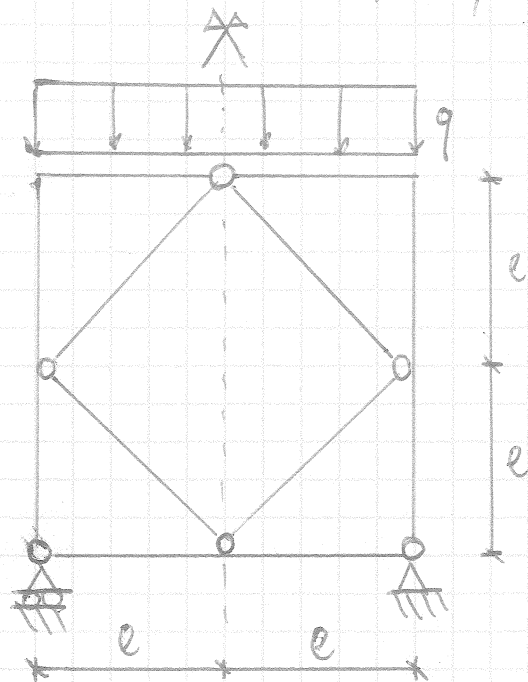
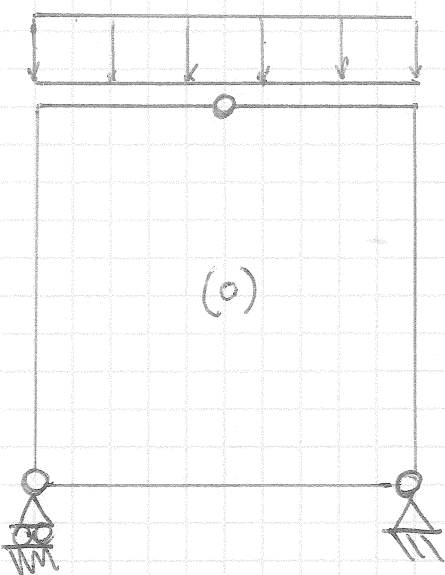


