

$$H_A = 0$$

$$B \uparrow - V_A l - 2ql^2 = 0 \rightarrow \boxed{V_A = -2ql}$$

$$D \uparrow V_E \cdot \frac{l}{2} - q \cdot \frac{l}{2} \cdot \frac{l}{4} = 0 \rightarrow \boxed{V_E = \frac{ql}{4}}$$

$$B \uparrow V_C \cdot l + V_E \cdot 2l - q \cdot \frac{3}{2}l = 0$$

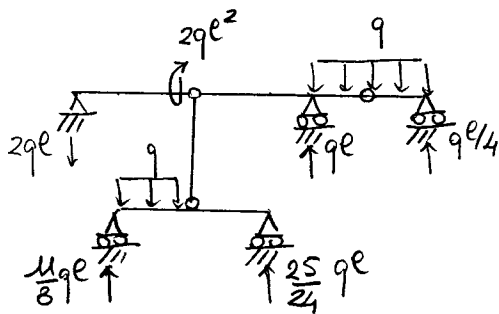
$$\rightarrow \boxed{V_C = \frac{3}{2}ql - 2V_E = ql}$$

$$B \uparrow - V_F \cdot \frac{2}{3}l + \frac{2}{3}q \cdot \frac{l}{3} + V_H \cdot \frac{2}{3}l = 0$$

$$V_F + V_H = \frac{2}{3}ql + ql + 2ql - ql - \frac{ql}{4}$$

$$= ql \frac{8+24-3}{12} = \frac{29}{12}ql$$

$$\rightarrow \begin{cases} -V_F + V_H = -\frac{ql}{3} \\ V_F + V_H = \frac{29}{12}ql \end{cases} \rightarrow \begin{cases} \boxed{V_F = \frac{11}{8}ql} \\ \boxed{V_H = \frac{25}{24}ql} \end{cases}$$



$$V_A = -2ql = -1000 \text{ kg}$$

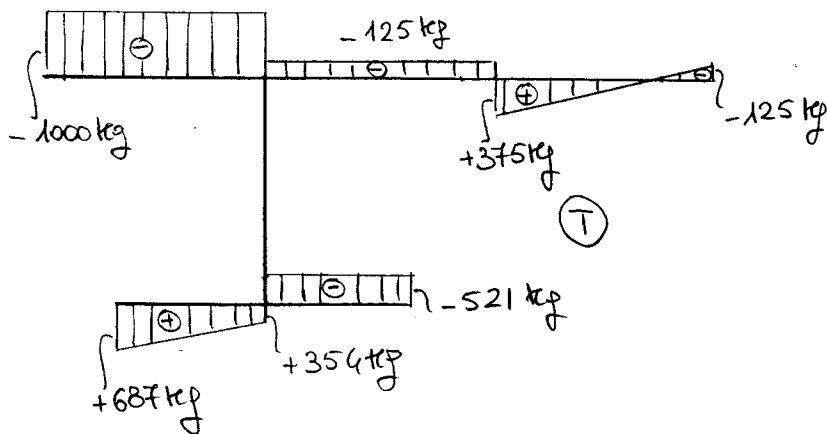
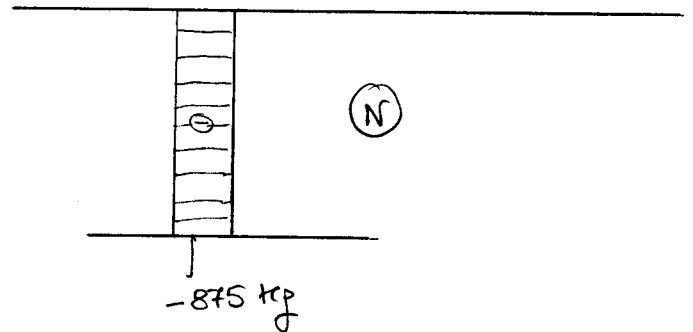
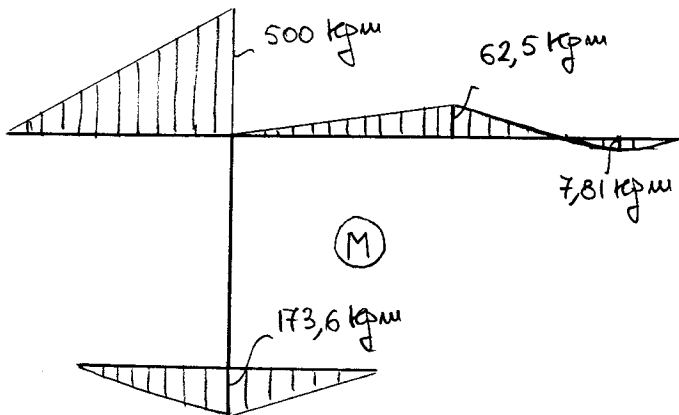
$$V_E = \frac{ql}{4} = 125 \text{ kg}$$

