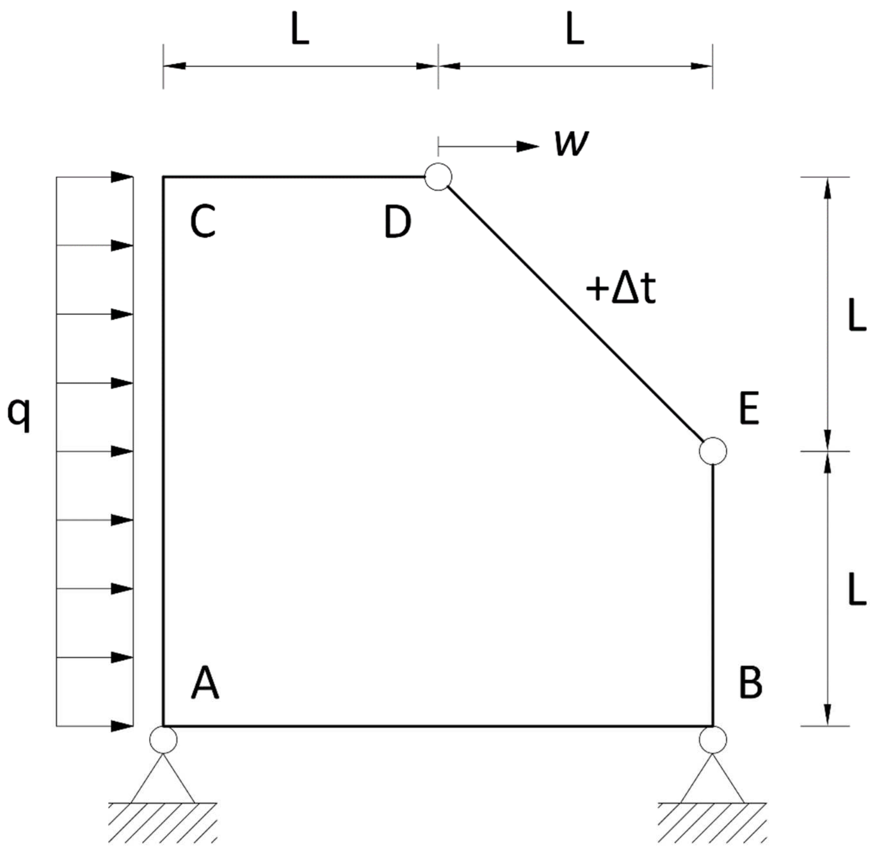
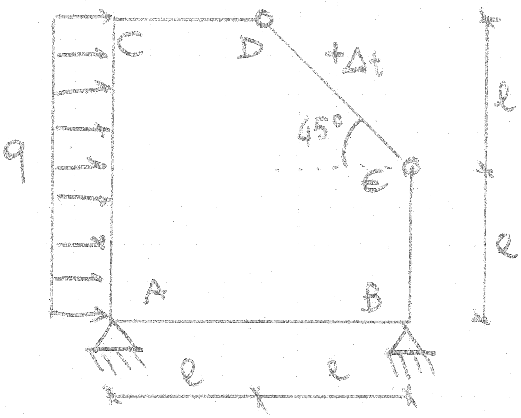
COMPITO 06/06/2017 A



1) Risolvere la struttura in figura mediante il metodo delle forze e disegnare i diagrammi M,N,T avendo posto L=3m, q=2000 N/m. In questa fase è lecito trascurare la deformazione assiale delle aste e il carico termico sull'asta DE.
2) Progettare il telaio alle tensioni ammissibili mediante profilati IPE assumendo un acciaio con tensione ammissibile di progetto pari a 390 MPa, modulo di Young E= 210000 MPa e considerando la tensione normale massima calcolata al punto 1.
3) Risolvere e determinare i diagrammi M,N,T tenendo conto della deformabilità assiale delle aste e considerando il carico termico Δt=30°C sull'asta DE (coefficiente di dilatazione termica α=1,2 x 10^{-5} °C^{-1}).
4) Calcolare lo spostamento orizzontale *w* del nodo D in assenza della deformabilità assiale.