COMPITO 15/09/2017

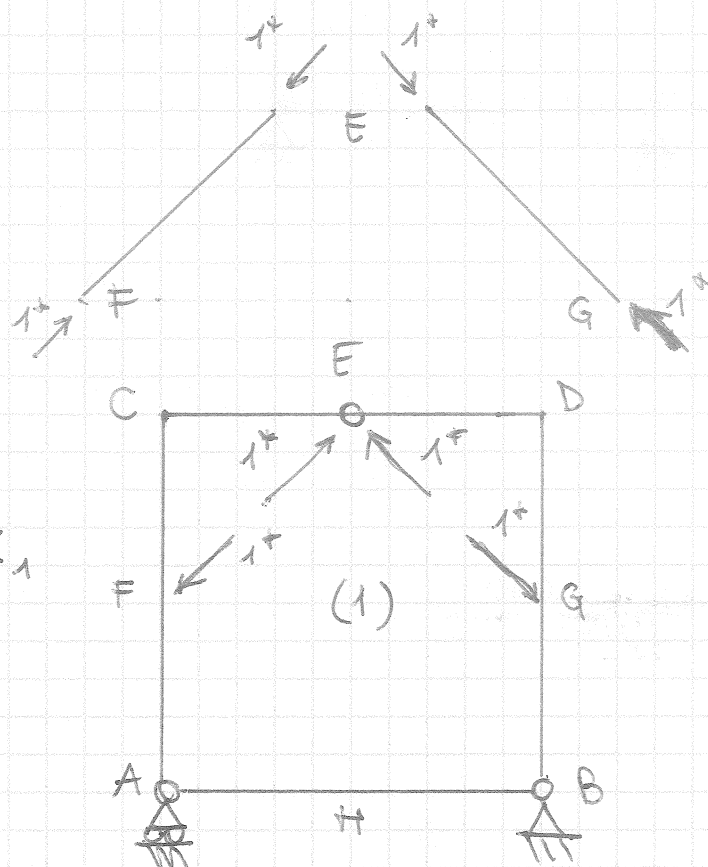


$$q = 2000 \text{ N/m}$$

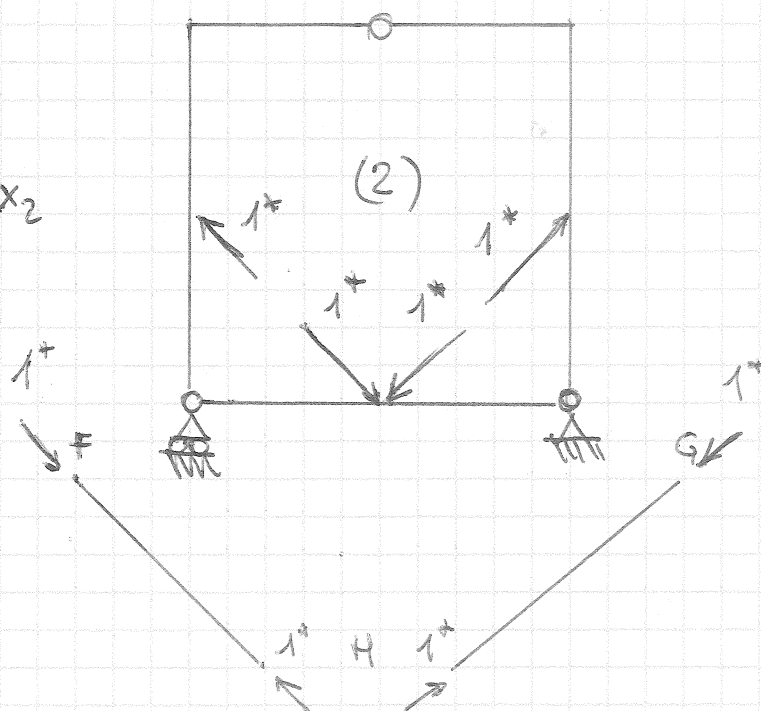
$$l = 2 \text{ m}$$



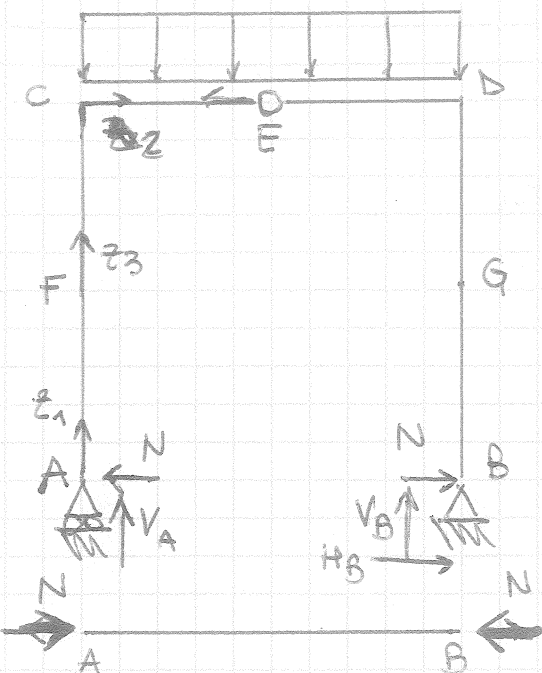
+ X_1



+ X_2



1) SISTEMA 0



$$\rightarrow H_B = 0$$

$$V_A = V_B \text{ per sym}$$

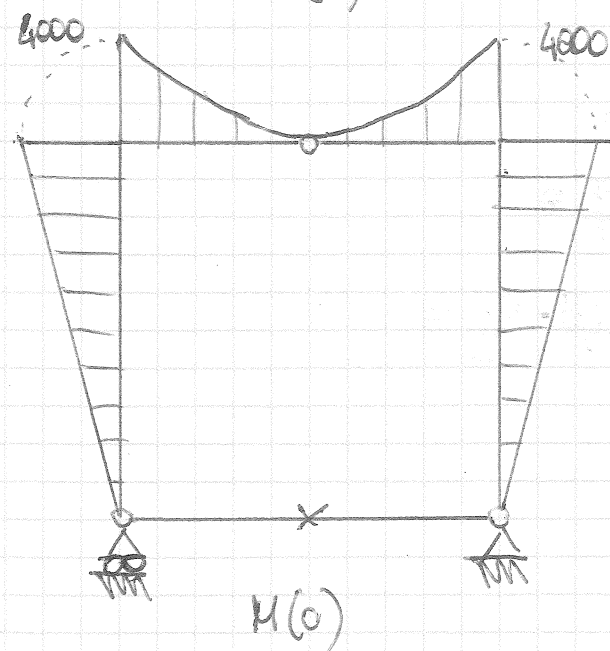
