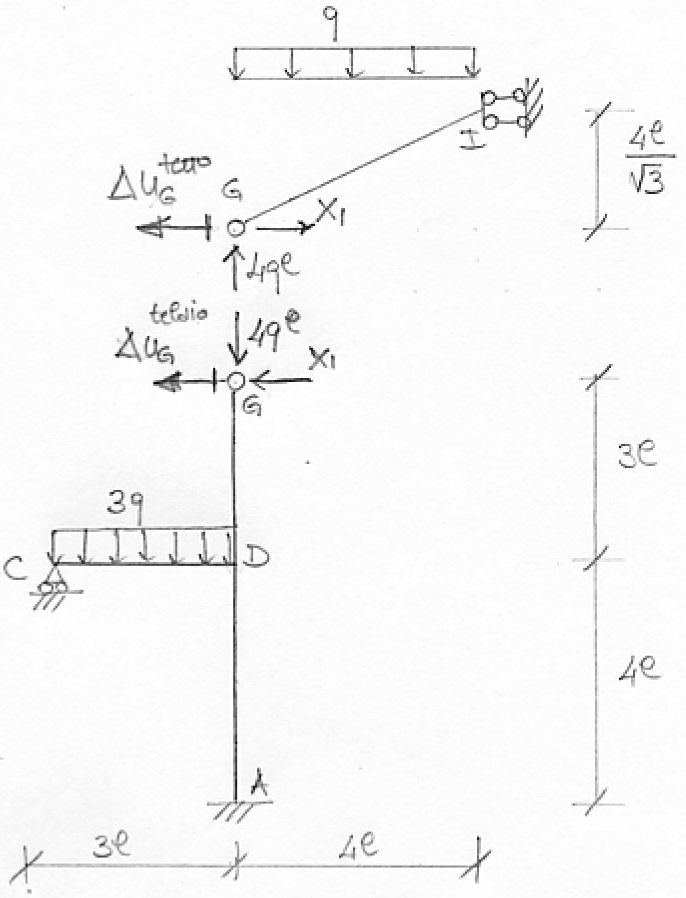
RISOLUZIONE CON IL METODO DELLE FORZE



$l = 1m$

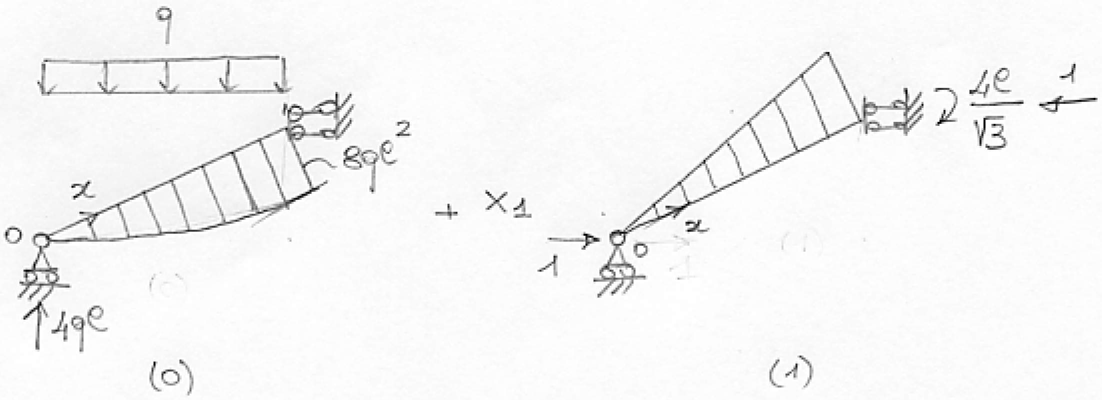
$q = 1000\ kg/m$

Sia Δu_G^{tetto} lo spostamento orizzontale del punto G pensato appartenere al "tetto" GI, e sia Δu_G^{telaio} lo spostamento orizzontale di G pensato appartenere al telaio ACDG.

Equazione di congruenza:

$$\Delta u_G^{tetto} = \Delta u_G^{telaio}$$

Per valutare Δu_G^{tetto}, consideriamo:

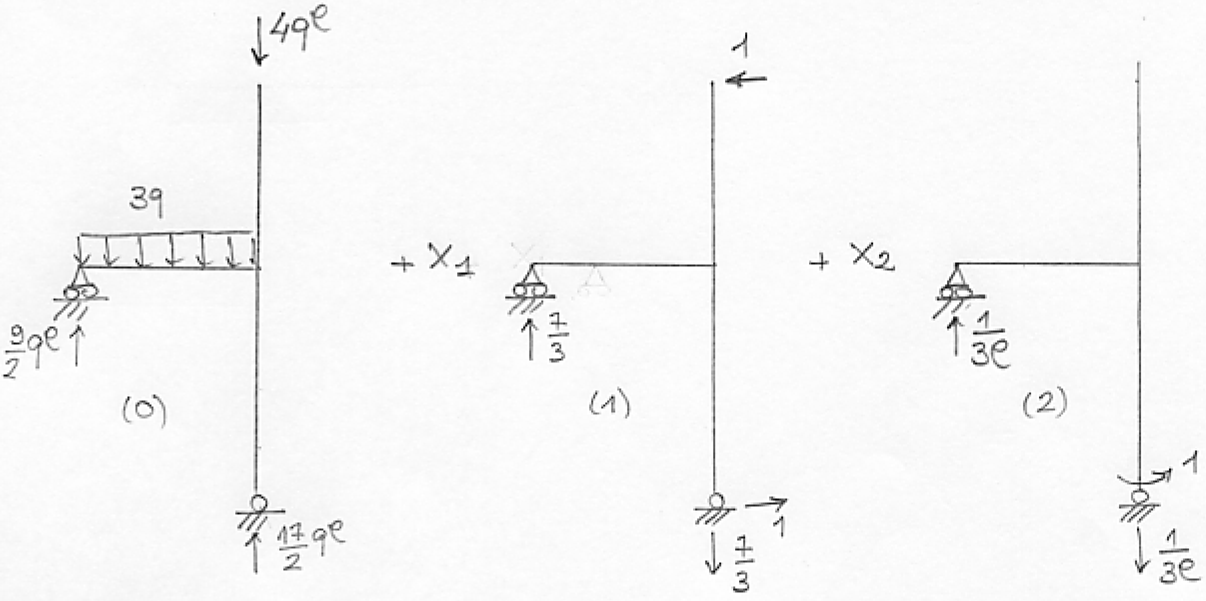


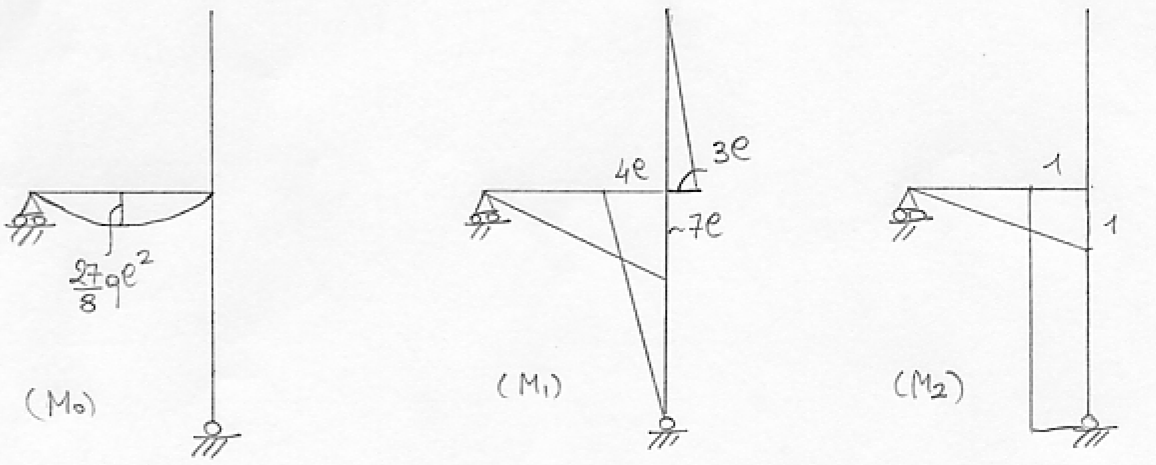
$$\eta_{10}^{tetto} + \eta_{11}^{tetto} X_1 = -\Delta u_G^{tetto}$$ (segno ⊖ perché Δu_G^{tetto} e' opposto al verso di X_1 applicata al tetto)

$$EJ\,\eta_{10}^{tetto} = \int_0^{\frac{8l}{\sqrt{3}}} \left(2ql\sqrt{3}\,x - \frac{3}{8}qx^2\right)\left(-\frac{x}{2}\right)dx = -\frac{320}{9}ql^4$$

$$EJ\,\eta_{11}^{tetto} = \int_0^{\frac{8l}{\sqrt{3}}} \left(-\frac{x}{2}\right)^2 dx = \frac{128\,l^3}{9\sqrt{3}}$$

Per valutare $\Delta u_G^{\text{teldio}}$, occorre introdurre una seconda incognita iperstatica, perché il teldio ACDG è una volta iperstatico.
Come incognita X_2 scegliamo il momento in A.





$$EJ\,\eta_{10}^{\text{teldio}} = \int_0^{3\ell} \left(\frac{9}{2}q\ell x - \frac{3qx^2}{2}\right)\left(\frac{7}{3}x\right)dx = \frac{189}{8}q\ell^4$$

$$EJ\,\eta_{20}^{\text{teldio}} = \int_0^{3\ell} \left(\frac{9}{2}q\ell x - 3q\frac{x^2}{2}\right)\left(\frac{x}{3\ell}\right)dx = \frac{27}{8}q\ell^3$$

$$EJ\,\eta_{11}^{\text{teldio}} = \frac{1}{3}\left[3\ell(3\ell)^2 + 3\ell\cdot(7\ell)^2 + 4\ell\cdot(4\ell)^2\right] = \frac{238\ell^3}{3}$$

$$EJ\,\eta_{12}^{\text{teldio}} = \left[\frac{1}{3}\cdot 3\ell\,(7\ell)\cdot 1 + \frac{1}{2}(4\ell)^2\right] = 15\ell^2$$

$$EJ\,\eta_{22}^{teldio} = \left[\frac{1}{3} \cdot 3\ell (1)^2 + 1 \cdot 4\ell \right] = 5\ell$$

