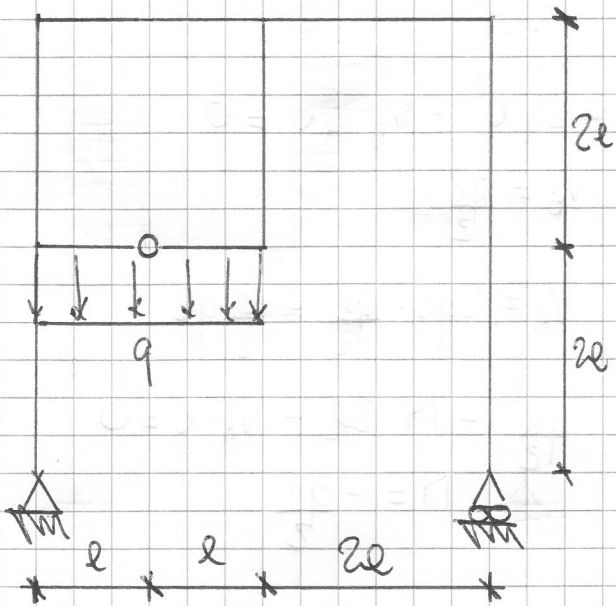


13/07/2018

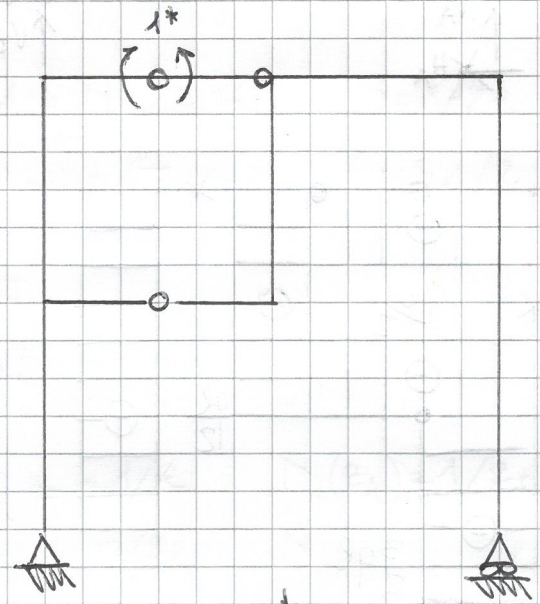
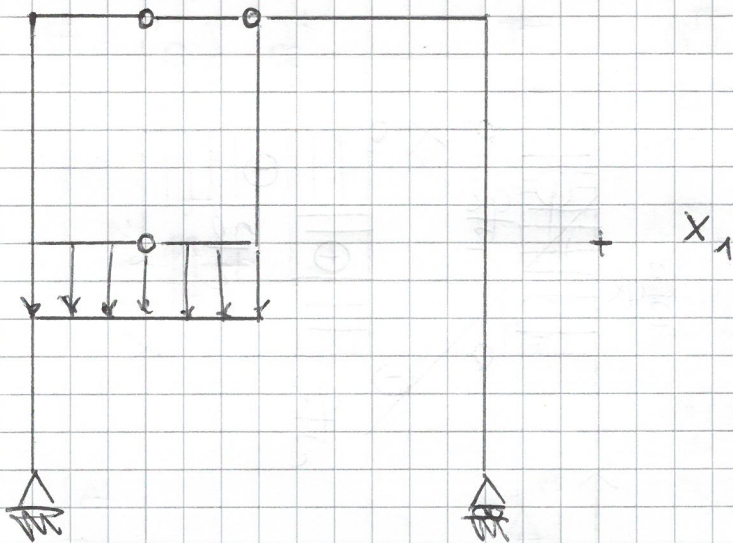