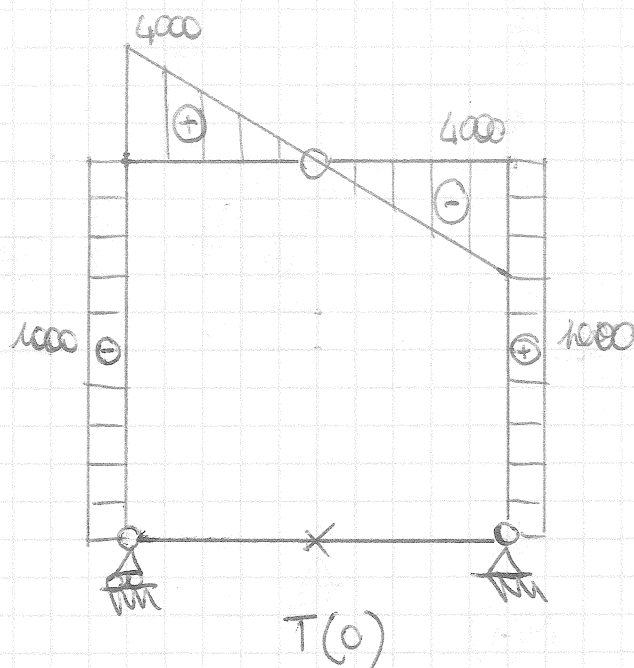
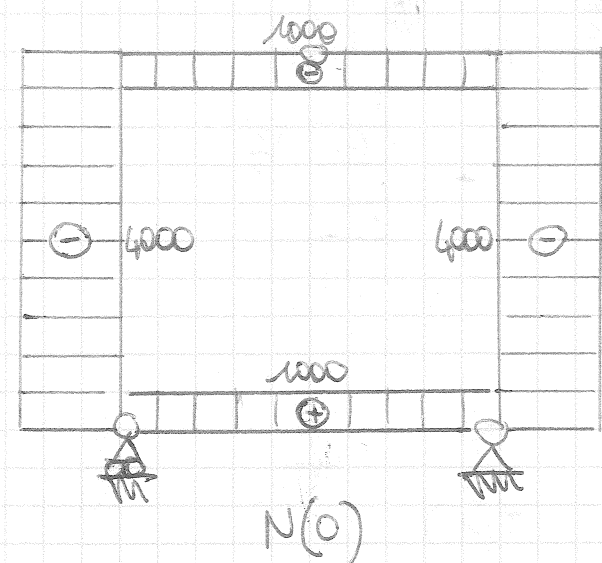
$$\uparrow) 2V_A - q \cdot 2l = 0$$

$$V_A = ql = V_B = 4000 \text{ N}$$

E) TRATTO ECA

$$\frac{ql^2}{2} - V_A \cdot l - N \cdot 2l = 0$$

$$N = \left(\frac{ql^2}{2} - ql^2 \right) \frac{1}{2l} = -\frac{ql^2}{2} \cdot \frac{1}{2l} = -\frac{ql}{4} = -1000 \text{ N}$$

