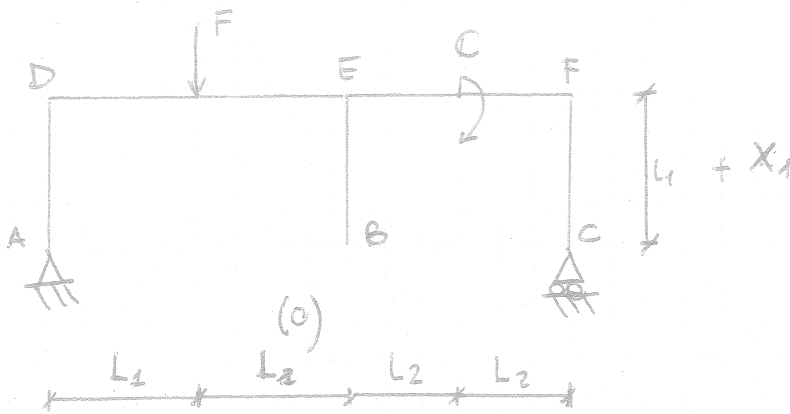
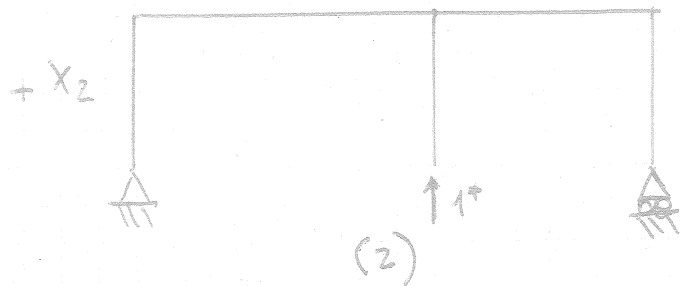
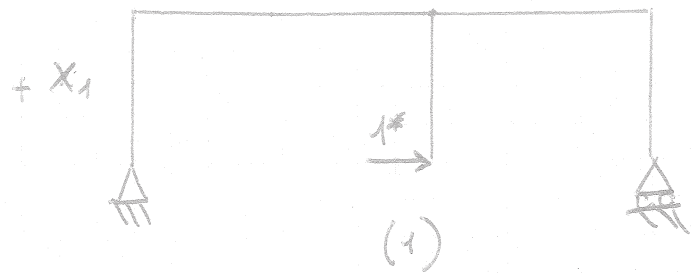




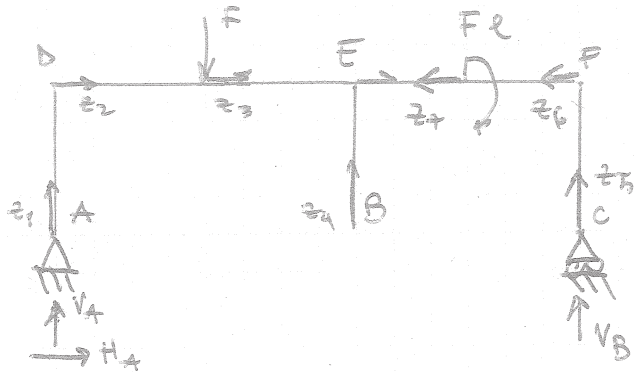
$$L_1 = 2m = l$$

$$L_2 = 1m = l/2$$

$$F = 100N \quad C = Fl = 200Nm$$



SISTEMA ○

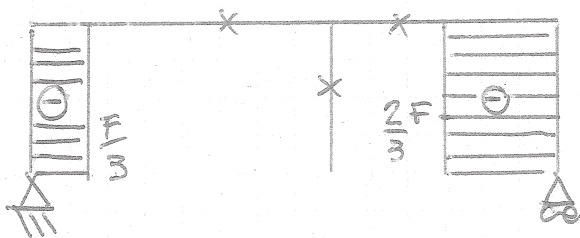


$$\rightarrow) H_A = 0$$

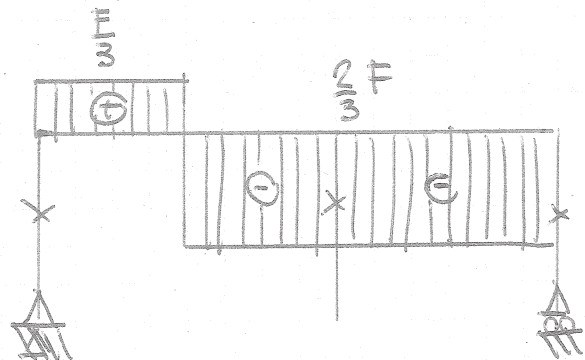
$$\uparrow) -Fl - Fl + V_B \cdot 3l = 0$$

$$V_B = \frac{2}{3}F$$

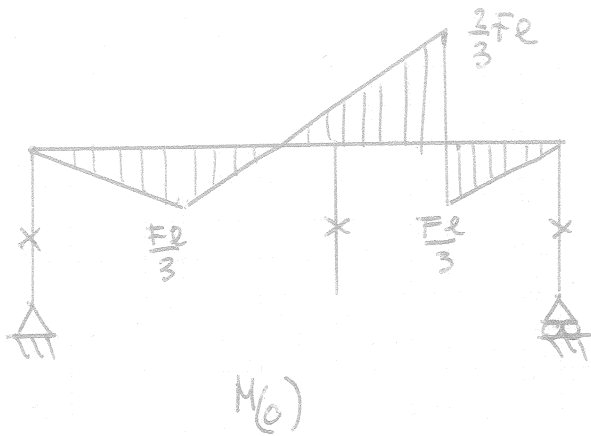
$$\uparrow) V_A + V_B - F = 0 \quad V_A = \frac{1}{3}F$$