$$q = 10000 \text{ N/m}$$

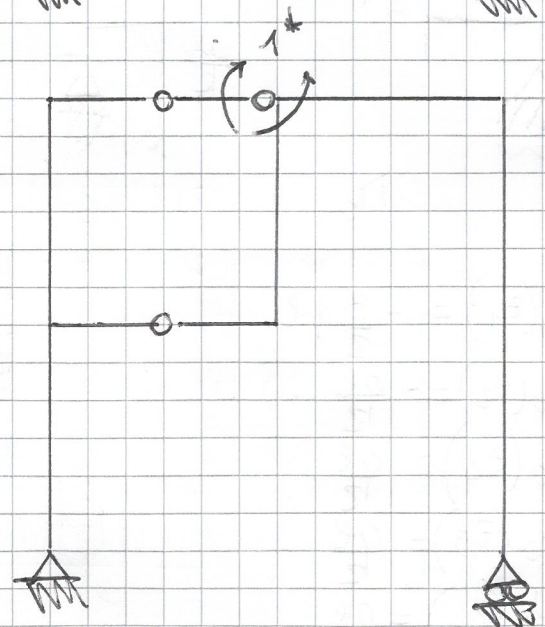
$$l = 2 \text{ m}$$

$$\sigma_{\text{adm}} = 120 \text{ MPa}$$

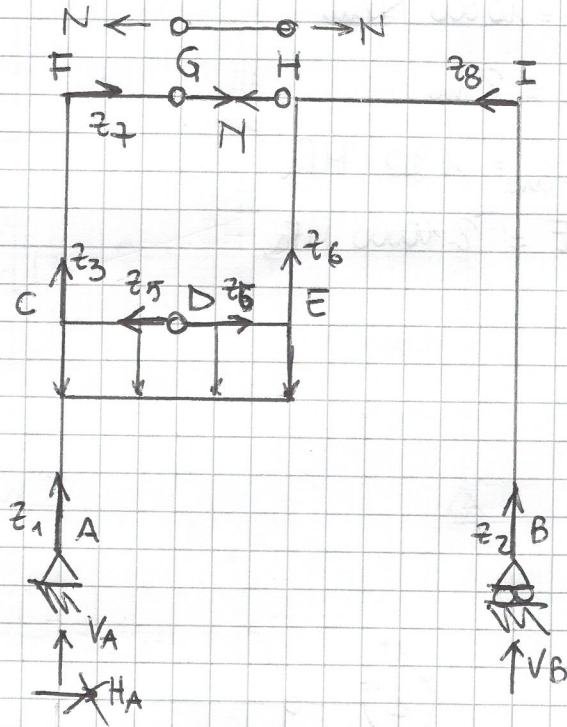
$$E = 210000 \text{ MPa}$$



$$+ X_2$$



1) SISTEMA (0)



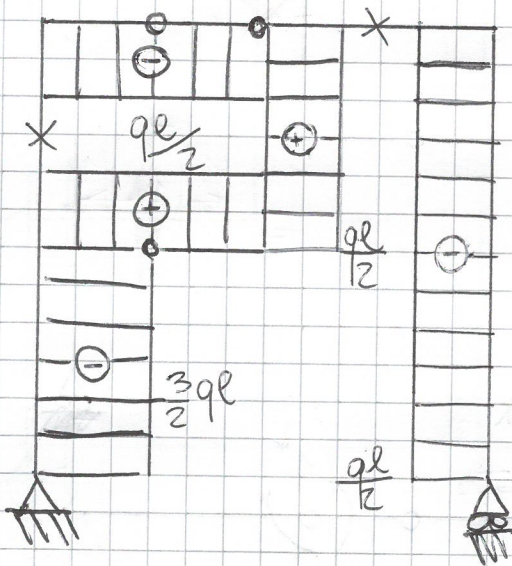
$$A) -2ql^2 + V_B \cdot 4l = 0$$

$$V_B = \frac{ql}{2}$$

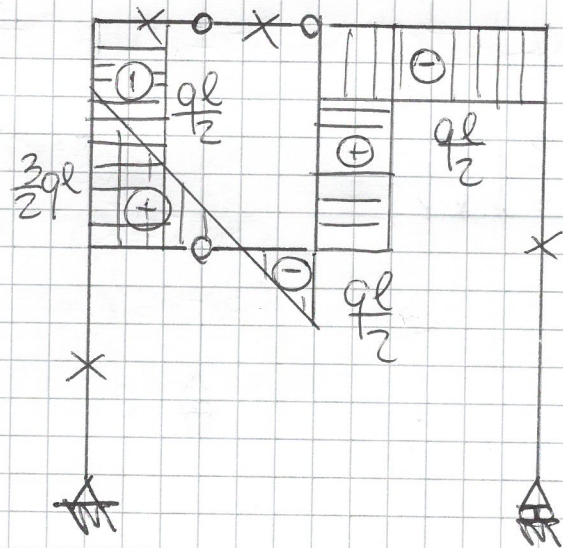
$$1) V_A = 2ql - \frac{ql}{2} = \frac{3ql}{2}$$