Equazioni di congruenza:

$$\begin{cases} \Delta u_G^{teldio} = \Delta u_G^{tetto} \\ \Delta \varphi_A = 0 \end{cases}$$

$$\begin{cases} \eta_{10}^{teldio} + \eta_{11}^{teldio} X_1 + \eta_{12}^{teldio} X_2 = - \eta_{10}^{tetto} - \eta_{11}^{tetto} X_1 \\ \eta_{20}^{teldio} + \eta_{12}^{teldio} X_1 + \eta_{22}^{teldio} X_2 = 0 \end{cases}$$

$$\begin{cases} (\eta_{11}^{teldio} + \eta_{11}^{tetto}) X_1 + \eta_{12}^{teldio} X_2 = - \eta_{10}^{tetto} - \eta_{10}^{teldio} \\ \eta_{12}^{teldio} X_1 + \eta_{22}^{teldio} X_2 = - \eta_{20}^{teldio} \end{cases}$$

Si trova:

$$\begin{cases} X_1 = 518 \text{ kg} \\ X_2 = - 2230 \text{ kgm} \end{cases}$$

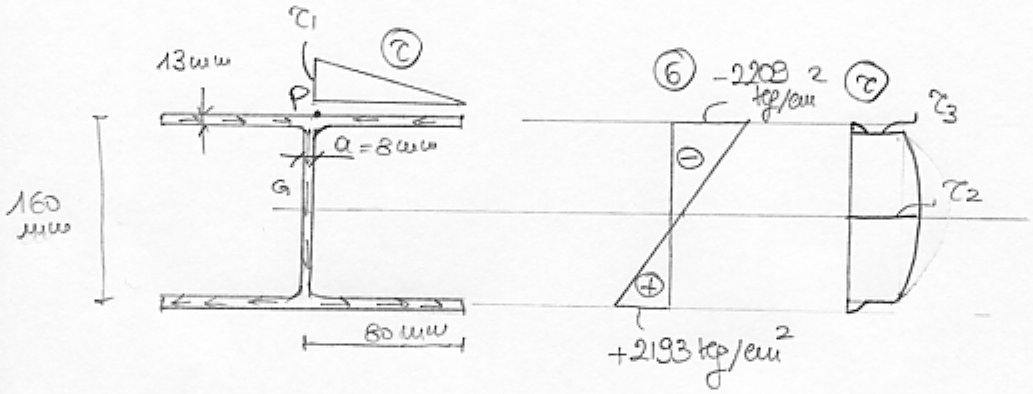
e i diagrammi delle azioni interne sono riportate nelle pagine seguenti.
La sezione più sollecitata a flessione è quella in I con: $M = 6847$ kgm

<u>Progetto</u>:

$$W_x \geq \frac{6847 \cdot 100}{2400} = 285 \text{ cm}^3$$

↳ HEB 160 $\begin{cases} W_x = 311 \text{ cm}^3 \\ J_x = 2492 \text{ cm}^4 \\ A = 54,3 \text{ cm}^2 \end{cases}$

Verifica nella sezione in I: $M_I = 6847$ kgm, $T_I = 259$ kg, $N_I = -449$ kg



$$\left.\begin{array}{l}\sigma_{min}\\ \sigma_{max}\end{array}\right\} = \pm\frac{684700}{311} \mp \frac{449}{54,3} = \pm 2201 - 8 = \begin{cases} 2193 \ \text{kg/cm}^2 \\ -2209 \ \text{kg/cm}^2 \end{cases}$$

$$\tau_1 = \frac{259}{2492} \quad \frac{8 \cdot \cancel{1,3}^2 \cdot}{\cancel{1,8}} = 6,6 \ \text{kg/cm}^2$$

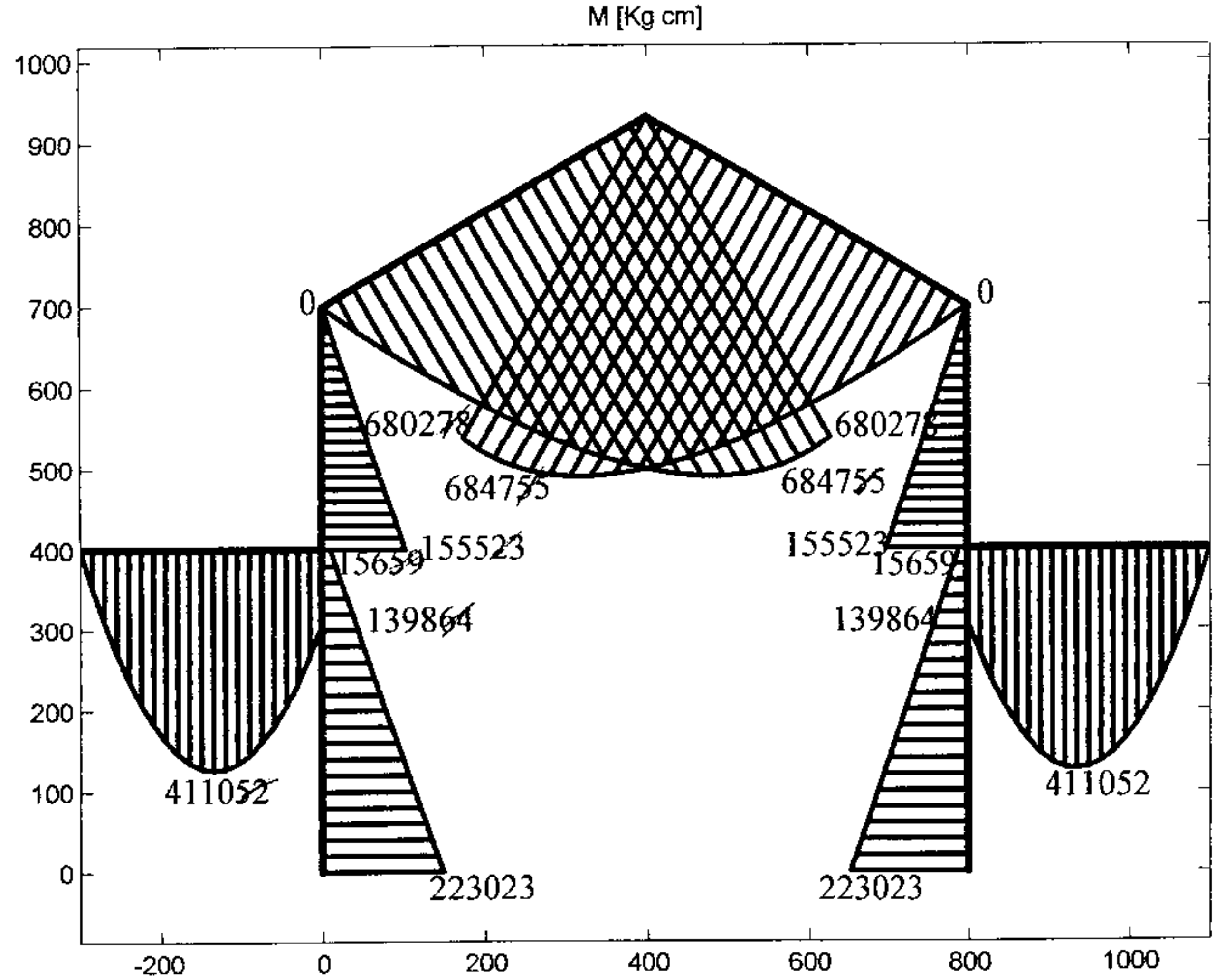
$$\tau_2 = \frac{259}{2492} \cdot 177 = 18,4 \ \text{kg/cm}^2$$

$$\tau_3 \cong 13,2 \ \text{kg/cm}^2$$

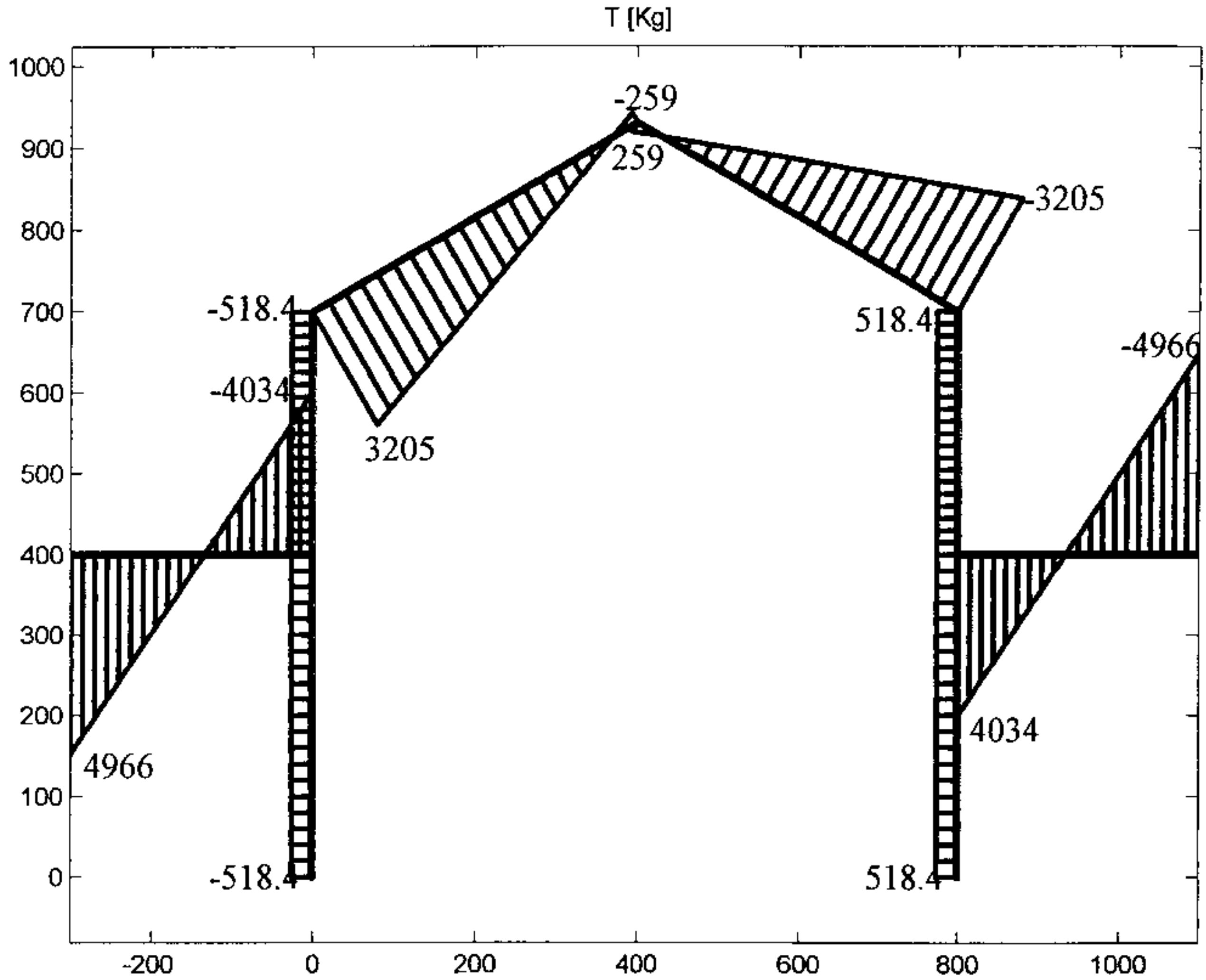
Il punto più sollecitato è P (vedi figura) con:

$$\sigma_{id} = \sqrt{(-2209)^2 + 3 \cdot (6.6)^2} \cong 2209 \ \text{kg/cm}^2 < 2400 \ \text{kg/cm}^2$$

PROVA A

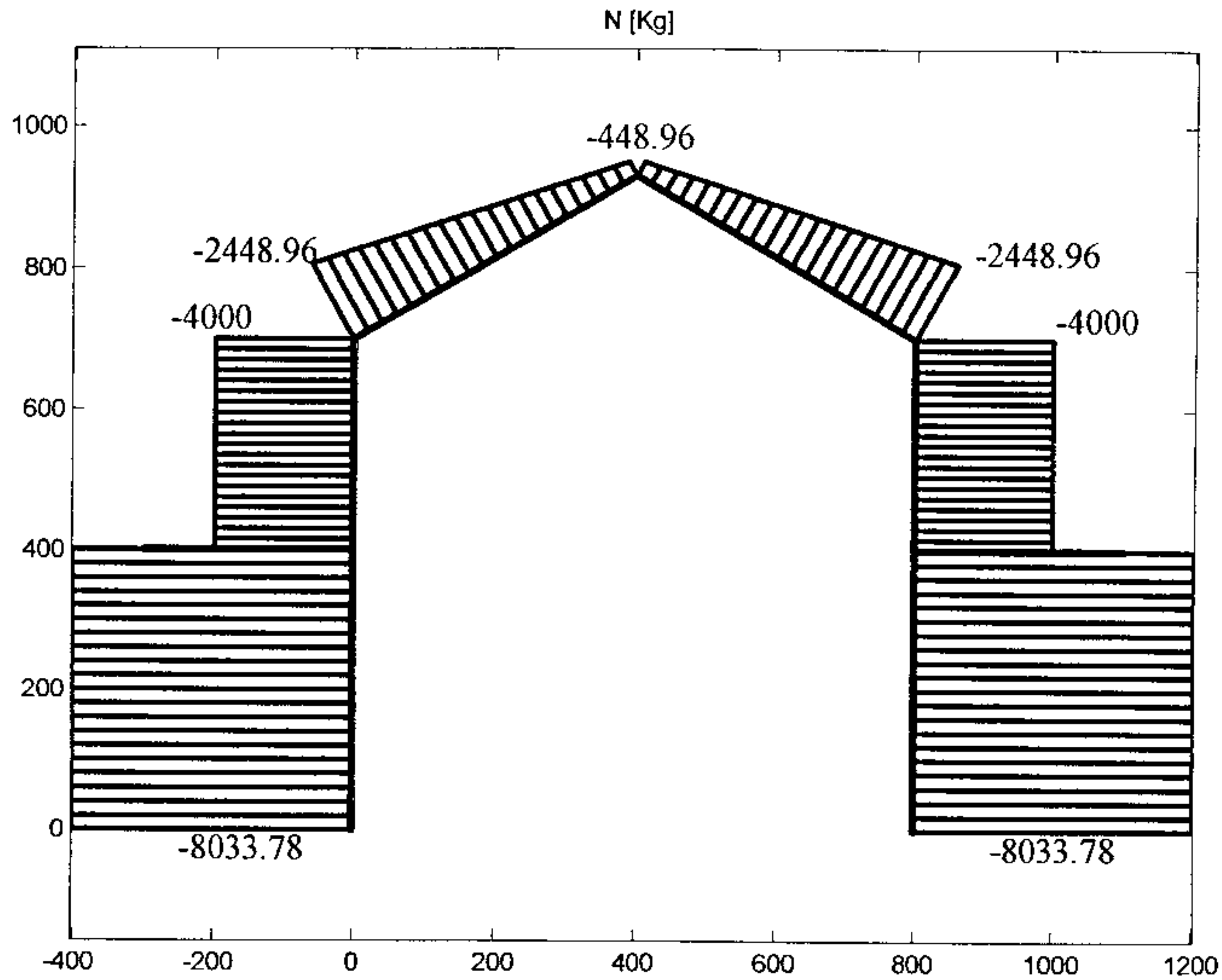

M [Kg cm]
1000
900
800
700
600
500
400
300
200
100
0
-200
0
200
400
600
800
1000
0
0
680278
680278
684755
684755
155523
155523
15659
15659
139864
139864
411052
411052
223023
223023

PROVA A

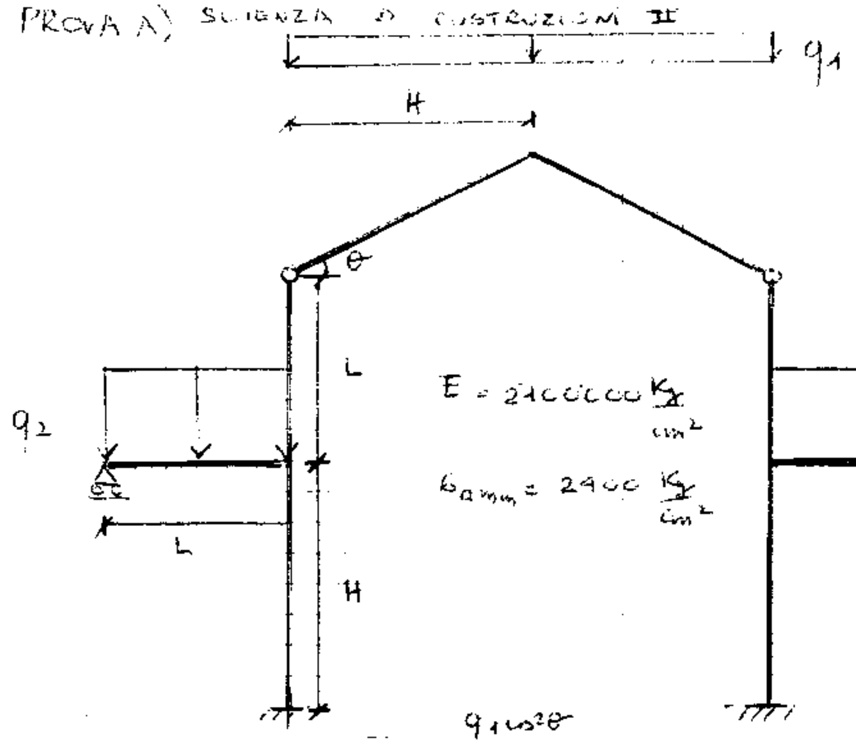


T [Kg]
-259
259
-3205
-518.4
518.4
-4966
-4034
3205
4966
4034
-518.4
518.4
1000
900
800
700
600
500
400
300
200
100
0
-200
0
200
400
600
800
1000

PROVA A


N [Kg]
-448.96
-2448.96
-2448.96
-4000
-4000
-8033.78
-8033.78
1000
800
600
400
200
0
-400
-200
0
200
400
600
800
1000
1200

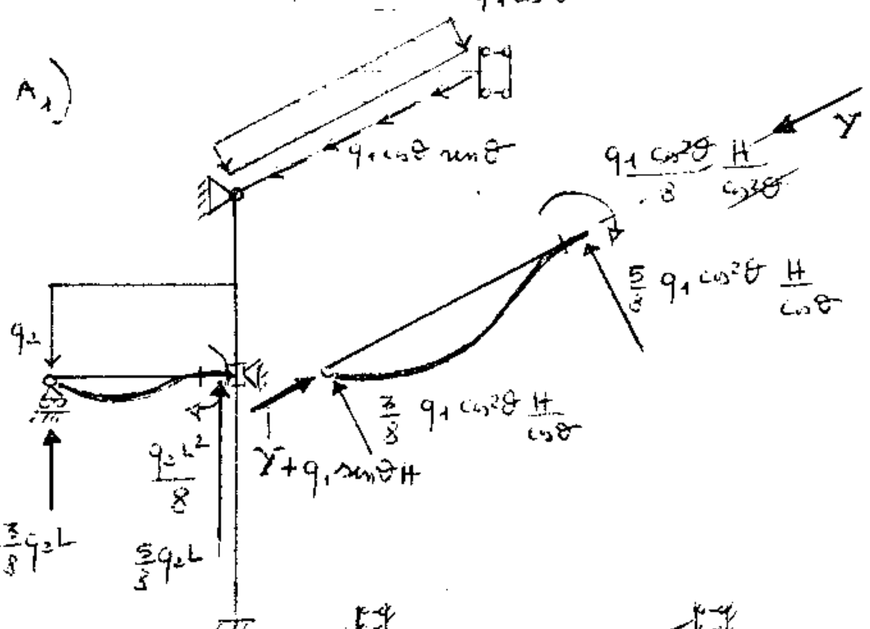
PROVA A) SCIENZA DI COSTRUZIONI II GIUGNO 2002

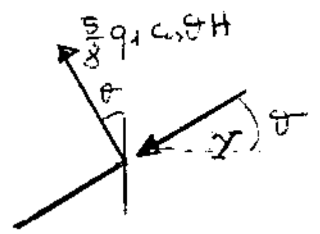


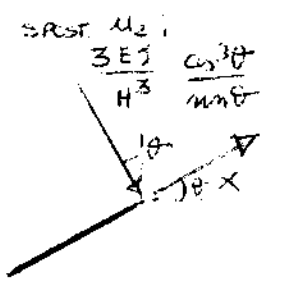
PROVA A	PROVA B
L = 300 cm	L = 400 cm
H = 400 cm	H = 300 cm
$\theta = 30°$	$\theta = 30°$
$q_1 = 1000$ Kg/m	$q_1 = 1500$ Kg/m
$q_2 = 3000$ Kg/m	$q_2 = 4000$ Kg/m

CARICHI.

