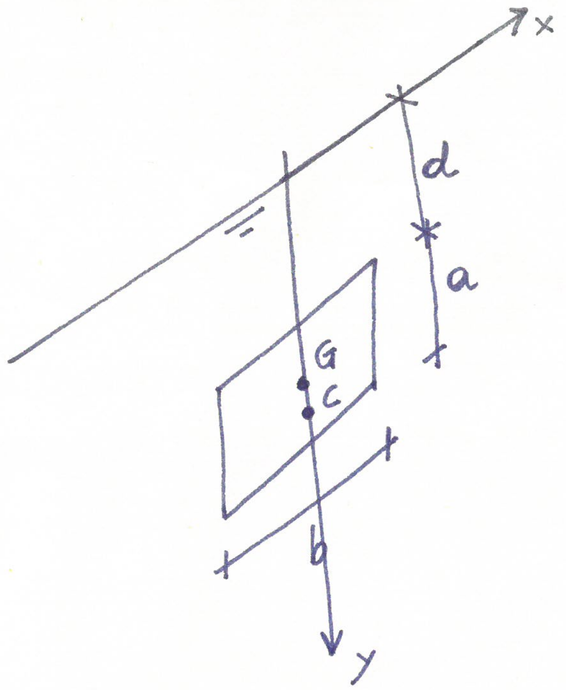
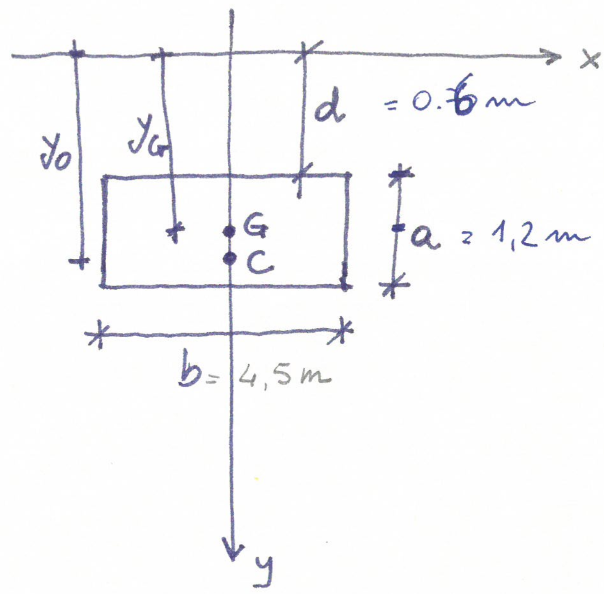
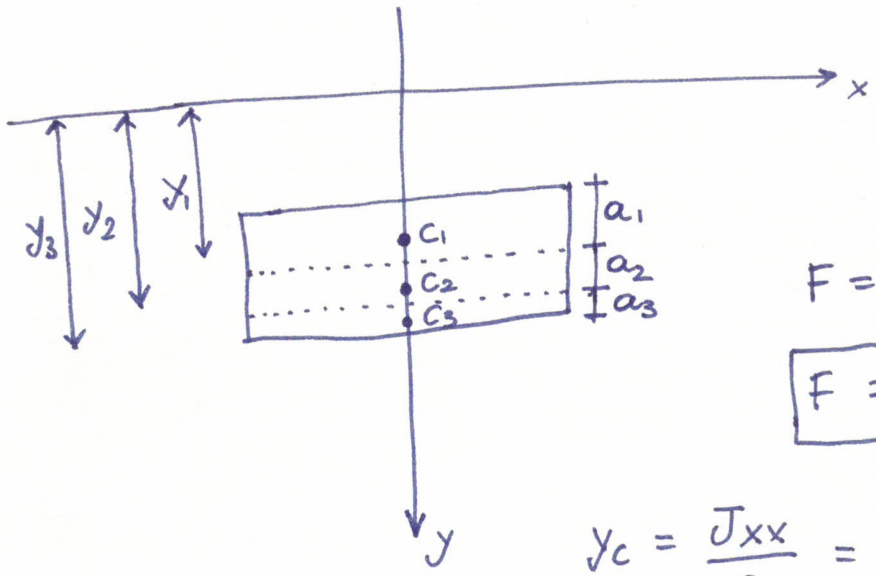
SOLUZIONE es. 3)
(esercitaz. n° 1 di Idraulica)