$N(x)$



$T(x)$



$$M_0(z_1) = 0$$

$$M_0(z_2) = \frac{Fz}{3}$$

$$M_0(z_3) = \frac{F}{3}(l+z) - Fz$$

$$M_0(z_4) = 0$$

$$M_0(z_6) = \frac{2Fz}{3}$$

$$M_0(z_5) = 0$$

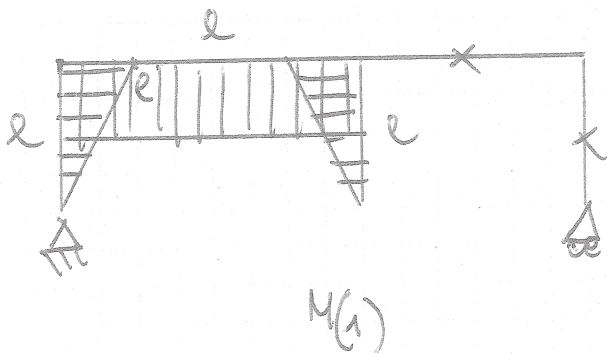
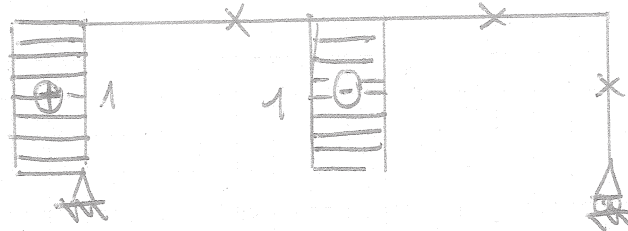
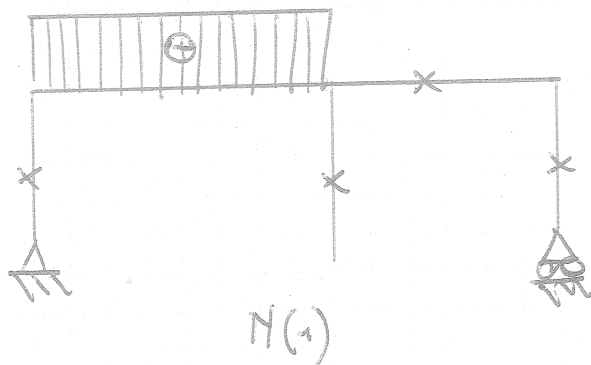
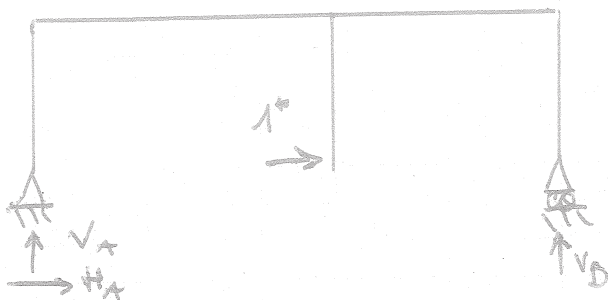
$$M_0(z_7) = \frac{2F}{3}\left(\frac{l}{2}+z\right) - Fe$$

SISTEMA 1

$$\rightarrow) H_A = -1$$

$$A) V_B = 0$$

$$\uparrow) V_A = 0$$



$$M_1(z_1) = 1^* \cdot z$$

$$M_1(z_2) = M_1(z_3) = l$$

$$M_1(z_4) = 1^* \cdot z$$

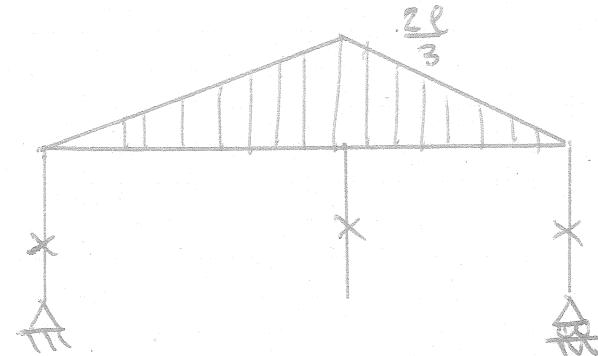
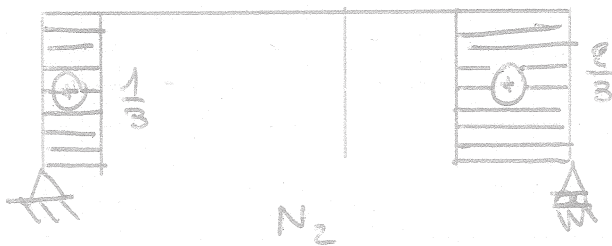
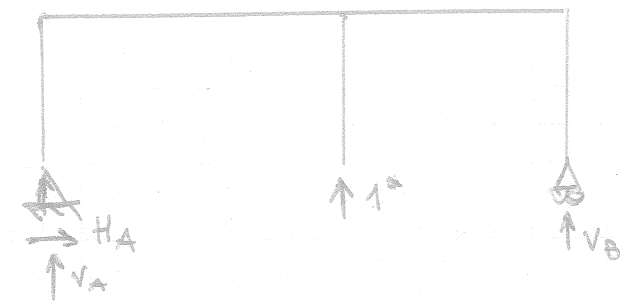
$$M_1(z_5) = M_1(z_6) = M_1(z_7) = 0$$

