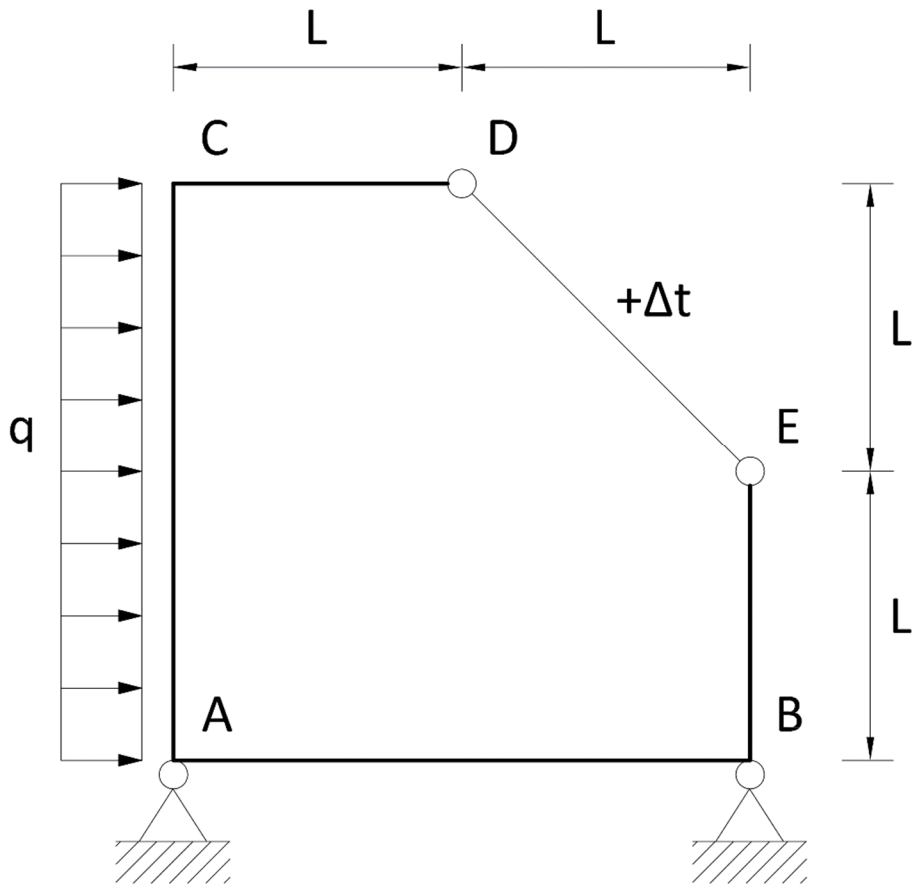
IV Parziale di Scienza delle Costruzioni 06/06/2017 A



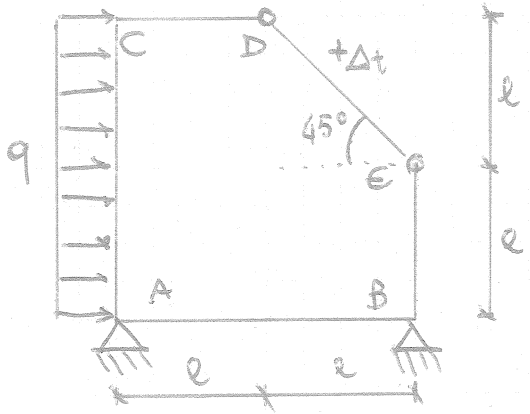
1) Risolvere la struttura in figura e disegnare i diagrammi M,N,T avendo posto L=3 m, q=2000 N/m, e considerando la deformazione assiale delle aste. In questa fase si trascuri il carico termico.
2) Risolvere e determinare i diagrammi M,N,T considerando anche un'azione termica Δt=30°C sull'asta DE (coefficiente di dilatazione termica α=1,2 x 10^{-5} °C^{-1}).