$$\gamma = 9810 \ \frac{N}{m^3}$$



$$F = p_G A = \gamma \left(d + \frac{a}{2}\right)(ab)$$

$$\boxed{F = 63.6 \text{ kN}}$$

$$y_C = \frac{J_{xx}}{S_x} = \frac{\frac{ba^3}{12} + (ab)\left(d + \frac{a}{2}\right)^2}{(ab)\ \left(d + \frac{a}{2}\right)}$$

$$\boxed{y_C = 1.3 \text{ m}}$$

$$\gamma \left(d + \frac{a_1}{2}\right)(a_1 b) = \frac{F}{3} \rightarrow \boxed{a_1 = .549 \text{ m}}$$

$$\gamma \left(d + a_1 + \frac{a_2}{2}\right)(a_2 b) = \frac{F}{3} \rightarrow \boxed{a_2 = .361 \text{ m}}$$

$$\gamma \left(d + a_1 + a_2 + \frac{a_3}{2}\right)(a_3 b) = \frac{F}{3} \rightarrow \boxed{a_3 = .290 \text{ m}}$$

$$y_i = \frac{J_{xx\,i}}{S_{x\,i}}$$

$$y_1 = d + \frac{a_1}{2} + \frac{a_1}{12\left(\frac{1}{2} + \frac{d}{a_1}\right)} \Rightarrow \boxed{y_1 = .903\,m}$$

$$y_2 = d + a_1 + \frac{a_2}{2} + \frac{a_2}{12\left(\frac{1}{2} + \frac{d+a_1}{a_2}\right)} \Rightarrow \boxed{y_2 = 1.338\,m}$$

$$y_3 = d + a_1 + a_2 + \frac{a_3}{2} + \frac{a_3}{12\left(\frac{1}{2} + \frac{d+a_1+a_2}{a_3}\right)} \Rightarrow \boxed{y_3 = 1.659\,m}$$

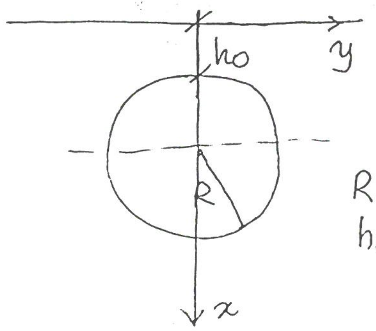
Verifica (equilibrio ai momenti rispetto al pelo libero)

$$F y_c = \underbrace{\frac{F}{3} \cdot y_1 + \frac{F}{3} \cdot y_2 + \frac{F}{3} y_3}_{82.6\ kNm}$$

$\Downarrow$

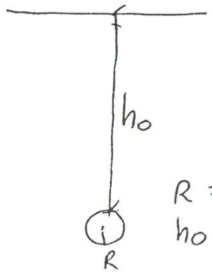
82.6 kNm

SPINTA IDR. PARAT. CIRC.



$R = 2m$

$h_0 = 0.5$



$R = 10cm$

$h_0 = 10m$

$F = \gamma z_G \cdot S$

$= \gamma \cdot (h_0+R)\pi R^2$

$\eta_C = \eta_G + \frac{I_{x_G x_G}}{\eta_G S}$

$\eta_G = z_G = (h_0+R)$

$S = \pi R^2$

$I_{x_G x_G} = \frac{\pi R^4}{4}$

$$J_{yyG} = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$$

$$S = \gamma z_G \Omega = \gamma (h_0+R)(\pi R^2)$$

$$z_G = x_G = (h_0+R)$$

$$z_C = x_C = (h_0+R) + \frac{\frac{\pi R^4}{4}}{\pi R^2 (h_0+R)}$$

$$z_C = (h_0+R) + \frac{R^2}{4(h_0+R)}$$

$$\frac{z_C - z_G}{z_G} = \frac{R^2}{4(h_0+R)^2} = \frac{1}{4}\frac{1}{(h_0/R+1)^2}$$