A₁)





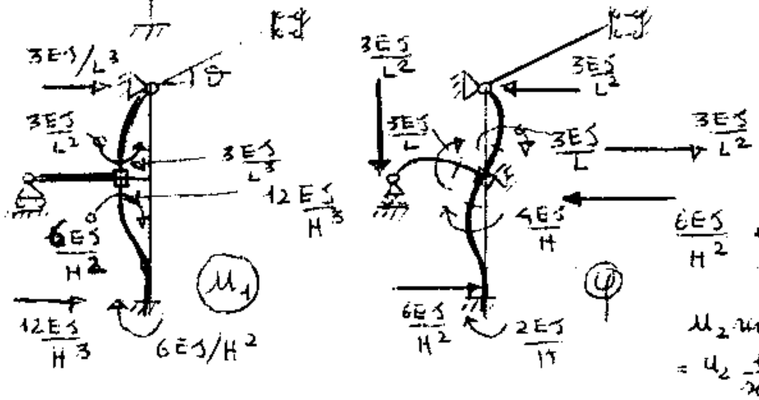


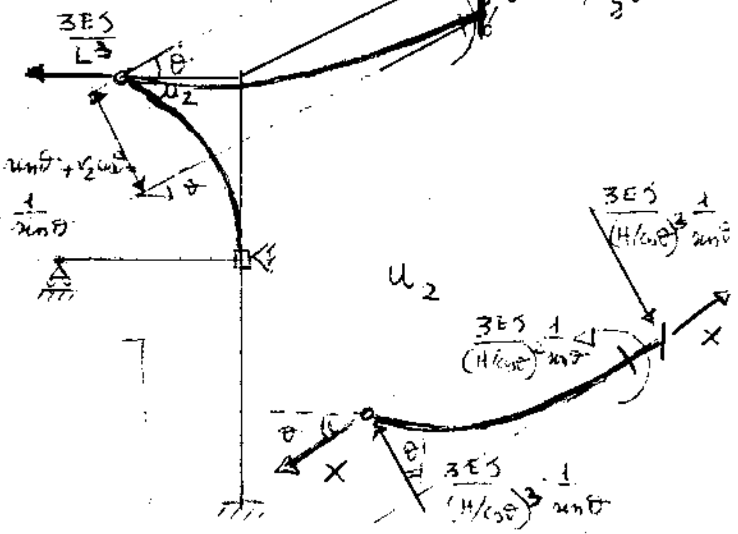
CONDIZIONE PER IL DOPPIO-PENDOLO:

$$Y \sin\theta = \frac{5}{8} q_1 H \cos^2\theta$$

$$Y = \frac{5}{8} q_1 H \frac{\cos^2\theta}{\sin\theta}$$

$$X \sin\theta = \frac{3EJ}{H^3} \frac{\cos^3\theta}{\sin\theta} \cos\theta$$





MATRICE DI RIGIDEZZA:

$$\underline{K} = EJ \begin{bmatrix} \frac{3}{L^3} + \frac{12}{H^3} & \frac{6}{H^2} - \frac{3}{L^2} & -\frac{3}{L^3} \\ \frac{6}{H^2} - \frac{3}{L^2} & \frac{4}{H} + \frac{6}{L} & \frac{3}{L^2} \\ -\frac{3}{L^3} & \frac{3}{L^2} & \frac{3}{H^3}\left(\frac{\cos^3\theta}{\sin\theta}\sin\theta + \frac{\cos^5\theta}{\sin^2\theta}\right) + \frac{3}{L^3} \end{bmatrix}$$

MATRICE DELLE FORZE:

$$\underline{F}^T = \left[\, 0 \;\middle|\; -\frac{q_2 L^2}{8} - \frac{3}{8} q_1 H \cos\theta \sin\theta + q_1 H \sin\theta \;\middle|\; + \frac{5}{8} q_1 H \cos^3\theta / \sin\theta \,\right]$$

$$\begin{bmatrix} \frac{3}{L^3}+\frac{12}{H^3} & \frac{6}{H^2}-\frac{3}{L^2} & -\frac{3}{L^3} \\ \frac{6}{H^2}-\frac{3}{L^2} & \frac{4}{H}+\frac{6}{L} & \frac{3}{L^2} \\ -\frac{3}{L^3} & \frac{3}{L^2} & \frac{3}{L^3}+\frac{3}{H^3}\frac{\cos^3\theta}{\sin^2\theta} \end{bmatrix} \begin{bmatrix} ESu_1 \\ ES\varphi \\ ESu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{q_2 L^2}{8} \\ \underbrace{\frac{5}{8}q_1 H\frac{\cos^3\theta}{\sin\theta}+\frac{5}{8}q_1 H\sin\theta\cos\theta}_{\frac{5}{8}q_1 H\frac{\cos\theta}{\sin\theta}} \end{bmatrix}$$

$$\Rightarrow \begin{cases} ESu_1 = q_1\cdot 1{,}23121\cdot 10^9\ \mathrm{Kg\,cm^3} \\ ES\varphi = q_1(-4{,}77364\cdot 10^6)\ \mathrm{Kg\,cm^2} \\ ESu_2 = q_1(3{,}12988\cdot 10^9)\ \mathrm{Kg\,cm^3} \end{cases} \qquad q_1 = 10\ \frac{\mathrm{Kg}}{\mathrm{cm}} \qquad q_2 = 30\ \frac{\mathrm{Kg}}{\mathrm{cm}}$$

MOMENTI:

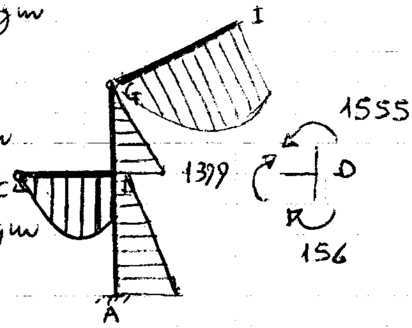
$$M_{AD} = \frac{6ES}{H^2}u_1 + \frac{2ES}{H}\varphi = +223022\ \mathrm{Kg\cdot cm} = 2230\ \mathrm{Kg\,m}$$

$$M_{DA} = \frac{6ES}{H^2}u_1 + \frac{4ES}{H}\varphi = -15660\ \mathrm{Kg\cdot cm} = -156{,}6\ \mathrm{Kg\,m}$$

$$M_{DC} = \frac{q_2 L^2}{8} + \frac{3ES}{L}\varphi = -139864\ \mathrm{Kg\cdot cm} = -1398{,}6\ \mathrm{Kg\,m}$$

$$M_{DG} = \frac{3ES}{L^2}u_1 - \frac{3ES}{L}\varphi - \frac{3ES}{L^2}u_2 = -155526\ \mathrm{Kg\cdot cm} = -1555\ \mathrm{Kg\,m}$$

$$M_{IG} = \frac{q_1 H^2}{8} - \frac{3ES}{H^2}\frac{\cos^2\theta}{\sin\theta}u_2 = -680279\ \mathrm{Kg\cdot cm} = -6802{,}79\ \mathrm{Kg\,m}$$



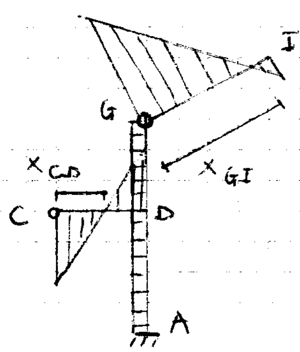
TAGLI:

$$T_{AD} = \frac{12}{H^3}ESu_1 + \frac{6}{H^2}ES\varphi = 518{,}4\ \mathrm{Kg}$$

$$T_{DA} = T_{AD}$$

$$T_{DC} = \frac{5q_2 L}{8} + \frac{3}{L^2}ES\varphi = 4033{,}79\ \mathrm{Kg}$$

$$T_{DG} = \frac{3}{L^3}ESu_1 - \frac{3}{L^2}ES\varphi - \frac{3}{L^3}ESu_2 = -518{,}42\ \mathrm{Kg}$$



$$T_{GI} = \frac{3}{8}q_1\cos\theta H + \frac{3ES}{H^3}u_2\frac{\cos^3\theta}{\sin\theta} = 3204{,}9\ \mathrm{Kg}$$

$$T_{IG} = \frac{5}{8}q_1\cos\theta H - \frac{3ES}{H^3}u_2\frac{\cos^3\theta}{\sin\theta} = 259{,}2\ \mathrm{Kg}$$

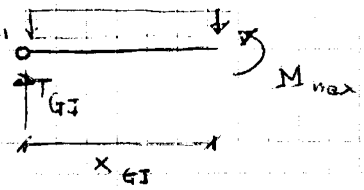
$$T_{CD} = \frac{3}{8}q_2 L - \frac{3}{L^2}ES\varphi = 40\ldots\ \mathrm{Kg}$$

MASSIMO MOMENTO IN CAMPATA:

a) CAMPATA GI:

$$X_{GI} = \left(T_{GI} + T_{IG}\right)^{-1} T_{GI} \frac{H}{\cos\theta} = 427 \text{ cm}$$

$$M_{max} = T_{GI} X_{GI} - \frac{q_1 \cos^2\theta \, X_{GI}^2}{2} = 684755 \text{ Kg cm}$$



