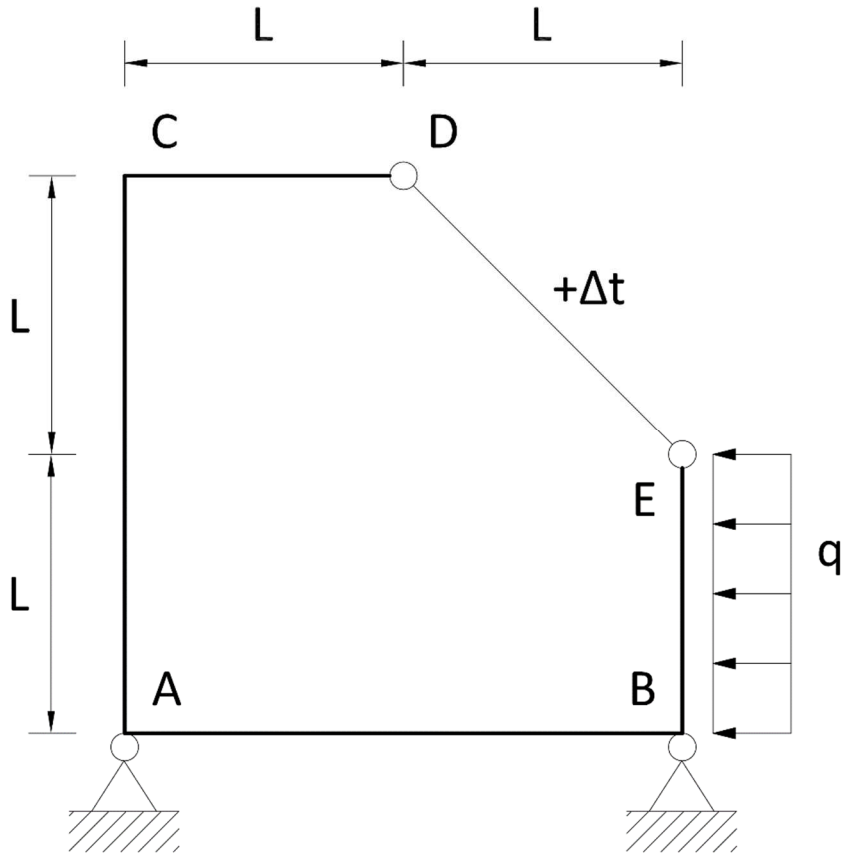
IV Parziale di Scienza delle Costruzioni 06/06/2017 B



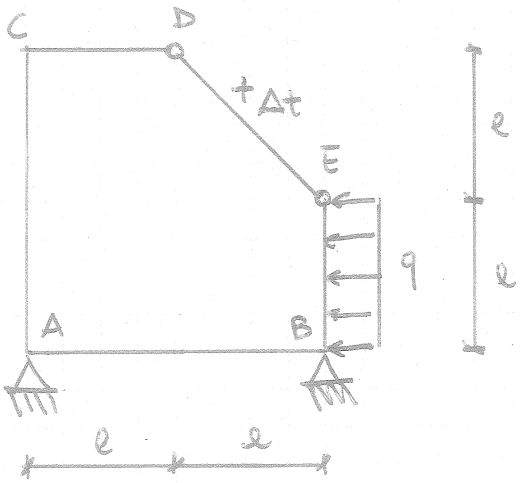
1) Risolvere la struttura in figura e disegnare i diagrammi M,N,T avendo posto L=3 m, q=4000 N/m, e considerando la deformazione assiale delle aste. In questa fase si trascuri il carico termico.
2) Risolvere e determinare i diagrammi M,N,T considerando anche un'azione termica Δt=30°C sull'asta DE (coefficiente di dilatazione termica α=1,2 x 10^{-5} °C^{-1}).

Le aste AB, BE, AC, CD sono realizzate con un profilato in acciaio IPE 160 (E= 210000 MPa, J_x=869 cm^4, A=20.1 cm^2, W=109 cm^3)

L'asta DE è realizzata con un tondino di acciaio di sezione circolare piena e diametro Ø=20 mm