Le aste AB, BE, AC, CD sono realizzate con un profilato in acciaio IPE 160 (E= 210000 MPa, J_x=869 cm^4, A=20.1 cm^2, W=109 cm^3)

L'asta DE è realizzata con un tondino di acciaio di sezione circolare piena e diametro Ø=20 mm

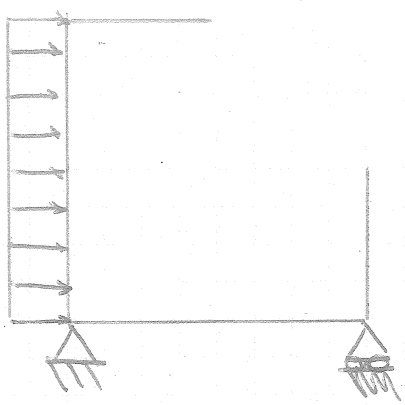
FILA A

06/06/2017

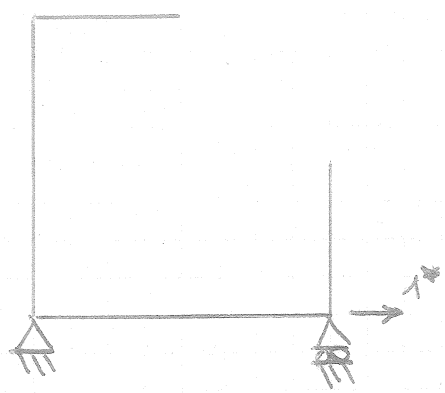


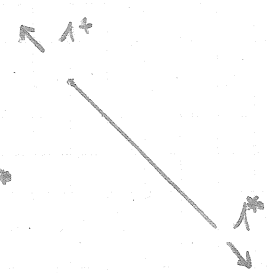
$\ell = 3m$

$q = 2000\ N/m$



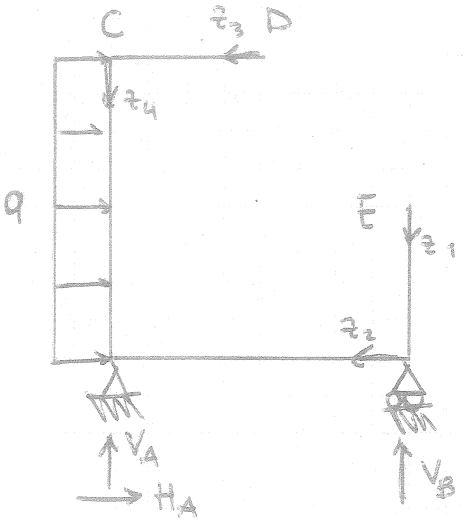
+ X_1





+ X_2

1) SISTEMA (0)

