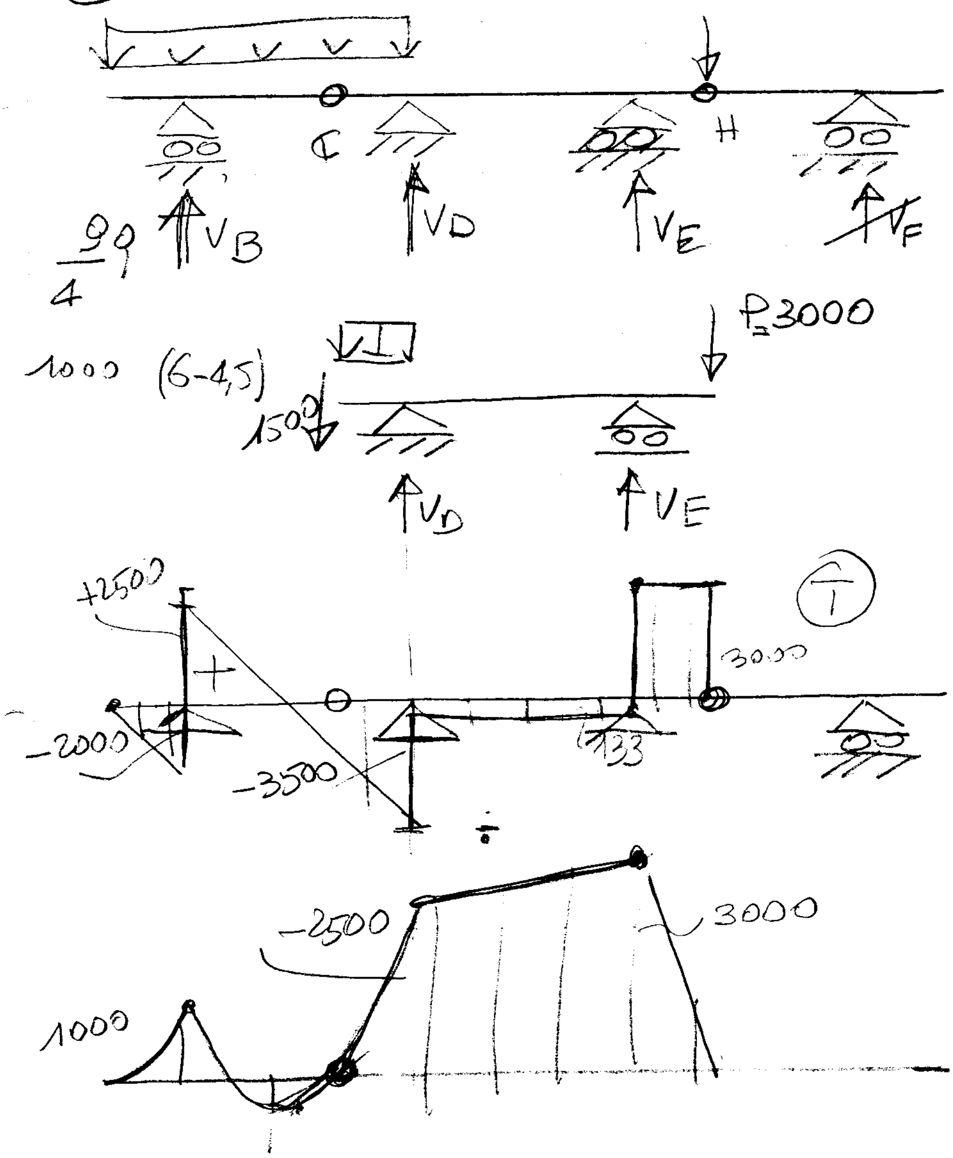
① A



→ $H_D = 0$

H↻ $V_F = 0$

C↺ $-V_B(\ell - a) + \frac{q\ell^2}{2} = 0$

⇓

$V_B = \frac{9}{4} q = 4500$

↑ $V_D + V_E = 6500$

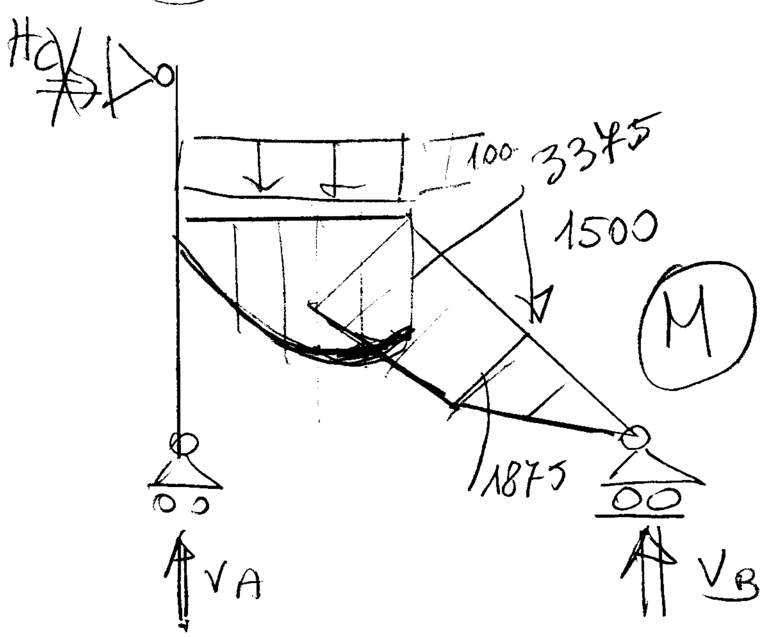
D↺ $-P(\ell + d) + V_E \ell +$
$1500 a + \frac{q a^2}{2} = 0$

$-12000 + 3V_E + 1500 + 1000 =$

$V_E = 9500/3$

$M_D = -q \cdot \frac{(\ell + a)^2}{}$
$+ \frac{9}{4} q \cdot 1.5^2 =$
$= -16000 + 13500 =$
-2500

(A2)



$$V_A + V_B = 4500$$

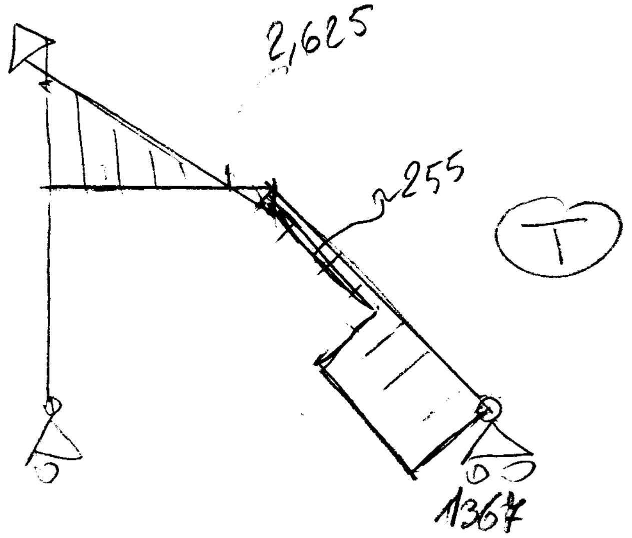
$$A) \; V_B \cdot 6 = P\left(\frac{3}{2}l\right) + \frac{q l^2}{2} =$$

$$V_B 6 = \cancel{4}\frac{3}{4} q l^2 + \frac{2 q l^2}{4} =$$

$$= \frac{5}{4} q l^2 = 11250$$

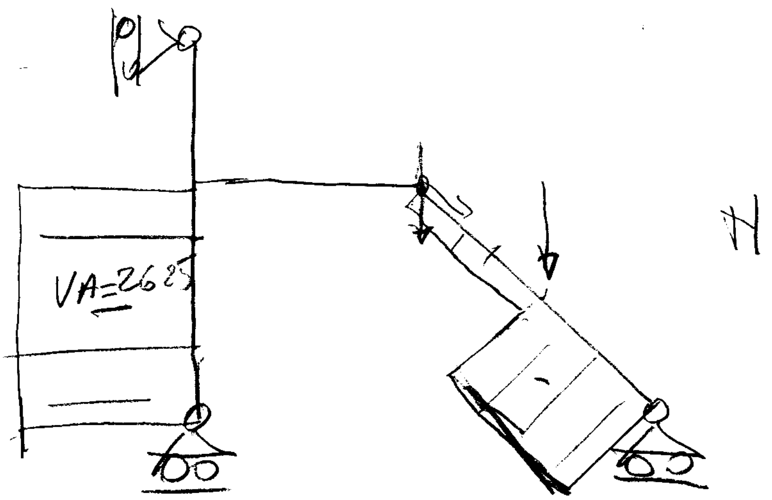
$$V_B = 1875$$

$$V_A = 2625$$

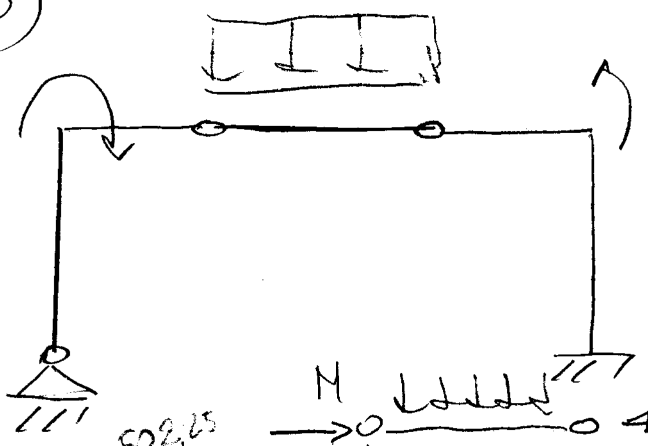


$$M_E = 7875 - 4500 = 3375$$

$$M_F = 2812.5$$



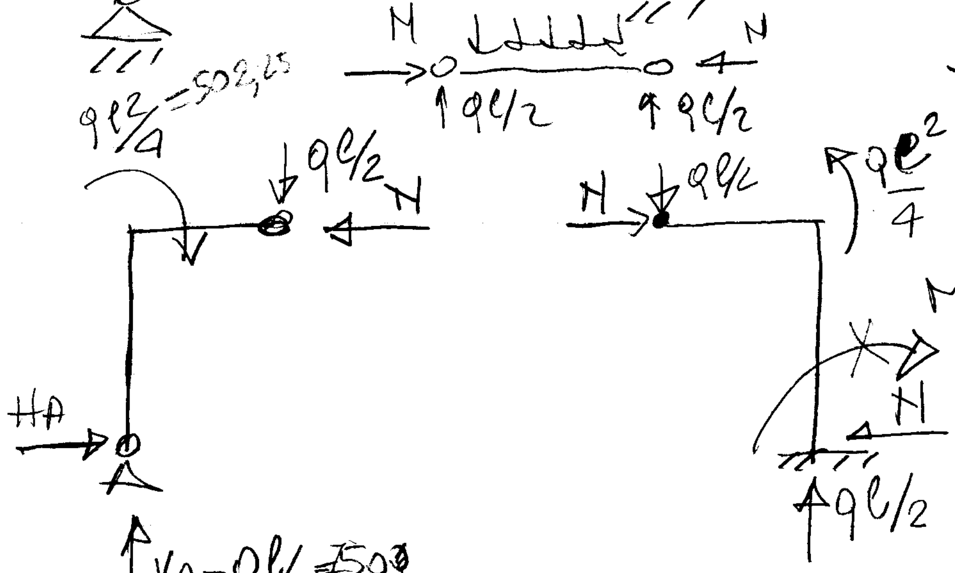
(13)