SISTEMA 2

$$\rightarrow H_A = 0$$

$$\uparrow 1^\circ - 2l + V_B \cdot 3l = 0 \quad V_B = 2/3$$

$$\uparrow 1) V_A = -1/3$$



$$M_2(z_1) = M_2(z_4) = M_2(z_5) = 0$$

$$M_2(z_2) = -z/3$$

$$M_2(z_3) = -1/3 (l+z)$$

$$M_2(z_6) = -2z/3$$

$$M_2(z_7) = -2/3 (l/2 + z)$$

$$\eta_{10} = \frac{1}{EI} \left[\int_0^l (l) \left(\frac{Fz}{3} \right) dz + \int_0^l (l) \left(\frac{F}{3}(l+z) - Fz \right) dz \right] = \frac{fl^3}{6EI}$$

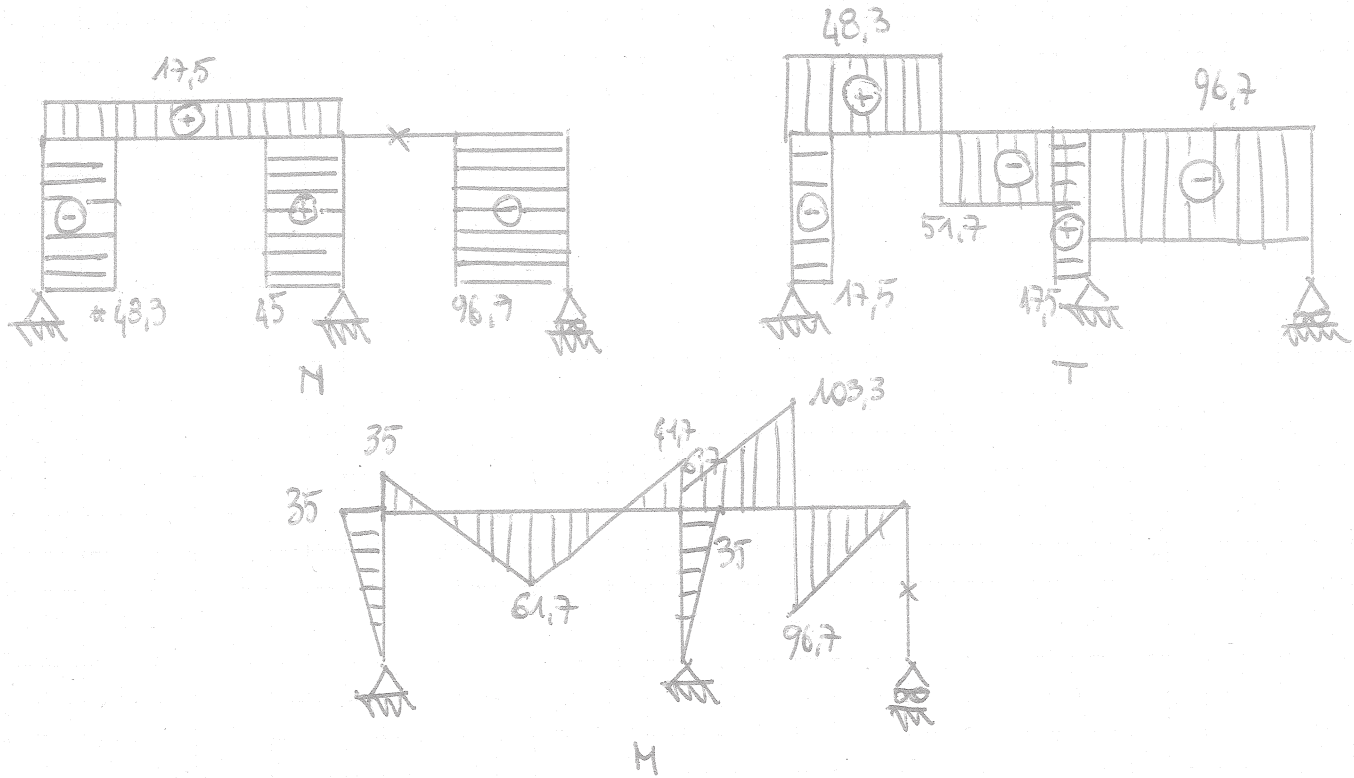
$$\eta_{20} = \frac{1}{EI} \left[\int_0^l \left(-\frac{z}{3} \right) \left(\frac{Fz}{3} \right) dz + \int_0^l \left(-\frac{(l+z)}{3} \right) \left(\frac{F}{3}(l+z) - Fz \right) dz + \int_0^{l/2} \left(-\frac{2z}{3} \right) \left(\frac{2Fz}{3} \right) dz + \int_0^{l/2} \left(-\frac{2(l/2+z)}{3} \right) \left(\frac{2F}{3} \left(\frac{l}{2} + z \right) - Fl \right) dz \right] = \frac{fl^3}{12EI}$$

$$\eta_{11} = \frac{1}{EI} \left[2 \int_0^l z^2 dz + \int_0^{2l} z^2 dz \right] = \frac{8l^3}{3EI}$$

$$\eta_{22} = \frac{1}{EI} \left[\int_0^{2l} \frac{1}{9} z^2 dz + \int_0^l \frac{4}{9} z^2 dz \right] = \frac{4l^3}{9EI}$$

$$M_{12} = \frac{1}{EI} \int_0^{2l} \left(-\frac{z}{3}\right) (l) dz = -\frac{2l^3}{3EI}$$

$$\begin{aligned} (1) \quad & \begin{cases} M_{11} \cdot X_1 + M_{12} \cdot X_2 = -M_{10} \\ (2) \quad & \begin{cases} M_{12} \cdot X_1 + M_{22} \cdot X_2 = -M_{20} \end{cases} \end{cases} \rightarrow \begin{aligned} X_1 &= -17,5 \text{ N} \\ X_2 &= -45 \text{ N} \end{aligned} \end{aligned}$$