$$D) \frac{ql^2}{2} - N \cdot 2l - V_A \cdot l = 0$$

$$N = -\frac{ql}{2}$$



N_0



T_0

$$N_0(z_1) = -\frac{3ql}{2}$$

$$N_0(z_2) = -\frac{ql}{2}$$

$$N_0(z_4) = \frac{ql}{2}$$

$$N_0(z_5) = \frac{ql}{2}$$

$$N_0(z_6) = \frac{ql}{2}$$

$$N_0(z_7) = \frac{ql}{2}$$

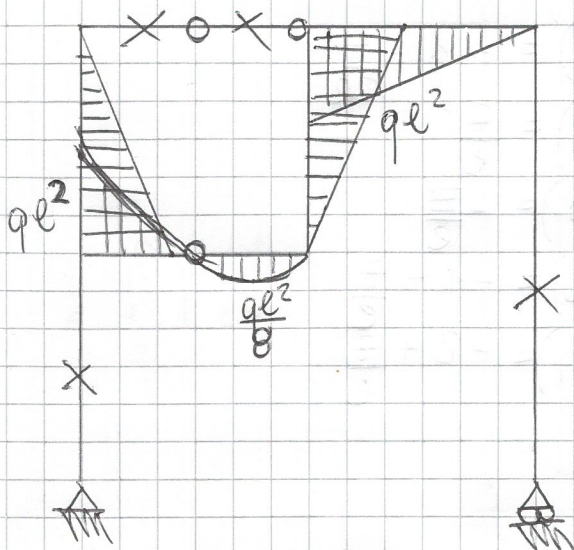
$$T_0(z_1) = T_0(z_2) = 0$$

$$T_0(z_3) = -\frac{ql^2}{2}$$

$$T_0(z_4) = \frac{ql^2}{2} + \frac{ql^2}{2} z$$

$$T_0(z_5) = \frac{ql^2}{2} - \frac{ql^2}{2} z$$

$$T_0(z_6) = \frac{ql^2}{2}$$



$$M(z_3) = ql^2 - \frac{qlz}{2}$$

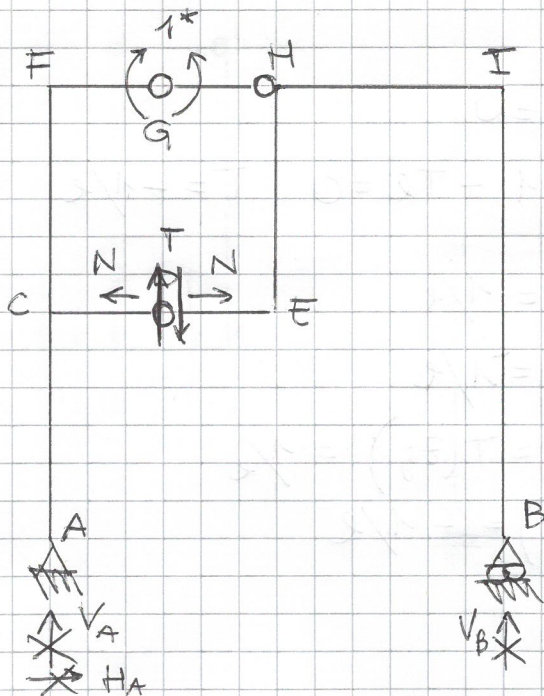
$$M(z_4) = -\frac{ql^2}{2} - \frac{qlz}{2}$$

$$M(z_5) = \frac{qlz}{2} - \frac{ql^2}{2}$$

$$M(z_6) = \frac{qlz}{2}$$

$$M(z_7) = \frac{qlz}{2}$$

SISTEMA (1)