$\rightarrow N = H_A$

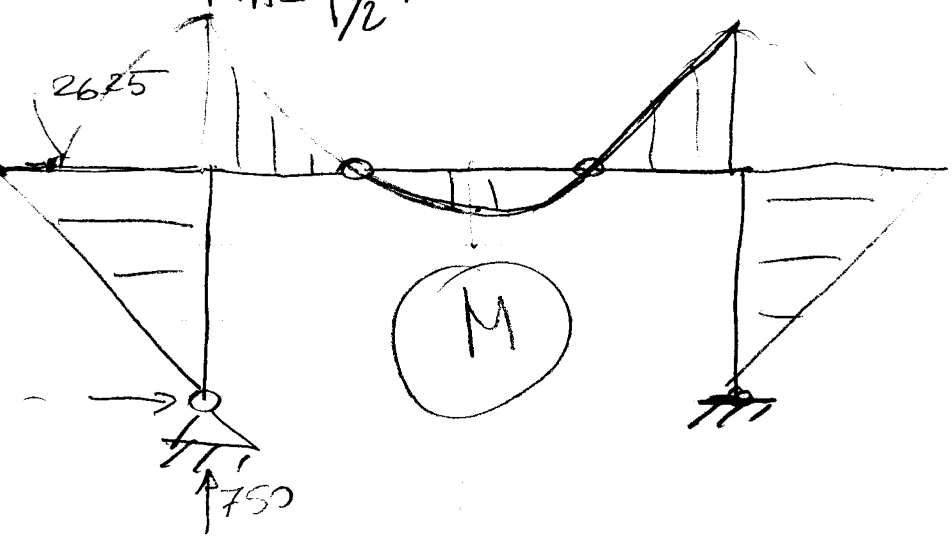
A) $N\ell = \dfrac{d q \ell}{2} + \dfrac{q\ell^2}{4} = 875$

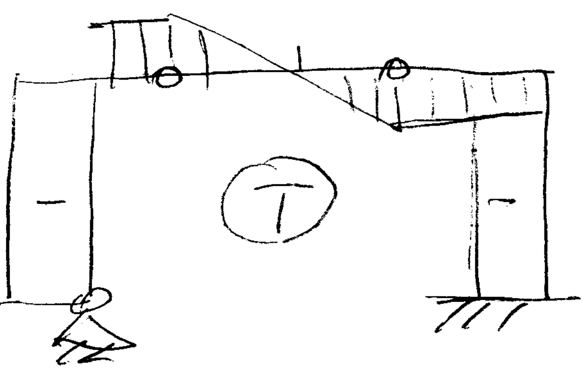
B) $-M_B - N\ell + q\dfrac{\ell}{2} d + \dfrac{q\ell^2}{4} = 0$

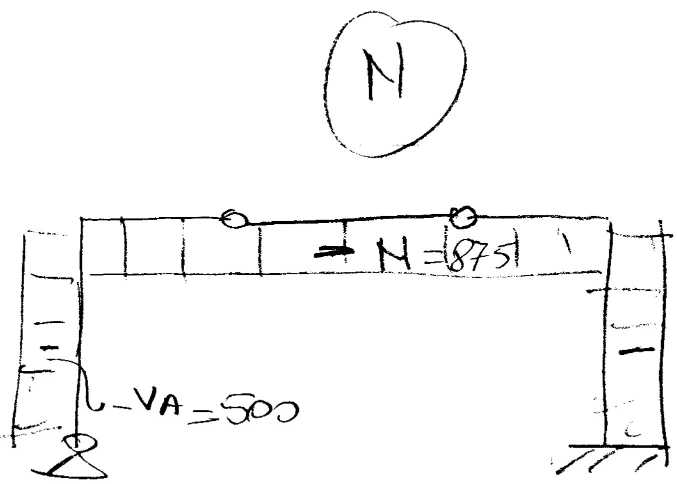


$\Rightarrow M_B = 0$

$H_A = \dfrac{1500 + \frac{0}{8}\,1000}{3}$

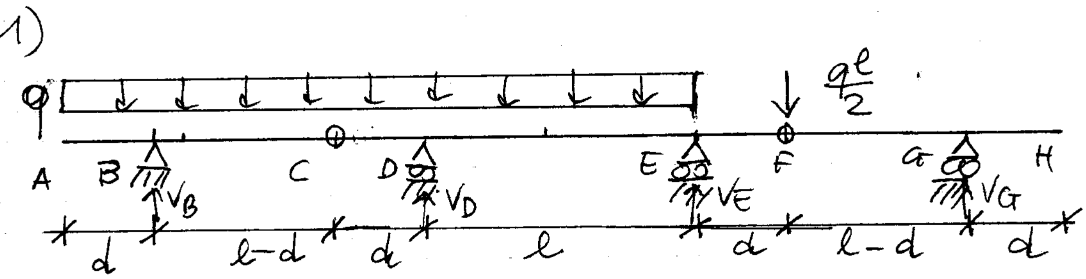






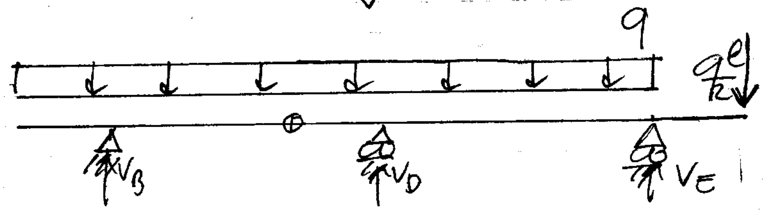
PROVA (B) 4/12/02 I PROVA PARZIALE

1)



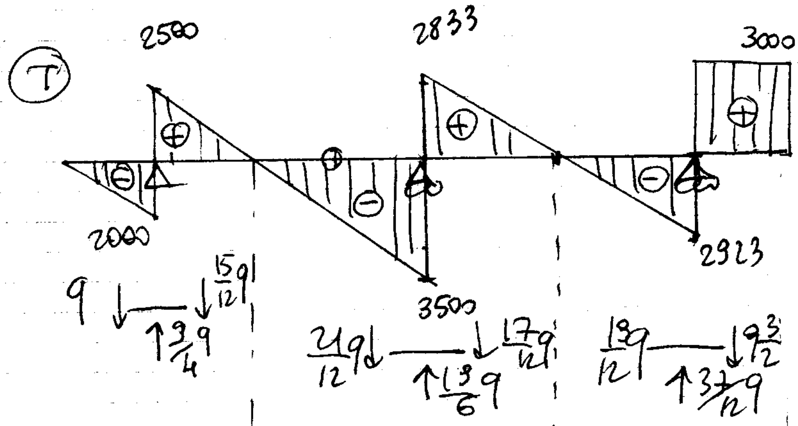
$d = 1$ m
$l = 3$ m
$q = 2000$ kg/m

$F\circlearrowleft \Rightarrow V_G(l-d) = 0$
tratto FH scarico



$\uparrow)\quad V_B + V_D + V_E = q(2l+d) + q\frac{l}{2} = \frac{17}{2}q$

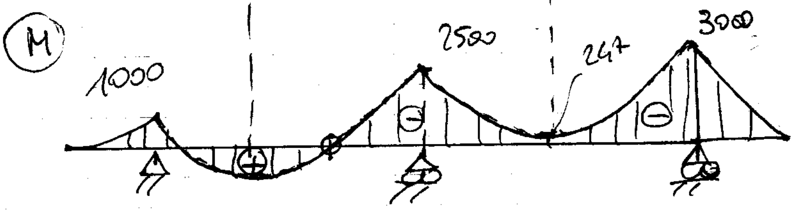
$C\circlearrowleft \quad -V_B(l-d) + \frac{ql^2}{2} = 0 \Rightarrow \boxed{V_B = \frac{9}{4}q}$



$D\circlearrowleft \quad -V_B l + q(2l+d)\left(l+d-\frac{2l+d}{2}\right) - q\frac{l}{2}(l+d) + V_E l = 0$

$\boxed{V_E = \frac{37}{12}q}$

$\boxed{V_D} = -V_A - V_E + \frac{17}{2}q = \boxed{\frac{19}{6}q}$



Controllo

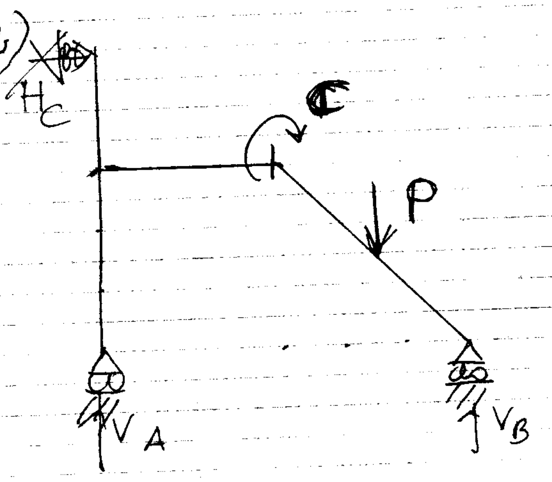
$E\circlearrowleft \quad -\frac{ql}{2}d - V_D l - V_B 2l + q\frac{(2l+d)^2}{2} =$
$= -\frac{3}{2}q - \frac{19}{2}q - \frac{27}{2}q + \frac{49}{2}q = 0$ ok!!

oppure

$C\circlearrowleft \quad \frac{37}{12}q\cdot 4 + \frac{19}{6}q\cdot - 4q\cdot 2 - \frac{15}{2}q = 0$ ok!!

$V_B = \frac{9}{4}q = 4500$ kg
$V_E = \frac{37}{12}q = 6167$ kg
$V_D = \frac{19}{6}q = 6333$ kg

2)



$l = 3m$; $h = 2m$

$\mathbb{C} = \frac{ql^2}{4} = 2250$ kgm

$P = \frac{ql}{2} = 1500$ kg

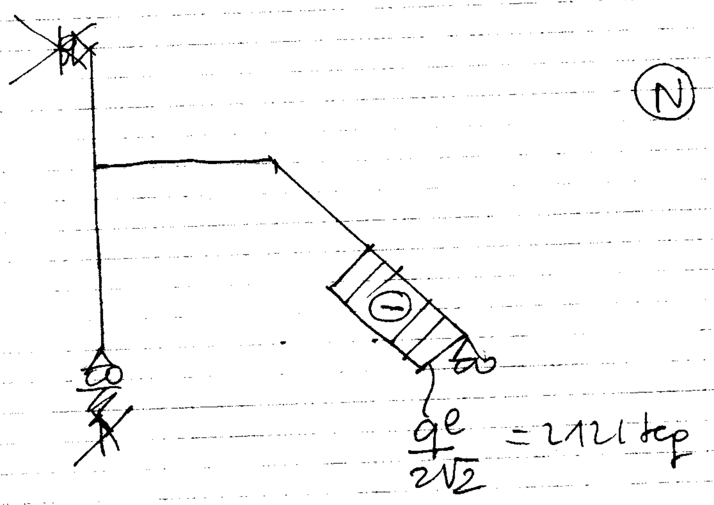
$q = 1000$ kg/m

$V_A + V_B = \frac{ql}{2}$

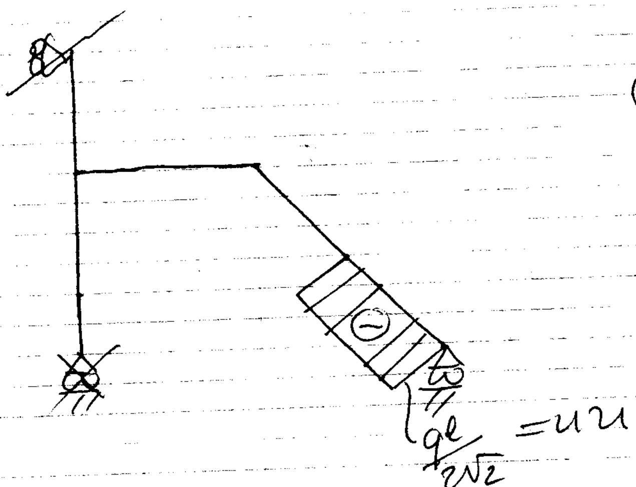
$B \circlearrowleft \; -V_A 2l - \frac{ql^2}{4} + \frac{ql^2}{4} = 0$