2) Deformabilități axiale

$$M_{10}^N = 0$$

$$M_{20}^N = \frac{1}{EA} \left[\int_0^l \left(\frac{1}{3}\right) \left(-\frac{E}{3}\right) dz + \int_0^l \left(\frac{2}{3}\right) \left(-\frac{2E}{3}\right) dz \right] = -\frac{5Fl}{9EA}$$

$$M_{11}^N = \frac{1}{EA} \left[\int_0^{2l} 1 dz \right] = \frac{2l}{EA}$$

$$M_{22}^N = \frac{1}{EA} \left[\int_0^l \frac{1}{9} dz + \int_0^l \frac{4}{9} dz \right] = \frac{5l}{9EA}$$

$$M_{12}^N = 0$$

$$M_{10}^{Tot} = M_{10}$$

$$M_{20}^{Tot} = M_{20} + M_{20}^N$$

$$\begin{aligned} M_{11}^{Tot} &= M_{11} + M_{11}^N \\ M_{22}^{Tot} &= M_{22} + M_{22}^N \end{aligned}$$

$$M_{12}^{Tot} = M_{12}$$

cedimento $\Delta_A = 1 \text{ cm} = 0,01 \text{ m}$

$$(1) L_{VE} = 1 \cdot 0 + 0 \cdot (-0,01) = 0$$

$$(2) L_{VE} = 1 \cdot 0 + \left(-\frac{1}{3}\right) \cdot (-0,01) = \frac{0,01}{3}$$