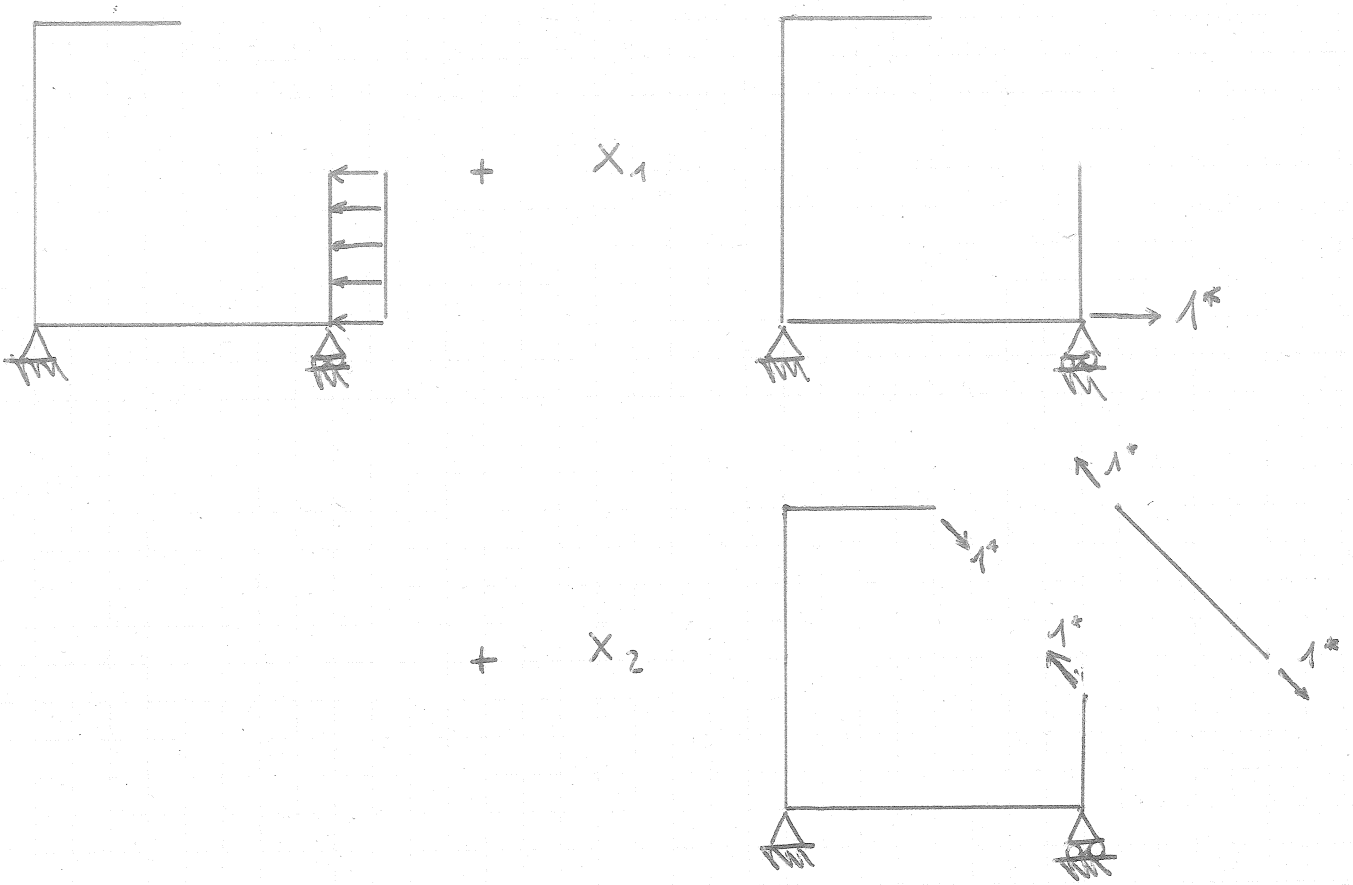
FILA B

06/06/2017

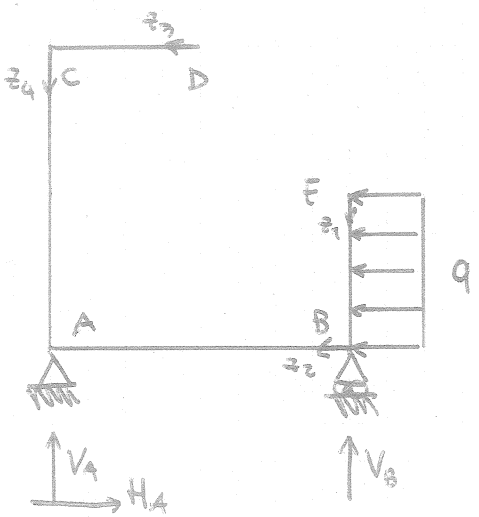


$\ell = 3m$

$q = 4000\ N/m$



1) SISTEMA (0)