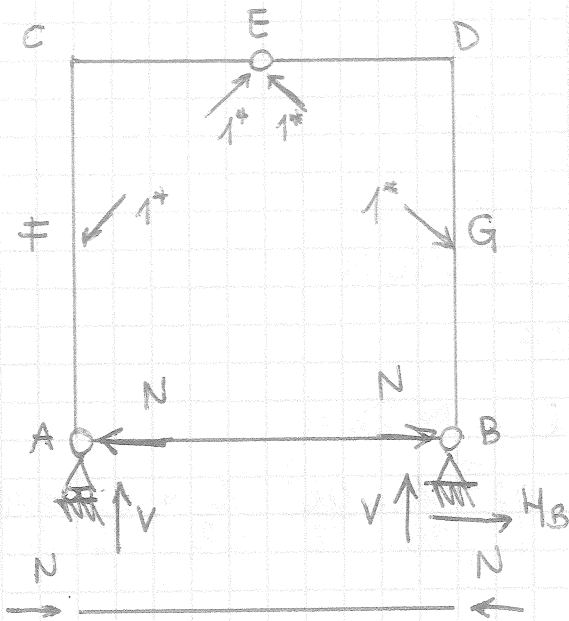


$$M_0(z_1) = N \cdot z = -\frac{ql}{4} z$$

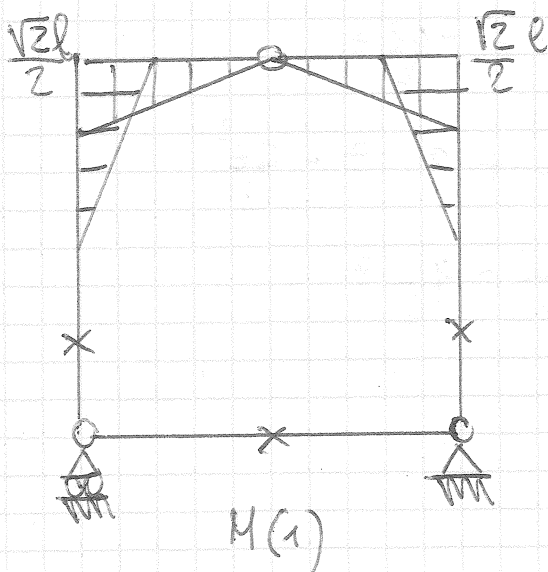
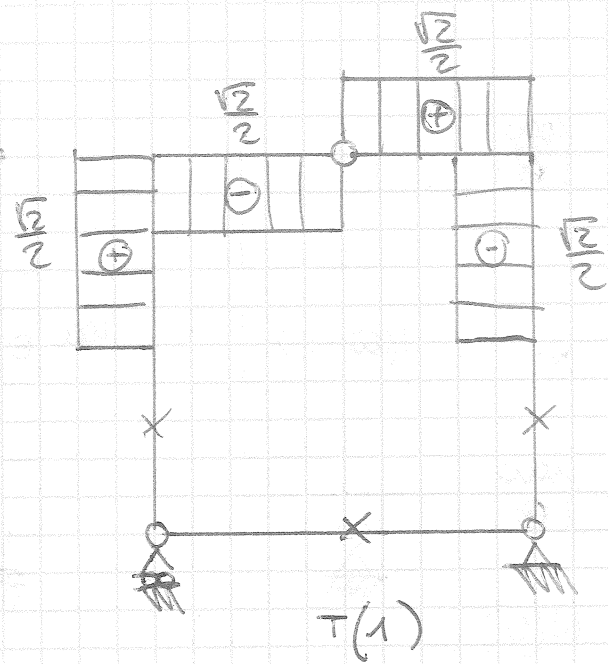
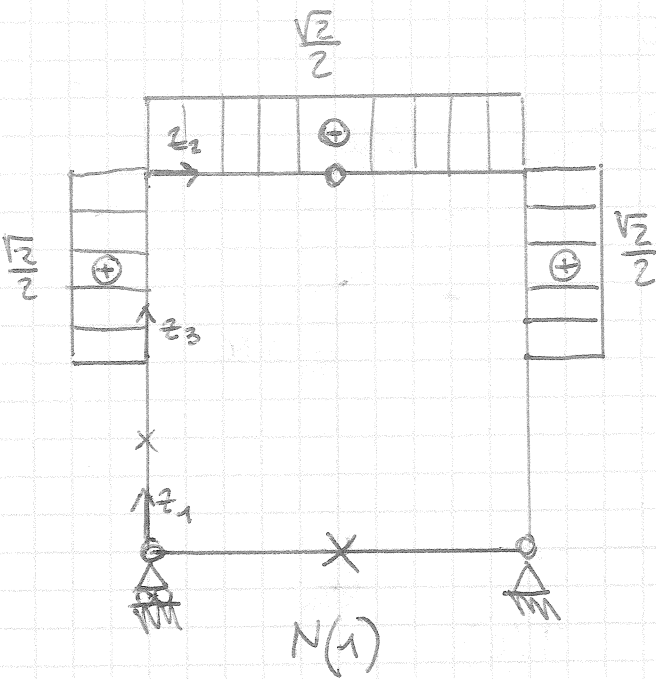
$$M_0(z_2) = -\frac{ql^2}{2} + ql \cdot z - \frac{qz^2}{2}$$

$$M_0(z_3) = -\frac{ql^2}{4} - \frac{ql}{4} z$$

SISTEMA 1

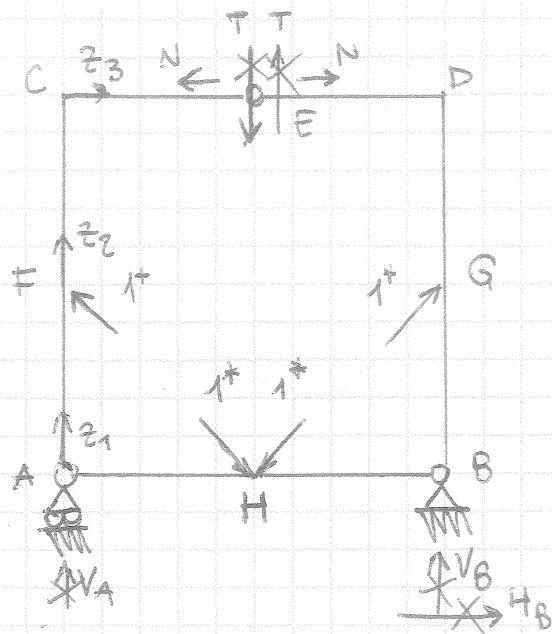


$$\begin{aligned} \rightarrow) H_B &= 0 \\ \uparrow) V &= 0 \\ E) \text{ TRATTO ECFA} \\ N &= 0 \end{aligned}$$