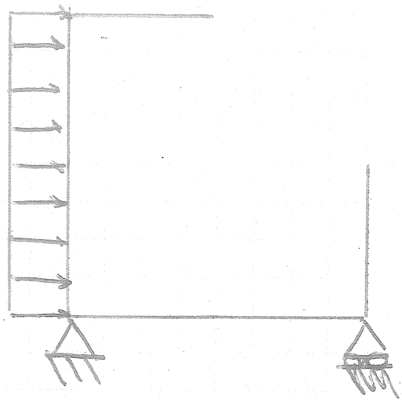
FILA A

06/06/2017

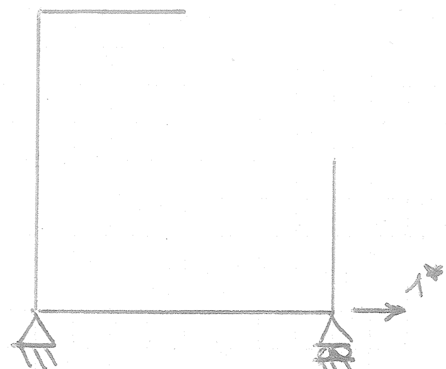


$l = 3\,m$

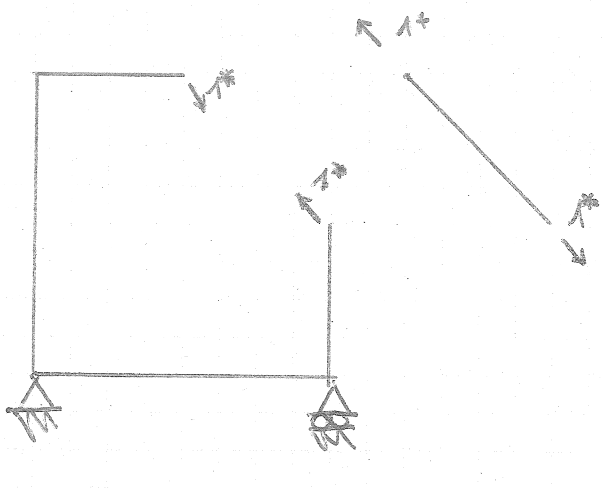
$q = 2000\ N/m$



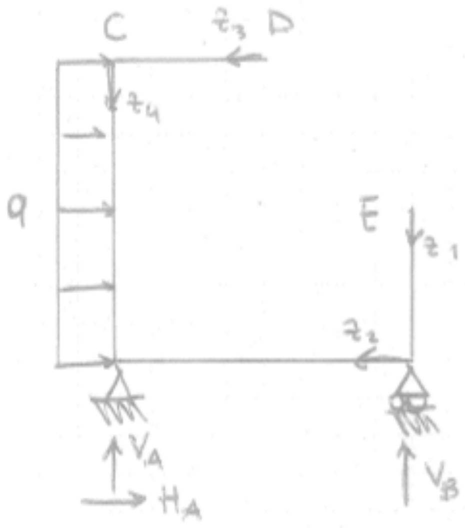
$+\ X_1$



$+\ X_2$



1) SISTEMA (0)

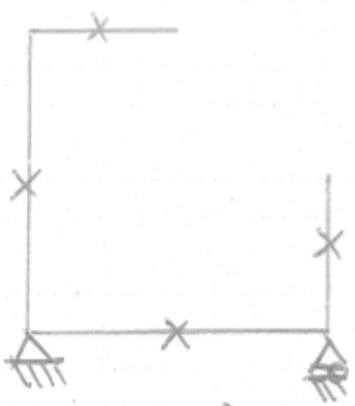


$\rightarrow)\ H_A = -2q\ell$

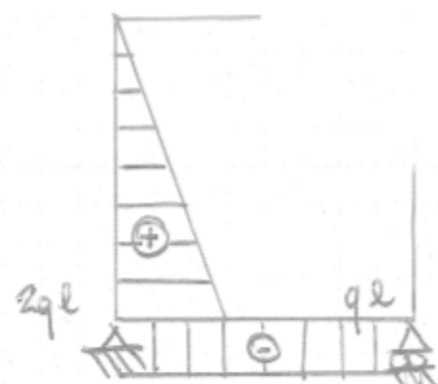
$A\circlearrowleft)\ V_B \cdot 2\ell - q2\ell^2 = 0$