$S = 3.08 \cdot 10^5 N$

$z_G = 2.5 m$

$z_C = 2.9 m$

$\frac{z_C - z_G}{z_G} = 0.16$ (16%)

$S = 3.11 \cdot 10^3 N$

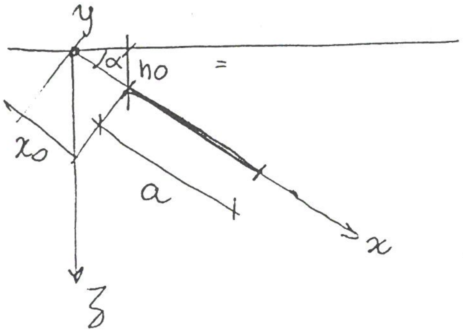
$z_G = 10.1 m$

$z_C = 10.10025 m$

$\frac{z_C - z_G}{z_G} = 2.48 \cdot 10^{-5}$

(0.0025%)

Par. rett.

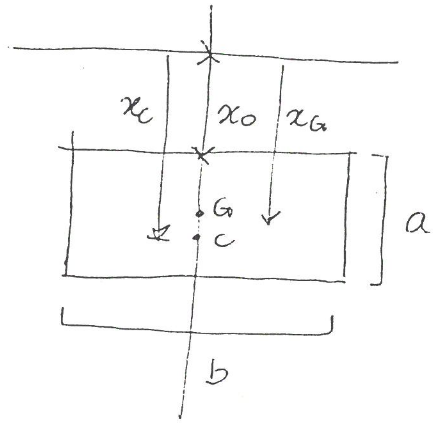


$\alpha = 30^\circ$

$h_0 = 1m$

$b = 4m$

$a = 2m$



$$x_0 = \frac{h_0}{\sin\alpha} = 2m$$

$$x_G = x_0 + \frac{a}{2} = 3m$$

$$x_C = \left(x_0 + \frac{a}{2}\right) + \frac{1}{12}\,\frac{ba^3}{ab\left(x_0 + \frac{a}{2}\right)} = 3.11m$$

$$S = \gamma z_G \Omega = \gamma (x_G \sin\alpha)(ab)$$

$$= \gamma \left(x_0 + \frac{a}{2}\right)(ab)\sin\alpha = 117.672\ N$$

$$= 1.18 \cdot 10^5 N$$

$$S_1 = \gamma (x_0 + a_1/2) \sin\alpha\, (a_1 b) = S/3 \rightarrow a_1 = 0.828m$$

$$S_2 = \gamma \left(x_0 + a_1 + \frac{a_2}{2}\right) \sin\alpha\, (a_2 b) = S/3 \rightarrow a_2 = 0.636\ m$$

$$S_3 = \gamma \cdot (x_0 + a_1 + a_2 + a_3/2) \sin\alpha\, (a_3 b) = S/3 \rightarrow a_3 = 0.536\ m$$

$$a_1 + a_2 + a_3 = a \qquad OK$$

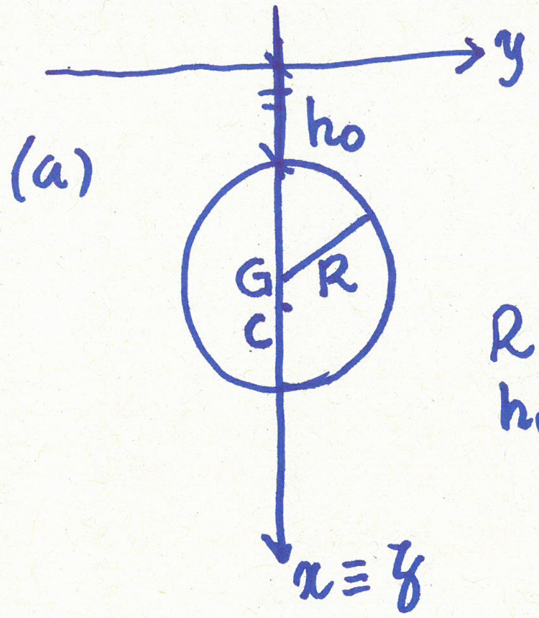
$$x_{c1} = \left(x_0 + \frac{a_1}{2}\right) + \frac{1}{12}\,\frac{a_1^2}{\left(x_0 + \frac{a_1}{2}\right)^2} = 2.438\ m$$