$$\begin{aligned} M_1(z_1) &= 0 \\ M_1(z_3) &= \frac{\sqrt{2}}{2} z \\ M_1(z_2) &= \frac{\sqrt{2}l}{2} - \frac{\sqrt{2}}{2} z \end{aligned}$$

SISTEMA 2



$T=0$ per sim

$$\rightarrow) H_B = 0$$

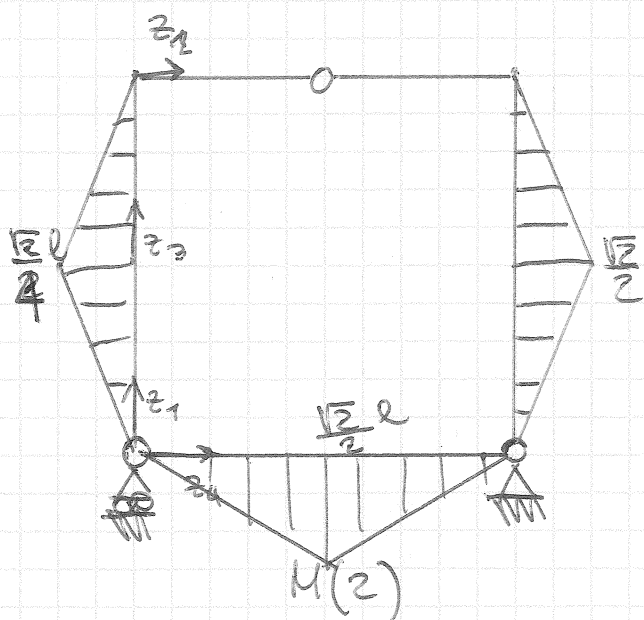
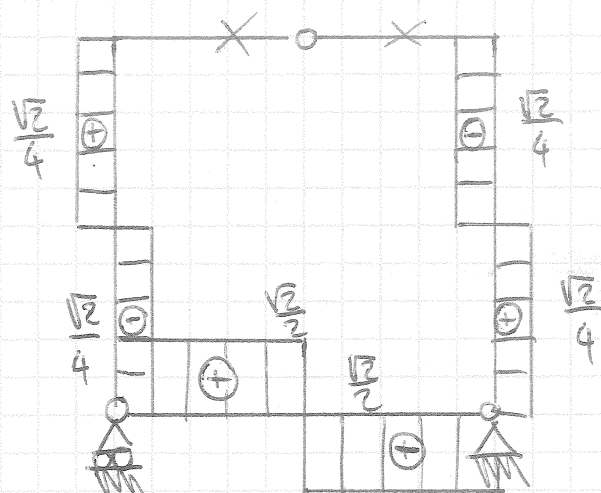
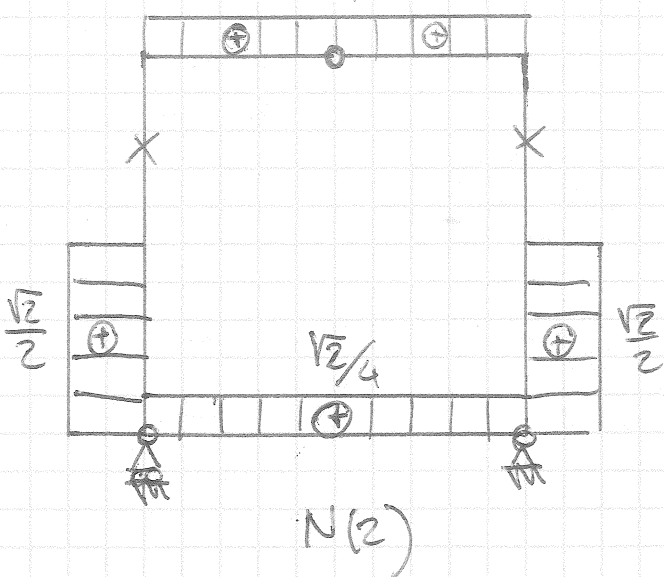
$$\uparrow) V_A = V_B = 0$$