$$V_C = ql = 500 \text{ kg}$$

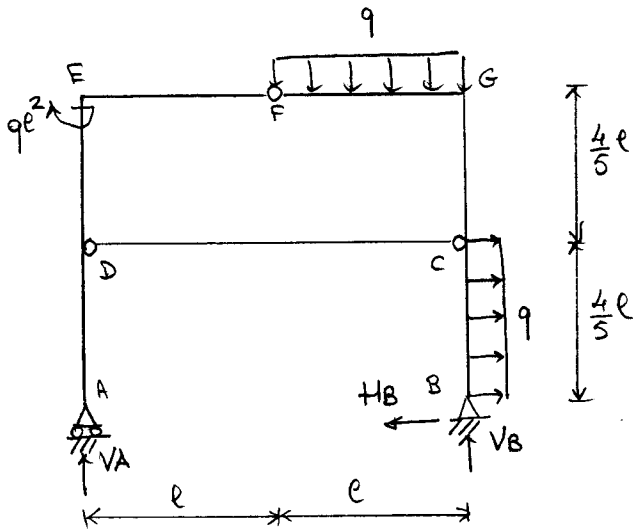
$$V_F = \frac{11}{8}ql = 687 \text{ kg}$$

$$V_H = \frac{25}{24}ql = 521 \text{ kg}$$

Diagrammi quotati:



## Risoluzione Es. 2

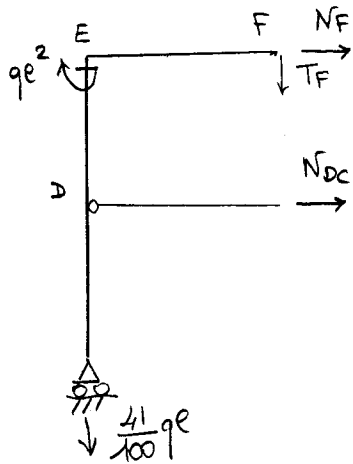


$$H_B = \frac{4}{5} q l = 4000 \text{ kg}$$

$$\sum \curvearrowright V_B \cdot 2l - \frac{4}{5} q l \frac{2l}{2} - q l \frac{3l}{2} - q l^2 = 0$$

$$\rightarrow V_B = q l \left( \frac{4}{25} + \frac{3}{4} + \frac{1}{2} \right) = \frac{141}{100} q l = 7050 \text{ kg}$$