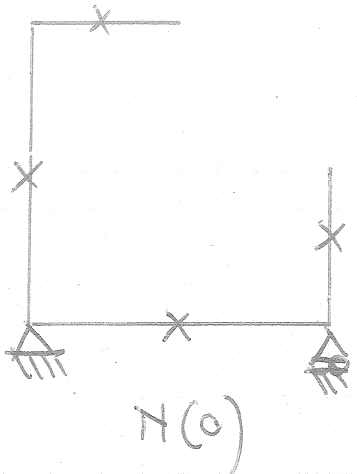


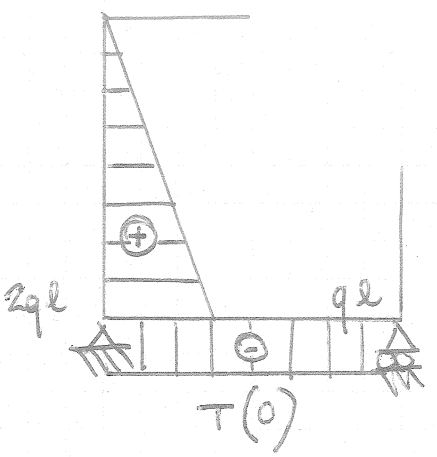
$\rightarrow) \; H_A = -2ql$

$A\curvearrowright) \; V_B \cdot 2l - q2l^2 = 0$

$V_B = ql$

$\uparrow) \; V_A = -V_B = -ql$



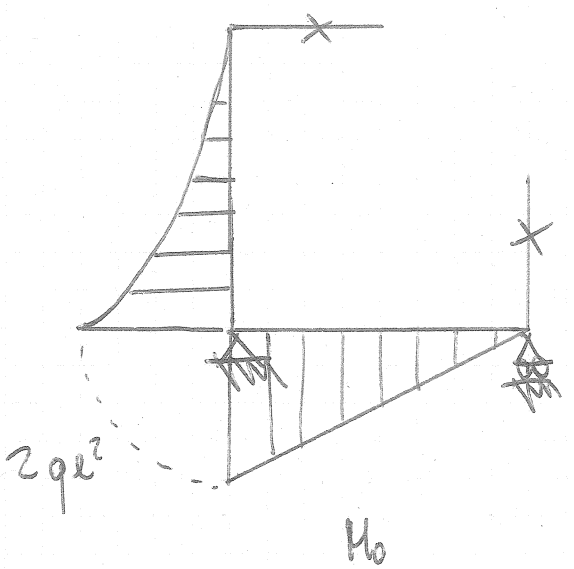


$M_0(z_1) = 0$

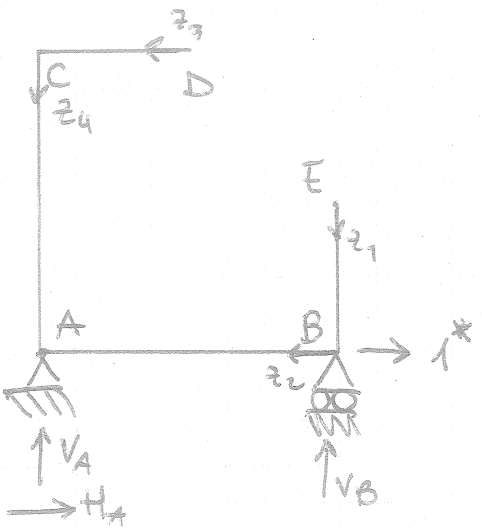
$M_0(z_2) = V_B \cdot z = qlz$

$M_0(z_3) = 0$

$M_0(z_4) = -q\frac{z^2}{2}$



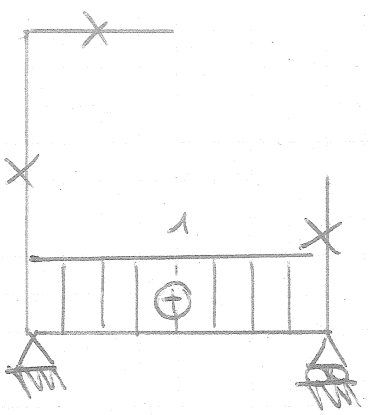
SISTEMA (1)



