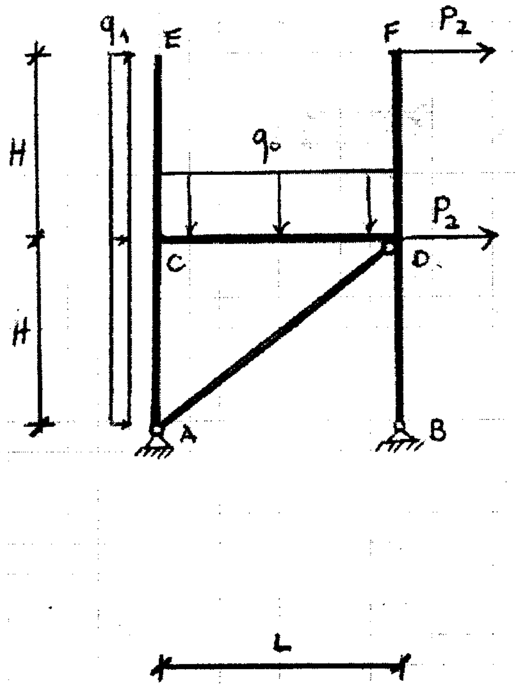
PROVA 22 MAGGIO 2002



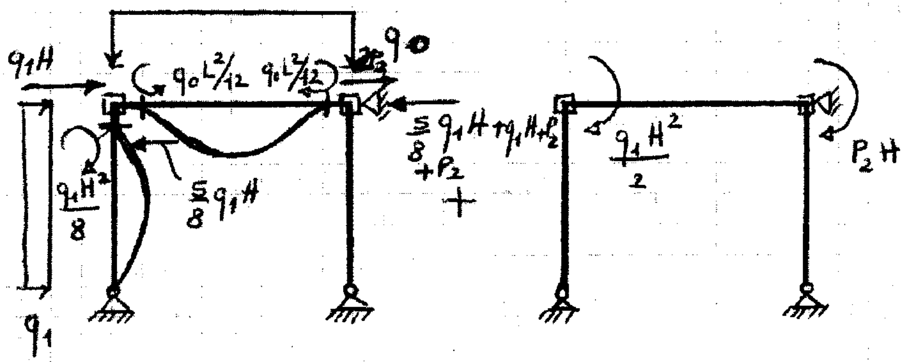
DATI,

GEOMETRIA:
$L = 400\,cm$
$H = 300\,cm$

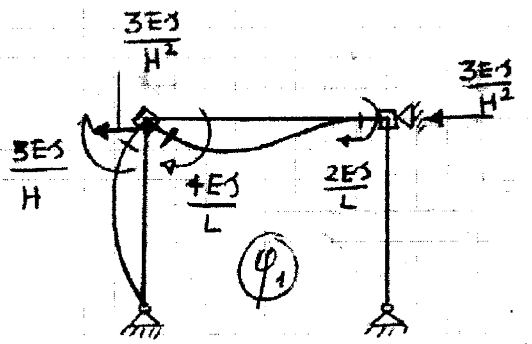
CARICHI:
$q_1 = 500\,\frac{Kg}{m} = 5\,\frac{Kg}{cm}$
$q_0 = 2000\,\frac{Kg}{m} = 20\,\frac{Kg}{cm}$
$P_2 = 1000\,Kg$

MATERIALI
$E = 2100000\,\frac{Kg}{cm^2}$
$\sigma_{am} = 2400\,\frac{Kg}{cm^2}$

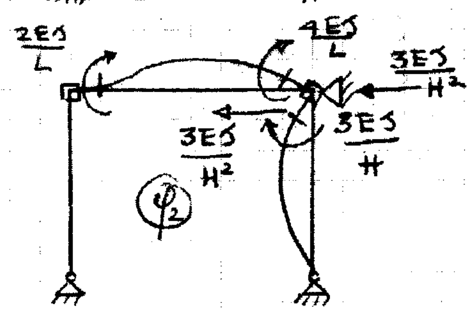
A1) PROGETTO: TRAVI EF E FD ISOSTATICHE



CARICHI NODALI



$$\begin{bmatrix} \frac{4}{L}+\frac{3}{H} & \frac{2}{L} \\ \frac{2}{L} & \frac{4}{L}+\frac{3}{H} \end{bmatrix} \cdot \begin{bmatrix} EJ\varphi_1 \\ EJ\varphi_2 \end{bmatrix} = \begin{bmatrix} \frac{q_0L^2}{12} + \frac{q_1H^2}{2} - \frac{q_1H^2}{8} \\ -\frac{q_0L^2}{12} + P_2H \end{bmatrix}$$



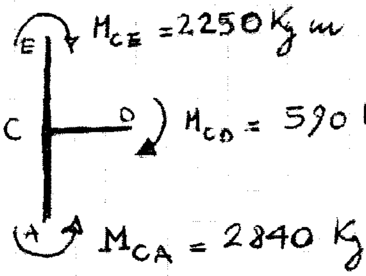
$$\Rightarrow \begin{cases} EJ\varphi_1 = 2{,}2\overline{7} \cdot 10^7\ Kg\,cm^2 \\ EJ\varphi_2 = -4{,}02\overline{7} \cdot 10^6\ Kg\,cm^2 \end{cases}$$

MOMENTI:

$$M_{CA} = \frac{3}{H} ES\varphi_1 + \frac{q_1 H^2}{8} = 284028 \ Kg\,cm = 2840 \ Kg\,m$$

$$M_{CD} = \frac{4}{L} ES\varphi_1 + \frac{2}{L} ES\varphi_2 - \frac{q_0 L^2}{12} = -59028 \ Kg\,cm = -590 \ Kg\,m$$

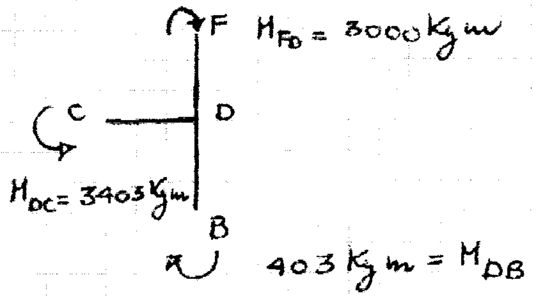
$$M_{CE} = -\frac{q_1 H^2}{2} = -225000 \ Kg\,cm = -2250 \ Kg\,m$$



$$M_{DB} = \frac{3}{H} ES\varphi_2 = -40277{,}7 \ Kg\,cm = -403 \ Kg\,m$$

$$M_{DC} = \frac{4}{L} ES\varphi_2 + \frac{2}{L} ES\varphi_1 + \frac{q_0 L^2}{12} = 340278 \ Kg\,cm = 3403 \ Kg\,m$$

$$M_{FD} = -P_2 H = -300000 \ Kg\,cm = -3000 \ Kg\,m$$



PROGETTO:

$$M_{max} = M_{DC} = 340278 \ Kg\,cm$$

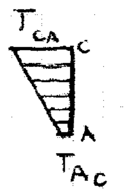
$$W_{min} = \frac{M_{max}}{\sigma_{am}} = 142 \ cm^3 \longrightarrow \text{HEA 140}$$