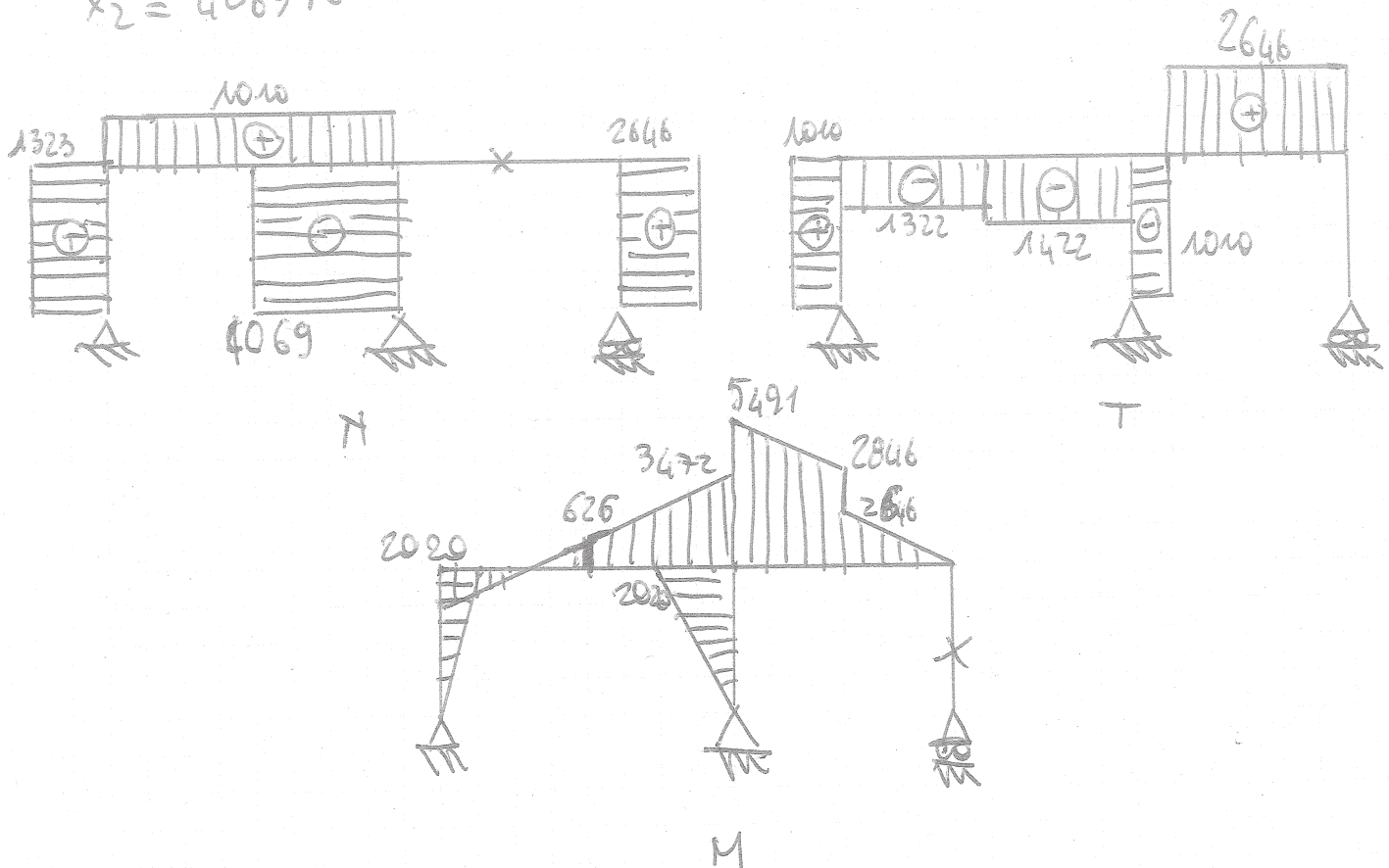
cedimento + Deformabilità anisale

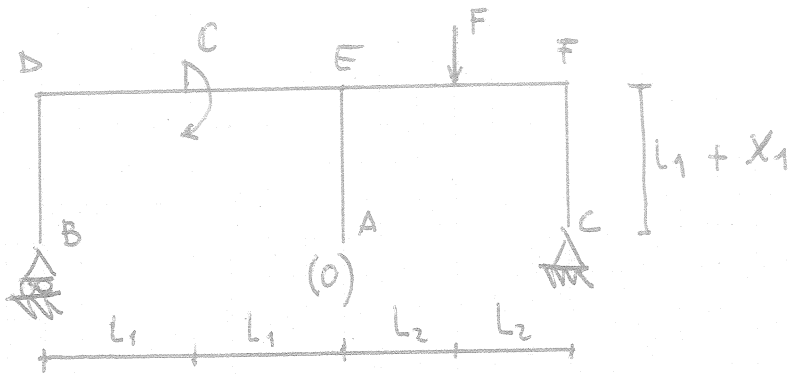
$$(1) \begin{cases} L_{VE} = 0 = M_{11}^{Tot} \cdot X_1 + M_{12}^{Tot} \cdot X_2 + M_{10}^{Tot} \end{cases}$$

$$(2) \begin{cases} L_{VE} = \left(-\frac{1}{3}\right)(0,001) = M_{12}^{Tot} \cdot X_1 + M_{22}^{Tot} \cdot X_2 + M_{20}^{Tot} \end{cases}$$