$$x_{c2} = \left(x_0 + a_1 + a_2/2\right) + \frac{1}{12}\,\frac{\cancel{b}a_2^{\cancel{3}}\ a_2^2}{\cancel{a_2 b}\left(x_0 + a_1 + \frac{a_2}{2}\right)} = 3.157\ m$$

$$x_{c3} = \left(x_0 + a_1 + a_2 + a_3/2\right) + \frac{1}{12}\,\frac{a_3^2}{x_0 + a_1 + a_2 + \frac{a_3}{2}} = 3.738\ m$$

$$\frac{x_{c1} + x_{c2} + x_{c3}}{3} = x_c \Leftrightarrow OK$$

(a)

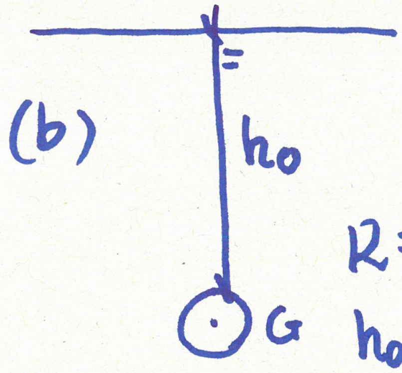


$\alpha = \dfrac{\pi}{2}$

$R = 2m$

$h_0 = 0.5m$

(b)

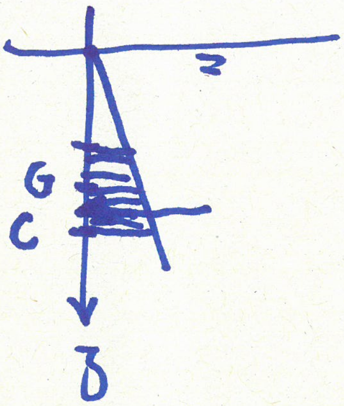


$R = 0.1m$

$h_0 = 10m$

$J_{xy} = 0$

$\cancel{x \equiv z} \; y_G = y_C = 0$



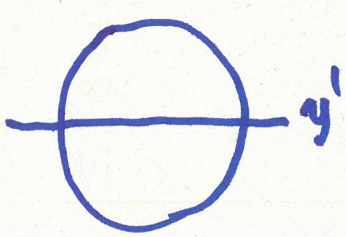
$$\bar{F} = F\bar{n}$$

$$F = \overset{9806\,N/m^3}{\gamma}\, z_G\, \Omega_0 = \gamma (h_0 + R)(\pi R^2)$$

(a) $F = 3.08 \cdot 10^5 N = 308\,kN$

(b) $F = 3.11 \cdot 10^3 N = 3.11\,kN$

$$z_G = h_0 + R \begin{cases} 2.5\,m \\ \cancel{2.9\,m}\; 10.1\,m \end{cases}$$

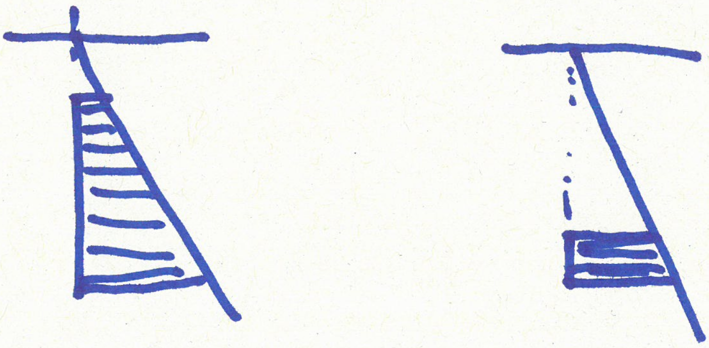


$$J_{y'y'} = J_{yyG} = \frac{\pi R^4}{4}$$

$$z_C = \overset{h_0+R}{z_G} + \frac{J_{yyG}}{\pi R^2 \underset{h_0+R}{z_G}}$$

$$y_c = h_0 + R + \frac{\frac{\pi R^4}{4}}{\pi R^2 (h_0+R)}$$

$$y_c = h_0 + R + \frac{R^2}{4(h_0+R)} \left\langle \begin{array}{l} 2.9\ m \\ 10.10025\ m \end{array} \right.$$