A	W	I
31,4	155	1033
cm^2	cm^3	cm^4

TAGLI:

$$\leftarrow T_{AC} = \frac{3}{8} q_1 H - \frac{3}{H^2} ES\varphi_1 = -197 \ Kg$$

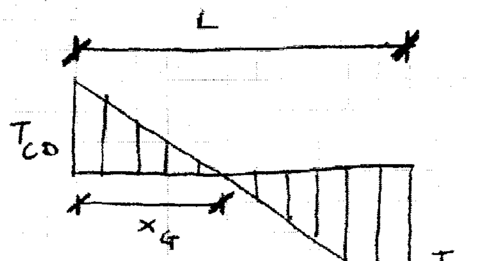
$$\leftarrow T_{CA} = \frac{3}{H^2} ES\varphi_1 + \frac{5}{8} q_1 H = 1697 \ Kg$$

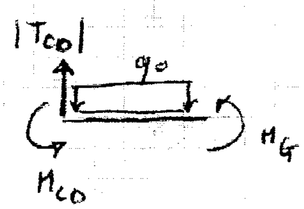


$$\leftarrow T_{EC} = q_1 H = 1500 \ Kg$$

$$\downarrow T_{CD} = \frac{6}{L^2} ES\varphi_1 + \frac{6}{L^2} ES\varphi_2 - \frac{q_0 L}{2} = -3297 \ Kg$$

$$\uparrow T_{DC} = \frac{6}{L^2} ES\varphi_1 + \frac{6}{L^2} ES\varphi_2 + \frac{q_0 L}{2} = 4703 \ Kg$$





$$\leftarrow T_{FD} = P_2 = 1000 \ Kg$$

$$\leftarrow T_{DB} = \frac{3}{H^2} ES\varphi_2 = -134 \ Kg$$

MASSIMO MOMENTO IN CAMPATA $\overline{CD}$:

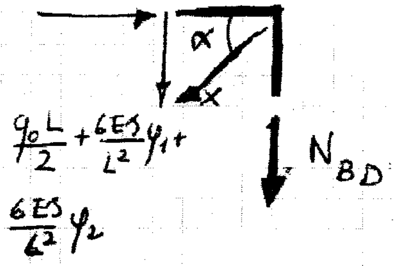
$$x_G = \frac{|T_{CD}|}{|T_{CD}| + |T_{DC}|} L = 164{,}85 \ cm \qquad M_G = -M_{CD} - \frac{q_0 x_G^2}{2} + T_{CD}\, x_G = 212727 \ Kg\,cm = 2127 \ Kg\,m$$

NORMALI :

$$N_{CA} = -\frac{q_0 L}{2} + \frac{6EJ}{L^2}\varphi_1 + \frac{6EJ}{L^2}\varphi_2 = -3297 \text{ Kg}$$

$$N_{CD} = \frac{5}{8}q_1 H + q_1 H + \frac{3}{H^2}EJ\varphi_1 = -3197 \text{ Kg}$$

$$\frac{5}{8}q_1 H + q_1 H + 2P_2 + \frac{3}{H^2}EJ\varphi_1 + \frac{3}{H^2}EJ\varphi_2$$



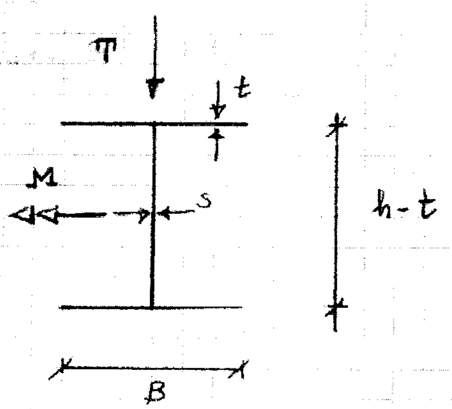
$$tg\,\alpha = \frac{H}{L}$$

$$X = \frac{\left(\frac{5}{8}q_1 H + q_1 H + 2P_2 + \frac{3}{H^2}EJ\varphi_1 + \frac{3}{H^2}EJ\varphi_2\right)}{\cos\alpha} = 6328 \text{ Kg}$$

$$N_{BD} = -X\sin\alpha - \Big[\underbrace{\frac{q_0 L}{2} + \frac{6}{L^2}\left(EJ\varphi_1 + EJ\varphi_2\right)}_{T_{DC}}\Big] = -8500 \text{ Kg}$$

A2) VERIFICA DELLO STATO TENSIONALE NELLA SEZIONE MAGGIORMENTE SOLLECITATA

$M_{DC} = 340278$ Kg·cm $\quad T_{DC} = 4703$ Kg $\quad N_{DC} = -3197$ Kg

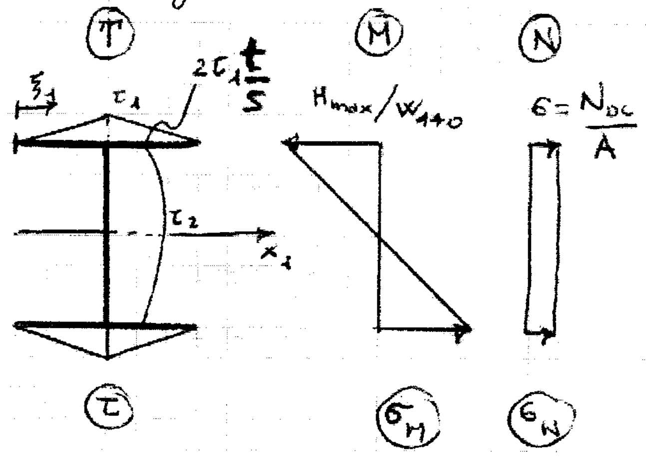


$h = 13,3$ cm

$B = 14$ cm

$t = 0,85$ cm

$s = 0,55$ cm



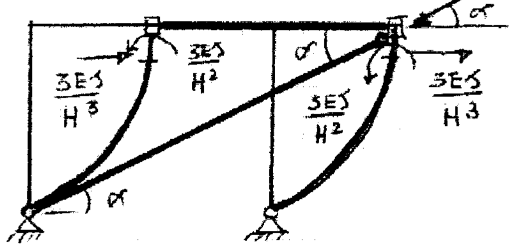
$$\sigma_{id} = \sqrt{\sigma^2 + 3\tau_1^2} = 2323 \frac{Kg}{cm^2} < \sigma_{amm} \longrightarrow \text{VERIFICATO}$$

$$\tau_1 = \frac{T\,\cancel{t}\,\frac{B}{2}\,\frac{h-t}{2}}{\cancel{t}\,J} = \frac{T(h-t)B}{4J} \qquad \tau_2 = \frac{T\left[B\frac{(h-t)}{2}t + s\frac{(h-t)}{2}\cdot\frac{(h-t)}{4}\right]}{sJ} = 701 \frac{Kg}{cm^2}$$