$\boxed{V_A = 0} \Rightarrow \boxed{V_B = \frac{ql}{2}}$

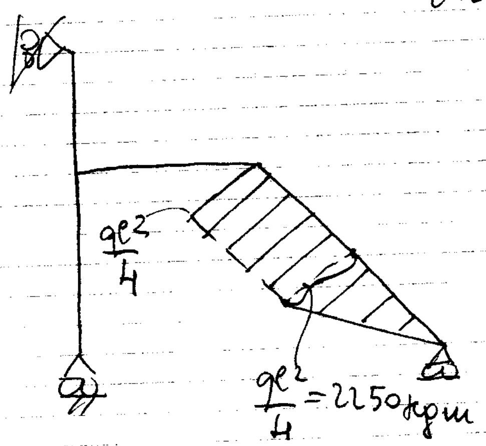
N



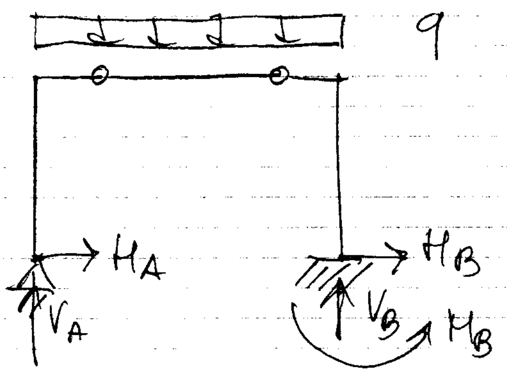
T

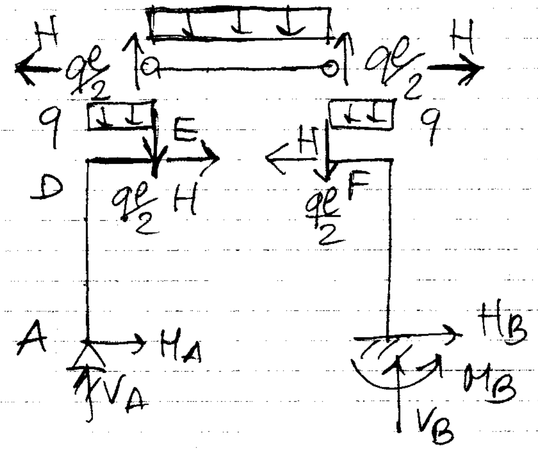


M



(8)





$d = 2\,m$

$l = 3\,m$

Equilibrio alla (↑) del tratto ADE

$V_A = qd + q\frac{l}{2} = 1750$ kg

Eq. alla (→) del tratto ADE

$H_A + H = 0 \Rightarrow$

Eq. alla rotazione (E↺) del tratto ADE

$E\circlearrowleft\; -V_A d + q\frac{d^2}{2} + H_A \cdot l = 0$

$H_A = 833$ kg $= \frac{5}{3}q$

$H = -H_A = -833$ kg

Equil. alla (→) di ~~tutta~~ la struttura

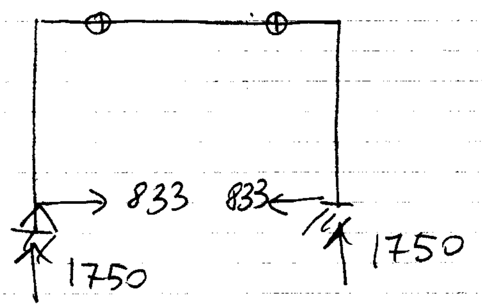
$H_B = -H_A = -833$ kg

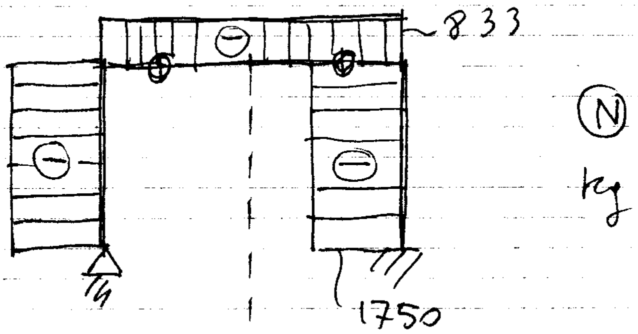
Poichè $V_B = V_A$

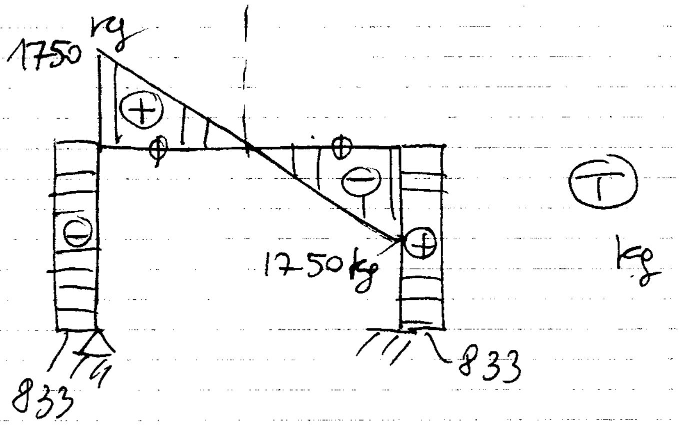
$H_A = -H_B$

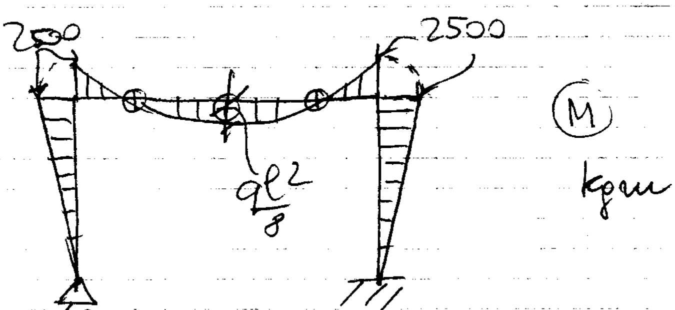
$M_{F} = 0 \Rightarrow \boxed{M_B = 0}$

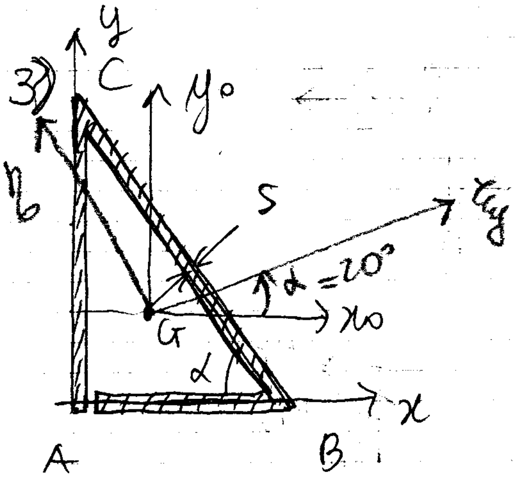
(Vedi equil. F↺ del tratto BF)











$\overline{AC} \equiv l = 20 cm$

$s = 2 cm$

$\alpha = 60°$

se lo spessore è molto sottile:

$G \simeq \left(\frac{AB}{3}, \frac{AC}{3}\right)$

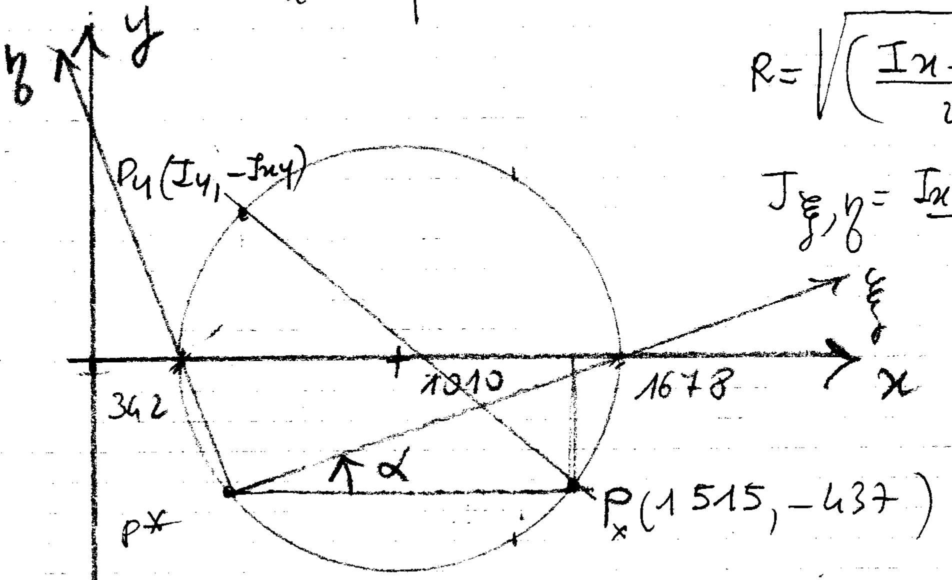
Per differenza tra i triangoli esterno ed interno

$$I_{x0} = \frac{AB \cdot (AC)^3}{36} - \frac{(AB-2s)(AC-2s)^3}{36} = 1515 \text{ cm}^4$$

$$I_{y0} = \frac{(AB)^3 \cdot AC}{36} - \frac{(AB-2s)^3(AC-2s)}{36} = 505 \text{ cm}^4$$

$$I_{x0y0} = -\frac{AB^2 \cdot AC^2}{72} + \frac{(AB-2s)^2(AC-2s)^2}{72} = -437,\bar{3} \text{ cm}^4$$

$$2\alpha = \operatorname{atan}\left(\frac{-2I_{xy}}{I_x - I_y}\right) \Rightarrow \alpha = 20° 44'$$



$$R = \sqrt{\left(\frac{I_x + I_y}{2}\right)^2 + I_{xy}^2} = 669$$

$$J_{\xi,\eta} = \frac{I_x + I_y}{2} \pm R \quad \begin{cases} 1678 \text{ cm}^4 \\ 342 \text{ cm}^4 \end{cases}$$

Metodo analitico,

tensore di inerzia $J = \begin{bmatrix} I_x & -I_{xy} \\ -I_{xy} & I_y \end{bmatrix}$

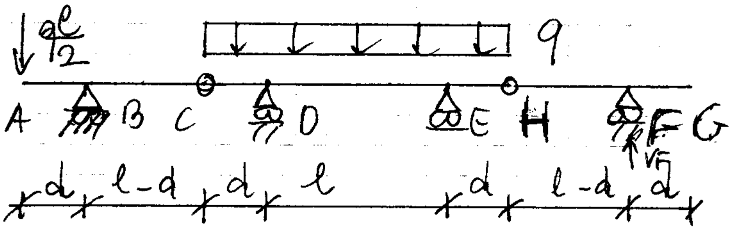
$J v = \lambda v$ se $\det \begin{bmatrix} I_x - \lambda & -I_{xy} \\ -I_{xy} & I_y - \lambda \end{bmatrix} = 0$