$$X_1 = 1010 \text{ N}$$

$$X_2 = 4069 \text{ N}$$

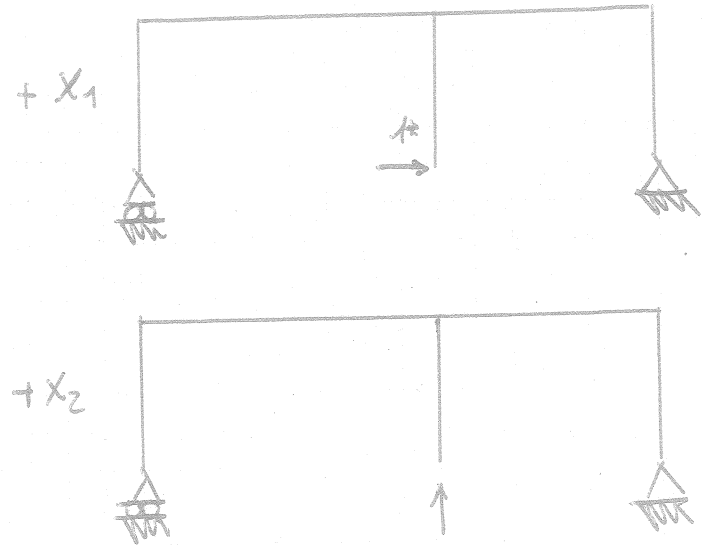




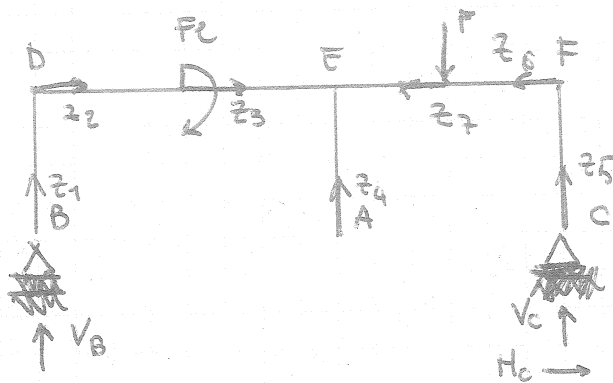
$$L_1 = 2 \text{ m} = l$$

$$L_2 = 1 \text{ m} = L_1/2 = l/2$$

$$F = 100 \text{ N} \quad C = F \cdot l = 200 \text{ Nm}$$



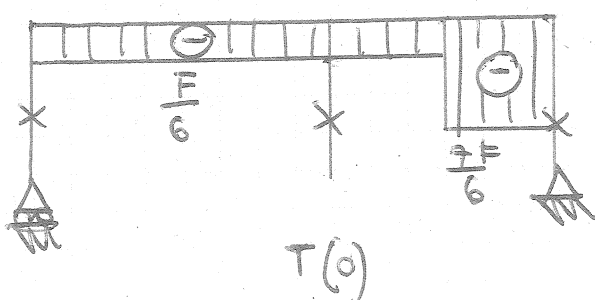
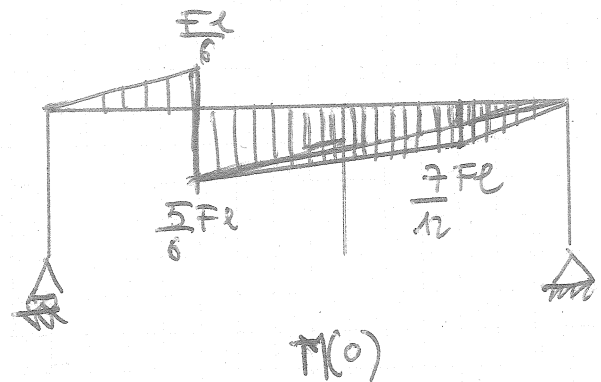
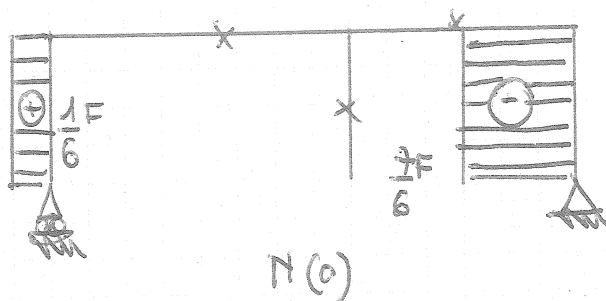
SISTEMA 0



$$\rightarrow H_A = 0$$

$$\uparrow -Fl - F \cdot \frac{5l}{2} + V_C \cdot 3l = 0 \quad V_C = \frac{7F}{6}$$

$$\uparrow V_B + V_C - F = 0 \quad V_B = -\frac{1}{6}F$$



$$M_0(z_1) = M_0(z_4) = M_0(z_5) = 0$$

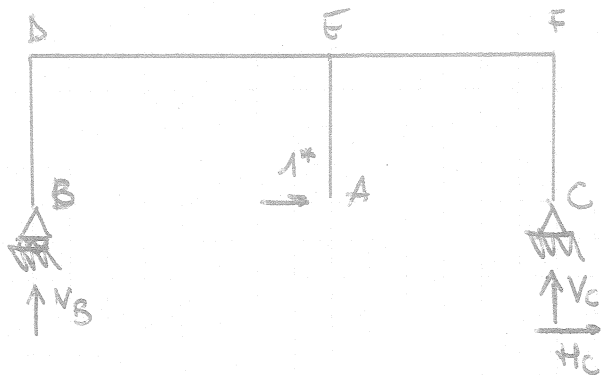
$$M_0(z_2) = -\frac{Fl}{6}$$

$$M_0(z_3) = -\frac{F(l+z)}{6} + Fl$$

$$M_0(z_6) = \frac{7Fl}{6}$$

$$M_0(z_7) = \frac{7F(l+z)}{6} - Fl$$

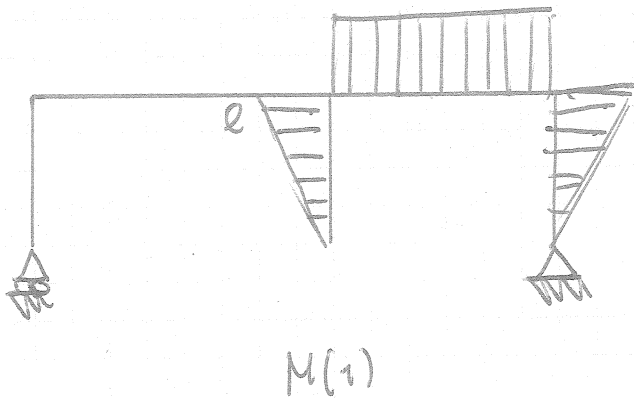
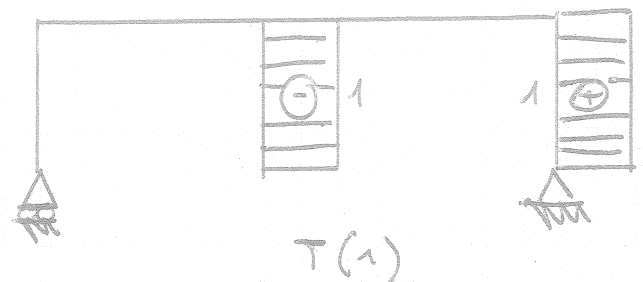
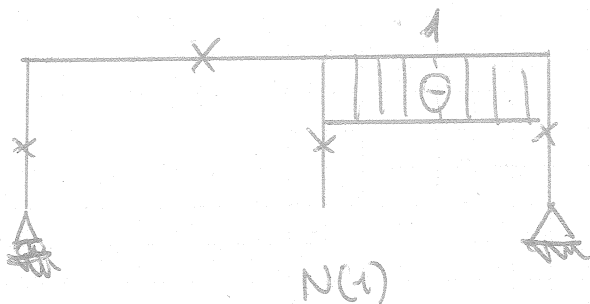
SISTEMA 1