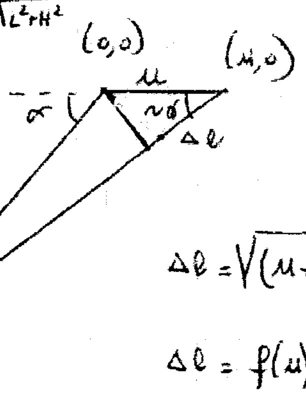
$$\sigma = \sigma_M + \sigma_N$$

B)

$$\frac{EA}{\sqrt{L^2+H^2}}\cos\alpha = \frac{A}{J}\frac{\cos\alpha}{\sqrt{L^2+H^2}}\,EJ$$



(u)

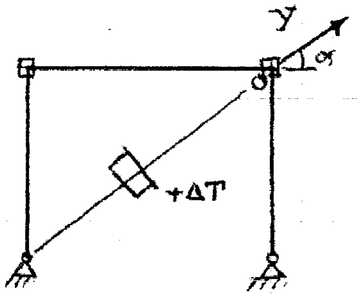


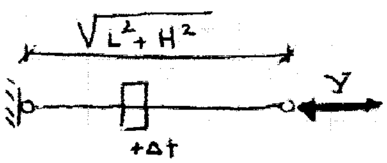
$$\cos\alpha = \frac{L}{\sqrt{L^2+H^2}}$$

$$\Delta l = \sqrt{(u+L)^2+H^2} - \sqrt{L^2+H^2}$$

$$\Delta l = f(u) \cong f(0) + \left.\frac{df}{du}\right|_0 \cdot u$$

$$= 0 + \left.\frac{u+L}{\sqrt{(u+L)^2+H^2}}\right|_{u=0} u = \frac{L}{\sqrt{L^2+H^2}}\,u = u\cos\alpha$$





$$-\frac{Y}{EA}\cancel{\sqrt{L^2+H^2}} + \alpha\Delta T\cancel{\sqrt{L^2+H^2}} = 0$$

$$Y = \alpha\Delta T EA = \Delta T(\alpha EA)$$

$$\frac{A}{J} = \frac{5\ cm^2}{1033\ cm^4} = 0{,}00484 \qquad \alpha EA = 1{,}2\cdot 10^{-5}\ 2100000\cdot 5$$

$$\begin{bmatrix} \frac{4}{L}+\frac{3}{H} & \frac{2}{L} & -\frac{3}{H^2} \\ \frac{2}{L} & \frac{4}{L}+\frac{3}{H} & -\frac{3}{H^2} \\ -\frac{3}{H^2} & -\frac{3}{H^2} & \frac{6}{H^3}+\frac{A}{J}\frac{\cos^2\alpha}{\sqrt{L^2+H^2}} \end{bmatrix} \begin{bmatrix} EJ\varphi_1 \\ EJ\varphi_2 \\ EJu \end{bmatrix} = \begin{bmatrix} \frac{q_0L^2}{12}+\frac{q_1H^2}{2}-\frac{q_1H^2}{8} \\ -\frac{q_0L^2}{12}+P_2H \\ \frac{5}{8}q_1H+q_1H+2P_2+Y\cos\alpha \end{bmatrix}$$

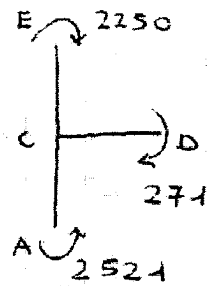
$$\Rightarrow \begin{cases} EJ\varphi_1 = 2{,}49061\ 10^7\ Kg\,cm^2 & \varphi_1 = 0{,}011481\ rad \\ EJ\varphi_2 = -1{,}89944\cdot 10^6\ Kg\,cm^2 & \varphi_2 = -0{,}000876\ rad \\ EJu = 1{,}59625\ 10^8\ Kg\,cm^3 & \Rightarrow u = 0{,}735838\ cm \end{cases}$$

MOMENTI:

$$M_{CA} = \frac{3}{H}EJ\varphi_1 - \frac{3}{H^2}EJu + \frac{q_1H^2}{8} = 252103\ Kg\,cm = 2521\ Kg\,m$$

$$M_{CD} = \frac{4}{L}EJ\varphi_1 + \frac{2}{L}EJ\varphi_2 - \frac{q_0L^2}{12} = -27103\ Kg\,cm = -271\ Kg\,m$$

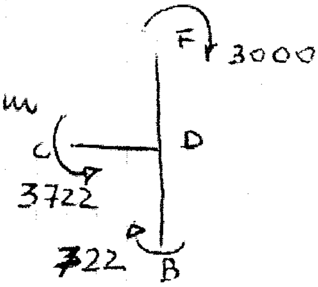
$$M_{CE} = -225000\ Kg\,cm = -2250\ Kg\,m$$



$$M_{DB} = \frac{3}{H}EJ\varphi_2 - \frac{3}{H^2}EJu = -72203\ Kg\,cm = -722\ Kg\,m$$

$$M_{DC} = \frac{4}{L}EJ\varphi_2 + \frac{2}{L}EJ\varphi_1 + \frac{q_0L^2}{12} = 372203\ Kg\,cm = 3722\ Kg\,m$$

$$M_{FD} = -3000\ Kg\,m$$



TAGLI:

$$-\ T_{AC} = \frac{3}{8} q_1 H - \frac{3}{H^2} EJ\varphi_1 + \frac{3}{H^3} EJu = -90{,}3\ \text{Kg}$$

$$-\ T_{CA} = \frac{3}{H^2} EJ\varphi_1 - \frac{3}{H^3} EJu + \frac{5}{8} q_1 H = 1590\ \text{Kg}$$

$$-\ T_{EC} = 1500\ \text{Kg}$$

$$\downarrow T_{CD} = \frac{6}{L^2} EJ\varphi_1 + \frac{6}{L^3} EJ\varphi_2 - \frac{q_0 L}{2} = -3137\ \text{Kg}$$

$$\uparrow T_{DC} = \frac{6}{L^2} EJ\varphi_1 + \frac{6}{L^2} EJ\varphi_2 + \frac{q_0 L}{2} = 4863\ \text{Kg}$$

$$-\ T_{FD} = 1000\ \text{Kg}$$

$$-\ T_{DB} = \frac{3}{H^2} EJ\varphi_2 - \frac{3EJu}{H^3} = -241\ \text{Kg}$$

MASSIMO MOMENTO IN CAMPATA $\overline{CD}$:

$$x_G = \frac{|T_{CD}|}{|T_{CD}| + |T_{DC}|} L = 156{,}86\ \text{cm}$$

$$M_G = -M_{CD} - \frac{q_0 x_G^2}{2} + T_{CD} x_G = 218919\ \text{Kg cm} = 2189\ \text{Kg m}$$