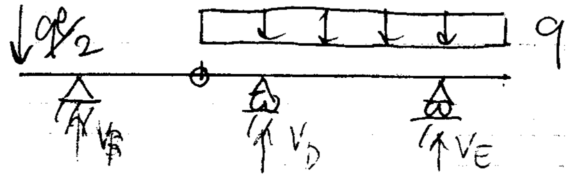
Risolvere

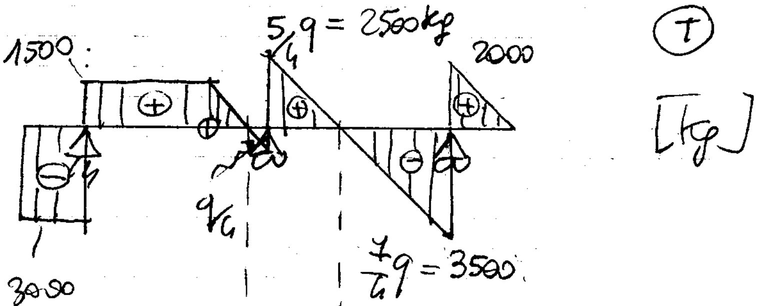
$$\lambda^2 - \operatorname{tr} J \, \lambda + \det J = 0$$

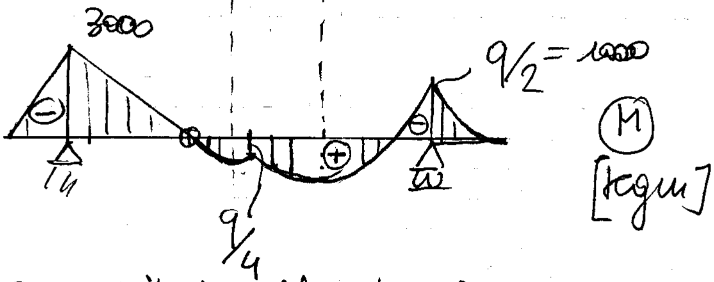
C 6/12/02

1)









$q = 2000$ kg/m

$H \circlearrowleft \; V_F(l-d) = 0 \Rightarrow V_F = 0$

$l = 3m \quad d = 1m$

$C \circlearrowleft \; -V_B(l-d) + q\frac{l^2}{2} = 0$

$\boxed{V_B = \frac{9}{4}q} = 4500$ kg

$E \circlearrowleft \; q\frac{l}{2}(2l+d) - V_B 2l - V_D l +$

$+ q(l+2d)\frac{l}{2} = 0$

$\hookrightarrow V_D = \frac{3}{2}q = 3000$ kg

$V_E = -V_B - V_D + q\frac{l}{2} + q(l+2d)$

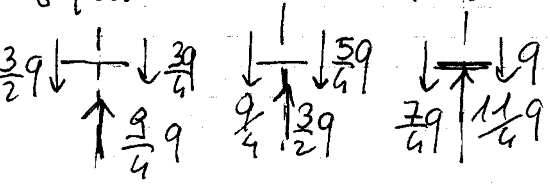
$= \frac{11}{4}q = 5500$ kg

controllo

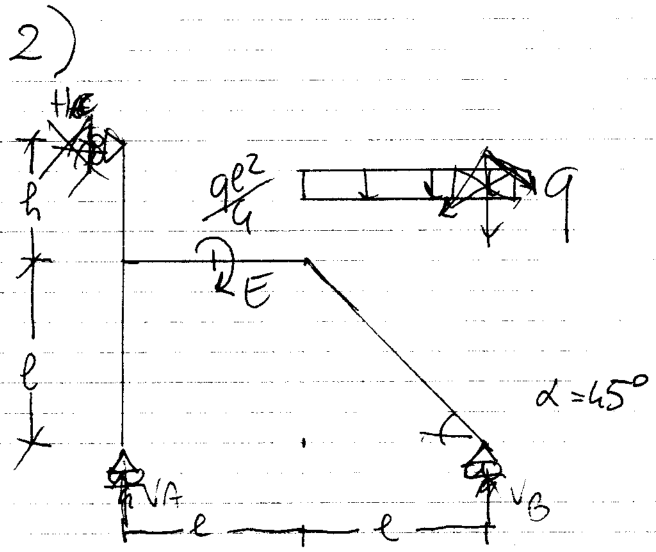
$C \circlearrowleft \; \frac{3}{2}q + \frac{11}{\cancel{4}}q\cdot\cancel{4} - 5q\cdot\frac{5}{2}$

$-\frac{22}{2}q + 11q = 0$ ok

Equilibrio alla traslazione dei nodi B,D,E



2)



$q = 1000$ kg/m
$l = 3$ m $h = 2$ m

$$V_A + V_B = ql$$

$$B \circlearrowleft \; -V_A\, 2l - \frac{ql^2}{4} + \frac{q l^3}{2} = 0$$

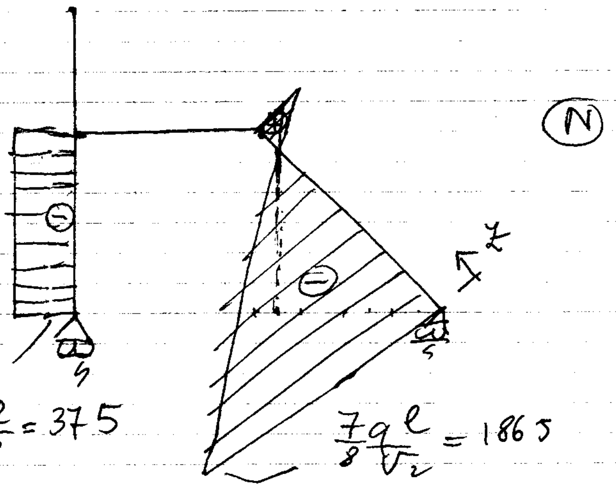
$$\hookrightarrow \boxed{V_A = \frac{ql}{8}} = 375 \text{ kg}$$

$$V_B = -V_A + ql = \frac{7}{8} ql = 2625 \text{ kg}$$

$$H_c = 0$$

Controllo

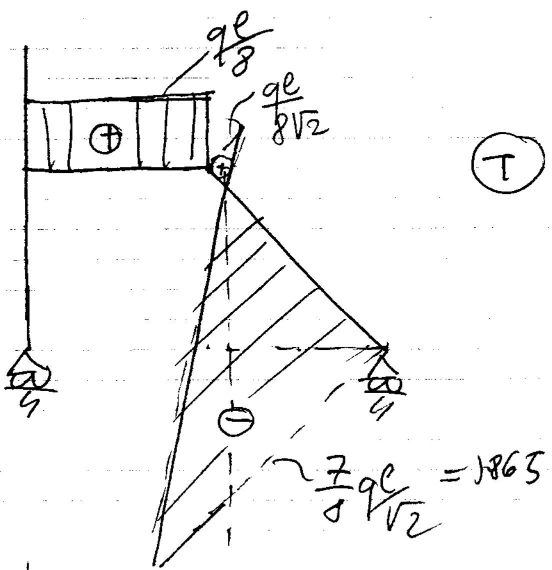
$$A \circlearrowleft \; \frac{7}{8} ql \cdot 2l - ql \frac{3}{2} l - \frac{ql^2}{4} = 0 \quad \text{ok}$$



$$N(z) = -\frac{7}{8} \frac{ql}{\sqrt{2}} + q z \cos\alpha \sin\alpha$$

$$= -\frac{7}{8} \frac{ql}{\sqrt{2}} + \frac{qz}{2}$$

$$N(\bar{z}) = 0 \rightarrow \quad \bar{z} = \frac{7}{8} ql\sqrt{2}$$



$$T(z) = -\frac{7}{8} \frac{ql}{\sqrt{2}} + q z \cos^2 45$$

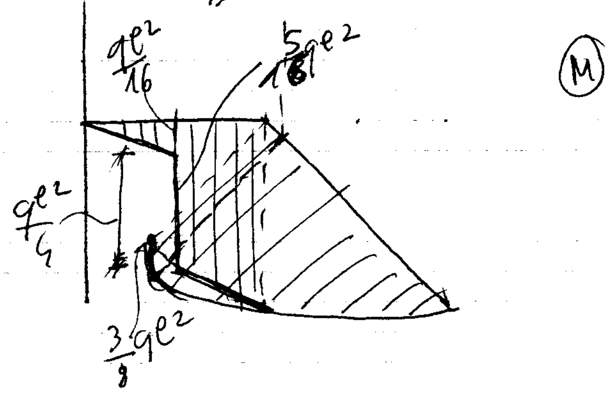
$$= -\frac{7}{8} \frac{ql}{\sqrt{2}} + q \frac{z}{2}$$

$$T(\bar{z}) = 0 \quad \bar{z} = \frac{7}{8} ql\sqrt{2}$$

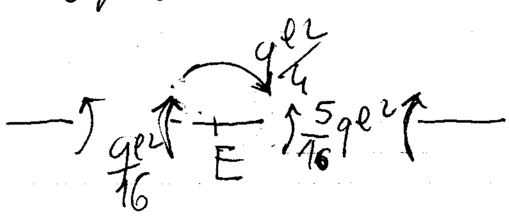
$$M(z) = \frac{7}{8} ql z \cos 45 - q \frac{z^2}{2} \cos^2 45$$

$$= \frac{7}{8} ql \frac{z}{\sqrt{2}} - q \frac{z^2}{4}$$

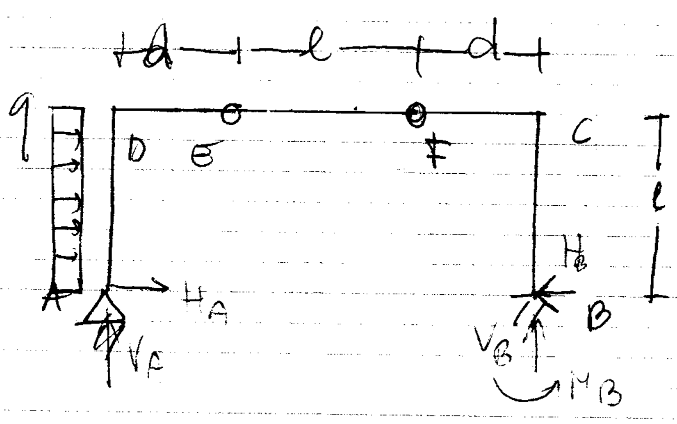
$$M(l\sqrt{2}) = \frac{7}{8} ql \frac{l\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} - q \frac{l^2}{2} = \frac{3}{8} ql^2$$



Equilibrio alla rotazione nodo E



3)