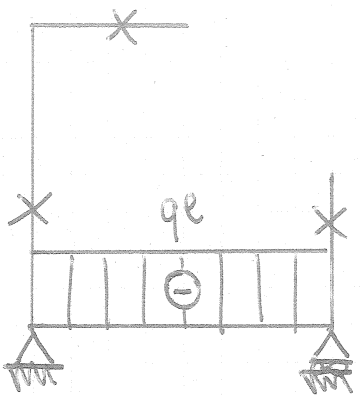


$\rightarrow) \; H_A = ql$

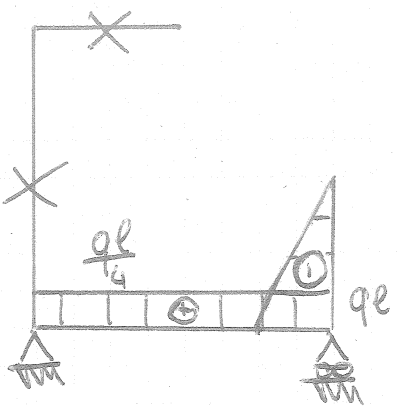
$A \curvearrowright \; V_B \cdot 2l + \frac{ql^2}{2} = 0$

$V_B = -\frac{ql}{4}$

$\uparrow) \; V_A = \frac{ql}{4}$



N(0)



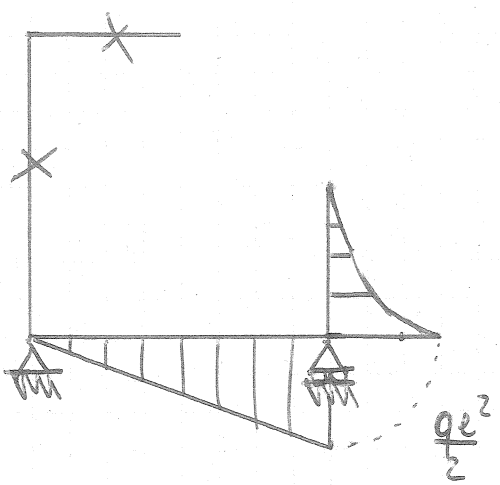
T(0)

$M_0(z_1) = -\frac{qz^2}{2}$

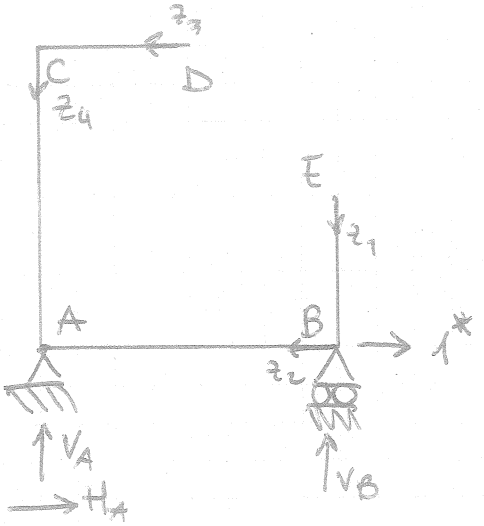
$M_0(z_2) = \frac{ql^2}{2} - \frac{qlz}{4}$

$M_0(z_3) = 0$

$M_0(z_4) = 0$



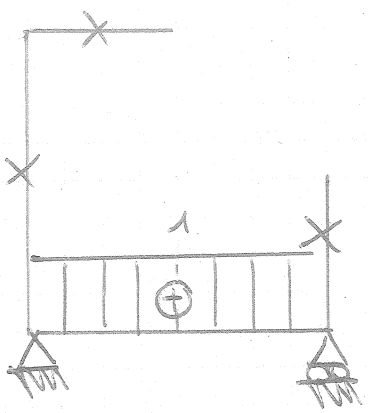
SISTEMA (1)