$$G \curvearrowright -1 + N \cdot 2l = 0 \quad N = 1/2l$$

$$H \curvearrowright N \cdot 2l + Tl = 0 \quad T = -1/l$$

$$A) V_A = 0$$

$$\uparrow) V_B = 0$$

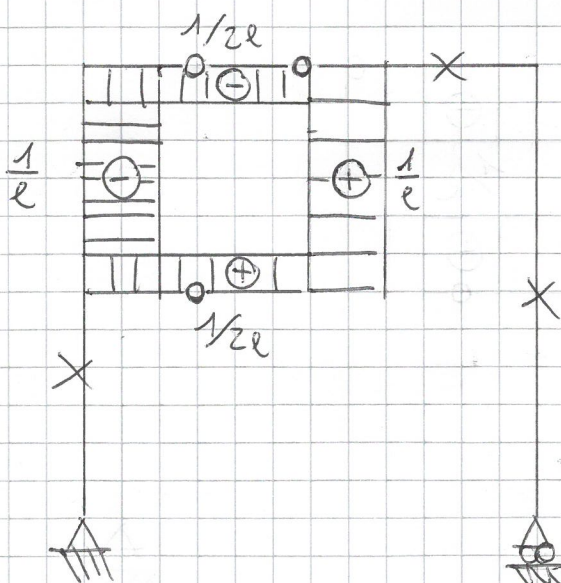
$$N_1(z_3) = -1/l$$

$$N_1(z_4) = 1/2l$$

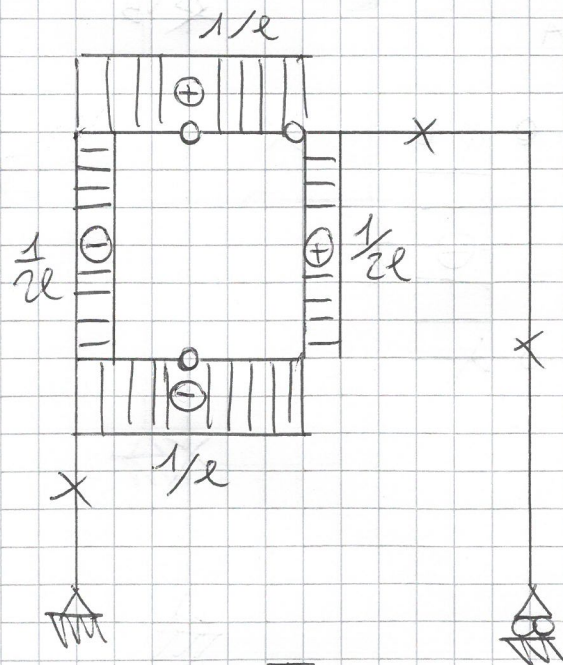
$$N_1(z_5) = 1/2l$$

$$N_1(z_6) = 1/l$$

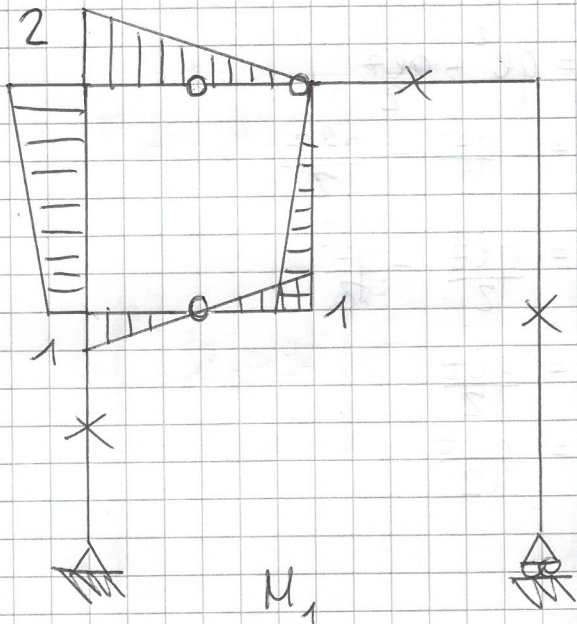
$$N_1(z_7) = -1/2l$$



N_1



T_1



$$M_1(z_3) = -1 - \frac{z}{2l}$$

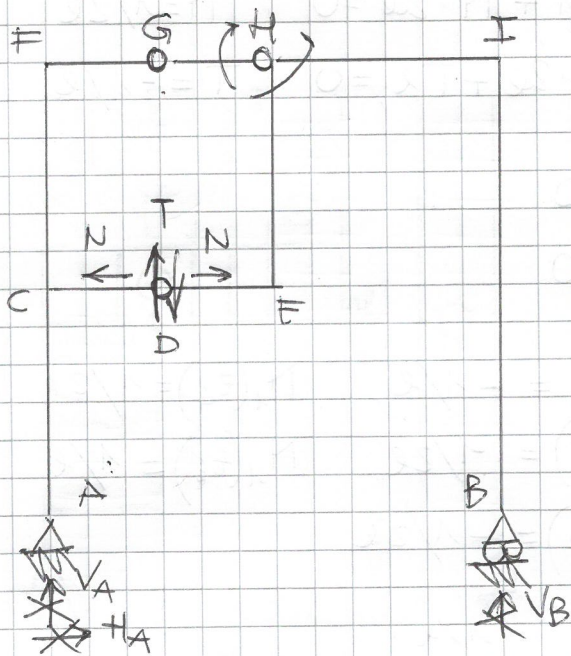
$$M_1(z_4) = \frac{z}{e}$$

$$M_1(z_5) = -\frac{z}{e}$$

$$M_1(z_6) = -1 + \frac{z}{e}$$

$$M_1(z_7) = -2 + \frac{z}{e}$$

SISTEMA (2)