$$\rightarrow) H_C = -1^*$$

$$B \uparrow) V_C = 0$$

$$\uparrow) V_B = 0$$



$$M_1(z_1) = M_1(z_2) = M_1(z_3) = 0$$

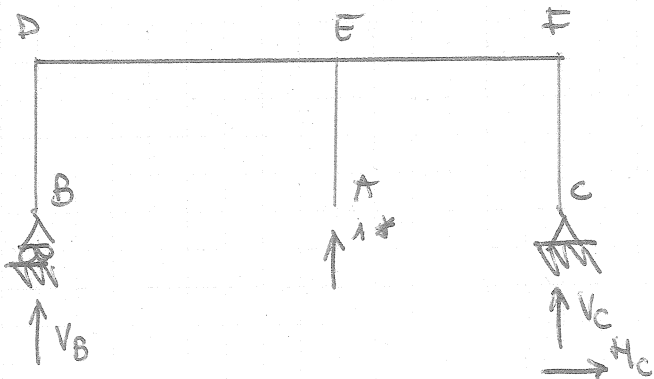
$$M_1(z_4) = -z$$

$$M_1(z_5) = -z$$

$$M_1(z_6) = -l$$

$$M_1(z_7) = -l$$

SISTEMA 2

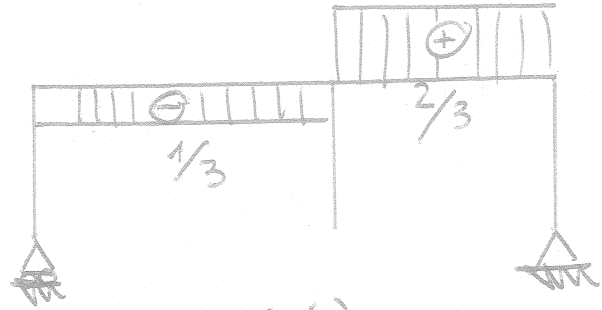
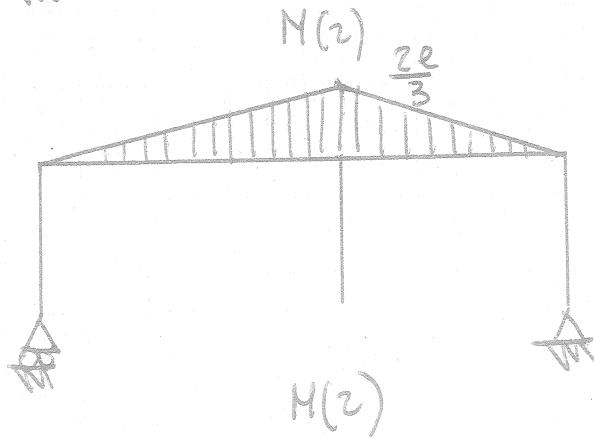
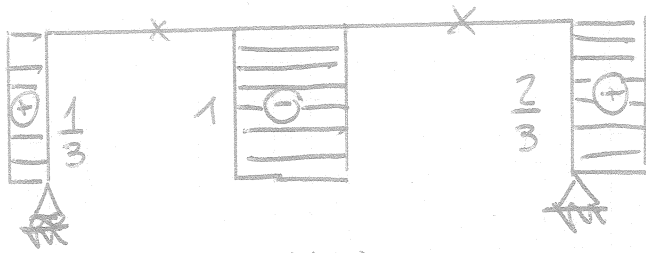


$$\rightarrow) H_C = 0$$

$$B \uparrow) 1 \cdot 2l + V_C \cdot 3l = 0$$

$$V_C = -\frac{2}{3}$$

$$\uparrow) V_B + V_C + 1 = 0 \quad V_B = -\frac{1}{3}$$



$T(z)$

$$M_2(z_1) = M_2(z_4) = M_2(z_5) = 0$$

$$M_2(z_2) = -\frac{z}{3}$$

$$M_2(z_3) = -\left(\frac{l+z}{3}\right)$$

$$M_2(z_6) = -\frac{2z}{3}$$

$$M_2(z_7) = -\frac{2}{3}\left(\frac{l}{2} + z\right)$$

$$\eta_{10} = \frac{1}{EI} \left[\int_0^{\frac{l}{2}} (-l) \left(\frac{7Fz}{6} \right) dz + \int_0^{\frac{l}{2}} (-l) \left(\frac{7F}{6} \left(\frac{l}{2} + z \right) - Fz \right) dz \right] = -\frac{11}{24} \frac{Fl^3}{EI}$$

$$\eta_{20} = \frac{1}{EI} \left[\int_0^l \left(-\frac{z}{3} \right) \left(-\frac{Fz}{6} \right) dz + \int_0^l \left(-\frac{l+z}{3} \right) \left(-\frac{F(l+z)}{6} + Fz \right) dz + \int_0^{\frac{l}{2}} \left(-\frac{2z}{3} \right) \left(\frac{7Fz}{6} \right) dz + \int_0^{\frac{l}{2}} \left(-\frac{2}{3} \left(\frac{l}{2} + z \right) \right) \left(\frac{7F}{6} \left(\frac{l}{2} + z \right) - Fz \right) dz \right] = -\frac{13}{24} \frac{Fl^3}{EI}$$

$$\eta_{11} = \frac{1}{EI} \left[z \int_0^l z^2 dz + \int_0^l l^2 dz \right] = \frac{5l^3}{3EI}$$

$$\eta_{22} = \frac{1}{EI} \left[\int_0^{2l} \frac{z^2}{9} dz + \int_0^l \frac{4}{9} z^2 dz \right] = \frac{4l^3}{9EI}$$