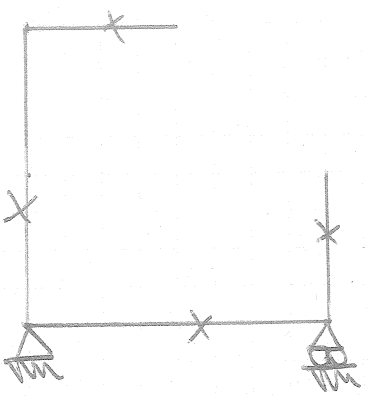
$\rightarrow) \quad H_A = -1$

$A\circlearrowleft \quad V_B = 0$

$\uparrow) \quad V_A = 0$



N_1



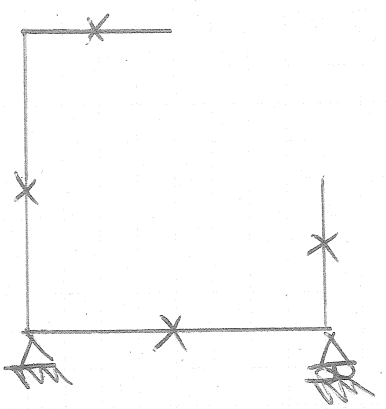
T_1

$M_1(z_1) = 0$

$M_1(z_2) = 0$

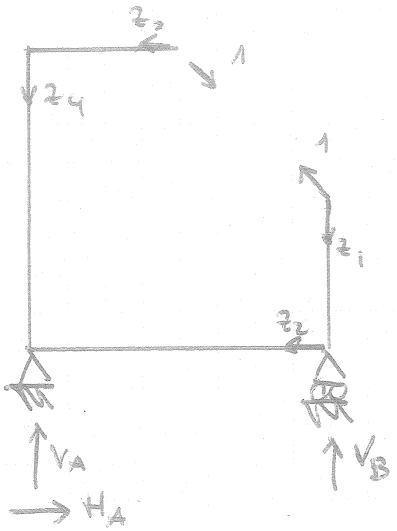
$M_1(z_3) = 0$

$M_1(z_4) = 0$



M_1

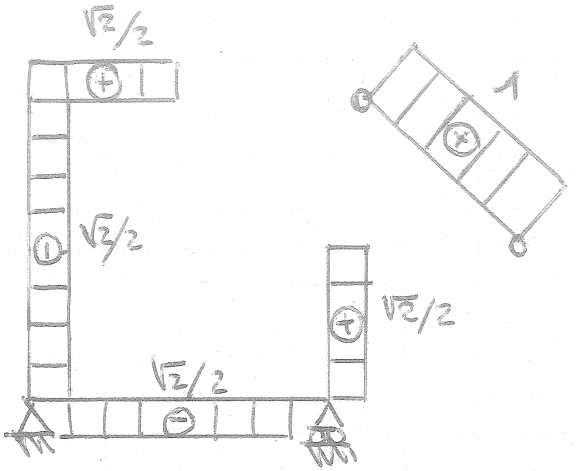
SISTEMA (2)