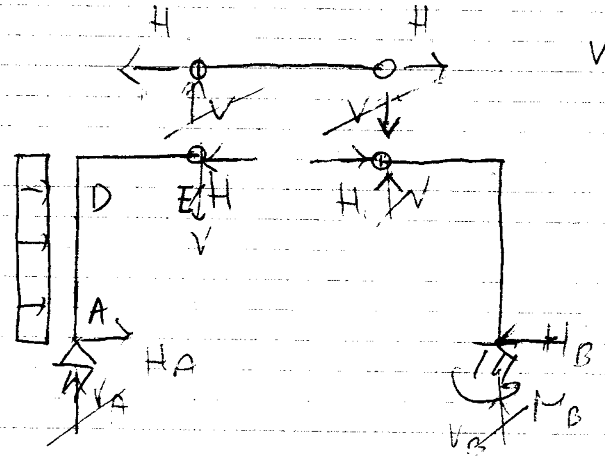


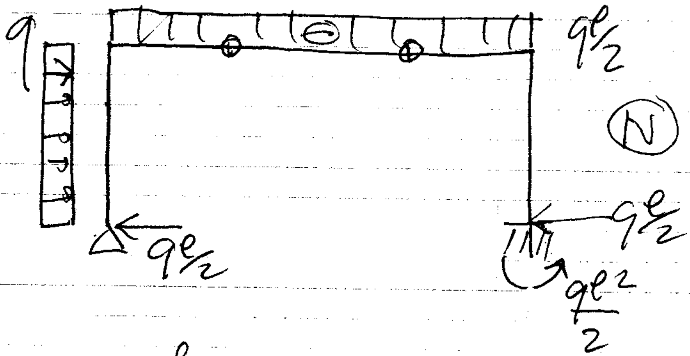
$d = 2$ m

$l = 3$ m

$q = 500$ kg/m

EF è una biella!! $(T=0)$





Equilibrio ADE

$V_A = 0 \qquad \Rightarrow V_B = 0$

E↺ $H_A l + q\frac{l^2}{2} = 0 \Rightarrow \boxed{H_A = -q\frac{l}{2}}$

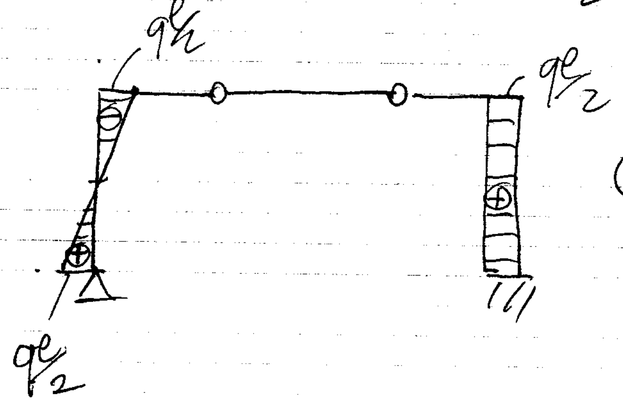
$\boxed{H} = -q\frac{l}{2} + ql = \boxed{q\frac{l}{2}}$

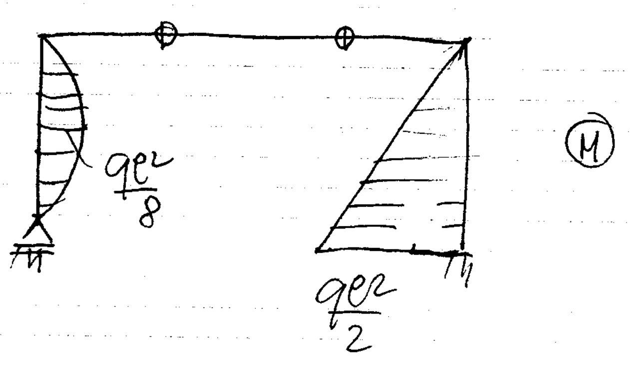
Equilibrio globale

$H_A - H_B + ql = 0 \Rightarrow \boxed{H_B = q\frac{l}{2}} = 750$ kg

$V_B = 0$

F↺ $\boxed{M_B} = q\frac{l^2}{2} = \boxed{2250 \text{ kgm}}$

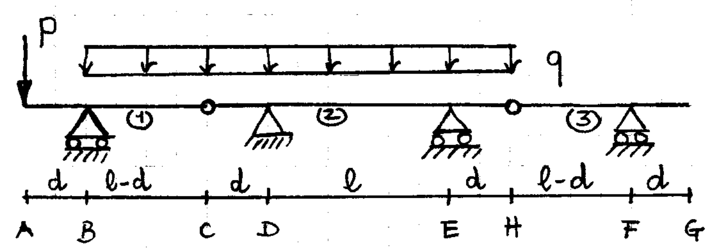




4) vedi prova B

D

esercizio 1)



DATI:

$l = 3\,m$

$d = 1\,m$

$q = 2000\ Kg/m$

$P = \frac{ql}{2} = 3000\ Kg$

Sol: SFORZO NORMALE N NULLO PER OGNI ASTA $(H_D = 0)$

a) REAZIONI VINCOLARI

$V_F = 0$ [DALL' EQUILIBRIO ALLA ROTAZIONE DELL' ASTA 3 CON POLO IN H]

$$V_B = \frac{Pl + q\frac{(l-d)^2}{2}}{(l-d)} = \frac{\frac{ql^2}{2} + \frac{q(l-d)^2}{2}}{(l-d)} = \frac{q}{2}\cdot\frac{l^2+(l-d)^2}{(l-d)} = 6500\ Kg$$ [DALL'EQ. ALLA ROTAZ. DELL'ASTA 1 CON POLO IN C]

$$V_D = \frac{P\cdot(2l+d) + q(2l+d)\cdot\left(l-\frac{d}{2}\right) - V_B\cdot 2l}{l} = 5666,\bar{6}\ Kg$$ [DALL' EQ. ALLA ROTAZIONE DELL' INTERA STRUTTURA CON POLO IN E]

$$V_E = P + q(2l+d) - (V_D + V_B + V_F) = 4833,\bar{3}\ Kg$$ [DALL' EQ. ALLA TRASLAZIONE VERTICALE DELL' INTERA STRUTTURA]

b) DIAGRAMMI DELLE AZIONI INTERNE

b-1) SFORZO NORMALE

assente

b-2) TAGLIO:

	L_i	T_1	T_2			$\xi = T_1/(T_1-T_2)\cdot L_i$
$\overline{AB}$	d	$-P$	$-P$	-3000	-3000	–
$\overline{BC}$	$l-d$	$-P+V_B$	$V_B-q(l-d)-P$	$+3500$	-500	$1,75$
$\overline{CD}$	d	$T_{2\overline{BC}}$	V_B-ql-P	-500	-2500	–
$\overline{DE}$	l	$V_D+T_{2\overline{CD}}$	$V_B+V_D-q\,2l-P$	$+3166.\bar{6}$	$-2833,\bar{3}$	$1,58$
$\overline{EH}$	d	$V_E+T_{2\overline{DE}}$	$V_B+V_D+V_E-q(2l+d)-P$	$+2000$	0	1

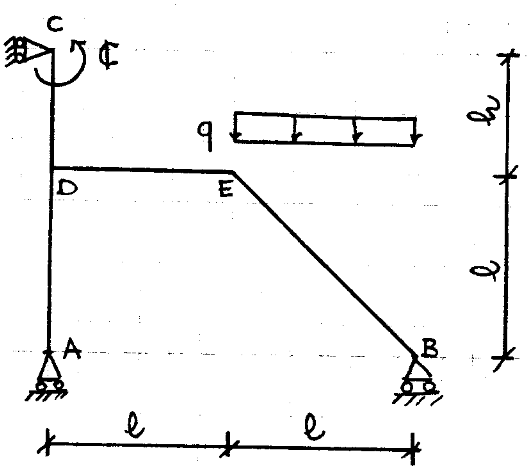
b-3) MOMENTO

		M_1	M_2	M_ξ			
$\overline{AB}$	d	0	$-Pd$	—	0	- 3000	—
$\overline{BC}$	$l-d$	$-Pd$	0	$M_1 - q\frac{\xi^2}{2} + T_{1_{BC}}\xi$	-3000	0	+62,5
$\overline{CD}$	d	0	$-\left[P(d+l)+\frac{ql^2}{2}-V_B l\right]$	—	0	- 1500	—
$\overline{DE}$	l	$M_{2\overline{CD}}$	$M_1 - \frac{ql^2}{2} + T_1 l$	$M_1 - q\frac{\xi^2}{2} + T_1\xi$	-1500	- 1000	+ 1007
$\overline{EH}$	d	$M_{2\overline{DE}}$	0	—	-1000	0	—

c) DISEGNI QUOTATI N, M, T [vedi fig]

D

esercizio 2)



DATI:

$l = 3$ m

$h = 2$ m

$q = 1000 \frac{Kg}{m}$

$C = \frac{ql^2}{4} = 2250$ Kg·m

) REAZIONI VINCOLARI:

$H_C = 0$ [EQ. ALLA TRASLAZIONE ORIZZONTALE]

$V_B = \frac{ql\left(l+\frac{l}{2}\right) - C}{2l} = 1875$ Kg [EQ. ALLA ROTAZ. CON POLO IN A]

$V_A = ql - V_B = 1125$ Kg [EQ. TRASLAZIONE VERTICALE]

) DIAGRAMMI DELLE AZIONI INTERNE:

) sforzo normale

$\overline{AD}$	$-V_A$	$-V_A$	-1125	-1125
$\overline{DC}$	0	0	±0	±0
$\overline{DE}$	0	0	±0	±0
$\overline{EB}$	$V_A \cos 45°$	$-V_B \cos 45°$	+795,49	- 1325,8

b2) TAGLIO

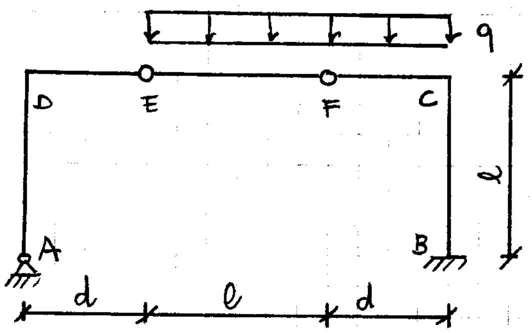
	L_i	T_{1is}	T_{2is}			ξ
$\overline{AD}$	l	0	0	± 0	± 0	
$\overline{DC}$	h	0	0	± 0	± 0	
$\overline{DE}$	l	$-N_{AD}$	$-N_{AD}$	$+1125$	$+1125$	
$\overline{EB}$	$l\sqrt{2}$	$V_A \cos 45°$	$-V_B \cos 45°$	$+795{,}49$	$-1325{,}8$	$\frac{T_1}{T_1+T_2} L_i = 1{,}59\,m$

b3) MOMENTO

	L_i	M_1	M_2	M_ξ			
$\overline{AD}$	l	0	0	–	± 0	± 0	–
$\overline{DC}$	h	C	C	–	$+2250$	$+2250$	–
$\overline{DE}$	l	C	$C - N_{AD} l$	–	$+2250$	-1125	–
$\overline{EB}$	$l\sqrt{2}$	M_{2DE}	$0 + H_A + T_1 \xi - \frac{q \cos^2 45 \, \xi^2}{2}$		1125	0	1758

c) D

esercizio 3)



DATI :

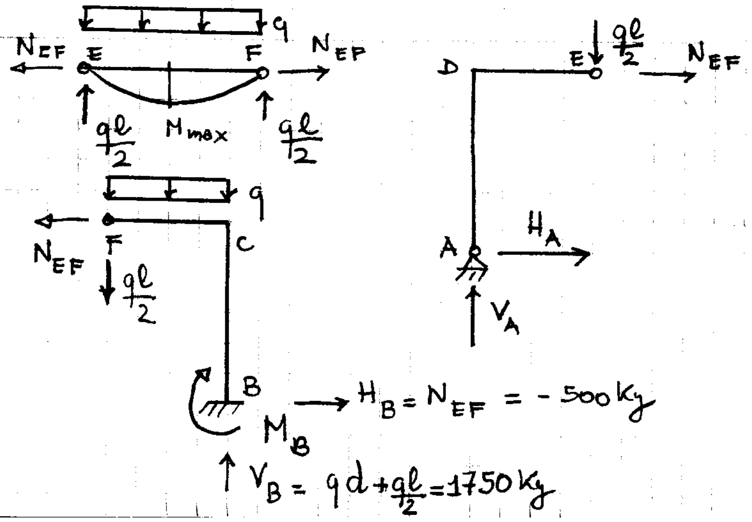
$l = 3$ m

$d = 2$ m

$q = 500$ kg/m

a) REAZIONI VINCOLARI

$H_{max} = ql^2/8 = 562{,}5$ kg m



$N_{EF} \cdot l + \frac{qld}{2} = 0 \rightarrow N_{EF} = -\frac{qd}{2} = -500$ kg

$\rightarrow H_A = -N_{EF} = 500$ kg

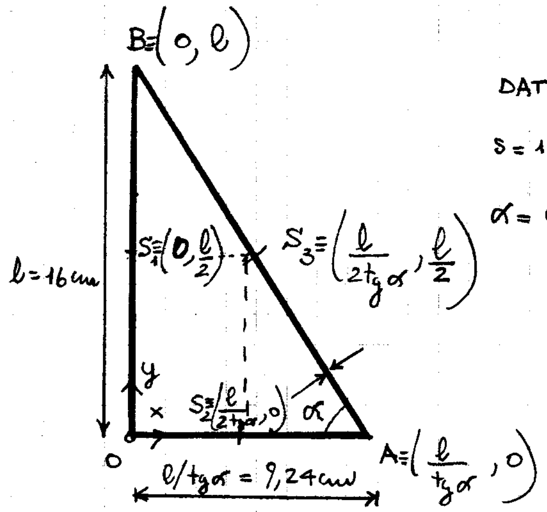
$\rightarrow V_A = \frac{ql}{2} = 750$ kg

$M_B = N_{EF} \cdot l + q\frac{d^2}{4} + \frac{qld}{2} = -\cancel{\frac{qdl}{2}} + \frac{qd^2}{2} + \cancel{\frac{qld}{2}} = 1000$ kgm

b) DIAGRAMMI QUOTATI DEI MOMENTI (vedi fig.)