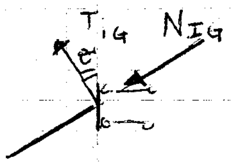
b) CAMPATA CD:

$$X_{CD} = \left(T_{CD} + T_{DC}\right)^{-1} T_{CD} \, L = 165,53 \text{ cm}$$

$$M_{max} = T_{CD} X_{CD} - \frac{q_2 X_{CD}^2}{2} = 411052 \text{ Kg cm}$$

SFORZI NORMALI:

$$N_{IG} = \frac{T_{IG}}{\tan\theta} = -448,95 \text{ Kg}$$

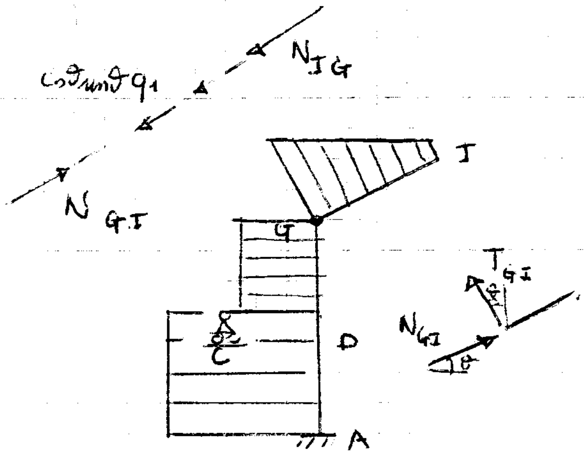


$$N_{IG} \sin\theta = T_{IG} \cos\theta$$

$$N_{GI} = q_1 H \sin\theta + N_{IG} = -2448,95 \text{ Kg}$$

$$N_{GD} = q_1 H = -4000 \text{ Kg}$$

$$\underline{\underline{|}} \; T_{GI} \cos\theta + N_{GI} \sin\theta$$



$$N_{CD} = 0$$

$$N_{DA} = N_{GD} + T_{DC} = 8033,79 \text{ Kg}$$

PROGETTO: $M_{max} = 684755 \text{ Kg} \cdot \text{cm}$

$$W_{min} = \frac{M_{max}}{\sigma_{adm}} = 285,3 \text{ cm}^3 \Rightarrow \boxed{\text{HEB } 160}$$ piena A

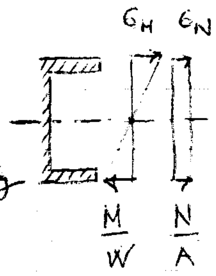
A2) VERIFICA NELLA SEZIONE MAGGIORMENTE SOLLECITATA A FLESSIONE:

A	W_x	
54,3 cm^2	311 cm^3	

$$M_{max} = 684755 \text{ Kg} \cdot \text{cm}$$

$$N = N_{IG} + \frac{\left(N_{GI} - N_{GI}\right)\left(\frac{H}{\cos\theta} - X_{GI}\right)}{\frac{H}{\cos\theta}} = 600 \text{ Kg}$$

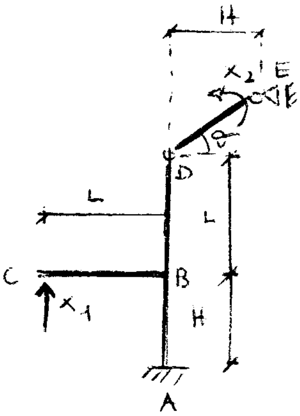
$$q_1 \cos\theta \sin\theta \left(\frac{H}{\cos\theta} - X_{GI}\right)$$

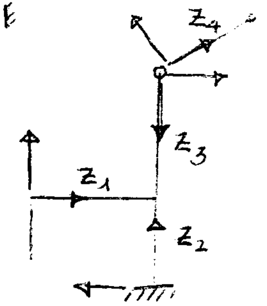


$$\sigma_{max} = \frac{M}{W_x} + \frac{N}{A} = 2213 \frac{\text{Kg}}{\text{cm}^2} < \sigma_{adm} \Rightarrow \text{VERIFICATO}$$

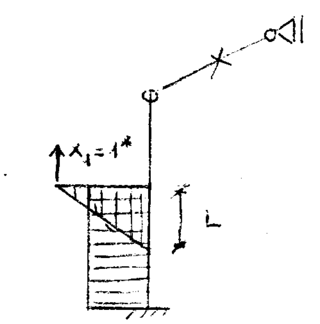
PROVA B) SCIENZA D COSTRUZIONI II — GIUGNO 2002

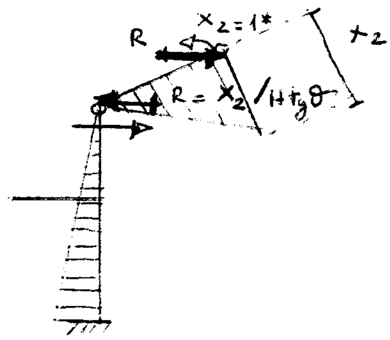
METODO DELLE FORZE:



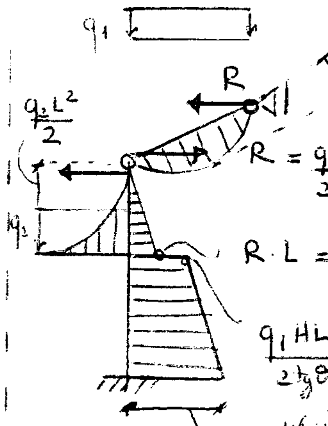


	PROVA A	PROVA B
$\theta = 30°$	$H = 4m$	$H = 3m$
$\sigma_{adm} = 2400\ Kg/cm^2$	$L = 3m$	$L = 4m$
$E = 2100000\ Kg/cm^2$	$q_1 = 1000\ Kg/m$	$q_1 = 1500\ Kg/m$
	$q_2 = 3000\ Kg/m$	$q_2 = 4000\ Kg/m$





$$R(H+L) = \frac{x_2}{tg\theta}\frac{H+L}{H}$$



$$I \quad \frac{q_1 \cancel{\cos^2\theta}}{8}\frac{H^2}{\cancel{\cos^2\theta}} = \frac{q_1 H^2}{8}$$

$$R = \frac{q_1 H}{2\,tg\theta} \qquad \frac{q_2 L^2}{2} \qquad \frac{q_1 HL}{2\,tg\theta}$$

$$R \cdot L = q_1 \frac{HL}{2\,tg\theta}$$

$$\frac{q_1 HL}{2\,tg\theta} + \frac{q_2 L^2}{2}$$

$$\frac{q_1 H(L+H)}{2\,tg\theta} + \frac{q_2 L^2}{2}$$

LUNGH. ASTA			
L	$M_1 = -z_1$	$M_1 = 0$	$M_1 = \frac{q_2 z_1^2}{2}$
H	$M_2 = L$	$M_2 = \frac{1}{tg\theta}\left(\frac{H+L}{H} - \frac{z_2}{H}\right)$	$M_2 = -\left[\frac{q_1 \cdot H(L+H)}{2\,tg\theta} + \frac{q_2 L^2}{2}\right] + \frac{q_1 H}{2\,tg\theta} z_2$
L	$M_3 = 0$	$M_3 = -\frac{1}{tg\theta}\frac{z_3}{H}$	$M_3 = \frac{q_1 H}{2\,tg\theta} z_3$
$\frac{H}{\cos\theta}$	$M_4 = 0$	$M_4 = -\frac{z_4}{H/\cos\theta}$	$M_4 = q_1 \cos^2\theta \cdot \left(\frac{z_4^2}{2} - \frac{z_4 H}{2\cos\theta}\right)$

$$ES\,\eta_{11} = \int_0^L z_1^2\, dz_1 + \int_0^H L^2 dz_2 = \frac{L^3}{3} + L^2 H$$

$$ES\,\eta_{22} = \int_0^H \frac{1}{tg^2\theta H^2}(H+L-z_2)^2 dz_2 + \int_0^L \frac{1}{tg^2\theta H^2} z_3^2 dz_3 + \int_0^{H/\cos\theta} \frac{z_4^2}{(H/\cos\theta)^2} dz_4 =$$

$$= \frac{(-1)}{tg^2\theta H^2} \cdot \left.\frac{(H+L-z_2)^3}{3}\right|_0^H + \frac{L^3}{3H^2 tg^2\theta} + \frac{H}{3\cos\theta} =$$

$$= \frac{1}{3H^2 tg^2\theta}\left(\cancel{L^3} + (H+L)^3 - \cancel{L^3}\right) + \frac{H}{3\cos\theta} =$$

$$= \frac{(H+L)^3}{3H^2 tg^2\theta} + \frac{H}{3\cos\theta}$$

$$ES\,\eta_{12} = \int_0^H \frac{L}{H\,tg\theta}(H+L-z_2)dz_2 = \frac{-L}{H\,tg\theta}\left.\frac{(H+L-z_2)^2}{2}\right|_0^H = \frac{L}{2H\,tg\theta}\left[(H+L)^2 - L^2\right] = \frac{HL/2 + L^2}{tg\theta}$$

$$ES\,\eta_{10} = \int_0^L -z_1 \cdot \frac{q_2 z_1^2}{2} dz_1 + \int_0^H L \cdot \left[\frac{q_1 H}{2tg\theta} z_2 - \left(\frac{q_1 H (L+H)}{2tg\theta} + \frac{q_2 L^2}{2} \right) \right] dz_2 =$$

$$= -\frac{q_2 L^4}{8} + L \cdot \left[\frac{q_1 H}{2tg\theta} \frac{z_2^2}{2} - z_2 \left(\frac{q_1 H (L+H)}{2tg\theta} + \frac{q_2 L^2}{2} \right) \right]_0^H =$$

$$= -\frac{q_2 L^4}{8} + L \left[\frac{q_1 H^3}{4tg\theta} - \frac{q_1 H^2 (L+H)}{2tg\theta} - \frac{q_2 L^2 H}{2} \right] =$$

$$= q_2 \left(-\frac{L^3 H}{2} - \frac{L^4}{8} \right) + \frac{q_1 H^2 L}{2tg\theta} \cdot \left(\frac{H}{2} - L - H \right) =$$