$$G \uparrow N = 0$$

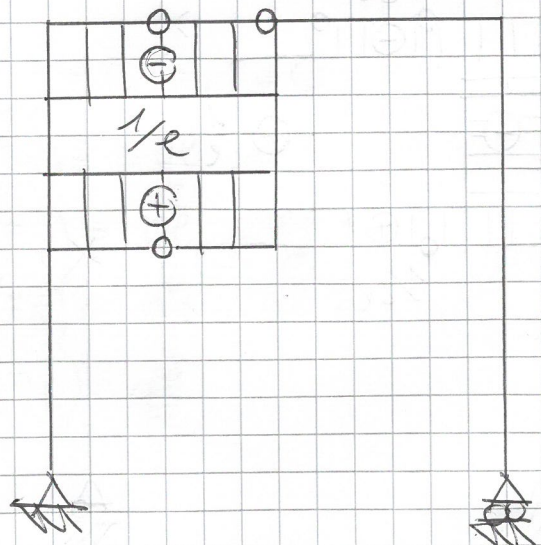
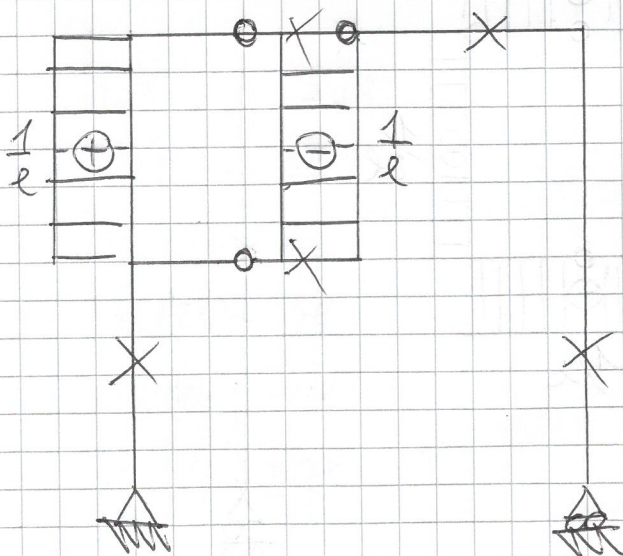
$$H \uparrow +1 + Tl = 0 \quad T = -1/l$$

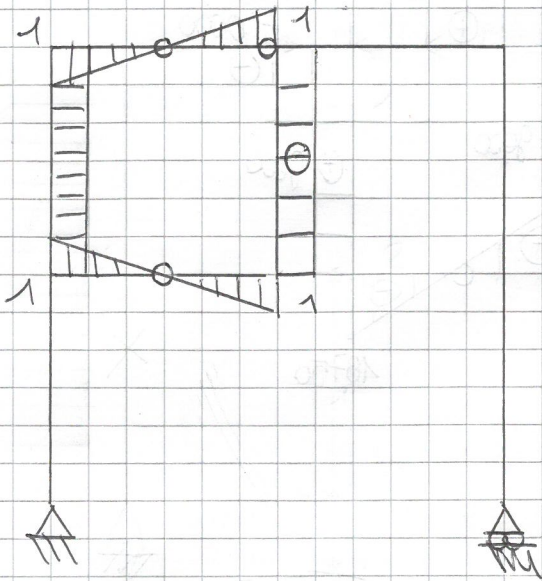
$$N_2(z_3) = 1/l$$

$$N_2(z_6) = -1/l$$

$$T(z_4) = T(z_5) = 1/l$$

$$T(z_7) = -1/l$$





$$M_2(z_3) = 1$$

$$M_2(z_4) = -\frac{z}{e}$$

$$M_2(z_5) = \frac{z}{e}$$

$$M_2(z_6) = 1$$

$$M_2(z_7) = 1 - \frac{z}{e}$$

$$\eta_{10} = \frac{1}{EI} \left[\int_0^{2l} \left(ql^2 - \frac{qlz}{2} \right) \left(-1 - \frac{z}{2l} \right) dz + \int_0^l \left(-\frac{qz^2}{2} - \frac{qlz}{2} \right) \left(\frac{z}{e} \right) dz + \int_0^l \left(-\frac{z}{e} \right) \left(\frac{qlz}{2} - \frac{qz^2}{2} \right) dz + \int_0^{2l} \left(\frac{qlz}{2} \right) \left(-1 + \frac{z}{2l} \right) dz \right] = -\frac{2ql^3}{EI}$$