$V_B = q\ell$

$\uparrow)\ V_A = -V_B = -q\ell$



N(0)



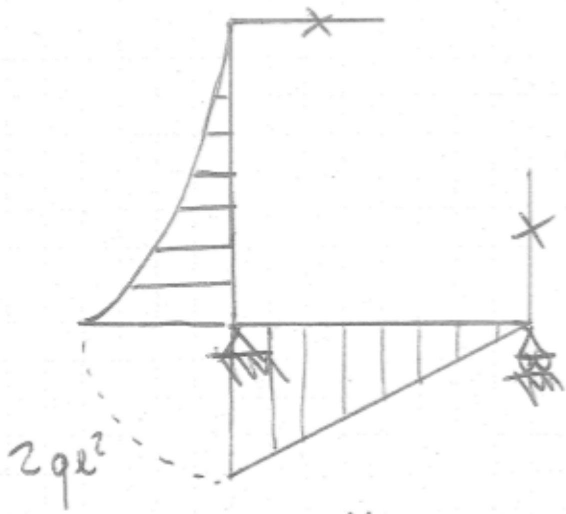
T(0)

$M_0(z_1) = 0$

$M_0(z_2) = V_B \cdot z = q\ell z$

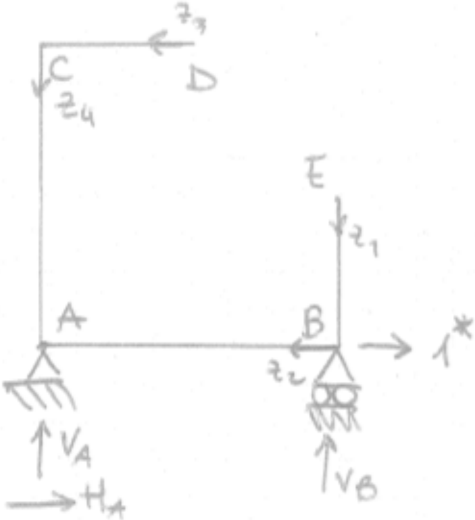
$M_0(z_3) = 0$

$M_0(z_4) = -q\frac{z^2}{2}$



M_0

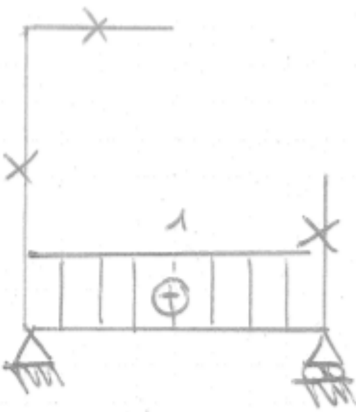
SISTEMA (1)