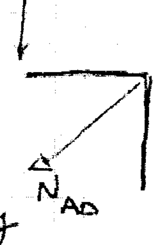
SFORZI NORMALI:

$$\uparrow N_{CA} = -\frac{q_0 L}{2} + \frac{6EJ}{L^2}\varphi_1 + \frac{6EJ}{L^2}\varphi_2 = -3137\ \text{Kg}$$

$$-N_{CD} = \frac{5}{8} q_1 H + q_1 H + \frac{3}{H^2} EJ\varphi_1 - \frac{3}{H^3} EJu = 3090\ \text{Kg}$$

$$T_{DC} = \frac{6}{L^2} EJ\varphi_1 + \frac{6}{L^2} EJ\varphi_2 + \frac{q_0 L}{2}$$

$$N_{AD} = \frac{A}{\cancel{J}} \frac{\cos\alpha}{\sqrt{L^2+H^2}} E\cancel{J} u^{-\gamma} = 6062\ \text{Kg}$$



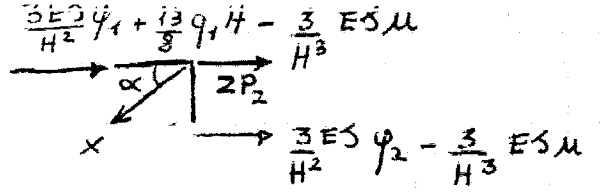
$$\downarrow N_{DB} = N_{AD} \sin\alpha + \underbrace{\frac{6}{L^2} EJ\varphi_1 + \frac{6}{L^2} EJ\varphi_2 + \frac{q_0 L}{2}}_{T_{DC}} = 8500\ \text{Kg}$$

c): SPOSTAMENTO NODI D e F:

$$u_D = u = 0{,}735838\ \text{cm}$$

$$u_F = u + \frac{P_2 H^3}{3EJ} + \varphi_2 H = 4{,}622\ \text{cm}$$

D)

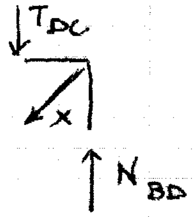


$$x\cos\alpha = \frac{3}{H^2}ES\varphi_1 + \frac{3}{H^2}ES\varphi_2 - \frac{6}{H^3}ES\mu + 2P_2 + \frac{13}{8}q_1 H \overset{!}{=} 0$$

$$\begin{array}{l} 1) \\ 2) \\ 3) \\ 4) \end{array}
\begin{bmatrix}
\frac{4}{L}+\frac{3}{H} & \frac{2}{L} & -\frac{3}{H^2} & 0 \\
\frac{2}{L} & \frac{4}{L}+\frac{3}{H} & -\frac{3}{H^2} & 0 \\
-\frac{3}{H^2} & -\frac{3}{H^2} & \frac{6}{H^3}+\frac{A}{S}\frac{\cos^3\alpha}{\sqrt{L^2+H^2}} & -EA\alpha\cos\alpha \\
-\frac{3}{H^2} & -\frac{3}{H^2} & +\frac{6}{H^3} & 0
\end{bmatrix}
\begin{bmatrix} ES\varphi_1 \\ ES\varphi_2 \\ \mu \\ \Delta T \end{bmatrix}
=
\begin{bmatrix} \frac{q_0 L^2}{12}+\frac{q_1 H^2}{2}-\frac{q_1 H^2}{8} \\ -\frac{q_0 L^2}{12}+P_2 H \\ \frac{5}{8}q_1 H + q_1 H + 2P_2 \\ 2P_2 + \frac{13}{8}q_1 H \end{bmatrix}$$

$\Rightarrow \quad \Delta T = 23337\ °C$

<u>N.B.</u> :



N_{BD} E' CALCOLABILE PER EQ. ALLA ROTAZIONE RISPETTO AD A DELL'INTERA STRUTTURA

$$N_{BD} = \frac{P_2\, 3H + q_1\, 2H^2 + q_0 L^2/2}{L} = 8500\,kg$$

$$T_{DC} = \frac{6}{L^2}ES\varphi_1 + \frac{6}{L^2}ES\varphi_2 + \frac{q_0 L}{2}$$

$$x\sin\alpha = N_{BD} - T_{DC} = 3P_2\frac{H}{L} + 2q_1\frac{H^2}{L} - \frac{6}{L^2}ES\varphi_1 - \frac{6}{L^2}ES\varphi_2 \overset{!}{=} 0 \quad (*)$$

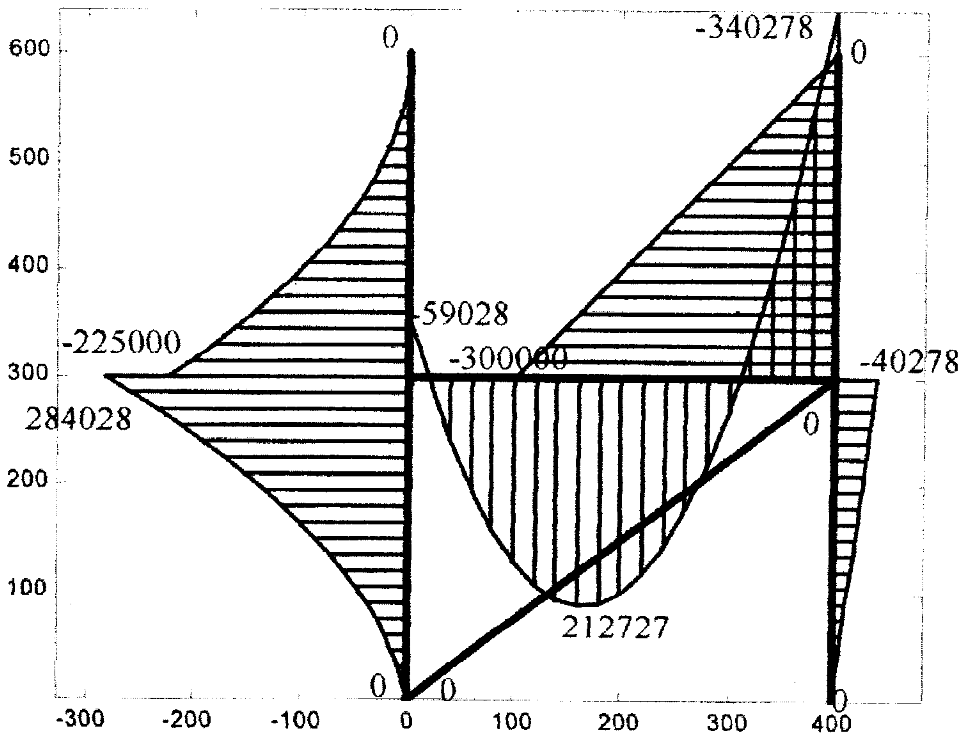
SOMMANDO ~~CON~~ 1) e 2) A (*) [MOLTIPLICATA PER L] SI OTTIENE LA 4) [MOLTIPLICATA PER H]

$$-\frac{6}{H^2}\mu + \left(\cancel{\frac{6}{L}}+\frac{3}{H}\right)ES\varphi_1 + \left(\cancel{\frac{6}{L}}+\frac{3}{H}\right)ES\varphi_2 - \frac{3}{8}q_1 H^2 - P_2 H + 3P_2\frac{H}{\cancel{L}} + 2q_1\frac{H^2}{\cancel{L}} - \cancel{\frac{6}{L}ES\varphi_1} - \cancel{\frac{6}{L}ES\varphi_2} = 0$$