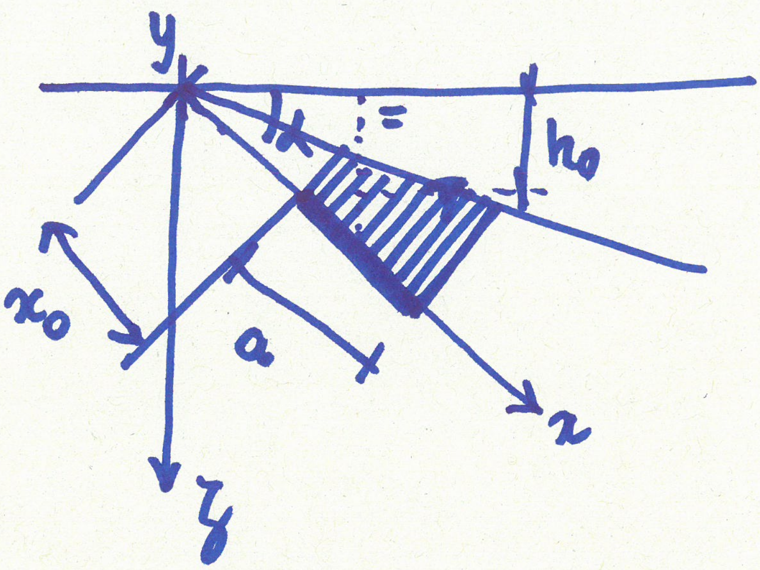
Differenza relativa tra y_c e y_G

$$\frac{y_c - y_G}{y_G} = \frac{R^2}{4(h_0+R^2)} = \frac{1}{4} \cdot \frac{1}{(h_0/R+1)^2}$$

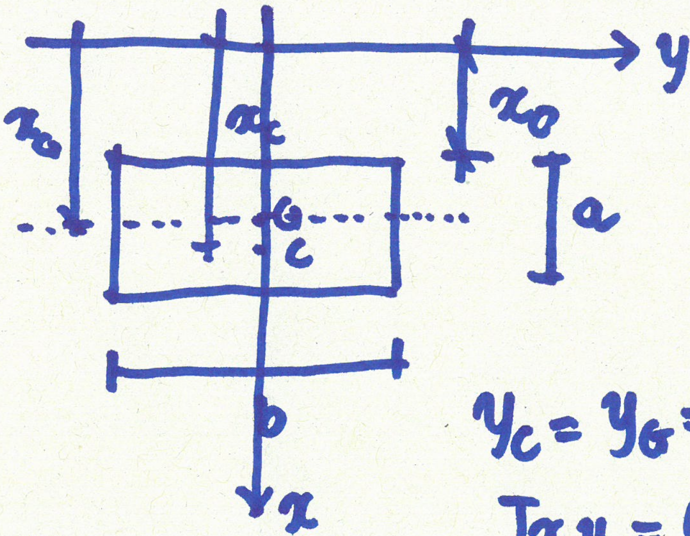
(a) $$\frac{y_c - y_G}{y_G} = 0.16 \quad (16\%)$$

(b) $$\frac{y_c - y_G}{y_G} = 2.48 \cdot 10^{-5}$$

(0.0025%)



$$x \sin\alpha = z$$



$h_0 = 1m$

$b = 4m$

$a = 2m$

$\alpha = 30°$

$$y_c = y_G = 0$$

$$J_{xy} = 0$$

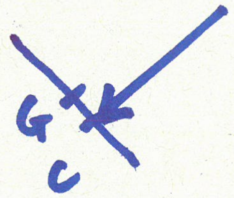
$$x_0 = \frac{h_0}{\sin\alpha} = 2m$$

$$x_G = x_0 + \frac{a}{2} = 3m$$

$$x_c = x_G + \frac{J_{yyG}}{S_y} = \overbrace{x_G}^{x_0 + a/2} + \frac{\frac{1}{12} b a^3}{(ab)(x_0 + a/2)}$$

$$x_c = \left(x_0 + \frac{a}{2}\right) + \frac{1}{12} \frac{a^2}{x_0 + a/2} = 3.11m$$

