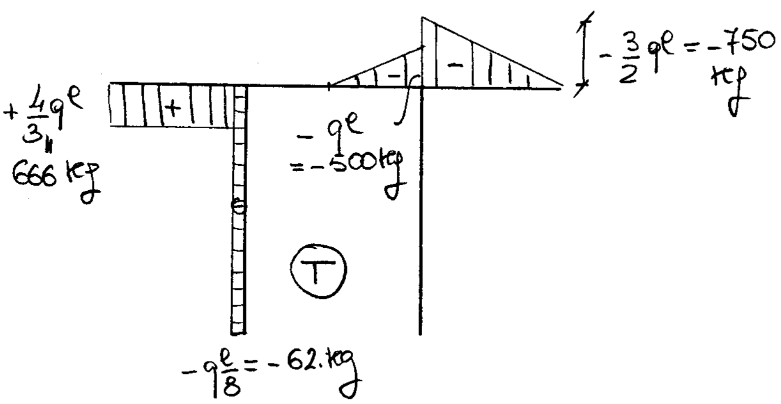
$$V_A + V_G = 0 \quad (T_c = 0)$$

$$\hookrightarrow \boxed{V_G = -\frac{4}{3}q\ell}$$

Diagrammi quotati :







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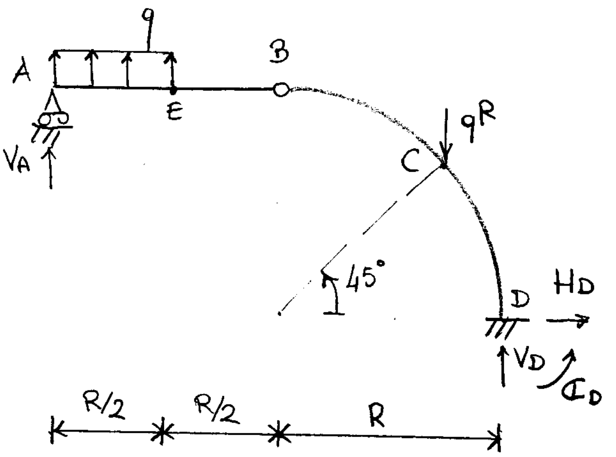
Risoluzione Es. 3



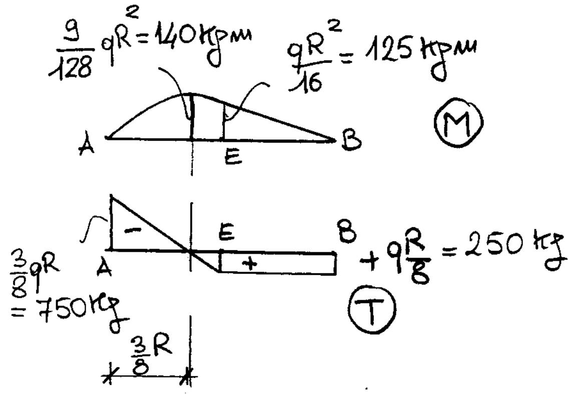
$H_D = 0$

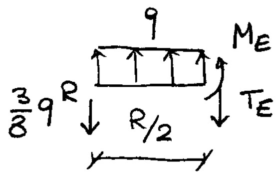
$\circlearrowleft B \quad V_A \cdot \cancel{R} + q\frac{R}{2} \cdot \frac{3}{4}\cancel{R} = 0 \rightarrow \boxed{V_A = -\frac{3}{8}qR}$

$\boxed{V_D} = -V_A - q\frac{R}{2} + qR = \boxed{\frac{7}{8}qR}$

$\circlearrowleft D \quad \mathbb{C}_D - V_A \cdot 2R - q\frac{R}{2}\frac{7}{4}R + qRR\left(1 - \frac{\sqrt{2}}{2}\right) = 0$

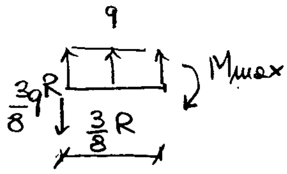
$\hookrightarrow \boxed{\mathbb{C}_D = -\left(\frac{7}{8} - \frac{\sqrt{2}}{2}\right)qR^2}$





$T_E = \left(\frac{1}{2} - \frac{3}{8}\right)qR = \frac{qR}{8}$

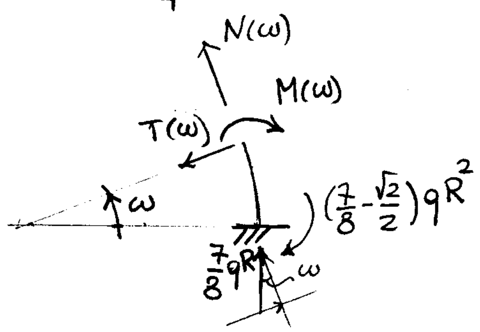
$M_E = q\frac{R}{2}\frac{R}{4} - \frac{3}{8}qR\frac{R}{2} = -\frac{qR^2}{16}$