$$\eta_{20} = \frac{1}{EI} \left[\int_0^{2l} \left(-\frac{qlz}{2} + ql^2 \right) dz + \int_0^l \left(-\frac{qz^2}{2} - \frac{qlz}{2} \right) \left(-\frac{z}{e} \right) dz + \int_0^l \left(\frac{qlz}{2} - \frac{qz^2}{2} \right) \left(\frac{z}{e} \right) dz + \int_0^{2l} \left(\frac{qlz}{2} \right) dz \right] = \frac{7 \cdot ql^3}{3EI}$$

$$\eta_{11} = \frac{1}{EI} \left[\int_0^{2l} \left(-1 - \frac{z}{2l} \right)^2 dz + 2 \int_0^l \left(\frac{z}{e} \right)^2 dz + \int_0^{2l} \left(-1 + \frac{z}{2l} \right)^2 dz + \int_0^{2l} \left(-2 + \frac{z}{l} \right) dz \right] = \frac{26l}{3EI}$$

$$\eta_{22} = \left[2 \int_0^{2l} dz + 2 \int_0^l \left(\frac{z^2}{e^2} \right) dz + \int_0^{2l} \left(1 - \frac{z}{e} \right)^2 dz \right] \frac{1}{EI} = \frac{16l}{3EI}$$

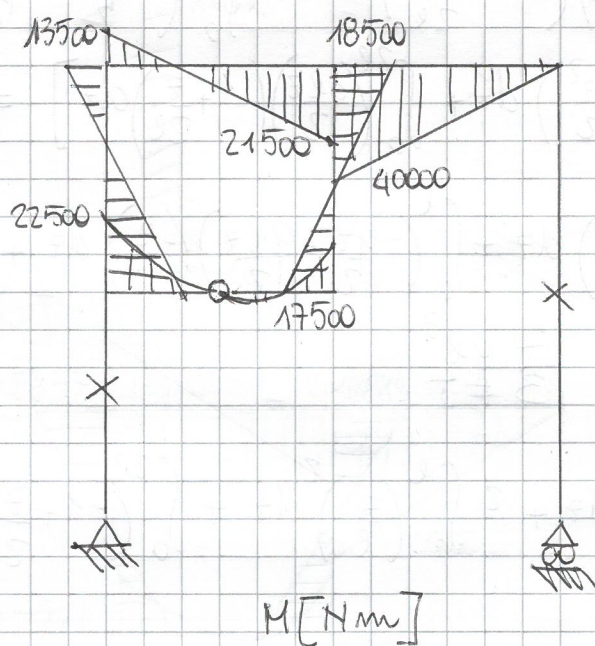
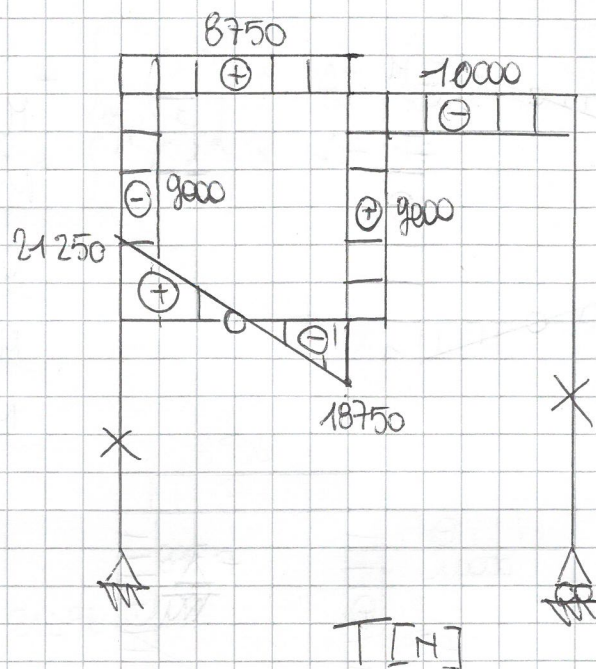
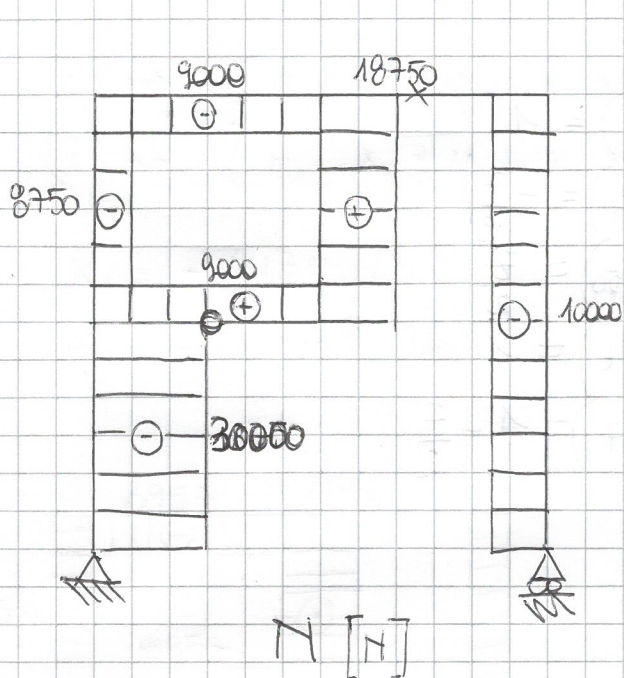
$$\eta_{12} = \frac{1}{EI} \left[\int_0^{2l} \left(-1 - \frac{z}{2l} \right) dz + 2 \int_0^l \left(-\frac{z^2}{e^2} \right) dz + \int_0^{2l} \left(-1 + \frac{z}{2l} \right) dz + \int_0^{2l} \left(-2 + \frac{z}{l} \right) \left(1 - \frac{z}{e} \right) dz \right] = -\frac{16l}{3EI}$$