D

esercizio 4)



DATI

$s = 1.2$ cm $\qquad s << \ell$ (linea sottile)

$\alpha = 60° = \frac{\pi}{3}$

$x_{s_1} = 0 \qquad y_{s_1} = 8$ cm

$x_{s_2} = 4,6188$ cm $\qquad y_{s_2} = 0$ cm

$x_{s_3} = 4,6188$ cm $\qquad y_{s_3} = 8$ cm

$A_1 = 19,2\,cm^2 \quad A_2 = 11,09\,cm^2 \quad A_3 = 22,17\,cm^2 \qquad \ell_1 = 16$ cm $\quad \ell_2 = 9,24$ cm $\quad \ell_3 = 18,475$ cm

POSIZIONE DEL BARICENTRO G

$$x_G = \frac{x_{s_1} \cdot A_1 + x_{s_2} A_2 + x_{s_3} A_3}{A_1 + A_2 + A_3} = \frac{x_{s_1} + \frac{x_{s_2}}{tg\alpha} + \frac{x_{s_3}}{\sin\alpha}}{1 + \frac{1}{tg\alpha} + \frac{1}{\sin\alpha}} = 2,928 \text{ cm } r$$

① $A_1 = \ell s \qquad$ ② $A_2 = \frac{\ell s}{tg\alpha} \qquad$ ③ $A_3 = \frac{\ell}{\sin\alpha} s$

$$y_G = \frac{y_{s_1} \cdot A_1 + y_{s_2} A_2 + y_{s_3} A_3}{A_1 + A_2 + A_3} = \frac{y_{s_1} + \frac{y_{s_2}}{tg\alpha} + \frac{y_{s_3}}{\sin\alpha}}{1 + \frac{1}{tg\alpha} + \frac{1}{\sin\alpha}} = 6,309 \text{ cm } r$$

MOMENTI D'INERZIA RISPETTO A G

$$I^{①}_{xx_G} = \frac{s\ell^3}{12} + A_1 \cdot (y_{s_1} - y_G)^2 = 464,5\,cm^4 \qquad I^{①}_{yy_G} = 0 + A_1 \cdot (x_{s_1} - x_G)^2 = 164,605\,cm^4$$

$$I^{②}_{xx_G} = 0 + A_2\, y_G^2 = 441,42\,cm^4 \qquad I^{②}_{yy_G} = \frac{s\left(\frac{\ell}{tg\alpha}\right)^3}{12} + A_2 \cdot (x_G - x_{s_2})^2 = 110,638\,cm^4$$

$$I^{③}_{xx_G} = I^{③}_{xx_{S_3}} + A_3 \cdot (y_{s_3} - y_G)^2 = 536,194\,cm^4 \qquad I^{③}_{yy_G} = I^{③}_{yy_{S_3}} + A_3 (x_{s_3} - x_G)^2 = 221,119\,cm^4$$

con: $I^{③}_{xx_{S_3}} = \frac{s\overline{AB}^3}{12} \sin^2(180° - \alpha) = 472,95\,cm^4$ $\qquad$ con: $I^{③}_{yy_G} = \frac{s\overline{AB}^3}{12} \cos^2(180° - \alpha) = 157,65\,cm^4$

$$I^{①}_{xy_G} = 0 + A_1 \cdot (y_{s_1} - y_G) \cdot (x_{s_1} - x_G) = -95,064\,cm^4$$

$$I^{②}_{xy_G} = 0 + A_2 (y_{s_2} - y_G) \cdot (x_{s_2} - x_G) = -118,384\,cm^4$$

$$I^{③}_{xy_G} = I^{③}_{xy_{S_3}} + A_3 \cdot (y_{s_3} - y_G) \cdot (x_{s_3} - x_G) = -209,625\,cm^4$$

con $I^{③}_{xy_{S_3}} = \frac{s\overline{AB}^3}{12} \sin(180-\alpha) \cdot \cos(180-\alpha) = -273,06\,cm^4$

$$I_{xx_G} = \sum_i I^i_{xx_G} = 1442,18\,cm^4 \; r$$

$$I_{yy_G} = \; . \quad . \; = 496,45\,cm^4 \; r$$

$$I_{xy_G} = \; . \quad . \; = -423,01\,cm^4 \; r$$

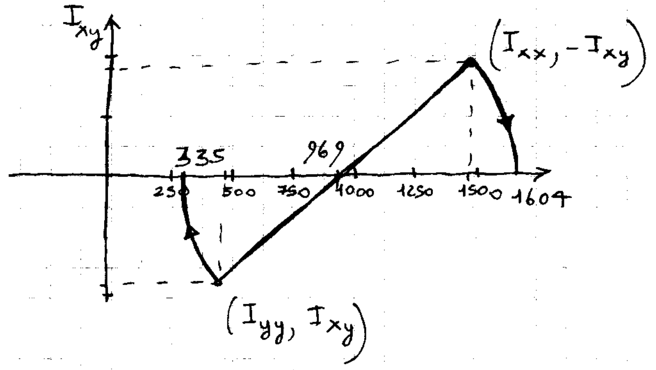
- MOMENTI PRINCIPALI D'INERZIA (METODO ANALITICO)

$$I_\xi = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = 1603{,}77 \text{ cm}^4$$

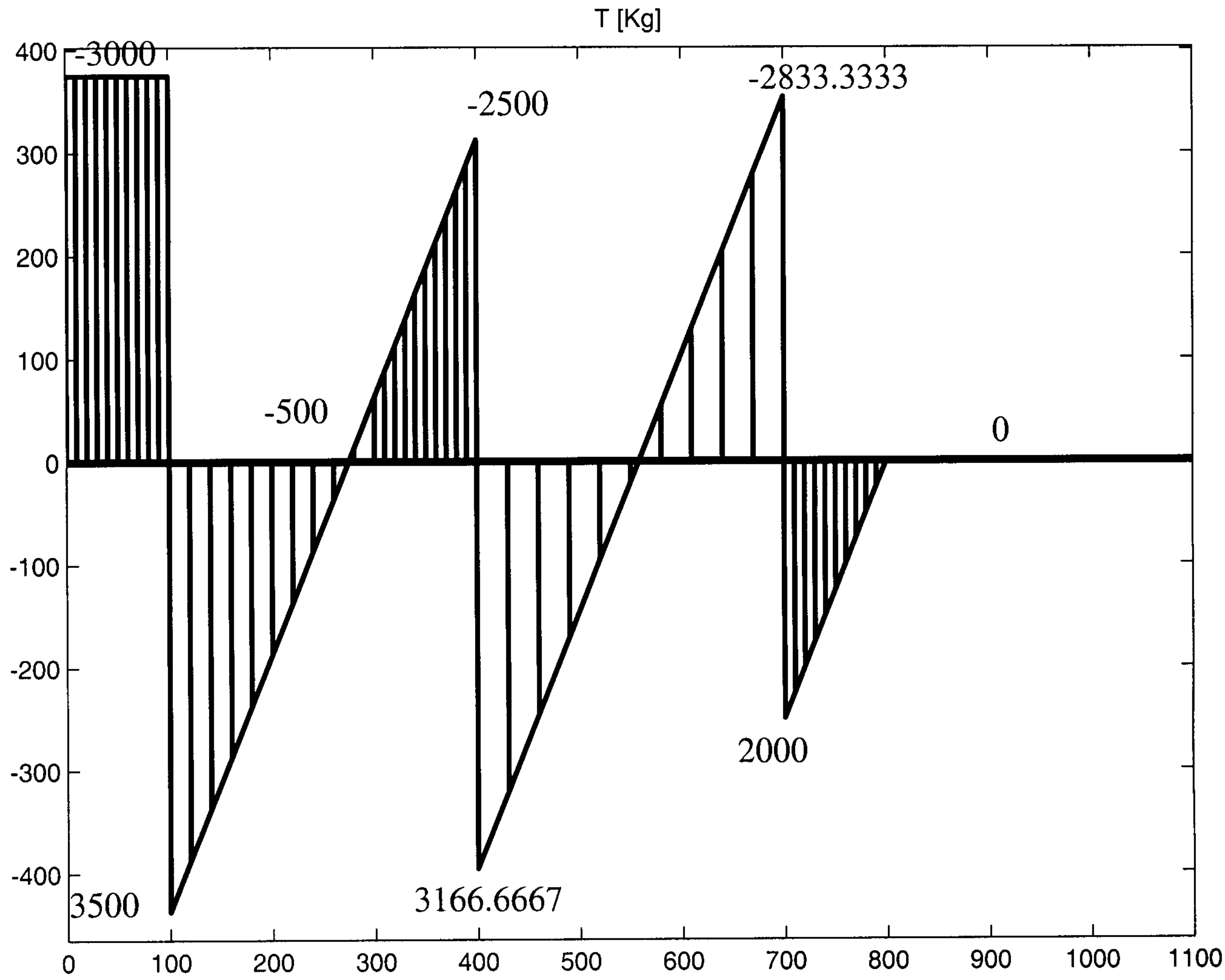
$$I_\eta = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = 334{,}856 \text{ cm}^4$$

$$\alpha_\xi = \frac{\operatorname{arctg}\left(\frac{-2 I_{xy}}{I_x - I_y}\right)}{2} \cong 20{,}91^\circ$$