$M_{max} = -\frac{3}{8}qR\frac{3}{16}R + \frac{3}{8}qR \cdot \frac{3}{8}R \overset{!}{=} \frac{9}{64}qR^2\left(1 - \frac{1}{2}\right) = \frac{9qR^2}{128}$

Azioni interne sull'arco

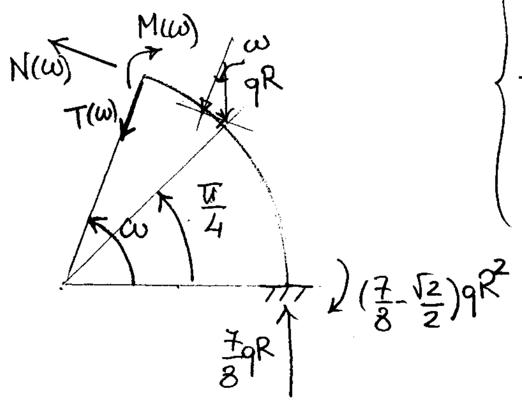
$0 \leq \omega \leq \frac{\pi}{4}$



$$\begin{cases} N(\omega) = -\frac{7}{8}qR\cos\omega \\ T(\omega) = \frac{7}{8}qR\,\text{sen}\,\omega \\ M(\omega) = \frac{7}{8}qR^2(\cancel{1} - \cos\omega) - \cancel{\frac{7}{8}qR^2} + \frac{\sqrt{2}}{2}qR^2 \end{cases}$$

$\overset{!}{=} \frac{\sqrt{2}}{2}qR^2 - \frac{7}{8}qR^2\cos\omega$

$\frac{\pi}{4} \leq \omega \leq \frac{\pi}{2}$



$$\begin{cases} N(\omega) = -\frac{7}{8}qR\cos\omega + qR\cos\omega = q\frac{R}{8}\cos\omega \\ T(\omega) = \frac{7}{8}qR\,\text{sen}\,\omega - qR\,\text{sen}\,\omega = -q\frac{R}{8}\,\text{sen}\,\omega \\ M(\omega) = \frac{7}{8}qR^2(\cancel{1} - \cos\omega) - \cancel{\frac{7}{8}qR^2} + \cancel{\frac{\sqrt{2}}{2}qR^2} - qR^2\left(\cancel{\frac{\sqrt{2}}{2}} - \cos\omega\right) \end{cases}$$

$\overset{!}{=} q\frac{R^2}{8}\cos\omega$

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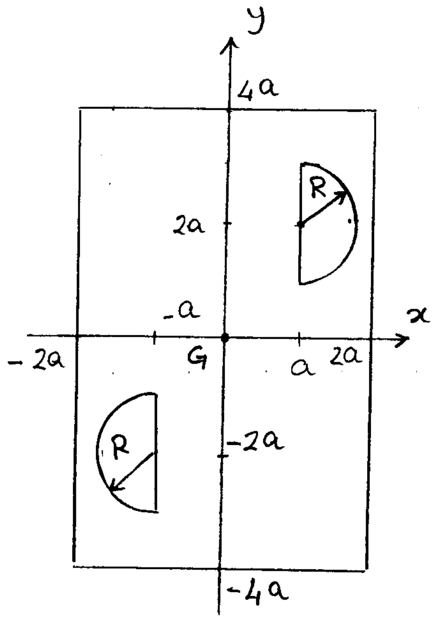
Risoluzione Es. 4



$R = \frac{3}{4}a$, $a = 5$ cm.

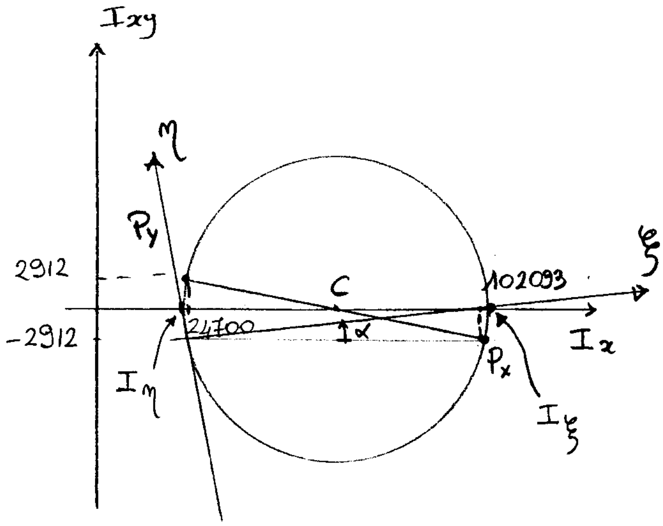
Per ragioni di simmetria, il baricentro G della figura cade nel centro del rettangolo.