$$= q_2 \frac{L^3}{2} \cdot \left(-H - \frac{L}{4} \right) - \frac{q_1 H^2 L}{2tg\theta} \left(L + \frac{H}{2} \right)$$

$$ES\,\eta_{20} = \int_0^H \frac{1}{Htg\theta} \cdot (H+L-z_2) \cdot \left[\frac{q_1 H}{2tg\theta} z_2 - \left(\frac{q_1 H(L+H)}{2tg\theta} + \frac{q_2 L^2}{2} \right) \right] dz_2 +$$

$$+ \int_0^L \left(-\frac{1}{tg\theta} \frac{z_3}{\cancel{H}} \right) \frac{q_1 \cancel{H}}{2tg\theta} z_3 \, dz_3 + \int_0^{H/\cos\theta} \left(-\frac{z_4}{H/\cos\theta} \right) q_1 \cos^2\theta \left(\frac{z_4^2}{2} - \frac{z_4 H}{2\cos\theta} \right) dz_4 =$$

$$= \frac{H+L}{Htg\theta} \cdot \left[\frac{q_1 H}{4tg\theta} z_2^2 - z_2 \cdot \left(\frac{q_1 H (L+H)}{2tg\theta} + \frac{q_2 L^2}{2} \right) \right]_0^H - \frac{1}{Htg\theta} \cdot \left[\frac{q_1 H}{6tg\theta} z_2^3 - \frac{z_2^2}{2} \left(\frac{q_1 H (L+H)}{2tg\theta} + \frac{q_2 L^2}{2} \right. \right.$$

$$- \frac{q_1}{2tg^2\theta} \left. \frac{z_3^3}{3} \right|_0^L - \frac{q_1 \cos^3\theta}{H} \left[\frac{z_4^4}{8} - \frac{z_4^3 H}{6\cos\theta} \right]_0^{H/\cos\theta} =$$

$$= \frac{H+L}{Htg\theta} \cdot \left[\frac{q_1 H^3}{4tg\theta} - \frac{q_1 H^2 (L+H)}{2tg\theta} - \frac{q_2 L^2 H}{2} \right] - \frac{1}{Htg\theta} \cdot \left[\frac{q_1 H^4}{6tg\theta} - \frac{H^2}{2} \cdot \left(\frac{q_1 H (L+H)}{2tg\theta} + \frac{q_2 L^2}{2} \right) \right]$$

$$- \frac{q_1 L^3}{6tg^2\theta} - \frac{q_1 \cos^3\theta}{H} \cdot \left(\frac{H^4}{8\cos^4\theta} - \frac{H^4}{6\cos^4\theta} \right) =$$

$$= \frac{H+L}{Htg\theta} \cdot \left[-\frac{q_1 H^3}{4tg\theta} - \frac{q_1 H^2 L}{2tg\theta} - \frac{q_2 L^2 H}{2} \right] - \frac{H^2}{Htg\theta} \left[\frac{q_1 H^2}{6tg\theta} - \frac{q_1 H (L+H)}{4tg\theta} - \frac{q_2 L^2}{4} \right] -$$

$$- \frac{q_1 L^3}{6tg^2\theta} - \frac{q_1 H^3}{\cos\theta} \cdot \left(\frac{-1}{24} \right) =$$

$$E J \eta_{10} = \int_0^L -z_1 \cdot \frac{q_2 z_1^2}{2}\, dz_1 + \int_0^H L \cdot \left[\frac{q_1 H}{2 tg\theta} z_2 - \left(\frac{q_1 H (L+H)}{2 tg\theta} + \frac{q_2 L^2}{2} \right) \right] dz_2 =$$

$$= -\frac{q_2 L^4}{8} + L \cdot \left[\frac{q_1 H}{2 tg\theta} \frac{z_2^2}{2} - z_2 \left(\frac{q_1 H (L+H)}{2 tg\theta} + \frac{q_2 L^2}{2} \right) \right]_0^H =$$

$$= -\frac{q_2 L^4}{8} + L \left[\frac{q_1 H^3}{4 tg\theta} - \frac{q_1 H^2 (L+H)}{2 tg\theta} - \frac{q_2 L^2 H}{2} \right] =$$

$$= q_2 \left(-\frac{L^3 H}{2} - \frac{L^4}{8} \right) + \frac{q_1 H^2 L}{2 tg\theta} \cdot \left(\frac{H}{2} - L - H \right) =$$

$$= \frac{q_2 \cdot L^3}{2} \cdot \left(-H - \frac{L}{4} \right) - \frac{q_1 H^2 L}{2 tg\theta} \left(L + \frac{H}{2} \right)$$

$$E J \eta_{20} = \int_0^H \frac{1}{H tg\theta} \cdot (H + L - z_2) \cdot \left[\frac{q_1 H}{2 tg\theta} z_2 - \left(\frac{q_1 H (L+H)}{2 tg\theta} + \frac{q_2 L^2}{2} \right) \right] dz_2 +$$

$$+ \int_0^L \left(-\frac{1}{tg\theta} \frac{z_3}{\cancel{H}} \right) \frac{q_1 \cancel{H}}{2 tg\theta} z_3\, dz_3 + \int_0^{H/\cos\theta} \left(-\frac{z_4}{H/\cos\theta} \right) q_1 \cos^2\theta \left(\frac{z_4^2}{2} - \frac{z_4 H}{2 \cos\theta} \right) dz_4 =$$

$$= \frac{H+L}{H tg\theta} \cdot \left[\frac{q_1 H}{4 tg\theta} z_2^2 - z_2 \left(\frac{q_1 H (L+H)}{2 tg\theta} + \frac{q_2 L^2}{2} \right) \right]_0^H - \frac{1}{H tg\theta} \cdot \left[\frac{q_1 H}{6 tg\theta} z_2^3 - \frac{z_2^2}{2} \left(\frac{q_1 H (L+H)}{2 tg\theta} + \frac{q_2 L^2}{2} \right) \right]_0^H$$

$$- \frac{q_1}{2 tg^2\theta} \left. \frac{z_3^3}{3} \right|_0^L - \frac{q_1 \cos^3\theta}{H} \left[\frac{z_4^4}{8} - \frac{z_4^3 H}{6 \cos\theta} \right]_0^{H/\cos\theta} =$$

$$= \frac{H+L}{H tg\theta} \cdot \left[\frac{q_1 H^3}{4 tg\theta} - \frac{q_1 H^2 (L+H)}{2 tg\theta} - \frac{q_2 L^2 H}{2} \right] - \frac{1}{H tg\theta} \cdot \left[\frac{q_1 H^4}{6 tg\theta} - \frac{H^2}{2} \cdot \left(\frac{q_1 H (L+H)}{2 tg\theta} + \frac{q_2 L^2}{2} \right) \right]$$

$$- \frac{q_1 L^3}{6 tg^2\theta} - \frac{q_1 \cos^3\theta}{H} \cdot \left(\frac{H^4}{8 \cos^4\theta} - \frac{H^4}{6 \cos^4\theta} \right) =$$

$$= \frac{H+L}{H tg\theta} \cdot \left[-\frac{q_1 H^3}{4 tg\theta} - \frac{q_1 H^2 L}{2 tg\theta} - \frac{q_2 L^2 H}{2} \right] - \frac{H^2}{H tg\theta} \left[\frac{q_1 H^2}{6 tg\theta} - \frac{q_1 H (L+H)}{4 tg\theta} - \frac{q_2 L^2}{4} \right] -$$

$$- \frac{q_1 L^3}{6 tg^2\theta} - \frac{q_1 H^3}{\cos\theta} \cdot \left(\frac{-1}{24} \right) =$$

$$= \frac{q_1}{tg^2\theta}\left(-\frac{H^2}{\cancel{4}}\cancel{(H+L)} - \frac{HL}{2}(H+L) - \frac{H^3}{6} + \frac{H^2}{\cancel{4}}\cancel{(H+L)} - \frac{L^3}{6} + \frac{tg^2\theta\, H^3}{24\cos\theta}\right) +$$

$$+ \frac{q_2}{H\,tg\theta}\left(-\frac{L^2H}{2}\cdot(H+L) + \frac{H^2L^2}{4}\right) =$$

$$= \frac{q_1}{tg^2\theta}\left(-\frac{H^3+L^3+3H^2L+3HL^2}{6}\right) + \frac{q_2L^2\cancel{H}}{2\cancel{H}tg\theta}\left(\frac{H}{2}-(H+L)\right) + \frac{q_1H^3}{24\cos\theta} =$$

$$= -\frac{q_1(H+L)^3}{6\,tg^2\theta} - (L+H/2)\frac{q_2L^2}{2\,tg\theta} + \frac{H^3\cancel{tg^2\theta}}{24\cos\theta}\cdot\frac{q_1}{\cancel{tg^2\theta}}$$

$$\begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{12} & \eta_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\eta_{10} \\ -\eta_{20} \end{bmatrix}$$

PROVA A:

$X_1 = 4966,21$ Kg

$X_2 = 6802,78$ Kg m

PROVA B

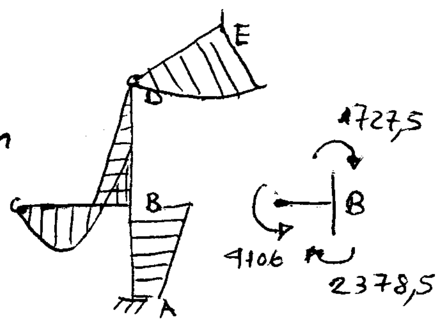
$X_1 = 6973,5$ Kg

$X_2 = 7498,03$ Kg m

PER IL CASO B:

MOMENTI:

$$M_{AB} = x_1L + \frac{x_2}{tg\theta}\frac{H+L}{H} - \left(\frac{q_1H(L+H)}{2tg\theta} + \frac{q_2L^2}{2}\right) = -1082,87 \text{ Kg m}$$



$$M_{BA} = M_{AB} + \frac{q_1H^2}{2tg\theta} - \frac{X_2}{tg\theta} = -2378,5 \text{ Kg m}$$

$$M_{BC} = x_1L - \frac{q_2L^2}{2} = -4106 \text{ Kg m}$$

$$M_{BD} = \frac{q_1HL}{2tg\theta} - \frac{X_2L}{H\,tg\theta} = -1727,5 \text{ Kg m}$$

$$M_{ED} = X_2 = 7498,03$$

TAGLI:

$$T_{DE} = \frac{X_2}{H/\cos\theta} + q_1\frac{\cos^2\theta}{2}\frac{H}{\cos\theta} = 4133 \text{ Kg}$$

$$T_{ED} = -\frac{X_2}{H/\cos\theta} + q_1\frac{\cos^2\theta}{2}\frac{H}{\cos\theta} = -216 \text{ Kg}$$

$$T_{CB} = X_1 = 6973,5 \text{ Kg}$$

$$T_{BC} = X_1 - q_2L = -9026 \text{ Kg}$$

$$T_{DB} = \frac{X_2}{H\,tg\theta} - \frac{q_1H}{2tg\theta} = 431,9$$

MASSIMO MOMENTO IN CAMPATA CB:

$$M_{max} = T_{CB}\, x_{max} - q_2 x_{max}^2/2 = 6077{,}84\ \mathrm{Kg}$$

$$x_{max} = \left[\frac{(T_{CB} + T_{BC})}{T_{CB}}\right]^{-1} L = 174{,}3\ \mathrm{cm}$$

SFORZI NORMALI:

$$N_{CB} = 0$$

$$N_{ED} = -\left(\frac{q_1 H}{2\,\mathrm{tg}\vartheta} - \frac{X_2}{H\,\mathrm{tg}\vartheta}\right)\cos\vartheta = 374\ \mathrm{Kg}$$

$$N_{DE} = N_{ED} - q_1 \cancel{\cos\vartheta}\,\mathrm{sen}\,\vartheta\,\frac{H}{\cancel{\cos\vartheta}} = 1876\ \mathrm{Kg}$$

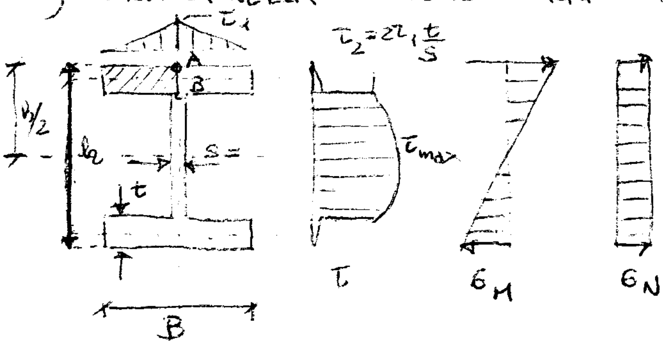
$$N_{DB} = q_1 H = 4500\ \mathrm{Kg}$$

$$N_{BA} = N_{DB} + T_{BC} = 13526\ \mathrm{Kg}$$

PROGETTO:

$M_{max} = 749803\ \mathrm{Kg\cdot cm}$ $\quad W_{min} = \dfrac{M_{max}}{\sigma_{adm}} = 312{,}4\ \mathrm{cm}^3 \Rightarrow$ HEB 180

A2) VERIFICA NELLA SEZIONE MAGGIORMENTE SOLLECITATA A FLESSIONE



$M = 749803\ \mathrm{Kg\cdot cm}$

$N = 374\ \mathrm{Kg}$

$T = 216\ \mathrm{Kg}$

HEB 180

A	h	B	t	s	J	W
65,3	18	18	1,4	0,85	3831	426
cm^2	cm	cm	cm	cm	cm^4	cm^3

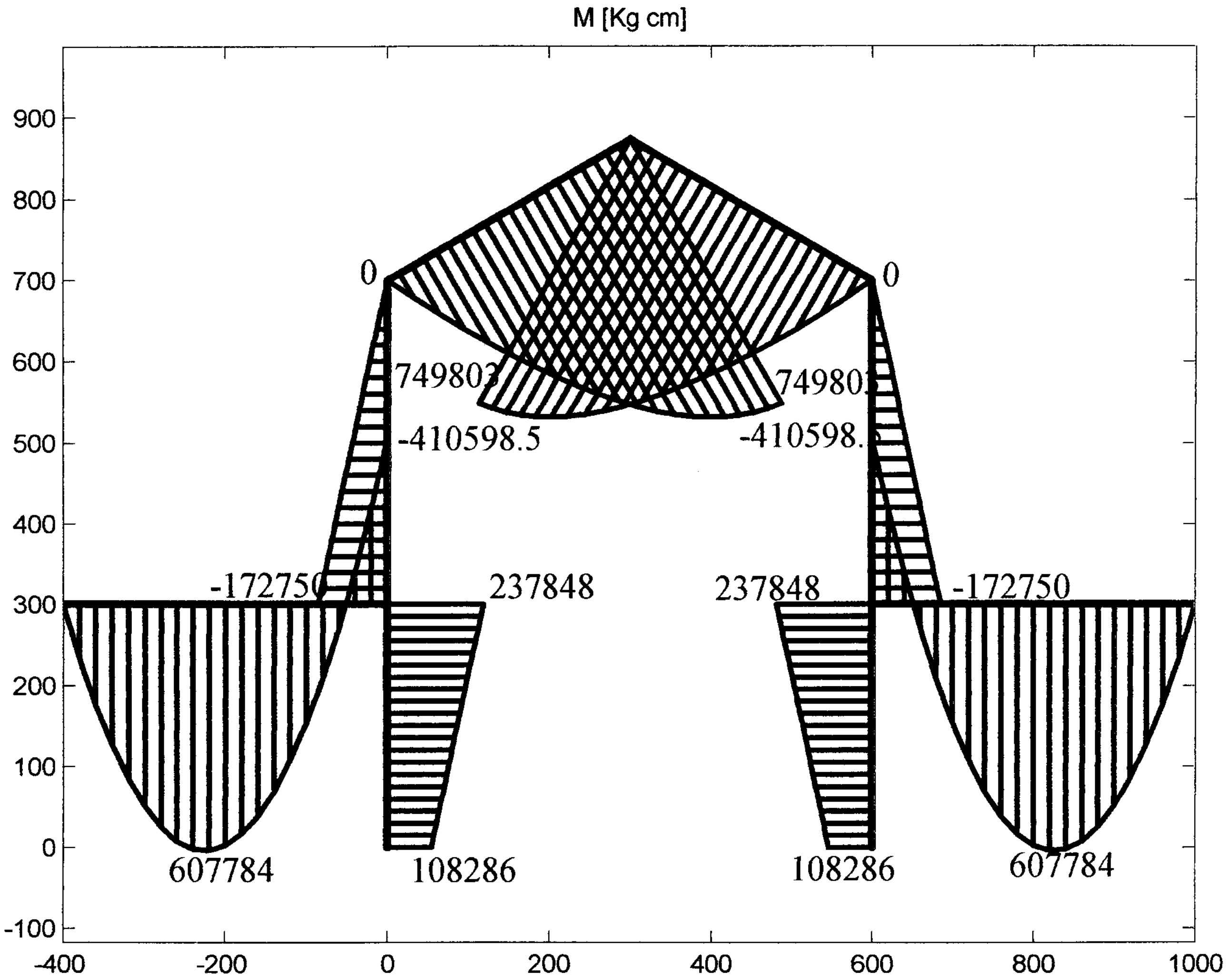
$$\tau_1 = \frac{\frac{Bt}{2}\,\frac{h-t}{2}\cdot T}{t\,J} \qquad \sigma_{M_1} = \frac{M}{W} \qquad \sigma_N = \frac{N}{A} \qquad (A)$$

$$\tau_2 = \frac{Bt\,\frac{h-t}{2}\,T}{s\,J} \qquad \sigma_{M_2} = \frac{M}{W}\cdot\frac{\frac{1}{2}h - t}{\frac{1}{2}h} \qquad \sigma_N = \frac{N}{A} \qquad (B)$$

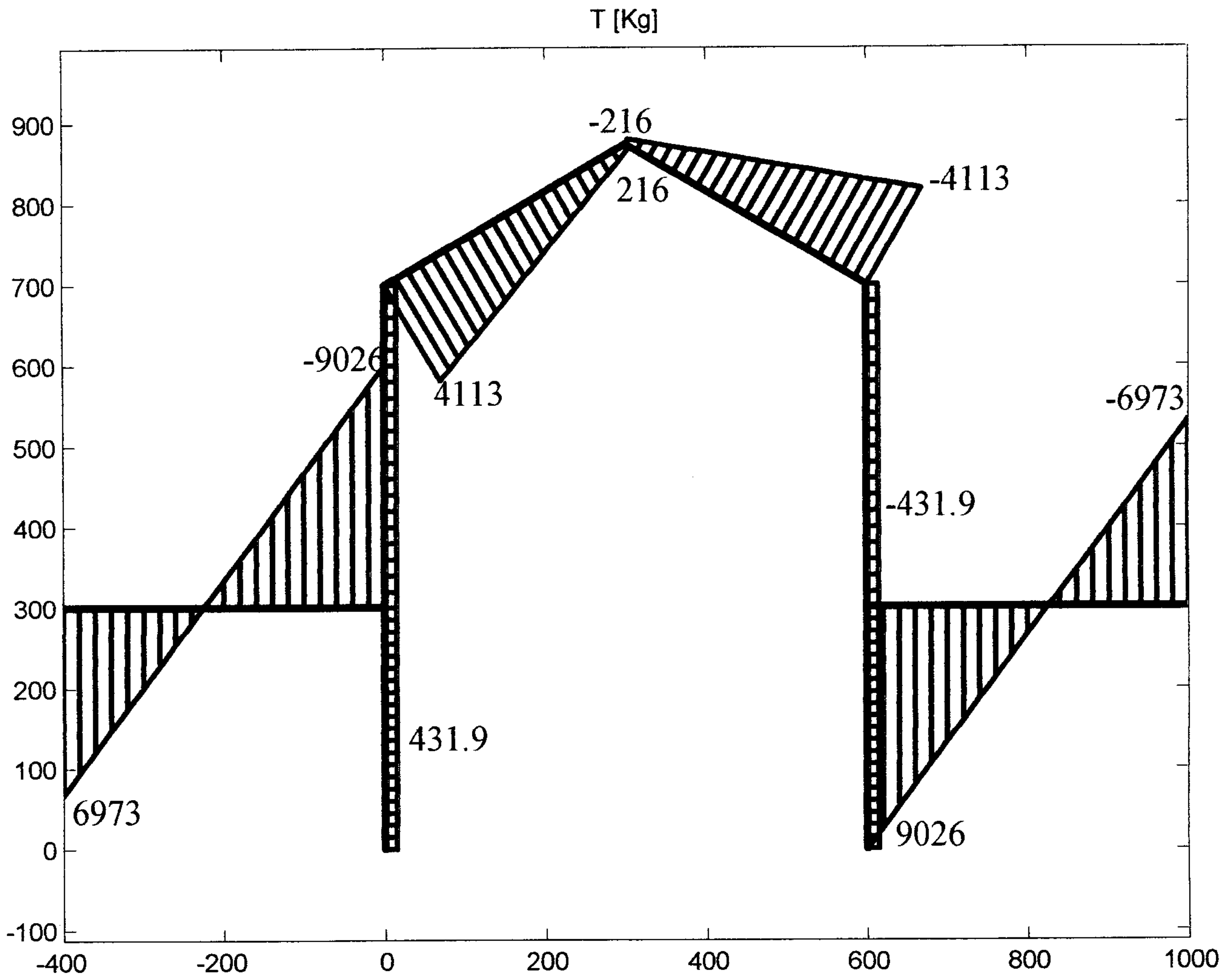
$$\sigma_{id}^{(1)} = \sqrt{(\sigma_{M_1} + \sigma_N)^2 + 3\tau_1^2} = 1766\ \frac{\mathrm{Kg}}{\mathrm{cm}^2} < \sigma_{adm} \Rightarrow \text{VERIFICATO}$$