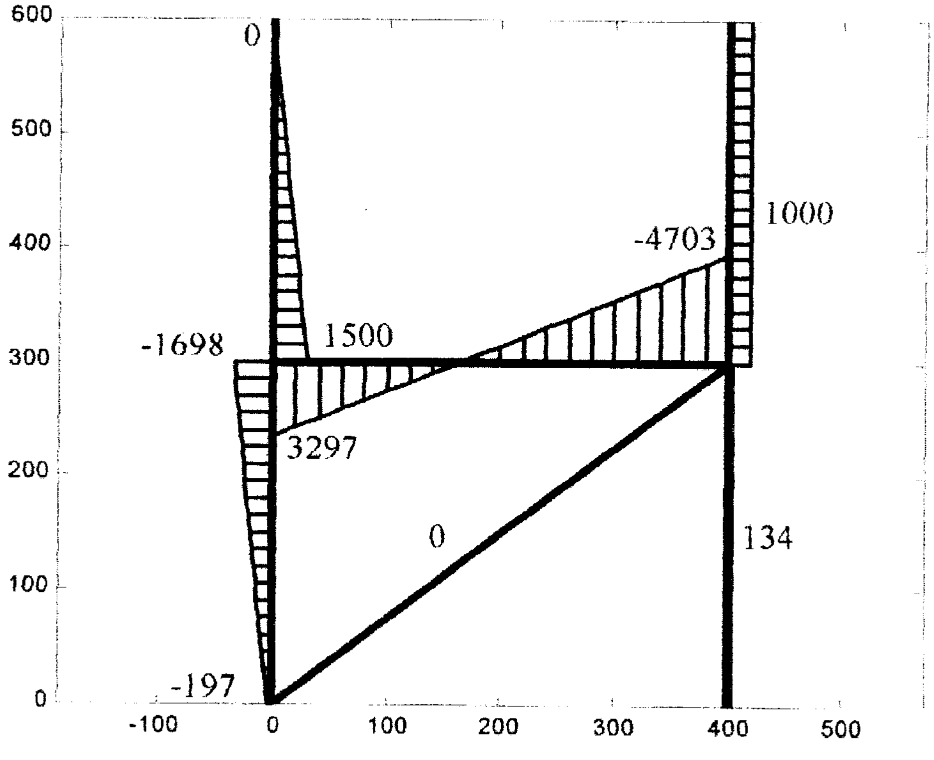
$$\Rightarrow -\frac{3}{H}ES\varphi_1 - \frac{3}{H}ES\varphi_2 + \frac{6}{H^2}\mu = \frac{13}{8}q_1 H^2 + 2P_2 H = 0$$

$$\Rightarrow -\frac{3}{H^2}ES\varphi_1 - \frac{3}{H^2}ES\varphi_2 + \frac{6}{H^3}\mu = \frac{13}{8}q_1 H + 2P_2$$

Punto A1 pag1/2

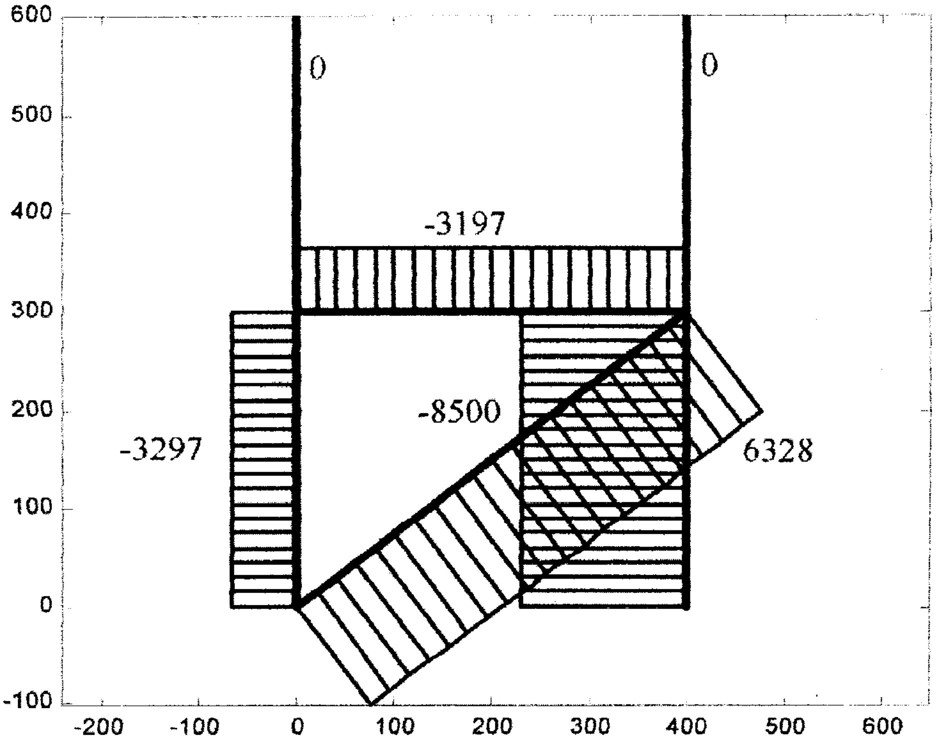


M [Kg cm]



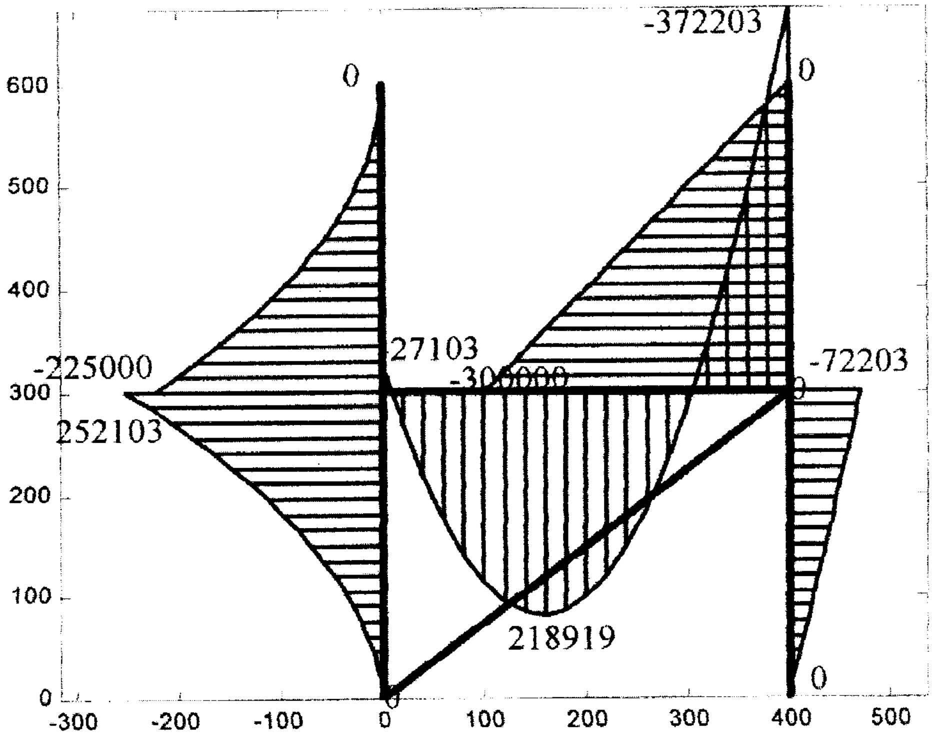
T [Kg]