A) TRATTO ACE

$$\frac{\sqrt{2}}{2} \cdot l + N \cdot 2l = 0$$

$$N = -\frac{\sqrt{2}}{4}$$

$$\frac{\sqrt{2}}{4}$$



$$M_2(z_1) = -\frac{\sqrt{2}}{4} \cdot z$$

$$M_2(z_3) = -\frac{\sqrt{2}}{4} (l - z)$$

$$M_2(z_2) = 0$$

$$M_2(z_4) = \frac{\sqrt{2}}{2} \cdot z$$

$$M_{10} = \frac{1}{EJ} \left[2 \int_0^l \left(\frac{\sqrt{2}}{2} z \right) \left(-\frac{ql}{4} (l+z) \right) dz + 2 \int_0^l \left(\frac{\sqrt{2}}{2} \cdot \left(\frac{l-z}{2} \right) \right) \left(-\frac{ql}{2} z^2 + qlz - \frac{ql^2}{2} \right) dz \right] =$$

$$= -\frac{\sqrt{2} ql^4}{3 EJ}$$

$$M_{20} = \frac{1}{EJ} \left[2 \int_0^l \left(-\frac{\sqrt{2}}{4} z \right) \left(-\frac{ql}{4} z \right) dz + 2 \int_0^l \left(-\frac{\sqrt{2}}{4} (l-z) \right) \left(-\frac{ql}{4} (l+z) \right) dz \right]$$

$$= \frac{\sqrt{2} ql^4}{8 EJ}$$

$$M_{11} = \frac{1}{EJ} \left[2 \int_0^l \frac{z^2}{2} dz + 2 \int_0^l \left(\frac{1}{2} l^2 + \frac{z^2}{2} - lz \right) dz \right] = \frac{2 l^3}{3 EJ}$$

$$M_{22} = \frac{1}{EJ} \left[2 \int_0^l \frac{z^2}{8} dz + 2 \int_0^l \left(\frac{l^2}{8} + \frac{z^2}{8} - \frac{lz}{4} \right) dz + 2 \int_0^l \frac{z^2}{2} dz \right] =$$

$$= \frac{l^3}{2 EJ}$$

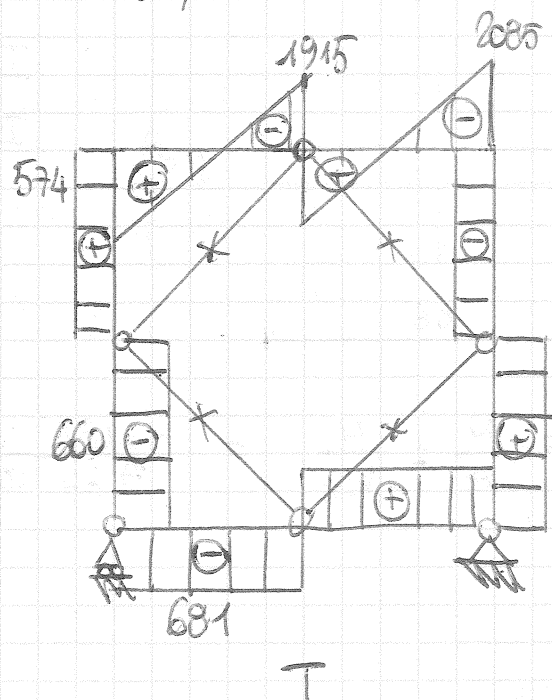
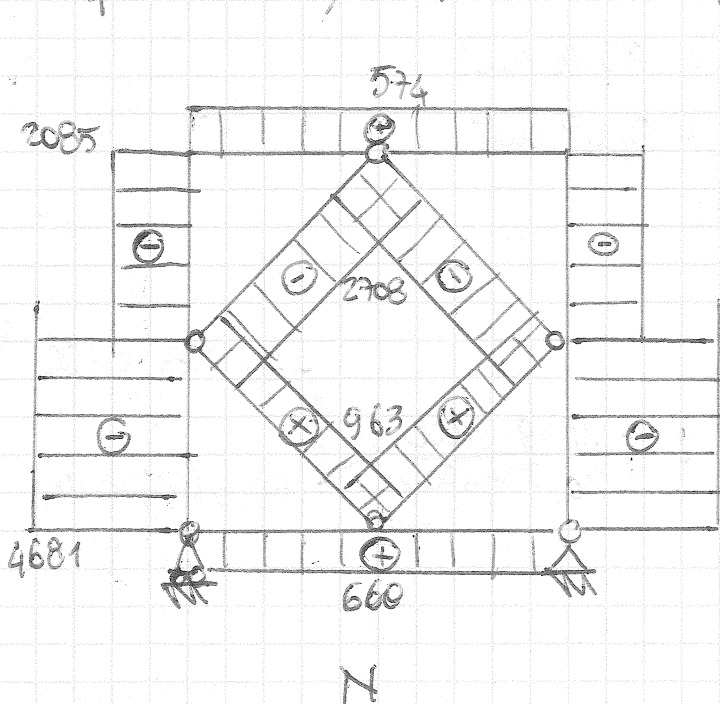
$$M_{12} = \frac{2}{EJ} \int_0^l \left(\frac{\sqrt{2}}{2} z \right) \left(-\frac{\sqrt{2}}{4} (l-z) \right) dz = -\frac{l^3}{12 EJ}$$