- CIRCOLI DI MOHR

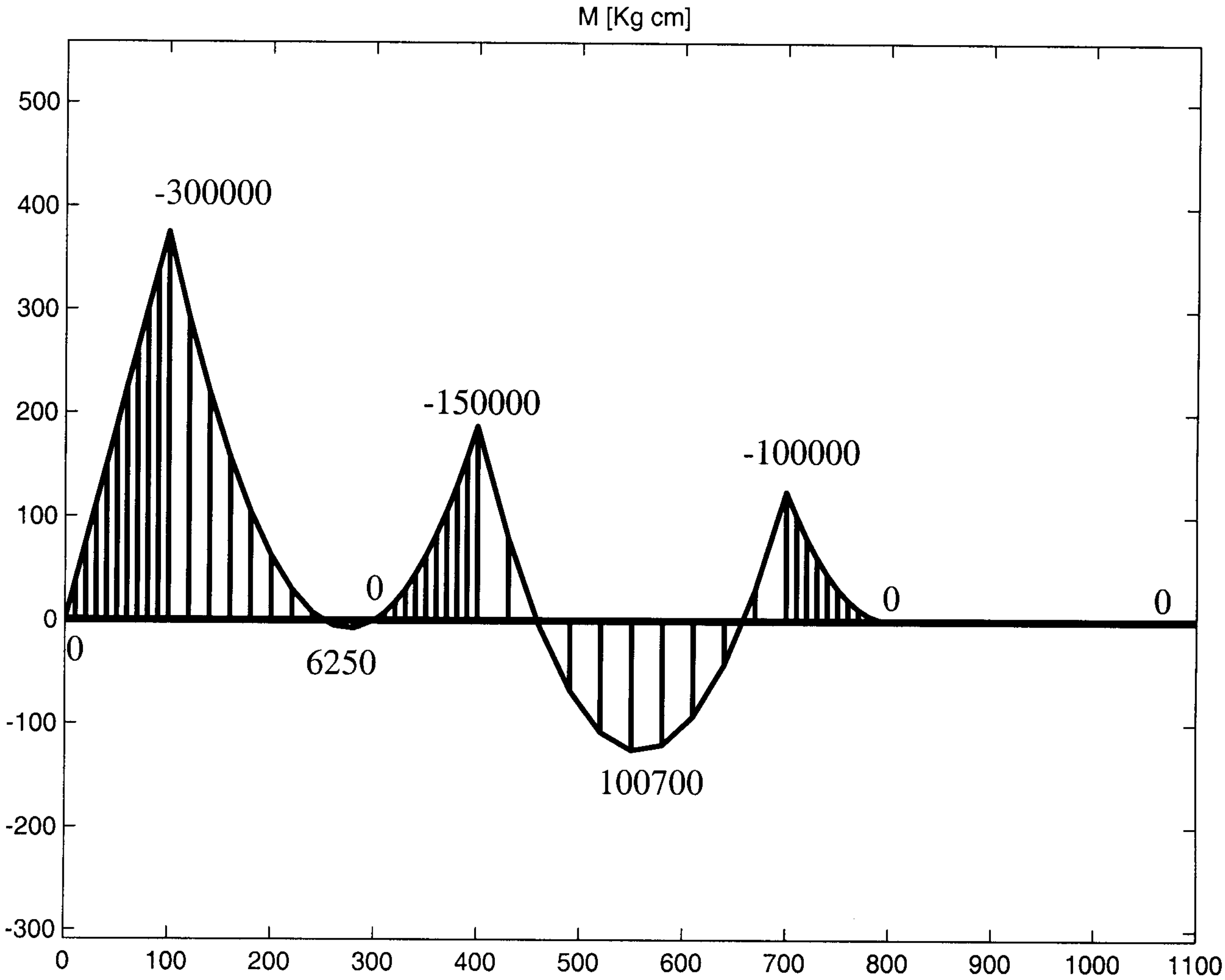


D1



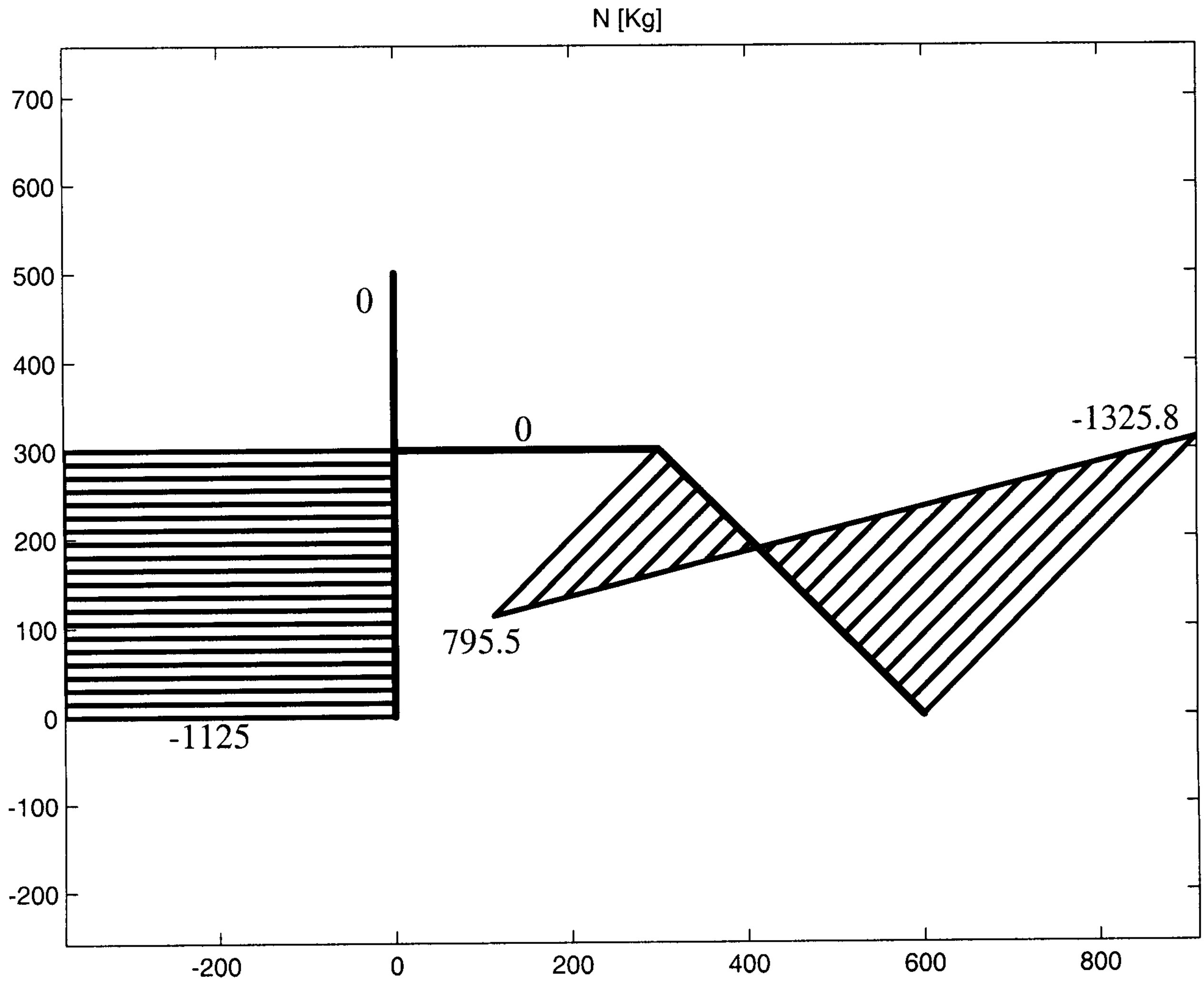
T [Kg]
-3000
-2500
-2833.3333
-500
0
2000
3500
3166.6667
400
300
200
100
0
-100
-200
-300
-400
0
100
200
300
400
500
600
700
800
900
1000
1100

D1



M [Kg cm]
-300000
-150000
-100000
0
0
0
0
6250
100700
500
400
300
200
100
0
-100
-200
-300
0
100
200
300
400
500
600
700
800
900
1000
1100

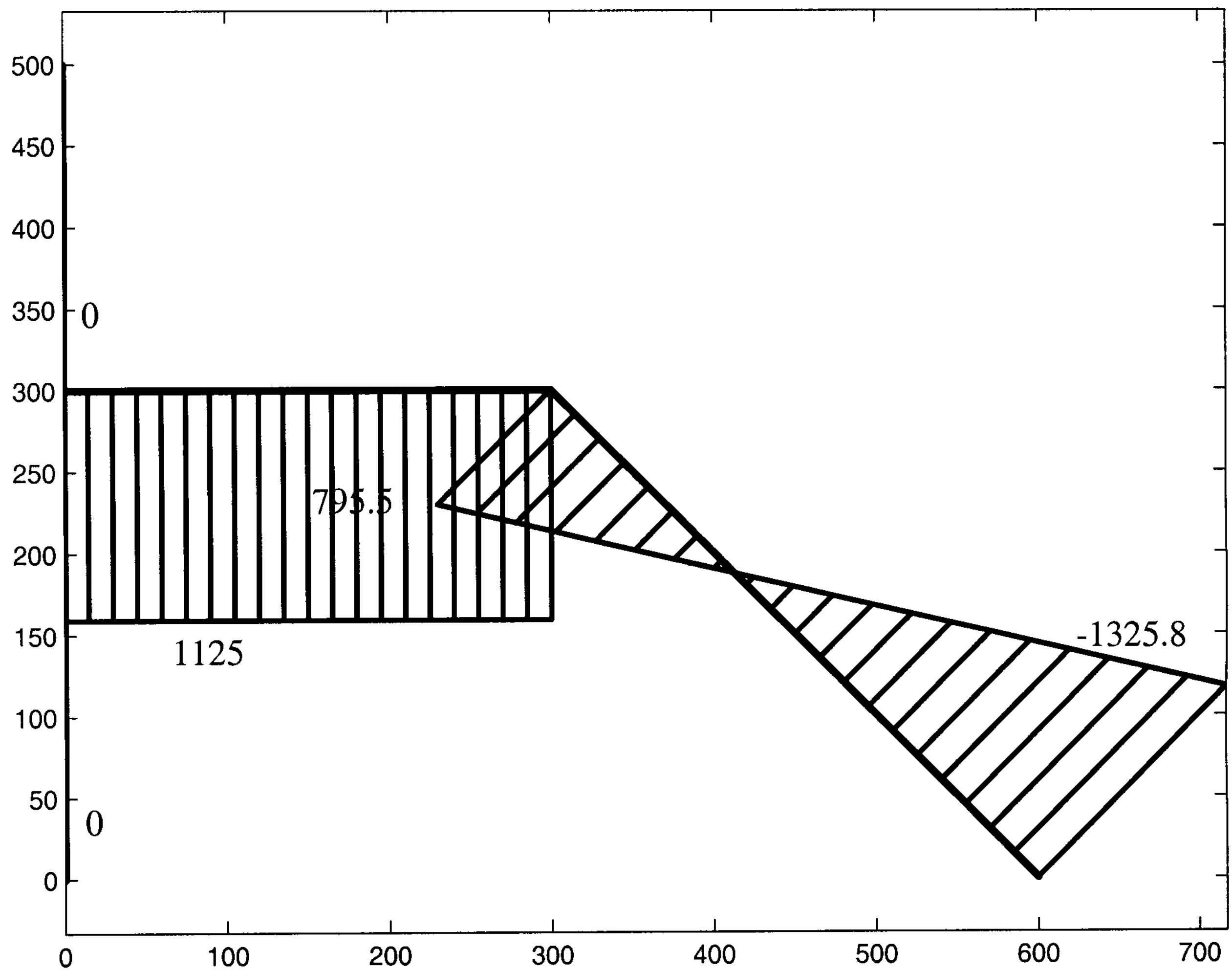
D2



N [Kg]
700
600
500
400
300
200
100
0
-100
-200
-200
0
200
400
600
800
0
0
-1325.8
795.5
-1125