$$A = 4a \cdot 8a - \pi R^2 = 32a^2 - \pi \frac{9}{16}a^2 = 30{,}23\,a^2 = 756 \text{ cm}^2$$

$$I_x = \frac{1}{12} 4a(8a)^3 - 2\left[\frac{\pi R^4}{8} + \frac{\pi R^2}{2} 4a^2\right] = 163{,}35\,a^4 = 102093 \text{ cm}^4$$

$$I_y = \frac{1}{12} 8a(4a)^3 - 2\left[\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4 + \frac{\pi R^2}{2}\left(a + \frac{4R}{3\pi}\right)^2\right] = 39{,}52\,a^4 = 24700 \text{ cm}^4$$

$$I_{xy} = -2\pi\frac{R^2}{2}\left(\frac{4R}{3\pi} + a\right)2a = -4{,}65\,a^4 = -2912 \text{ cm}^4$$



$$\alpha = \frac{1}{2}\,\text{arctg}\left(-\frac{2I_{xy}}{I_x - I_y}\right) = 0{,}037 = 2{,}14^\circ$$

$$\left.\begin{matrix} I_\xi \\ I_\eta \end{matrix}\right\} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \begin{cases} 102202 \text{ cm}^4 \\ 24590 \text{ cm}^4 \end{cases}$$

Risoluzione Es. 5

$$\begin{cases} u_1 = \frac{\alpha}{3}x_1 - \beta x_2 x_3 \\ u_2 = \beta x_1 x_3 - \frac{\alpha}{3}x_2 \\ u_3 = \alpha x_3 \end{cases}$$

$$\nabla \underset{\sim}{u} = \begin{pmatrix} \frac{\alpha}{3} & -\beta x_3 & -\beta x_2 \\ \beta x_3 & -\frac{\alpha}{3} & \beta x_1 \\ 0 & 0 & \alpha \end{pmatrix}, \quad \underset{\sim}{E} = \begin{pmatrix} \frac{\alpha}{3} & 0 & -\frac{\beta}{2}x_2 \\ 0 & -\frac{\alpha}{3} & \beta\frac{x_1}{2} \\ -\beta\frac{x_2}{2} & \beta\frac{x_1}{2} & \alpha \end{pmatrix}$$

$$\underset{\sim}{W} = \nabla\underset{\sim}{u} - \underset{\sim}{E} = \begin{pmatrix} 0 & -\beta x_3 & -\frac{\beta}{2}x_2 \\ \beta x_3 & 0 & \beta\frac{x_1}{2} \\ +\frac{\beta}{2}x_2 & -\beta\frac{x_1}{2} & 0 \end{pmatrix}$$

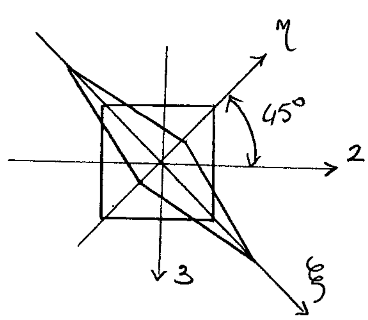
In $P = (+R, 0, 0)$ per $\alpha = 1$, $\beta = \frac{1}{10R}$, si ha: $\underset{\sim}{E} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{1}{20} \\ 0 & \frac{1}{20} & 1 \end{pmatrix}$.

$$\boxed{\varepsilon_{mm}} = \underset{\sim}{E}\,\underset{\sim}{m}\cdot\underset{\sim}{m} = \frac{1}{3}(1,1,1)\cdot\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{1}{20} \\ 0 & \frac{1}{20} & 1 \end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = \frac{1}{3}(1,1,1)\cdot\begin{pmatrix} 1/3 \\ -17/60 \\ 21/20 \end{pmatrix}$$

$$= \frac{1}{3}\left[\frac{1}{3} - \frac{17}{60} + \frac{21}{20}\right] = 0,36 = \boxed{36\%}$$

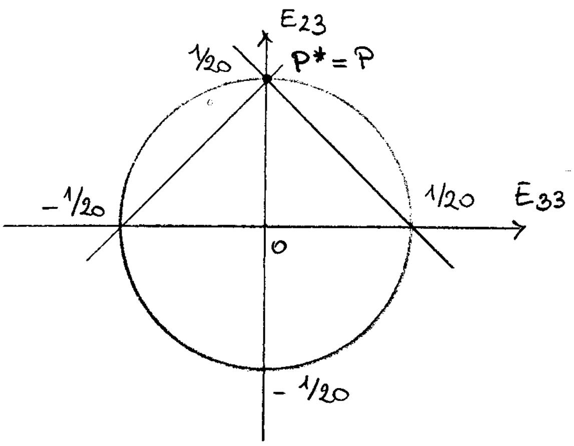
Dilatazioni principali per $\alpha = 0$, $\beta = \frac{1}{10R}$:

$$\underset{\sim}{E} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1/20 \\ 0 & 1/20 & 0 \end{pmatrix}$$



$\varepsilon_\xi = 1/20$

$\varepsilon_\eta = -1/20$



Le direzioni delle dilatazioni principali sono inclinate di $\pm 45°$ rispetto agli assi 2 e 3.