$$V_A = q l - V_B = -\frac{41}{100} q l = -2050 \text{ kg}$$



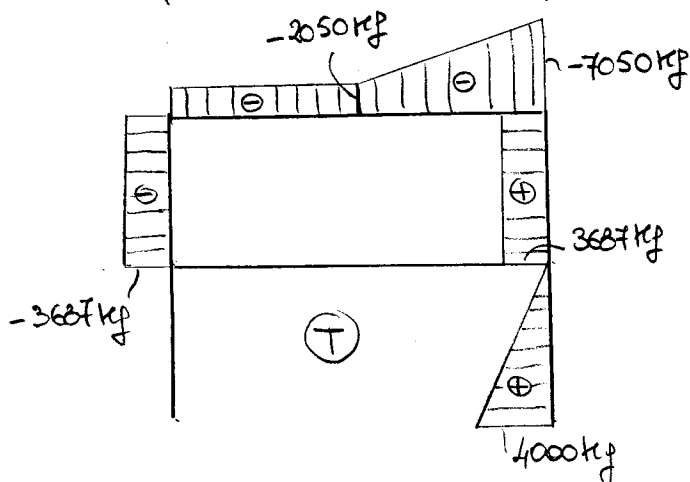
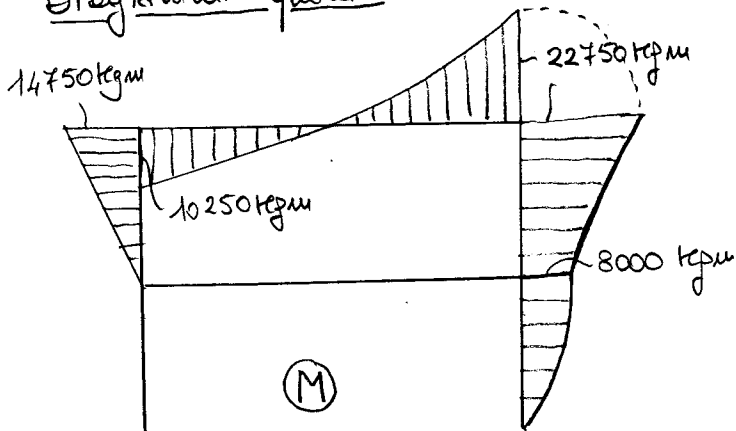
$$T_F = -\frac{41}{100} q l = -2050 \text{ kg}$$

$$\sum \curvearrowright N_{Dc} \cdot \frac{4l}{5} - q l^2 + \frac{41}{100} q l^2 = 0$$

$$\rightarrow \frac{4}{5} N_{Dc} = q l - \frac{41}{100} q l = \frac{59}{100} q l \rightarrow N_{Dc} = \frac{59}{80} q l = 3687 \text{ kg}$$

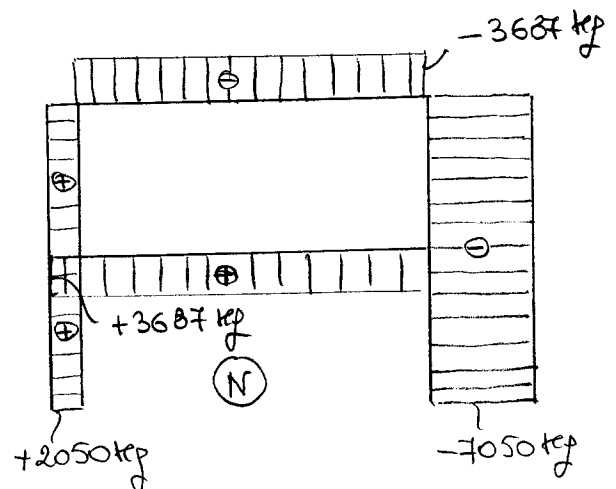
$$N_F = -N_{Dc} = -\frac{59}{80} q l = -3687 \text{ kg}$$

Diagrammi quotati:



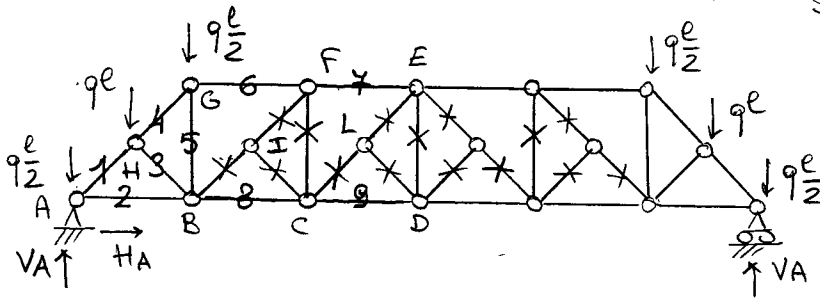
Calcoli

$$q l^2 \curvearrowright \frac{41}{100} q l^2 \quad M_{ED} = \left(1 - \frac{41}{100}\right) q l^2 = \frac{59}{100} q l^2$$