Punto A1 pag 2/2

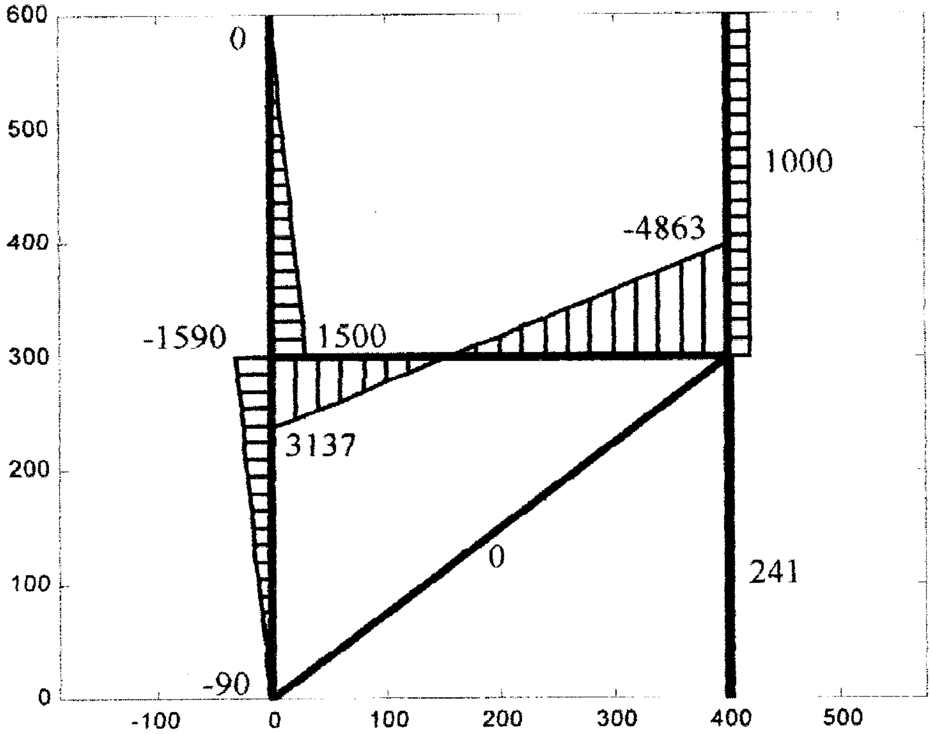


N [Kg]

Punto B pag 1/2

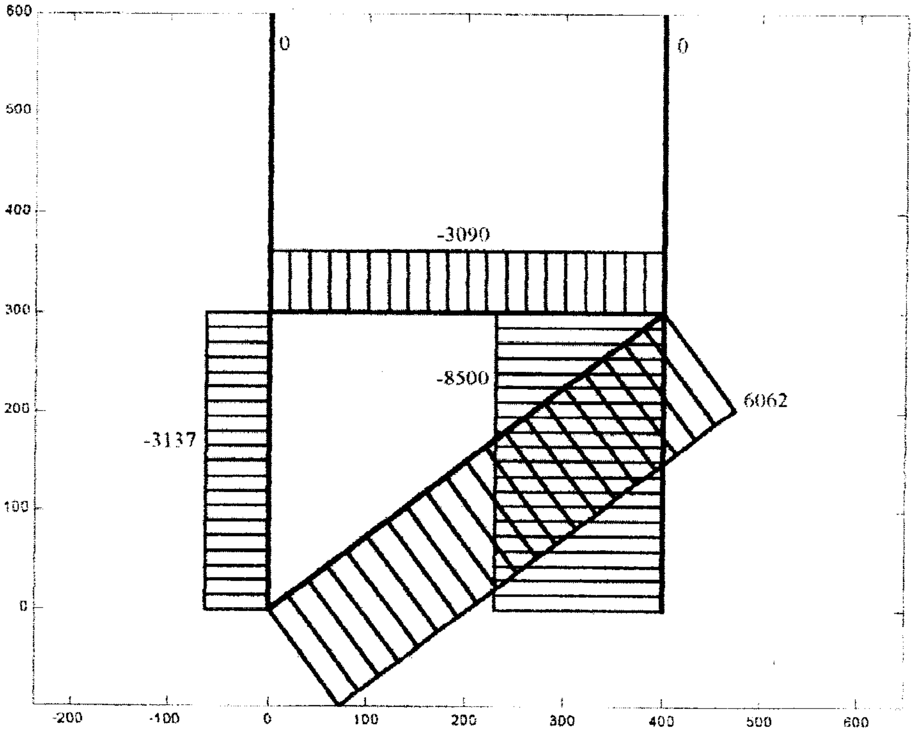


M [Kg cm]



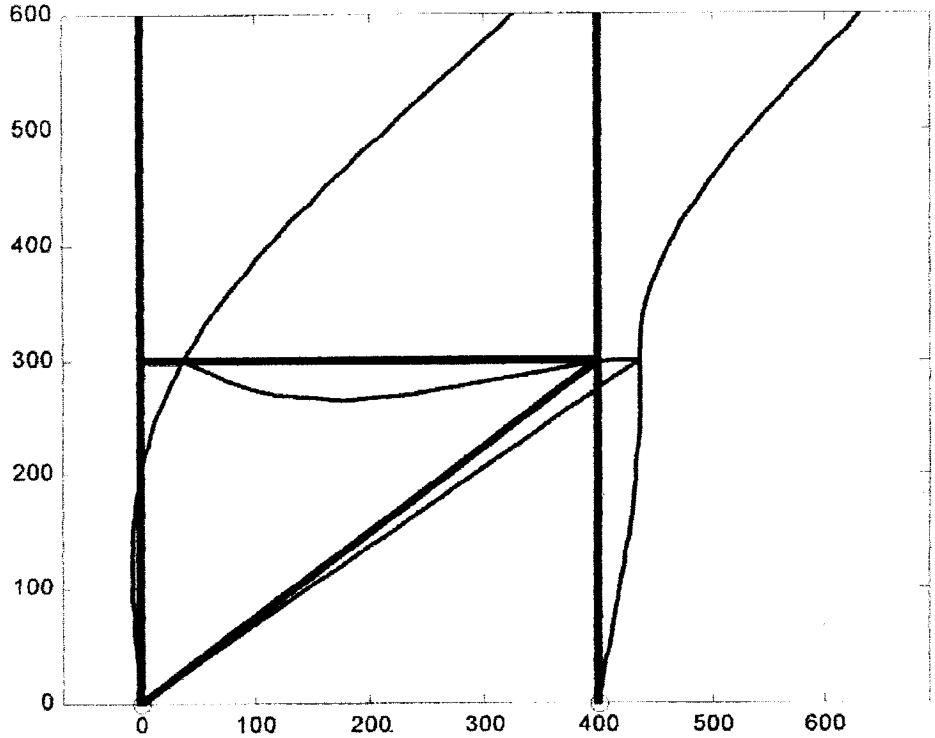
T [Kg]