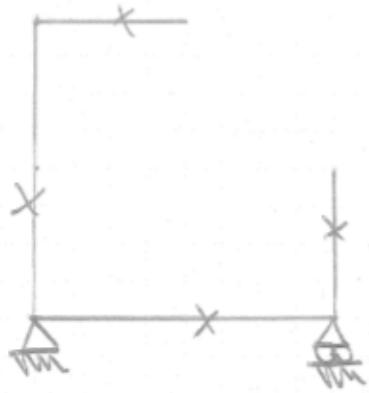
$\rightarrow)\quad H_A = -1$

$A \circlearrowleft)\quad V_B = 0$

$\uparrow)\quad V_A = 0$



N_1



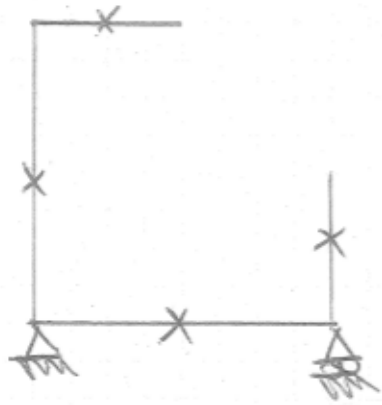
T_1

$M_1(z_1) = 0$

$M_1(z_2) = 0$

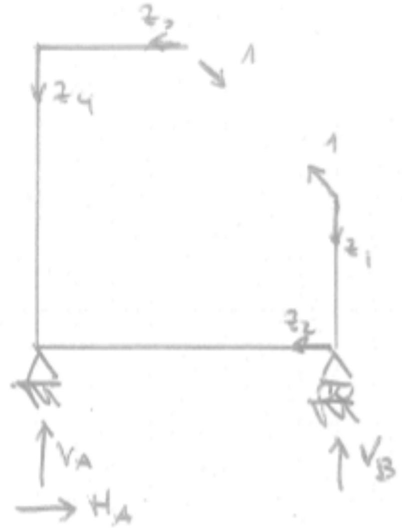
$M_1(z_3) = 0$

$M_1(z_4) = 0$



M_1

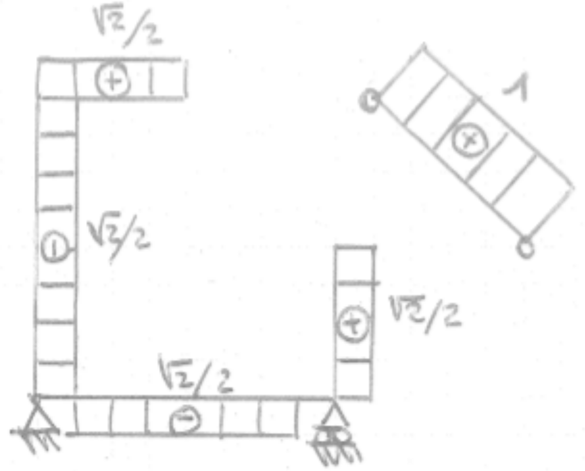
SISTEMA (2)



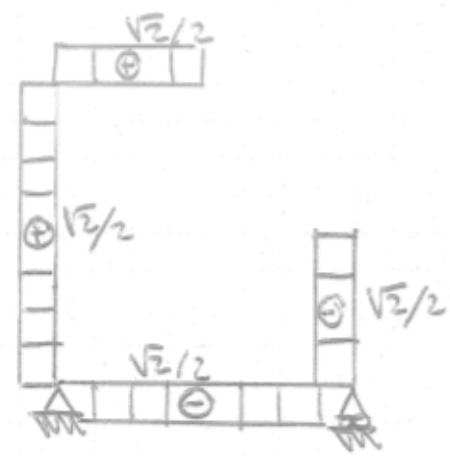
$\rightarrow)\ H_A = 0$

$A \curvearrowleft)\ V_B = 0$

$\uparrow)\ V_A = 0$

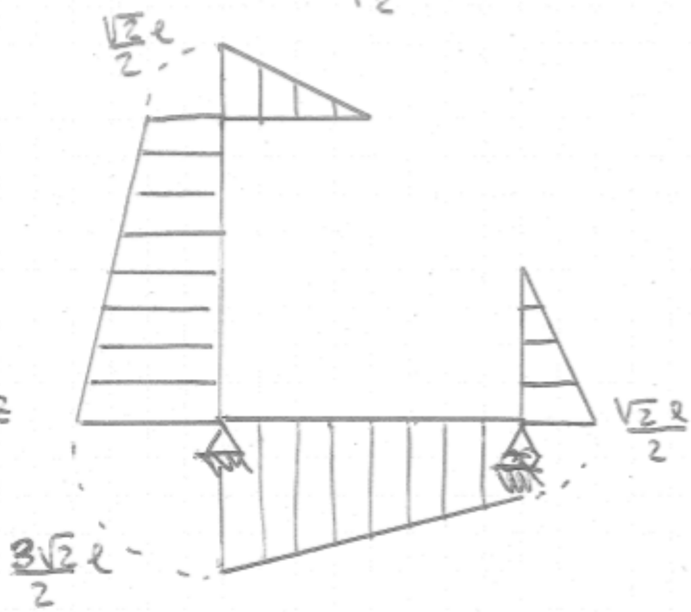


N_2



T_2

$$M_2(z_1) = (\sqrt{2}/2) \cdot z$$
$$M_2(z_2) = (\sqrt{2}\,l/2) + (\sqrt{2}/2) \cdot z$$
$$M_2(z_3) = -(\sqrt{2}/2)\, z$$
$$M_2(z_4) = -(\sqrt{2}\,l/2) - (\sqrt{2}/2)\, z$$