$$(1) \rightarrow \eta_{11} \cdot x_1 + \eta_{12} \cdot x_2 = -\eta_{10}$$

$$(2) \rightarrow \eta_{12} \cdot x_1 + \eta_{22} \cdot x_2 = -\eta_{20}$$

$$x_1 = -\frac{ql^2}{10} = -4000 \text{ Nm}$$

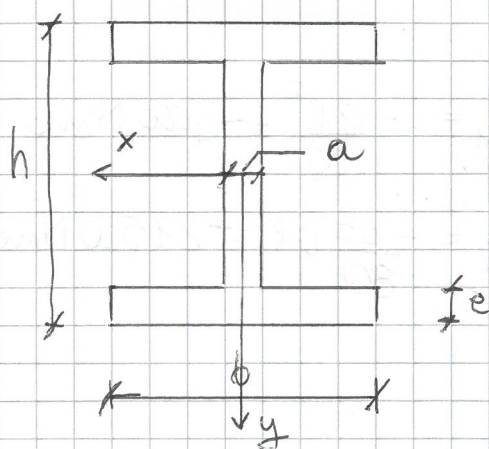
$$x_2 = -\frac{43}{80} ql^2 = -21500 \text{ Nm}$$



2) SEMIPROGETTO

$$M = -22500 \text{ Nm} \quad N = 9000 \text{ N} \quad T = 21250 \text{ N}$$

$$W_{\min} = \frac{M}{\sigma_{\text{am}}} = 118 \text{ cm}^3 \rightarrow \text{Adatto IPE 180}$$