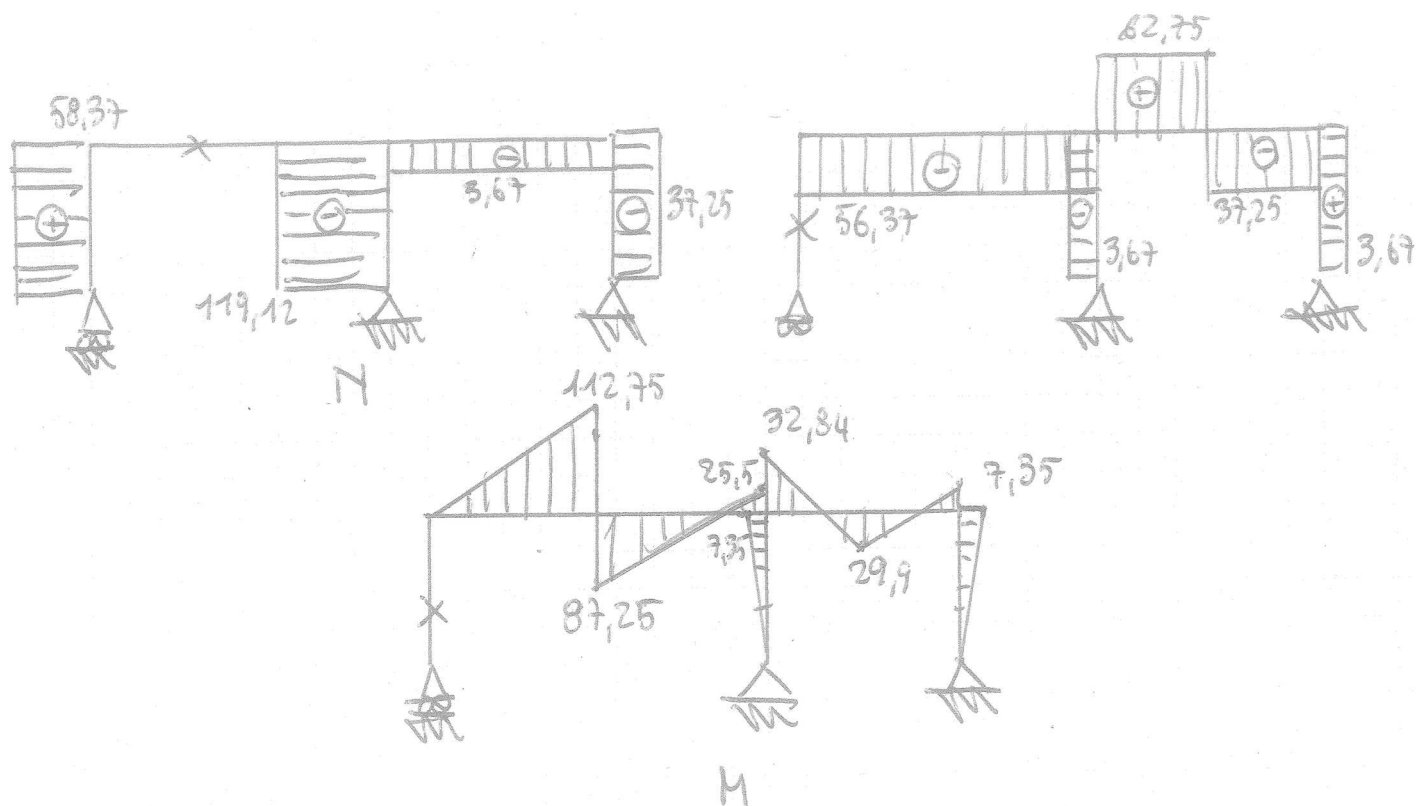
$$\eta_{12} = \frac{1}{EI} \left[\int_0^l \left(-\frac{2z}{3} \right) (-l) dz \right] = \frac{l^3}{3EI}$$

$$(1) \begin{cases} \eta_{11} \cdot x_1 + \eta_{12} \cdot x_2 = -\eta_{10} \end{cases}$$

$$(2) \begin{cases} \eta_{12} \cdot x_1 + \eta_{22} \cdot x_2 = -\eta_{20} \end{cases}$$

$$x_1 = 3,68 \text{ N}$$

$$x_2 = 119,12 \text{ N}$$



2) Deformabilită axială

$$\eta_{10}^N = 0$$

$$\eta_{20}^N = \frac{1}{EA} \left[\int_0^l \left(\frac{1}{3} \right) \left(\frac{1}{6} F \right) dz + \int_0^l \left(\frac{2}{3} \right) \left(-\frac{7}{6} F \right) dz \right] = -\frac{13 Fl}{18 EA}$$

$$\eta_{11}^N = \frac{1}{EA} \left[l \right] = \frac{l}{EA}$$

$$\eta_{22}^N = \frac{1}{EA} \left[\frac{l}{9} + l + \frac{4l}{9} \right] = \frac{14 l}{9 EA}$$

$$\eta_{12}^N = 0$$

$$\eta_{ij}^{TOT} = \eta_{ij} + \eta_{ij}^N$$

Cădere în A $\Delta_A = -1 \text{ cm} = -0,01 \text{ m}$

$$\begin{cases} (1) & 0 = \eta_{10}^{TOT} + \eta_{11}^{TOT} \cdot X_1 + \eta_{12}^{TOT} \cdot X_2 \\ (2) & 1 \cdot (-0,01) = \eta_{20}^{TOT} + \eta_{21}^{TOT} \cdot X_1 + \eta_{22}^{TOT} \cdot X_2 \end{cases}$$

$$X_1 = 1821,4 \text{ N}$$

$$X_2 = -8977,4 \text{ N}$$