$$\eta_{10} = 0$$

$$\eta_{20} = \frac{1}{EJ}\int_0^{2l}\left(\frac{\sqrt{2}}{2}(l+z)\right)(qlz)\,dz + \frac{1}{EJ}\int_0^{2l}\left(-\frac{\sqrt{2}}{2}(l+z)\right)\left(-\frac{qz^2}{2}\right)dz \overset{!}{=} \frac{4\sqrt{2}}{EJ}\,ql^4$$

$$\eta_{11} = 0$$

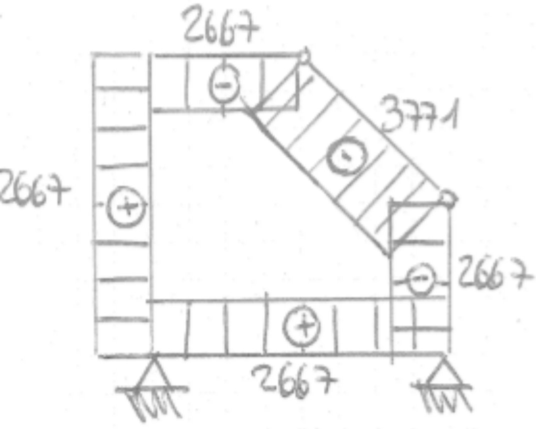
$$\eta_{22} = \frac{2}{EJ}\int_0^{l}\left(\frac{\sqrt{2}\,z}{2}\right)^2 dz + \frac{2}{EJ}\int_0^{2l}\left(\frac{\sqrt{2}}{2}(l+z)\right)^2 dz = \frac{9\,l^3}{EJ}$$

$$\eta_{12} = 0$$

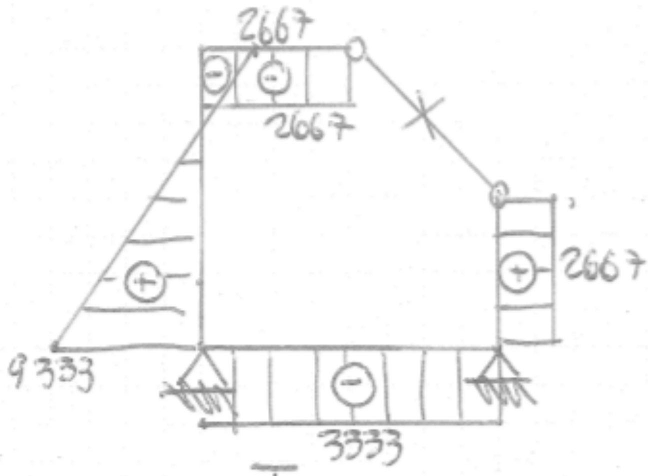
$$\begin{array}{ll} (1) \rightarrow & \left\{ \begin{array}{l} \eta_{11}\cdot x_1 + \eta_{12}\cdot X_2 = -\eta_{10} \\ \eta_{12}\cdot x_1 + \eta_{22}\cdot X_2 = -\eta_{20} \end{array} \right. \\ (2) \rightarrow & \end{array}$$