$$A = 23,95 \text{ cm}^2$$

$$J_x = 1317 \text{ cm}^4$$

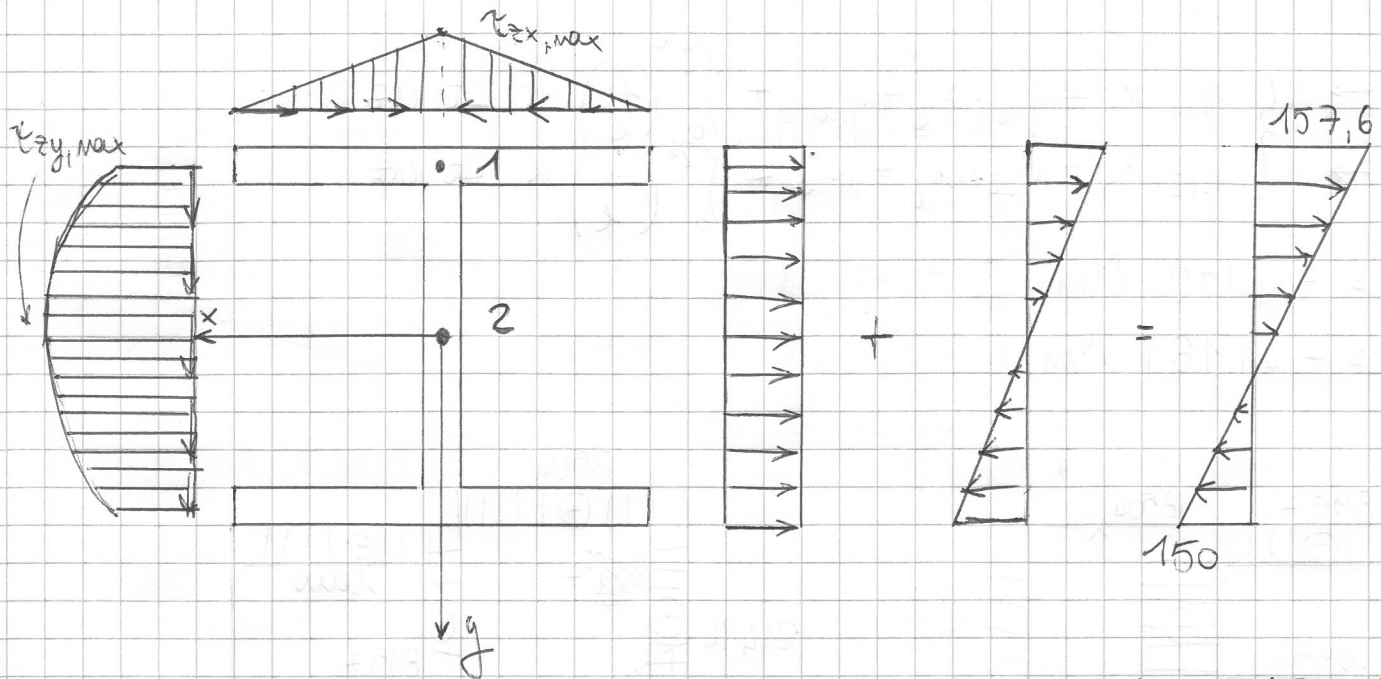
$$W_x = 146,3 \text{ cm}^3$$

$$h = 180 \text{ mm}$$

$$b = 91 \text{ mm}$$

$$a = 5,3 \text{ mm}$$

$$e = 8,0 \text{ mm}$$



$$\sigma_z = \frac{N}{A} + \frac{M}{W} = \frac{9000}{2395} + \frac{-22500 \cdot 1000}{146,3 \cdot 1000} = \begin{cases} \sigma_{z,1} = -150,03 \\ \sigma_{z,2} = 157,55 \end{cases}$$

$$\tau_{z,y} = - \frac{T_y \cdot S_x^*}{b_y \cdot I_x}$$

$$\tau_{z,x,max} = - \frac{21250 \cdot 8 \cdot (180+8) \cdot 0,5 \cdot 91 \cdot 0,5}{8 \cdot 1317 \cdot 10000} = 6,3 \text{ MPa}$$

$$\tau_{z,y,max} = - \frac{21250 \cdot (8 \cdot 91 \cdot (-180+8) \cdot 0,5 + 5,3 \cdot (90-8)^2 \cdot 0,5)}{5,3 \cdot 1317 \cdot 10000} = 24,48 \text{ MPa}$$

$$\sigma_{id,1} = \sqrt{\sigma_{z,max}^2 + 3 \tau_{z,x,max}^2} = 158 < 190 \text{ MPa}$$

$$\sigma_{id,2} = \sqrt{\sigma_{z,1}^2 + 3 \tau_{z,y,max}^2} = 43,4 < 190 \text{ MPa}$$

$$3) \quad M_{ij}^{TOT} = M_{ij}^H + M_{ij}^N$$

$$M_{10}^N = \frac{ql}{2EA} + \frac{ql}{EA} + \frac{ql}{2EA} = \frac{2ql}{EA}$$

$$M_{20}^N = \frac{ql}{EA}$$

$$M_{11}^N = \frac{4}{2EA} + \frac{1}{2EA} = \frac{5}{2EA}$$

$$M_{22}^N = \frac{4}{2EA} \quad M_{12}^N = -\frac{4}{2EA}$$

$$\begin{aligned}
 (1) & \rightarrow \begin{cases} M_{11} \cdot x_1 + M_{12} \cdot x_2 + M_{10} + \int_0^{2l} \left(\frac{1}{l} \right) \times \Delta t \, dz \\
 (2) & \rightarrow \begin{cases} M_{12} \cdot x_1 + M_{22} \cdot x_2 + M_{20} + \int_0^{2l} \left(-\frac{1}{l} \right) \times \Delta t \, dz
 \end{cases}
 \end{aligned}$$

$$x_1 = -4172 \text{ Nm}$$

$$x_2 = -21181 \text{ Nm}$$