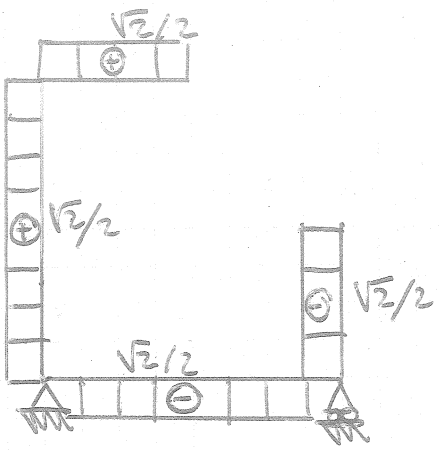
$\rightarrow) \; H_A = 0$

$A) \; V_B = 0$

$\uparrow) \; V_A = 0$



N_2



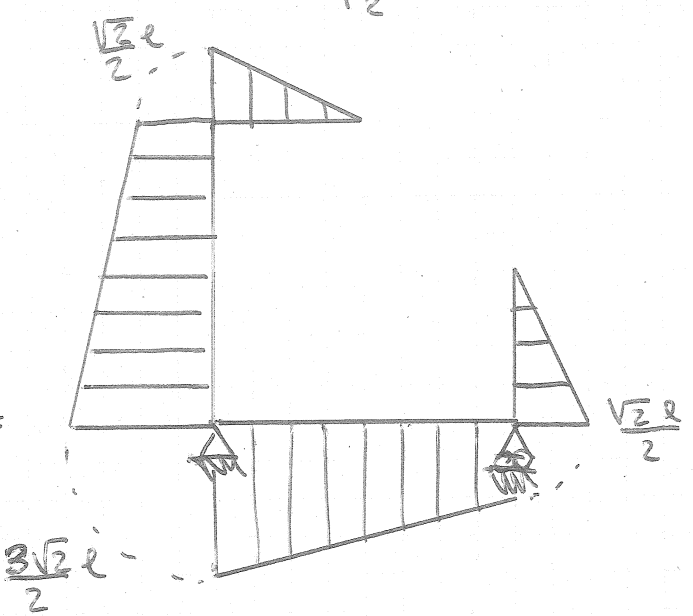
T_2

$$M_2(z_1) = (\sqrt{2}/2) \cdot z$$

$$M_2(z_2) = (\sqrt{2}\,\ell/2) + (\sqrt{2}/2) \cdot z$$

$$M_2(z_3) = -(\sqrt{2}/2)\, z$$

$$M_2(z_4) = -(\sqrt{2}\,\ell/2) - (\sqrt{2}/2)\, z$$



$E = 2 \cdot 10^5$ MPa $\quad A_{IPE} = 20{,}1\ cm^2 \quad J_{IPE} = 869\ cm^2$

$A_{TOND} = 3{,}14\ cm^2 \quad \Delta t = 0°C$

$$M_{10} = \frac{1}{EA_{IPE}} \int_0^{2\ell} (-q\ell)(1)\, dz = -\frac{2q\ell^2}{EA_{IPE}}$$

$$M_{20} = \frac{1}{EJ} \int_0^{\ell} \left(-\frac{\sqrt{2}z}{2}\right)\left(-\frac{qz^2}{2}\right) dz + \frac{1}{EJ} \int_0^{2\ell} \left(\frac{q\ell}{2}\left(\ell - \frac{z}{2}\right)\right)\left(\frac{\sqrt{2}}{2}\left(\ell + z\right)\right) dz +$$

$$+ \frac{1}{EA_{IPE}} \int_0^{2\ell} \left(-\frac{\sqrt{2}}{2}\right)(-q\ell)\, dz = \frac{23\sqrt{2}\, q\ell^4}{48\, EJ} + \frac{\sqrt{2}\, q\ell^2}{EA_{IPE}}$$

$$M_{11} = \frac{1}{EA_{IPE}} \int_0^{2\ell} 1^2\, dz = \frac{2\ell}{EA_{IPE}}$$

$$M_{22} = \frac{2}{EJ} \int_0^{\ell} \left(\frac{\sqrt{2}}{2} z\right)^2 dz + \frac{2}{EJ} \int_0^{2\ell} \left(\frac{\sqrt{2}}{2}\left(\ell + z\right)\right)^2 + \frac{6}{EA_{IPE}} \int_0^{\ell} \left(\frac{\sqrt{2}}{2}\right)^2 dz +$$

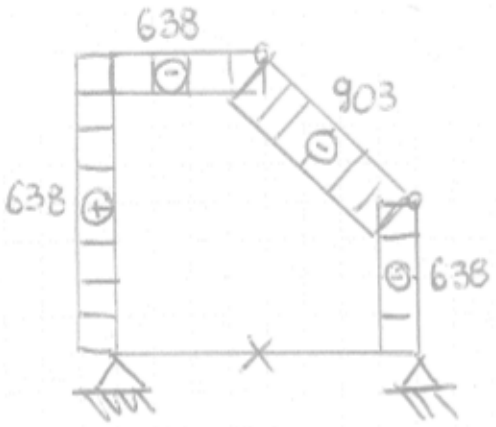
$$+ \frac{1}{EA_{TOND}} \int_0^{\sqrt{2}\,\ell} 1^2\, dz = \frac{9\ell^3}{EJ} + \frac{3\ell}{EA_{IPE}} + \frac{\sqrt{2}\,\ell}{EA_{TOND}}$$

$$M_{12} = \frac{1}{EA_{IPE}} \int_0^{2\ell} 1 \cdot \left(-\frac{\sqrt{2}}{2}\right) dz = -\frac{\sqrt{2}\,\ell}{EA_{IPE}}$$

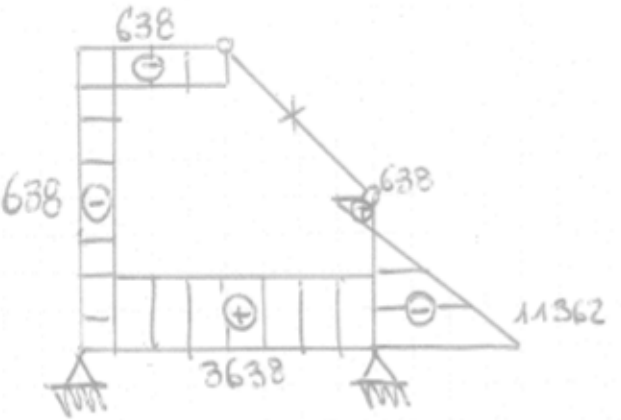
$$(1) \longrightarrow \quad M_{11} \cdot X_1 + M_{12} \cdot X_2 + M_{10} = 0$$