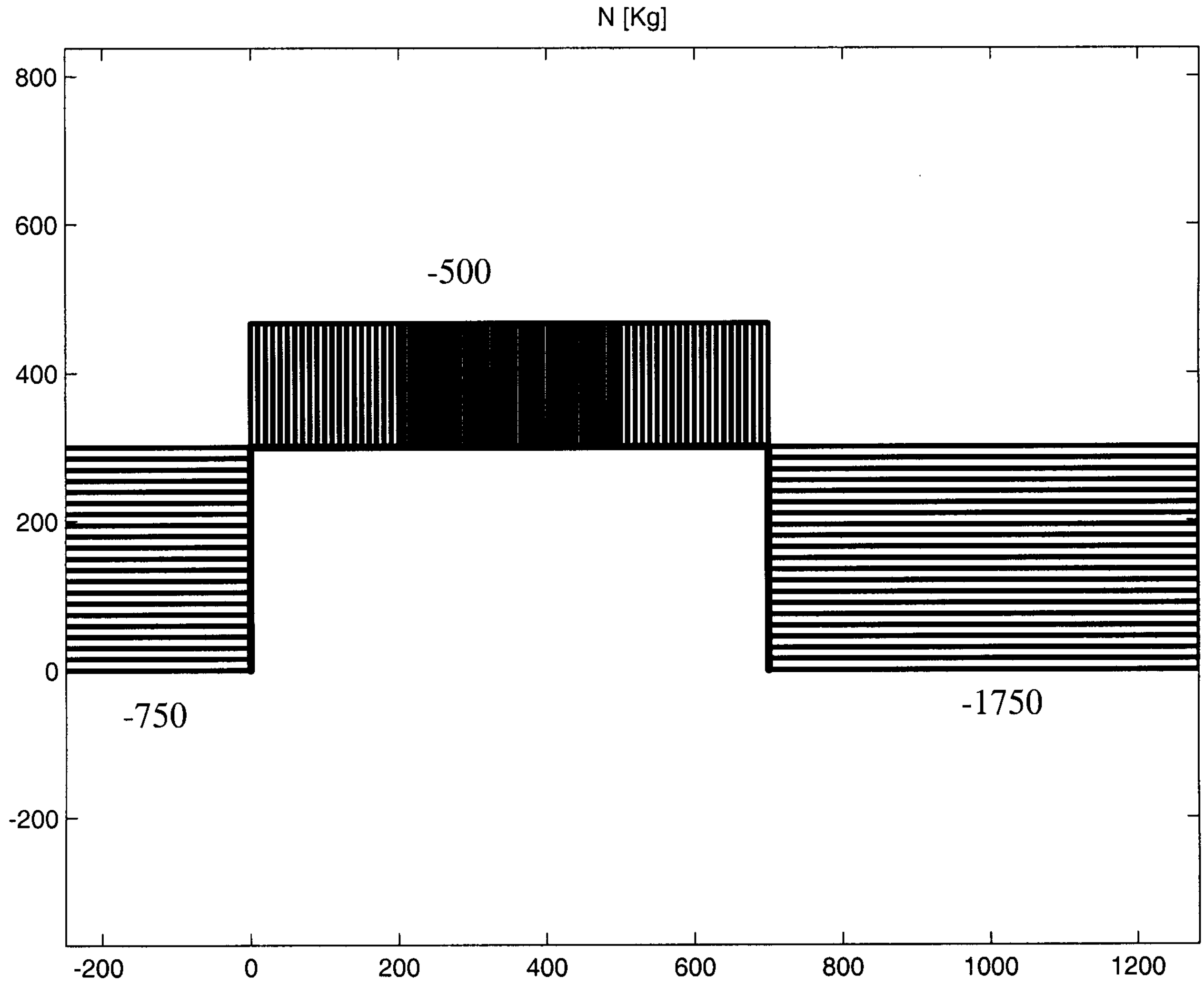
D2

T [Kg]



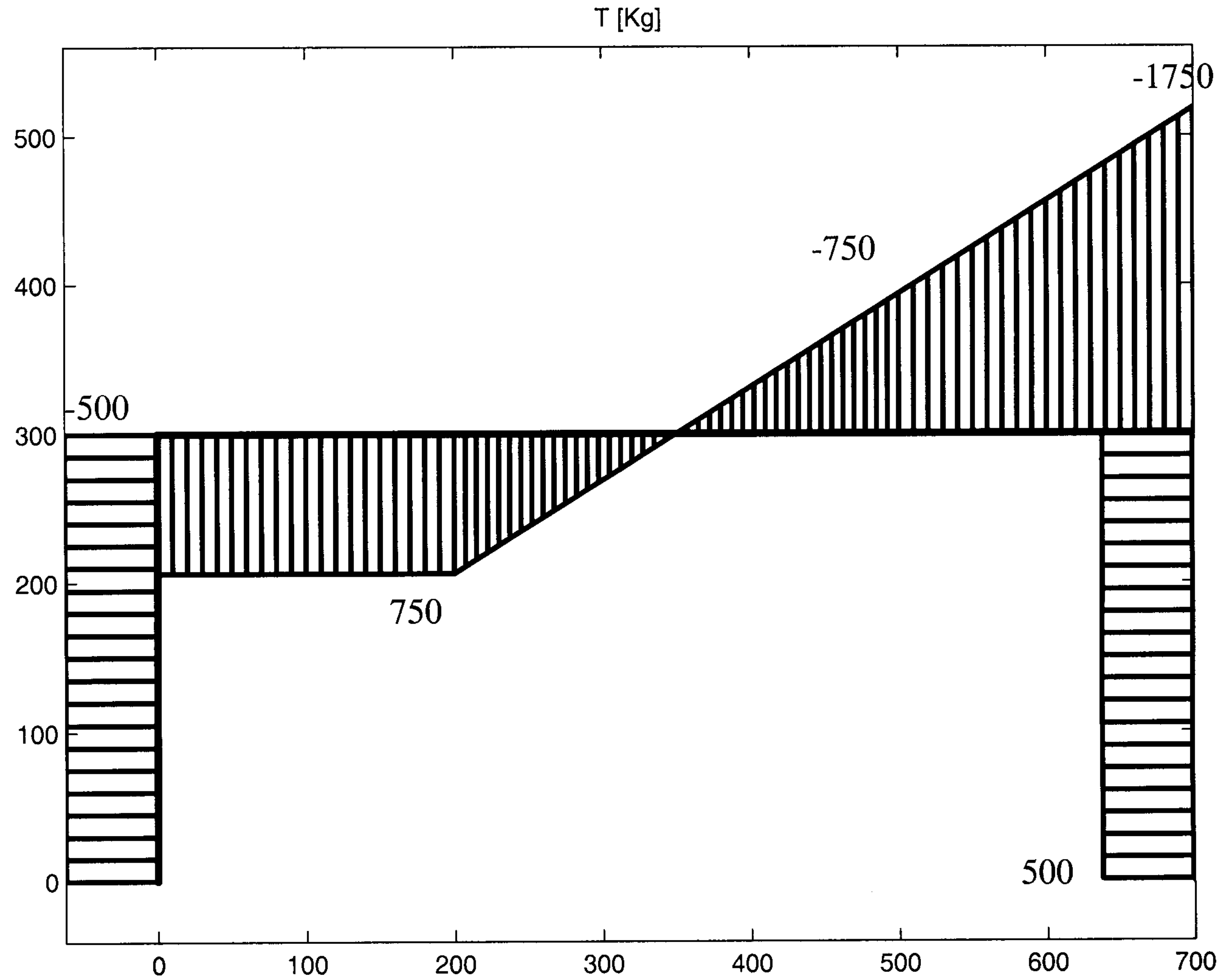
500
450
400
350
300
250
200
150
100
50
0
0
0
795.5
1125
-1325.8
0
100
200
300
400
500
600
700

63



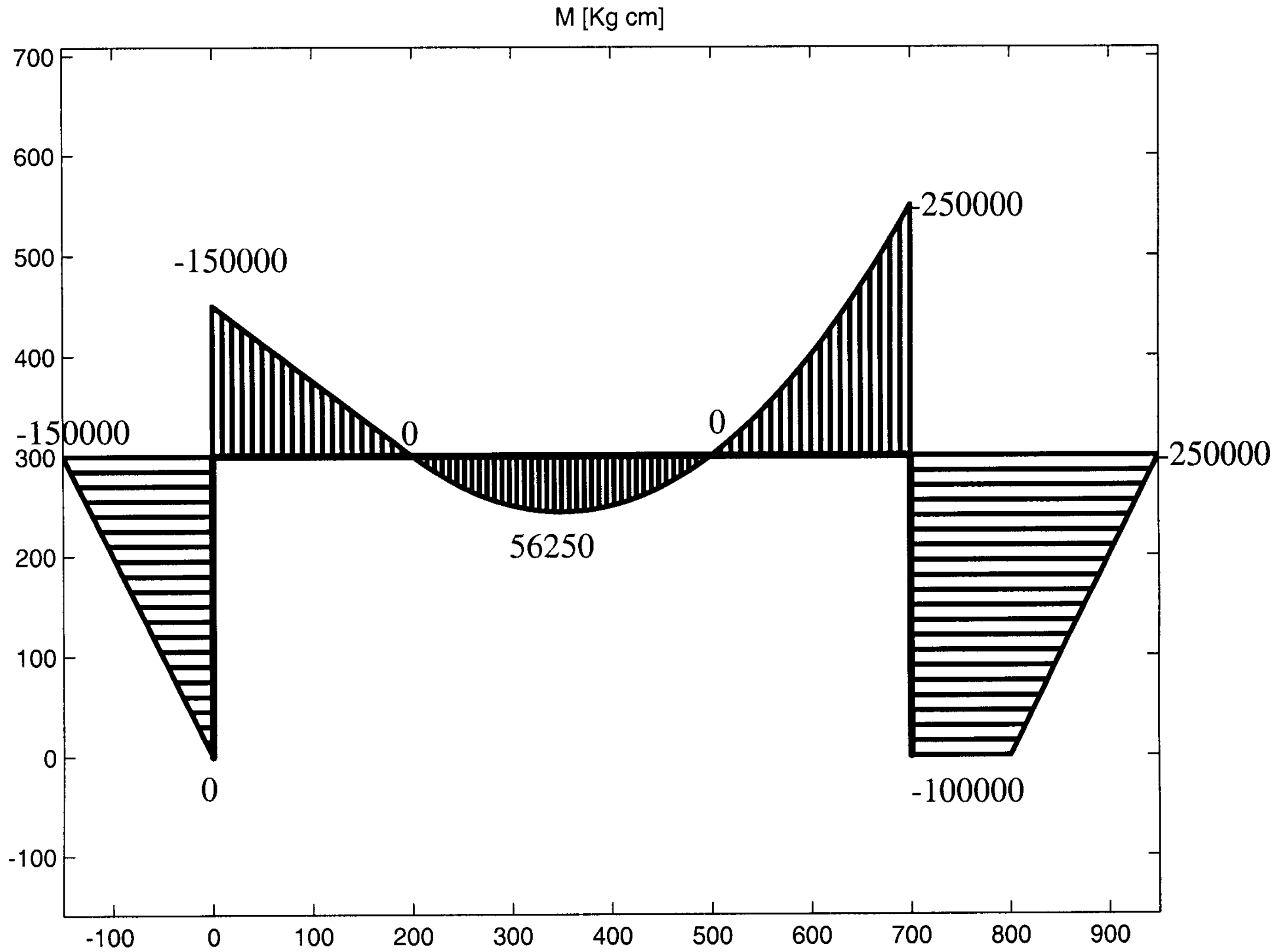
N [Kg]
800
600
400
200
0
-200
-500
-750
-1750
-200
0
200
400
600
800
1000
1200

t3



T [Kg]
-1750
-750
-500
750
500
500
400
300
200
100
0
0
100
200
300
400
500
600
700

D3



M [Kg cm]
700
600
500
400
300
200
100
0
-100
-100
0
100
200
300
400
500
600
700
800
900
-150000
-250000
-150000
0
0
-250000
56250
0
-100000