## Risoluzione Es. 3

Calcolo dello stato "primario" di sollecitazione:



$$V_A = 2q\ell \quad (\text{per simmetria})$$

- Osservazioni:
- le aste IC, LD (e le corrispondenti simmetriche) sono scarse per equilibrio dei nodi I e L (e dei nodi simmetrici)
  - di conseguenza, l'asta ED è scarsa, per equilibrio del nodo D.
  - utilizzando l'equilibrio del nodo E e la simmetria, si trova che anche l'asta EL (e la simmetrica) è scarsa (vedi figura a sinistra).
- $$\sum N_{EL} \frac{\sqrt{2}}{2} = 0$$
- per l'equilibrio di L (del nodo simmetrico), anche CL (e la simmetrica) è scarsa.
  - infine, l'equilibrio del nodo C impone che l'asta FC (e la sua simmetrica) sia scarsa. Analogamente, si ha che le aste IF, BI e le simmetriche sono scarse.

→ Sono cariche solo le aste indicate nelle figure. Per esse, si hanno i seguenti equilibri ai nodi:

A)  $\begin{cases} N_1 \frac{\sqrt{2}}{2} = -\frac{3}{2}q\ell \\ N_2 = -N_1 \frac{\sqrt{2}}{2} = +\frac{3}{2}q\ell \end{cases}$

H)  $\begin{cases} N_4 = q\ell \frac{\sqrt{2}}{2} - \frac{3}{2}\sqrt{2}q\ell \\ = -\sqrt{2}q\ell \\ N_3 = -q\ell \frac{\sqrt{2}}{2} \end{cases}$

G)  $\begin{cases} N_6 = -q\ell \\ N_5 = q\ell - q\ell \frac{\sqrt{2}}{2} \\ = \frac{1}{2}q\ell \end{cases}$

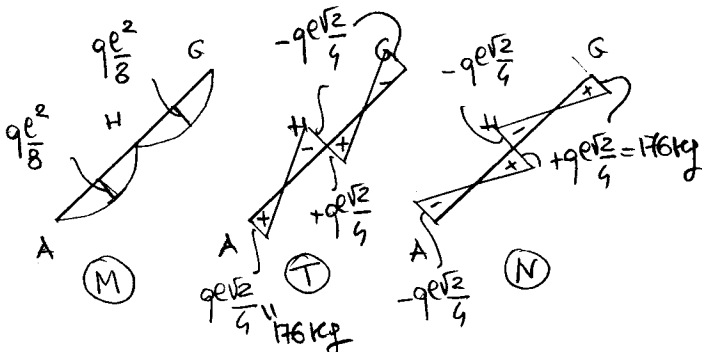
B)  $\begin{cases} N_3 = \frac{3}{2}q\ell - q\ell \\ = \frac{1}{2}q\ell \\ q\ell \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}q\ell \frac{\sqrt{2}}{2} \quad (*) \end{cases}$

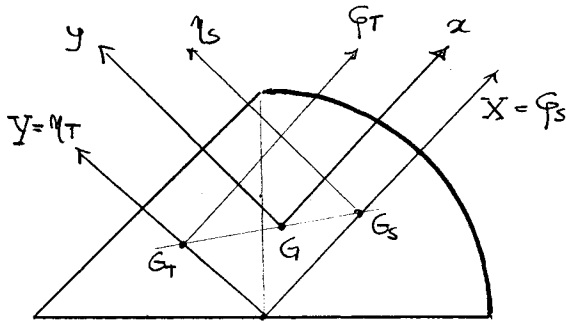
C)  $\begin{cases} N_8 = N_3 \\ N_9 = N_8 \end{cases}$  F)  $\begin{cases} N_7 = N_6 \\ N_6 \end{cases}$

Tabella dello stato primario

1	$-\frac{3\sqrt{2}}{2}q\ell$	-1060 kg	5	$q\ell/2$	250 kg
2	$+\frac{3}{2}q\ell$	750 kg	6	$-q\ell$	-500 kg
3	$-q\ell \frac{\sqrt{2}}{2}$	-353 kg	7	$-q\ell$	-500 kg
4	$-\sqrt{2}q\ell$	-707 kg	8	$+q\ell$	500 kg
			9	$+q\ell$	500 kg

Stato secondario di sollecitazione:



Risoluzione Es. 4

Dalle tabelle si ha:

$$X_{G_S} = \frac{4R}{3\pi} \sqrt{2}, \quad Y_{G_S} = 0$$

$$X_{G_T} = 0, \quad Y_{G_T} = \frac{R\sqrt{2}}{3}$$

$$I_{\xi_T} = \frac{R^4}{12}, \quad I_{\eta_T} = \frac{R^4}{24}$$

$$I_{\xi_S} = \frac{R^4}{4} \left( \frac{\pi}{4} - \frac{1}{2} \right), \quad I_{\eta_S} = \frac{R^4}{4} \left( \frac{\pi}{4} + \frac{1}{2} - \frac{32}{9\pi} \right)$$

Inoltre:  $A_T = \frac{R^2}{2}, \quad A_S = \frac{\pi R^2}{4}, \quad A = A_T + A_S$

Calcolo del baricentro:

$$X_G = \frac{A_T X_{G_T} + A_S X_{G_S}}{A} = \frac{4\sqrt{2}}{3\pi(2+\pi)} R = 1,16 \text{ cm}$$

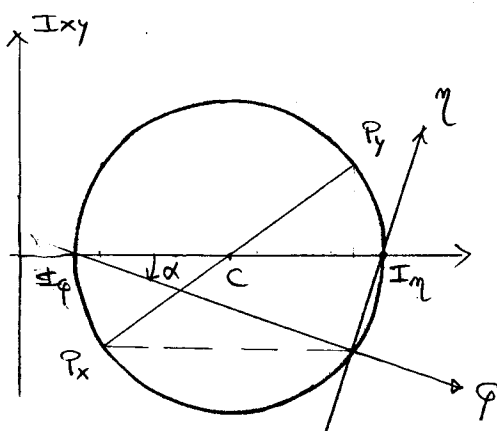
$$Y_G = \frac{A_T Y_{G_T} + A_S Y_{G_S}}{A} = \frac{2\sqrt{2}}{3(2+\pi)} R = 1,83 \text{ cm}$$

Momenti di inerzia nel riferimento centrale ( $G; x, y$ ):

$$I_x = I_{\xi_T} + \frac{R^2}{2} \left( \frac{R\sqrt{2}}{3} - Y_G \right)^2 + I_{\xi_S} + \frac{\pi R^2}{4} (Y_G)^2 = 1531 \text{ cm}^4$$

$$I_y = I_{\eta_T} + \frac{R^2}{2} X_G^2 + I_{\eta_S} + \frac{\pi R^2}{4} \left( \frac{4R\sqrt{2}}{3\pi} - X_G \right)^2 = 2709 \text{ cm}^4$$

$$I_{xy} = \frac{R^2}{2} \left( \frac{R\sqrt{2}}{3} - Y_G \right) (-X_G) + \frac{\pi R^2}{4} \left( \frac{4R\sqrt{2}}{3\pi} - X_G \right) (-Y_G) = -863 \text{ cm}^4$$

Cerchio di Mohr

$$\left. \begin{matrix} I_{\xi} \\ I_{\eta} \end{matrix} \right\} = \frac{(I_x + I_y)}{2} \pm \sqrt{\left( \frac{I_x - I_y}{2} \right)^2 + I_{xy}^2} = \begin{cases} 1075 \text{ cm}^4 \\ 3164 \text{ cm}^2 \end{cases}$$

$$\alpha = \frac{1}{2} \arctan \left( \frac{-2I_{xy}}{I_x - I_y} \right) = -0,486 \approx 27,8^\circ$$

Per via analitica

Eq. ne secondo:  $\lambda^2 - (1531 + 2709)\lambda + (1531 \cdot 2709 - 863^2) = 0 \rightarrow \lambda_{\xi} = 1075, \quad \lambda_{\eta} = 3164$

Autovettore di  $\lambda_{\xi}$ :

$$(1531 - 1075)u_1 + 863u_2 = 0$$

$$\rightarrow u_1 = -\frac{863}{456}u_2 = -1,89u_2$$

$$\rightarrow \underline{u}_{\xi} = \frac{1}{\sqrt{1+1,89^2}} (1; -1,89)$$

$$\underline{u}_{\eta} = \frac{1}{\sqrt{1+1,89^2}} (+1,89; 1) \quad (\text{direzione } \perp \text{ a } \underline{u}_{\xi})$$