$$(2) \longrightarrow \quad M_{12} \cdot X_1 + M_{22} \cdot X_2 + M_{20} = 0$$

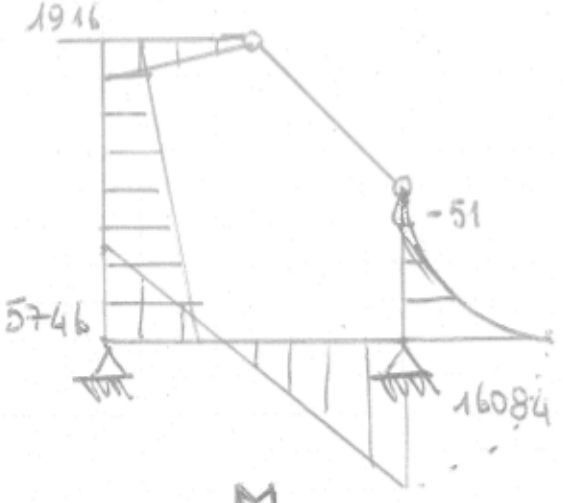
$X_1 = 11361{,}5\ N \qquad X_2 = -902{,}99\ N$



N



T



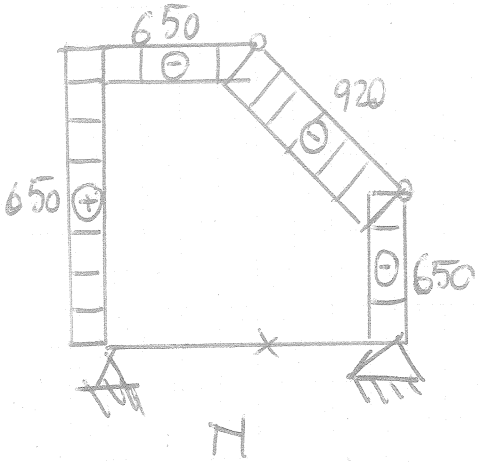
M

2) $\Delta t = 30°$ $\alpha = 1{,}2 \cdot 10^{-5}\,°C^{-1}$

$\varepsilon^t = \alpha \cdot \Delta t$

$$(1) \rightarrow \begin{cases} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{10} = 0 \\ (2) \rightarrow \quad \delta_{12} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{20} + \int_0^{\sqrt{2}\cdot\ell} (1) \cdot \alpha \cdot \Delta t \, dz = 0 \end{cases}$$

$X_1 = 11349{,}4\ N$ $\qquad X_2 = 920{,}2\ N$



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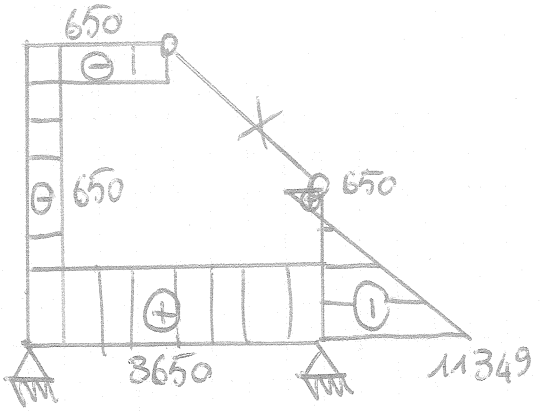

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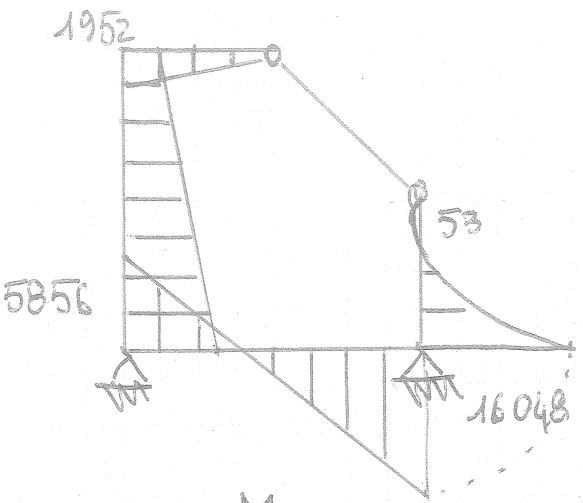

M