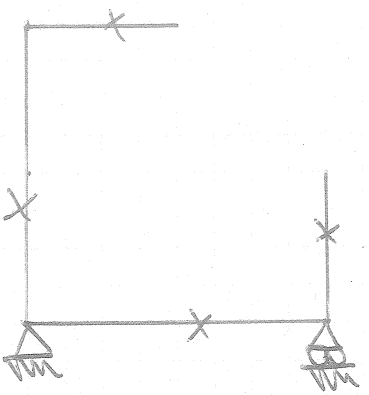
$\rightarrow) \quad H_A = -1$

$A\circlearrowleft \quad V_B = 0$

$\uparrow) \quad V_A = 0$



N_1



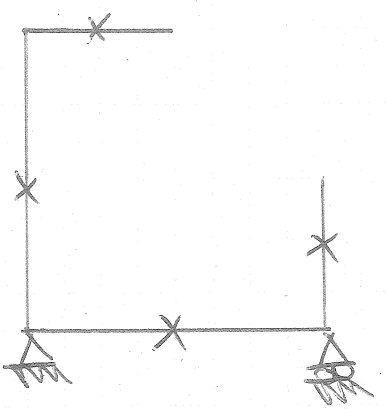
T_1

$M_1(z_1) = 0$

$M_1(z_2) = 0$

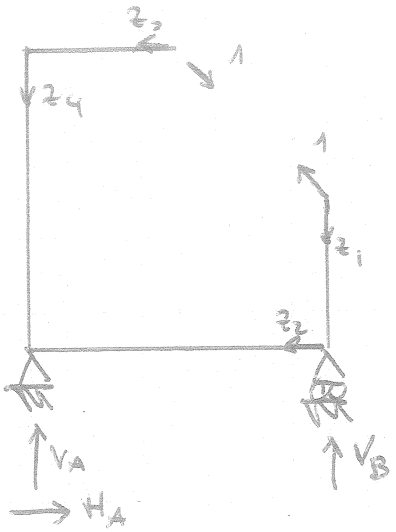
$M_1(z_3) = 0$

$M_1(z_4) = 0$



M_1

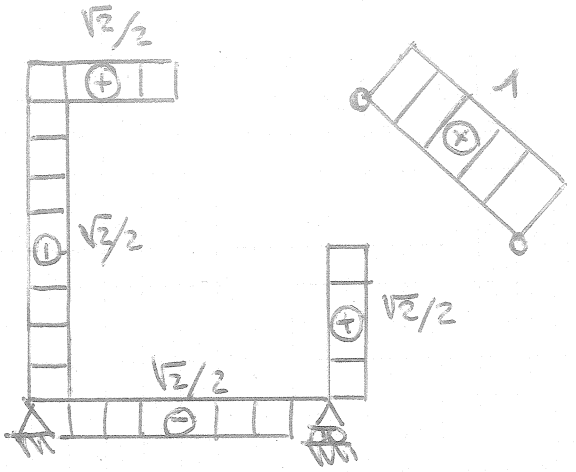
SISTEMA (2)



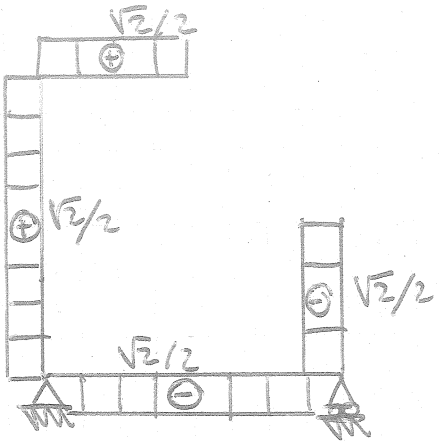
$\rightarrow) \; H_A = 0$

$A) \; V_B = 0$

$\uparrow) \; V_A = 0$



N_2



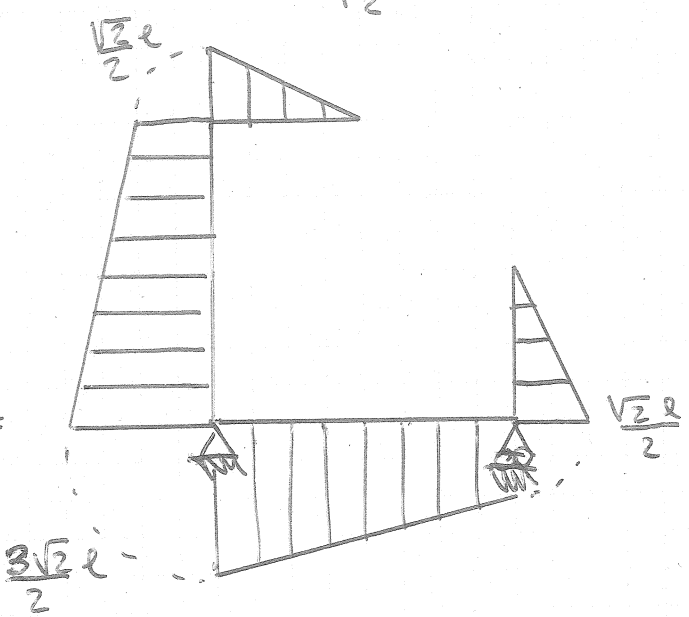
T_2

$$M_2(z_1) = (\sqrt{2}/2) \cdot z$$

$$M_2(z_2) = (\sqrt{2}\,\ell/2) + (\sqrt{2}/2) \cdot z$$

$$M_2(z_3) = -(\sqrt{2}/2)\, z$$

$$M_2(z_4) = -(\sqrt{2}\,\ell/2) - (\sqrt{2}/2)\, z$$



$E = 2 \cdot 10^5 \text{ MPa}$ $\quad A_{IPE} = 20,1 \text{ cm}^2$ $\quad J_{IPE} = 869 \text{ cm}^4$