Punto B pag 2/2

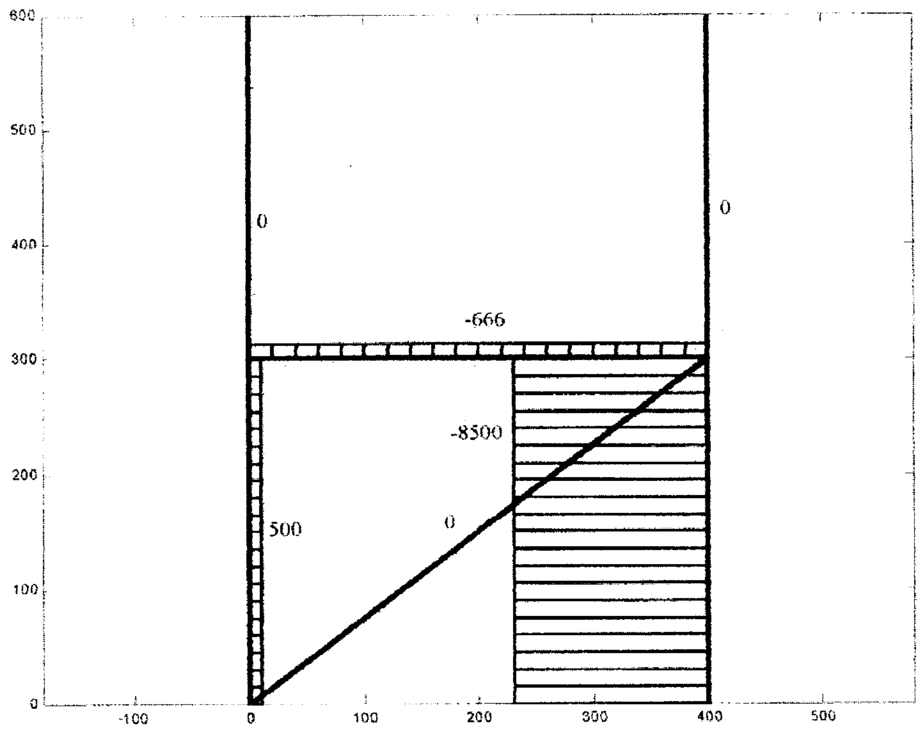


N [Kg]

Punti C e D pag 1/1

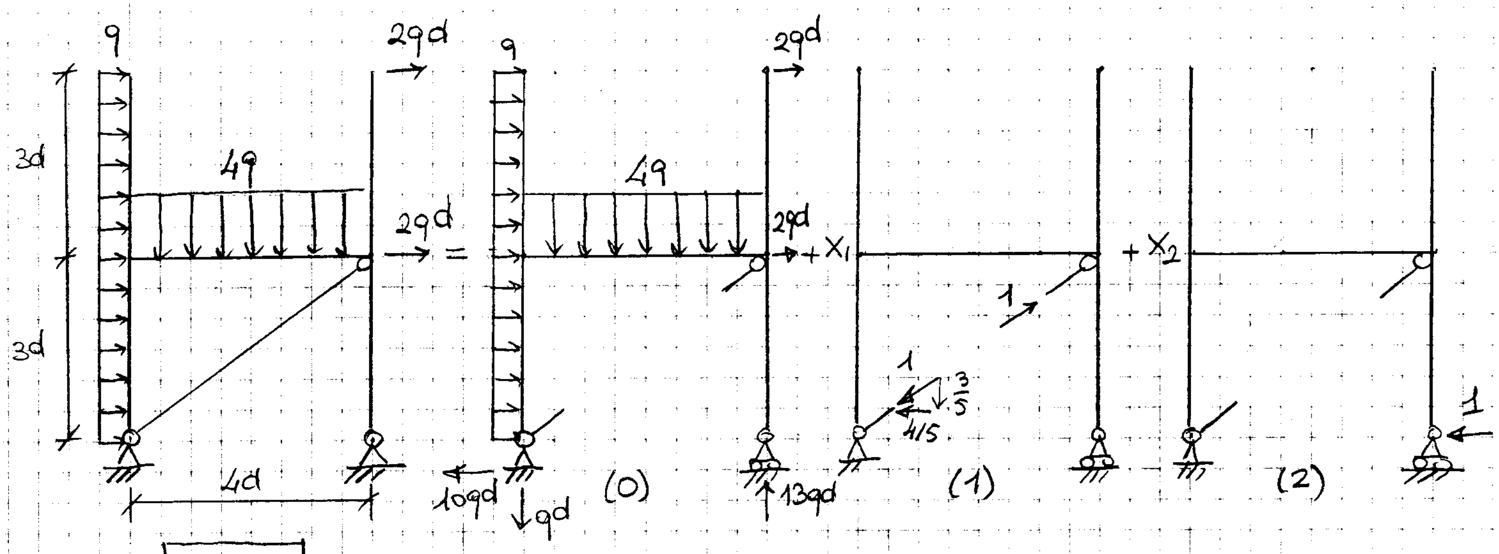


deformata

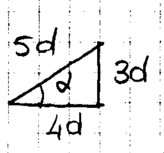


N [Kg]