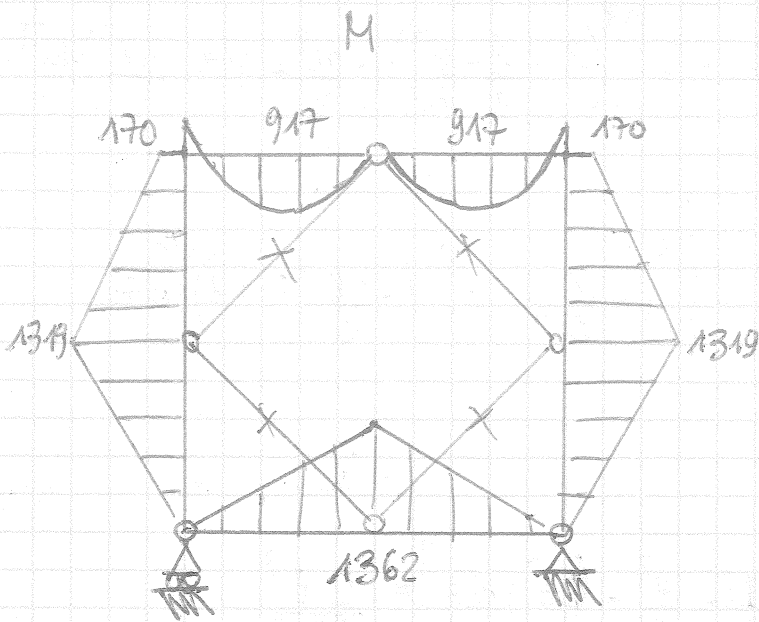
$$1^* \rightarrow \begin{cases} M_{11} \cdot X_1 + M_{12} \cdot X_2 = -M_{10} \\ M_{12} \cdot X_1 + M_{22} \cdot X_2 = -M_{20} \end{cases}$$

$$2^* \rightarrow \begin{cases} M_{12} \cdot X_1 + M_{22} \cdot X_2 = -M_{20} \end{cases}$$

$$X_1 = 2708,1 \text{ N}$$

$$X_2 = -962,9 \text{ N}$$

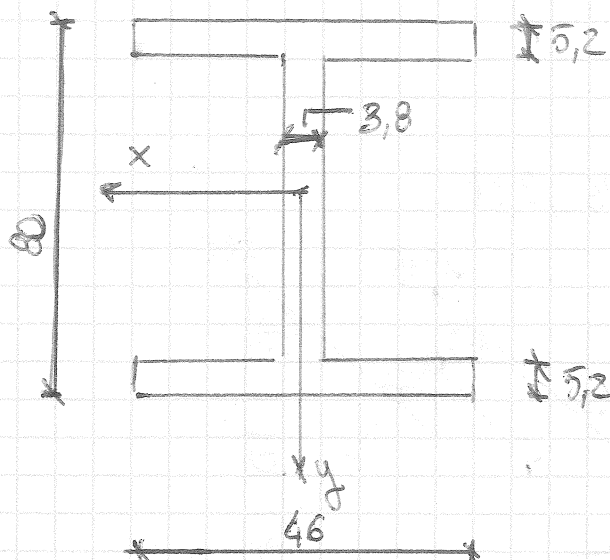




2) $M = 1319 \text{ Nm}$
 $N = 4681 \text{ N}$
 $T = -660 \text{ N}$

Progetto $W_{\min} = \frac{M}{\sigma_{\max}} = \frac{1319 \cdot 1000}{190} = 6942 \text{ mm}^3$