$A_{TOND} = 3,14 \text{ cm}^2$ $\quad \Delta t = 0\,^\circ C$

$$M_{10} = 0$$

$$M_{20} = \frac{1}{EJ}\int_0^{2\ell}\left(\frac{\sqrt{2}}{2}(\ell+z)\right)(q\ell z)\,dz + \frac{1}{EJ}\int_0^{2\ell}\left(-\frac{\sqrt{2}}{2}(\ell+z)\right)\cdot\left(-\frac{qz^2}{2}\right)dz$$

$$= \frac{4\sqrt{2}\,q\ell^4}{EJ}$$

$$M_{11} = \frac{1}{EA_{IPE}}\int_0^{2\ell}(1)^2\,dz = \frac{2\ell}{EA_{IPE}}$$

$$M_{22} = \frac{2}{EJ}\int_0^{\ell}\left(\frac{\sqrt{2}z}{2}\right)^2 dz + \frac{2}{EJ}\int_0^{2\ell}\left(\frac{\sqrt{2}}{2}(\ell+z)\right)^2 dz + \frac{6}{EA_{IPE}}\int_0^{\ell}\left(\frac{\sqrt{2}}{2}\right)^2 +$$

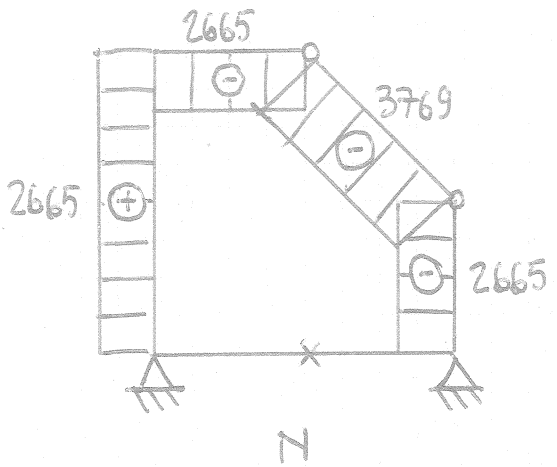
$$+ \frac{1}{EA_{tond}}\int_0^{\sqrt{2}\ell}(1)^2\,dz = \frac{9}{EJ}\ell^3 + \frac{3\ell}{EA_{IPE}} + \frac{\sqrt{2}\ell}{EA_{TOND}}$$

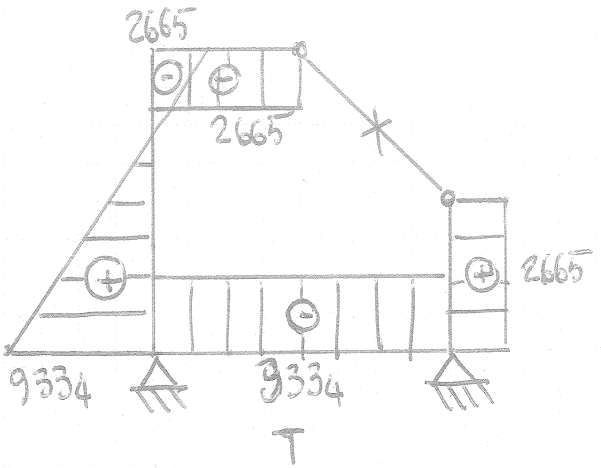
$$M_{12} = \frac{1}{EA_{IPE}}\int_0^{2\ell}(1)\left(-\frac{\sqrt{2}}{2}\right)dz = -\frac{\sqrt{2}\,\ell}{EA_{IPE}}$$