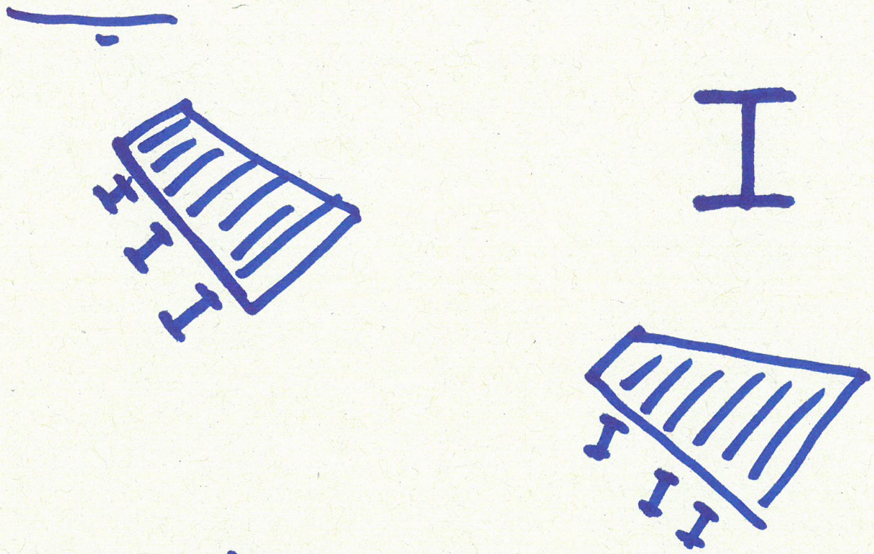
$$\bar{F} = F\bar{n}$$

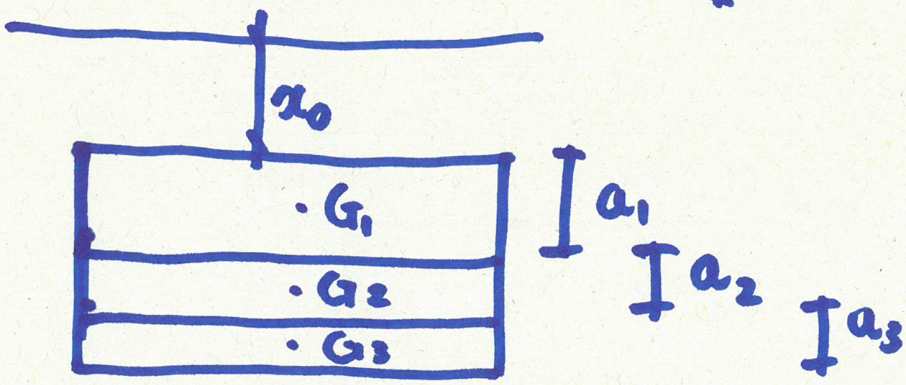


$$F = \gamma \xi_G \Omega =$$

$$= \gamma x_G \sin\alpha\,(ab) = 1.18 \cdot 10^5 N$$

118 kN





? a_1 a_2 a_3 t.c

$$F_1 = F_2 = F_3$$

$$F_1 = \gamma \left(x_0 + \frac{a_1}{2}\right) \sin\alpha \,(a_1 b)$$

$\downarrow F/3$

a_1 ?

eq.ne di 2° grado in a_1

$\rightsquigarrow \quad a_1 = 0.828\ m$

$$F_2 = \gamma \left(x_0 + a_1 + \frac{a_2}{2}\right) \sin\alpha \,(a_2 b)$$

$\downarrow F/3$

a_2 ?