ADOTTO IPE 80



$A = 7,64 \text{ cm}^2$

$W_x = 20,33 \text{ cm}^3$

$W_y = 3,69 \text{ cm}^3$

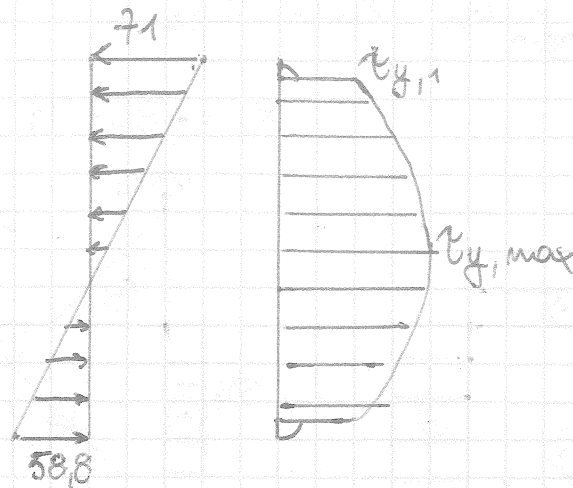
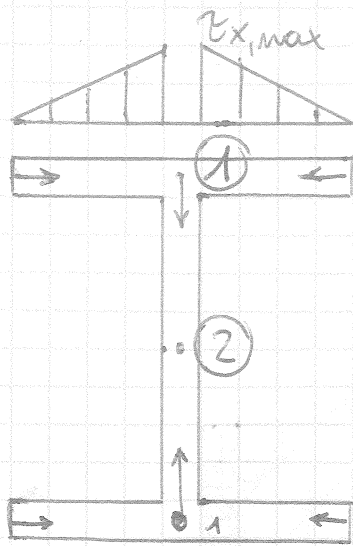
$I_x = 80,14 \text{ cm}^4$

$I_y = 8,49 \text{ cm}^4$

$$\sigma'_{\max} = \frac{N}{A} \pm \frac{M}{W_x} = \frac{-4681}{7,64 \cdot 100} \pm \frac{-1319 \cdot 1000}{20,33 \cdot 1000}$$

$-71 \text{ MPa} < 190$

$58,8 \text{ MPa} < 190$



$$\sigma_{x,max} = -\frac{T_y \cdot S_x^*}{I_x \cdot b} = -\frac{660 \cdot 5,2 \cdot (46 - 3,8) (80 - 5,2)}{4 \cdot 80,14 \cdot 10^4 \cdot 5,2} = 0,65 \text{ MPa}$$

$$\tau_{y,1} = -\frac{T_y \cdot S_x^*}{I_x \cdot b} = -\frac{660 \cdot 46 \cdot 5,2 \cdot (80 - 5,2)}{2 \cdot 80,14 \cdot 10^4 \cdot 3,8} = 1,80 \text{ MPa}$$

$$\tau_{y,max} = -\frac{T_y \cdot S_x^*}{I_x \cdot b} = -\frac{660 \cdot (46 \cdot 5,2 \cdot (80 - 5,2) + 3,8 \cdot (40 - 5,2)^2)}{2 \cdot 80,14 \cdot 10^4 \cdot 3,8} = 2,44 \text{ MPa}$$

$$\sigma'_{id,1} = \sqrt{\sigma_{z,1}^2 + 3\tau_{y,1}^2} = 71,1 \text{ MPa} < 190 \quad \text{OK}$$

$$\sigma'_{id,2} = \sqrt{\sigma_{z,2}^2 + 3\tau_{y,max}^2} = 5,20 \text{ MPa} < 190 \quad \text{OK}$$

$$3) \quad M_{ij}^{TOT} = M_{ij} + M_{ij}^N$$

$$M_{10}^N = 0$$

$$M_{20}^N = 0$$

$$M_{11}^N = \frac{2}{EA} \int_0^{2\sqrt{2}} 1 \, dz = \frac{2\sqrt{2} \cdot l}{EA}$$

$$M_{22}^N = \frac{2}{EA} \int_0^{2\sqrt{2}} 1 \, dz = \frac{2\sqrt{2} \cdot l}{EA}$$

$$M_{12}^N = 0$$

$$\begin{aligned} (1) & \rightarrow \begin{cases} M_{11}^{TOT} \cdot X_1 + M_{12}^{TOT} \cdot X_2 = -M_{10} \\ M_{12}^{TOT} \cdot X_1 + M_{22}^{TOT} \cdot X_2 = -M_{20} \end{cases} \\ (2) & \rightarrow \end{aligned}$$

$$X_1 = 2299 \text{ N}$$

$$X_2 = -827 \text{ N}$$