RISOLUZIONE CON IL METODO DELLE FORZE

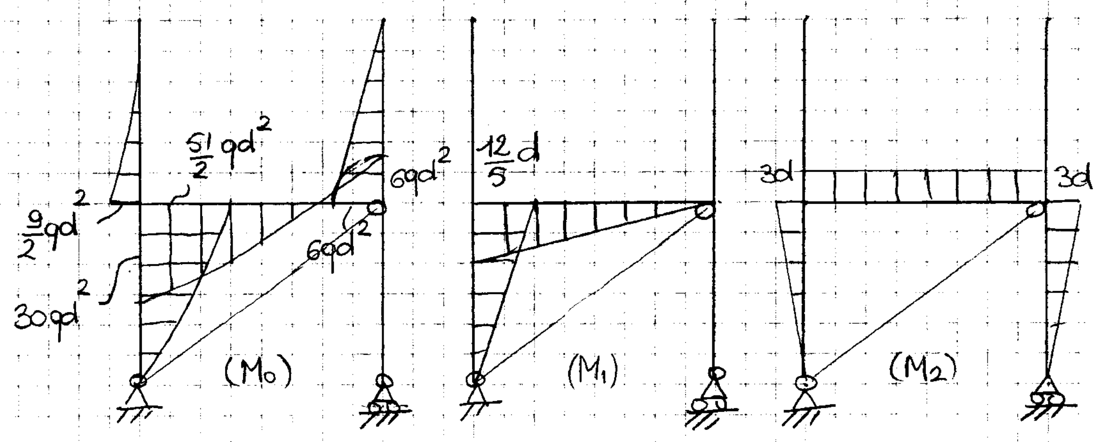


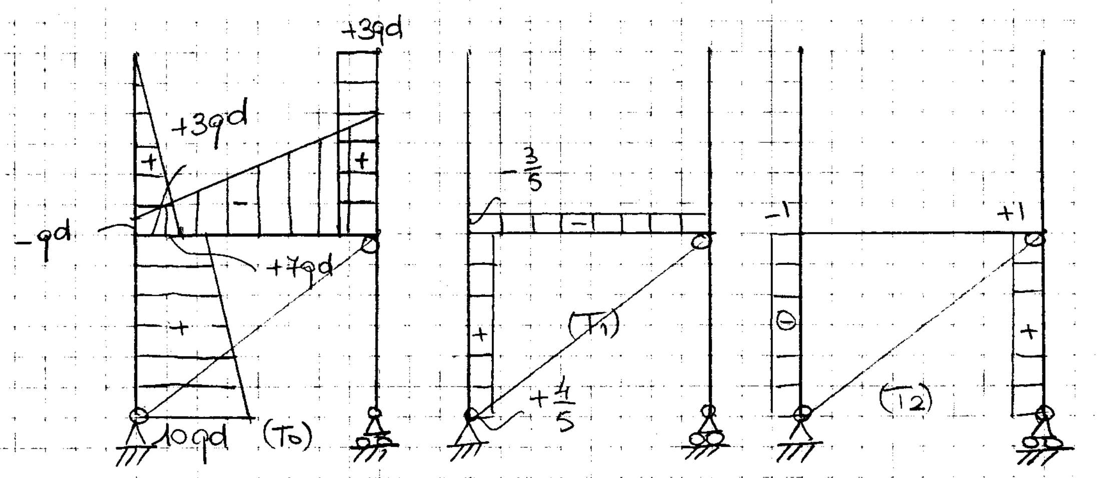
$\boxed{d = 1\,m}$

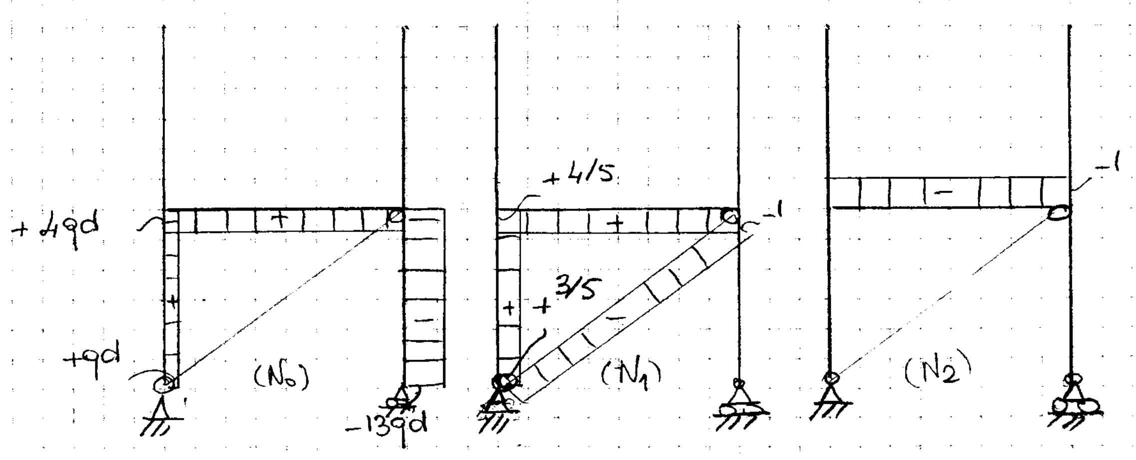


$\operatorname{sen} \alpha = \frac{3\cancel{d}}{5\cancel{d}} = \frac{3}{5}$

$\cos \alpha = \frac{4\cancel{d}}{5\cancel{d}} = \frac{4}{5}$







$$\eta_{10} = \frac{1}{EJ}\left[\int_0^{3d}\left(10qdx_3 - q\frac{x_3^2}{2}\right)\left(\frac{4}{5}x_3\right)dx_3 + \int_0^{4d}\left(30qd^2 - qdx_3 - 4q\frac{x_3^2}{2}\right)\left(\frac{12}{5}d - \frac{3}{5}x_3\right)dx_3\right]$$

$$= \frac{1759}{10}\,\frac{qd^4}{EJ}$$

$$\eta_{20} = \frac{1}{EJ}\left[\int_0^{3d}\left(10qdx_3 - q\frac{x_3^2}{2}\right)(-x_3)\,dx_3 + \int_0^{4d}\left(30qd^2 - qdx_3 - 2qx_3^2\right)(-3d)\,dx_3\right]$$

$$= -\frac{2303}{8}\,\frac{qd^4}{EJ}$$

$$\eta_{11} = \frac{1}{EJ}\left[\int_0^{3d}\left(\frac{4}{5}x_3\right)^2 dx_3 + \int_0^{4d}\left(\frac{3}{5}x_3\right)^2 dx_3\right] = \frac{336\,d^3}{25\,EJ}$$

$$\eta_{12} = \frac{1}{EJ}\left[\int_0^{3d}\left(\frac{4}{5}x_3\right)(-x_3)\,dx_3 + \int_0^{4d}(-3d)\left(\frac{3}{5}x_3\right)dx_3\right] = -\frac{108\,d^3}{5\,EJ}$$

$$\eta_{22} = \frac{1}{EJ}\left[2\int_0^{3d}(-x_3)^2\,dx_3 + (-3d)^2(4d)\right] = \frac{54\,d^3}{EJ}$$

$$\begin{bmatrix}\eta_{11} & \eta_{12}\\ \eta_{12} & \eta_{22}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} = \begin{bmatrix}-\eta_{10}\\ -\eta_{20}\end{bmatrix} \longrightarrow \begin{cases}X_1 = -6328\ \text{kg}\\ X_2 = +134\ \text{kg}\end{cases}$$

<u>Diagramma dei momenti, progetto e verifica</u>: vedi risoluzione con il metodo delle rigidezze