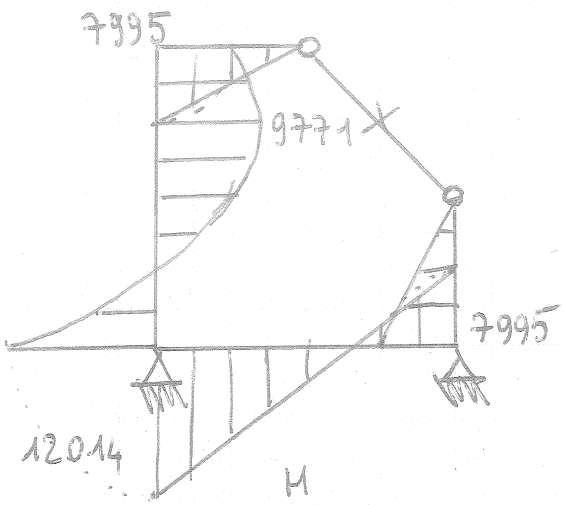
$$(1) \rightarrow \quad M_{11}\cdot X_1 + M_{12}\cdot X_2 = -M_{10}$$

$$(2) \rightarrow \quad M_{12}\cdot X_1 + M_{22}\cdot X_2 = -M_{20}$$

$X_1 = -2665,09 \text{ N}$ $\quad X_2 = -3769,01 \text{ N}$





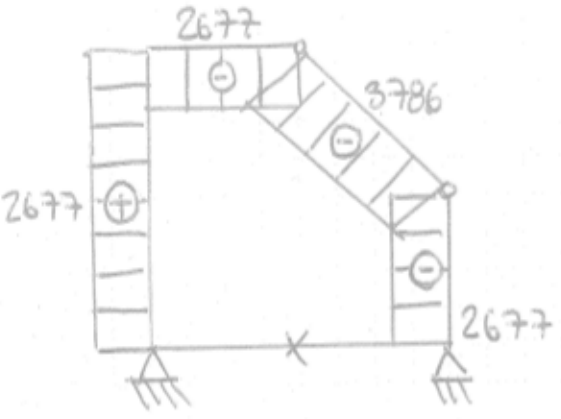


2) $\Delta t = 30°C \qquad \alpha = 1{,}2 \cdot 10^{-5}$

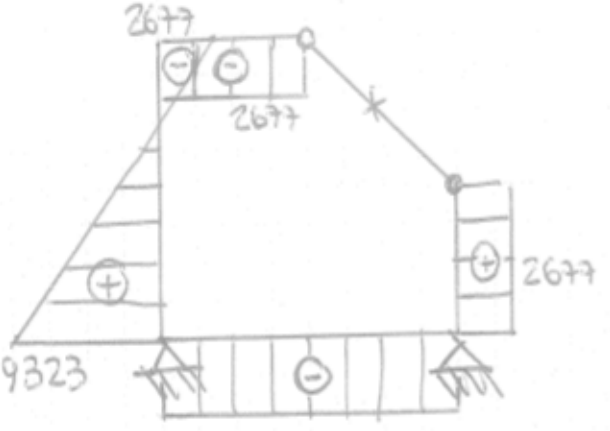
$\varepsilon^t = \alpha \Delta t$

$$(1) \rightarrow \begin{cases} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{10} = 0 \\ (2) \rightarrow \delta_{12} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{20} + \int_0^{\pi \cdot 2} (1)\, \alpha\, \Delta t\, dz = 0 \end{cases}$$

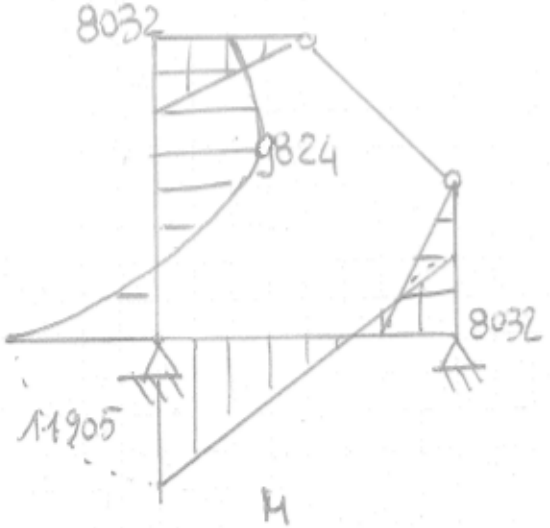
$X_1 = -2677{,}3 \qquad X_2 = -3786\ N$



N



T



M