$\Rightarrow$ IDENTITÀ Poniamo $x_1 = 0$

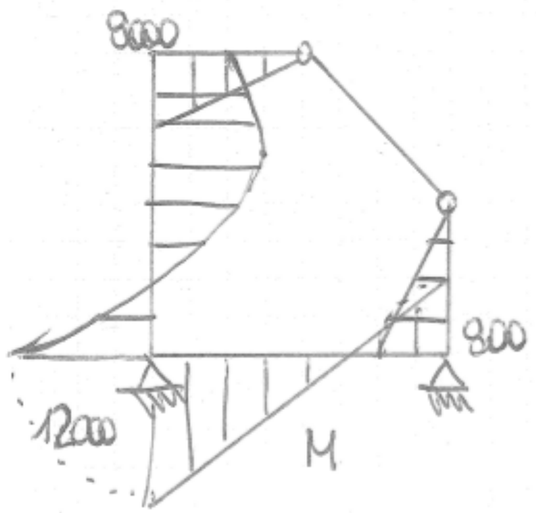
$X_2 = -3771{,}24$



N



T



M

2) PROGETTO

$M_{max} = M_A = 12000 \text{ Nm} \qquad N_A = 2667 \text{ N}$

$\sigma_{am} = 390 \text{ MPa} \qquad W_{min} = \frac{12000 \cdot 10^3}{390} = 30769 \text{ mm}^3$

ADOTTO IPE 120 $\qquad W_x = 52{,}96 \text{ cm}^3 \quad J_x = 317{,}8 \text{ cm}^4 \quad A = 13{,}21 \text{ cm}^2$

$E = 210000 \text{ MPa}$

$$\sigma_{max} = \frac{N}{A} + \frac{M_{max}}{W} = \frac{2667}{13{,}21 \cdot 10^2} + \frac{12000 \cdot 10^3}{52{,}96 \cdot 10^3} = 228{,}61 < 390 \text{ ok!}$$

3)

$$M_{11}^N = \frac{1}{EA}\int_0^{2l} (1)^2 \, dz = \frac{2l}{EA}$$

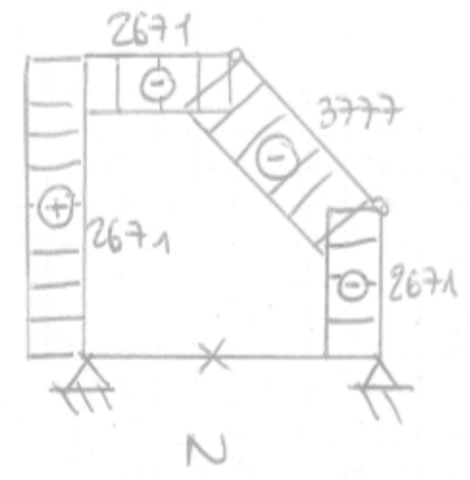
$$M_{22}^N = \frac{6}{EA}\int_0^{l}\left(\frac{\sqrt{2}}{2}\right)^2 dz + \frac{1}{EA}\int_0^{\sqrt{2}l} (1)^2 \, dz = \frac{(3+\sqrt{2})\,l}{EA}$$

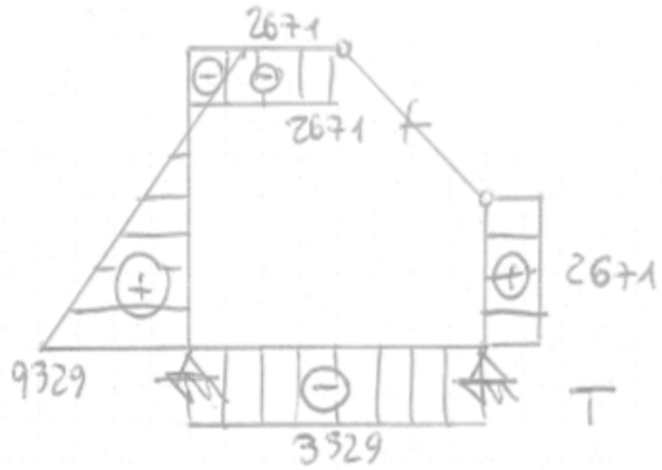
$$M_{12}^N = \frac{1}{EA}\int_0^{2l} (1)\left(-\frac{\sqrt{2}}{2}\right) dz = -\frac{\sqrt{2}\,l}{EA}$$