$$\sigma_{id}^{(2)} = \sqrt{(\sigma_{M2} + \sigma_N)^2 + 3\tau_2^2} < \sigma_{id}^{(1)}$$

PROVA B

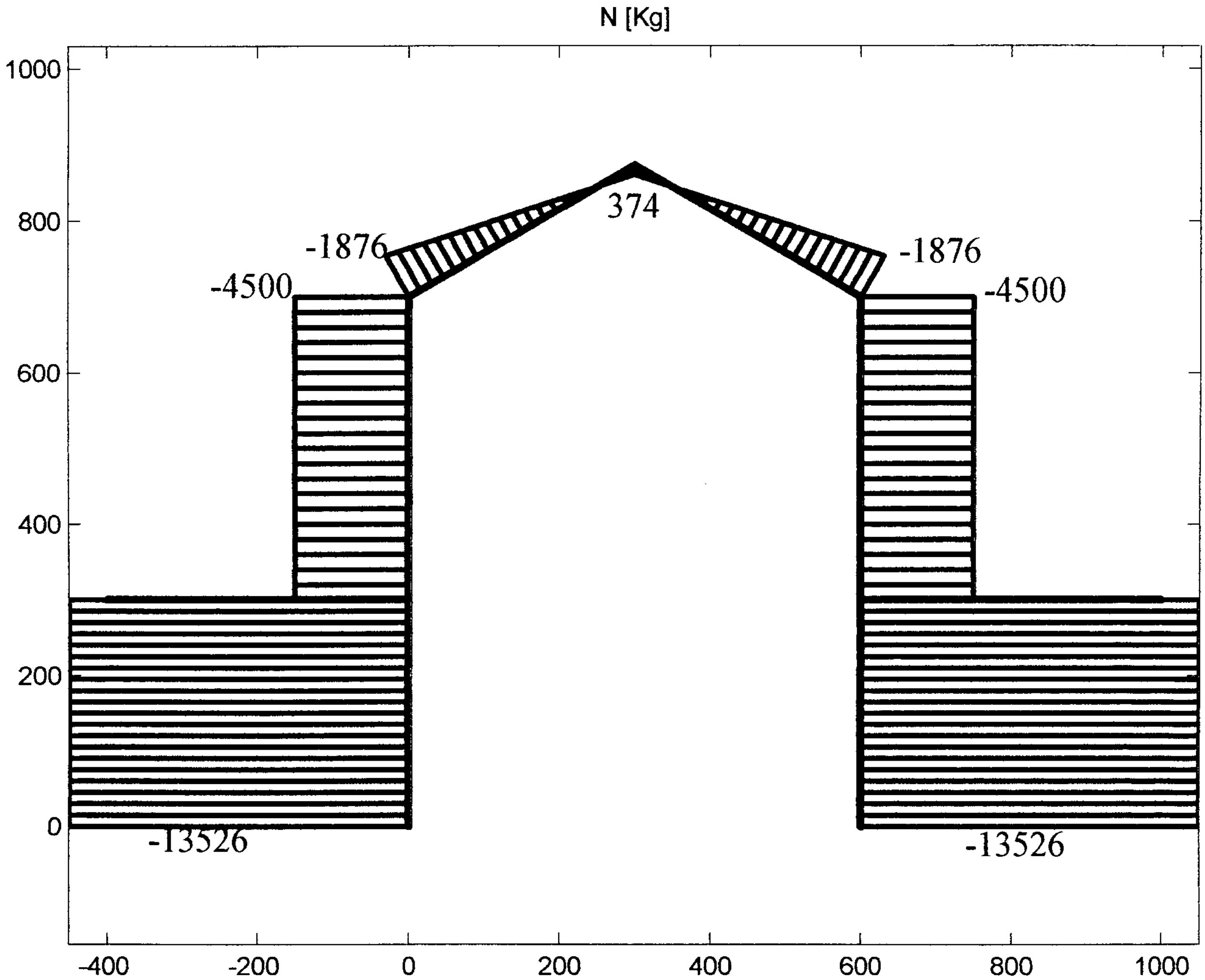

M [Kg cm]
900
800
700
600
500
400
300
200
100
0
-100
-400
-200
0
200
400
600
800
1000
0
0
749803
749803
-410598.5
-410598.5
-172750
237848
237848
-172750
607784
108286
108286
607784

PROVA B

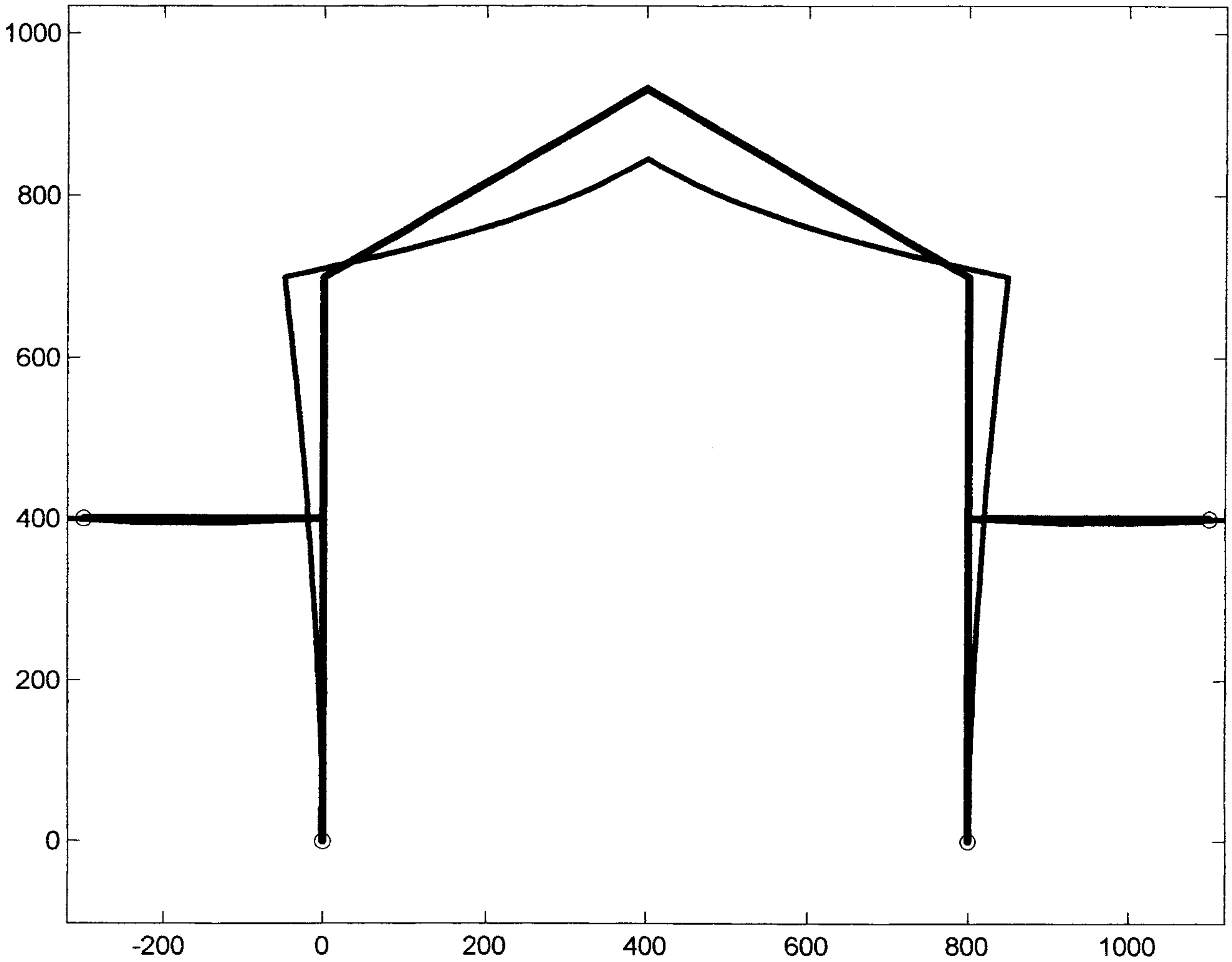


T [Kg]
-216
216
-4113
-9026
4113
-6973
-431.9
431.9
6973
9026
900
800
700
600
500
400
300
200
100
0
-100
-400
-200
0
200
400
600
800
1000

PROVA B



N [Kg]
1000
800
600
400
200
0
-400
-200
0
200
400
600
800
1000
374
-1876
-1876
-4500
-4500
-13526
-13526


1000
800
600
400
200
0
-200
0
200
400
600
800
1000