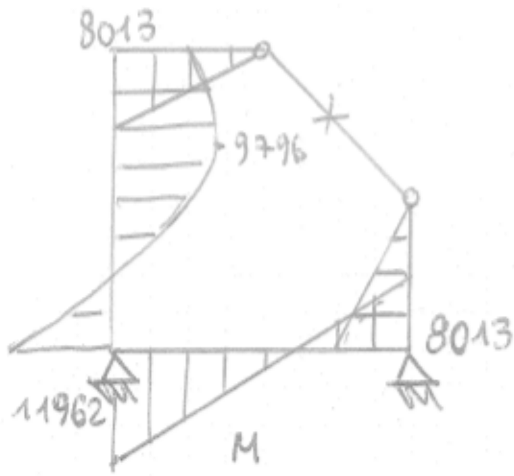
$$M_{ij}^{TOT} = M_{ij} + M_{ij}^N \qquad \alpha = 1{,}2 \cdot 10^{-5} \quad \Delta t = 30° \quad \varepsilon^t = \alpha \Delta t$$

$$\begin{aligned} (1) &\rightarrow \\ (2) &\rightarrow \end{aligned} \begin{cases} M_{11}^{TOT} \cdot x_1 + M_{12} \cdot x_2 + M_{10} \\ M_{12} \cdot x_1 + M_{22} \cdot x_2 + M_{20} + \int_0^{\sqrt{2}l} (1) \cdot \alpha \, \Delta t \, dz = 0 \end{cases}$$

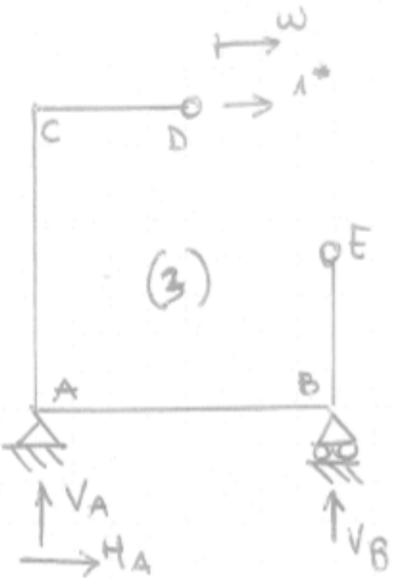
$x_1 = 2670{,}85 \text{ N} \qquad x_2 = 3777{,}15 \text{ N}$


2671
3777
2671
2671
N


2671
2671
2671
9329
3329
T


8013
9796
8013
11962
M

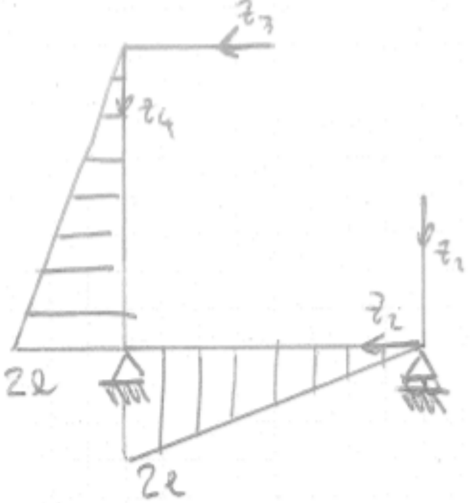
4)



$\rightarrow)\ H_A = -1$

$A \curvearrowleft)\ V_B = 1$

$\uparrow)\ V_A = -1$



$M_3(z_1) = 0$

$M_3(z_2) = z$

$M_3(z_3) = 0$

$M_3(z_4) = -z$

Utilizzando X_1 e X_2 del punto 3

$$1^* \cdot \omega = \frac{1}{EJ}\int_0^{2l}(z)(qlz)\,dz + \frac{1}{EJ}\int_l^{2l}(-z)\left(-\frac{qz^2}{2}\right)dz + X_1\left[0\right] +$$
$$+ X_2\left[\frac{2}{EJ}\int_0^{l}(z)\left(\frac{\sqrt{2}}{2}(l+z)\right)dz\right] = 0{,}1247\ m$$