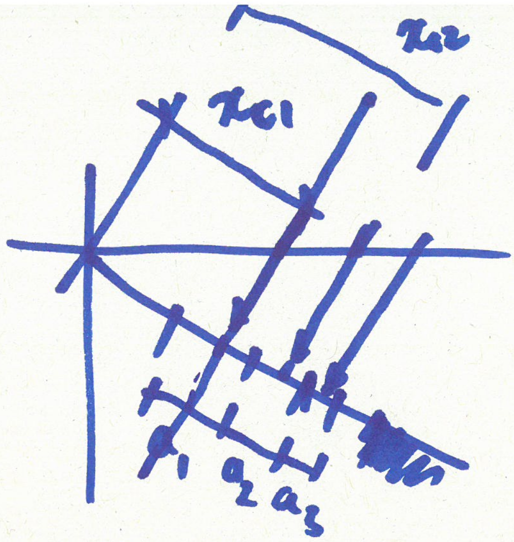
$\rightsquigarrow \quad a_2 = 0.636\ m$

$$F_3 = \gamma \left(x_0 + a_1 + a_2 + a_3/2\right) \sin\alpha \,(a_3 b)$$

$\downarrow F/3 \quad \rightsquigarrow \quad a_3 = 0.536\ m$

$a_1 + a_2 + a_3 = a$ (riprova)

<u>OK</u>



$C_1 \quad F/3$

$C_2 \quad F/3$

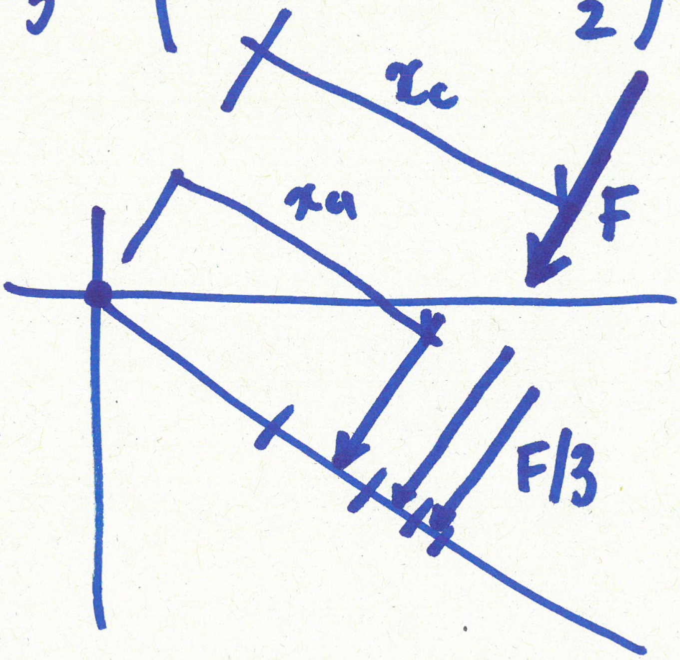
$C_3 \quad F/3$

$$x_{c1} = \left(x_0 + \frac{a_1}{2}\right) + \frac{1}{12}\,\frac{a_1^2}{\left(x_0 + \frac{a_1}{2}\right)} = 2.438\ m$$

$$x_{c2} = \left(x_0 + a_1 + \frac{a_2}{2}\right) + \frac{1}{12}\,\frac{a_2^2}{\left(x_0 + a_1 + \frac{a_2}{2}\right)} = 3.157\ m$$

$$x_{c3} = \left(x_0 + a_1 + a_2 + \frac{a_3}{2}\right) + \frac{1}{12}\,\frac{a_3^2}{x_0 + a_1 + a_2 + a_3/2}$$

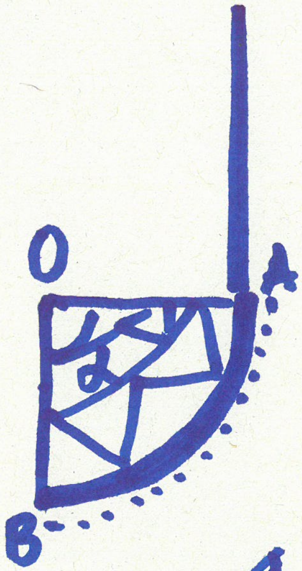
$$= 3.738\ m$$

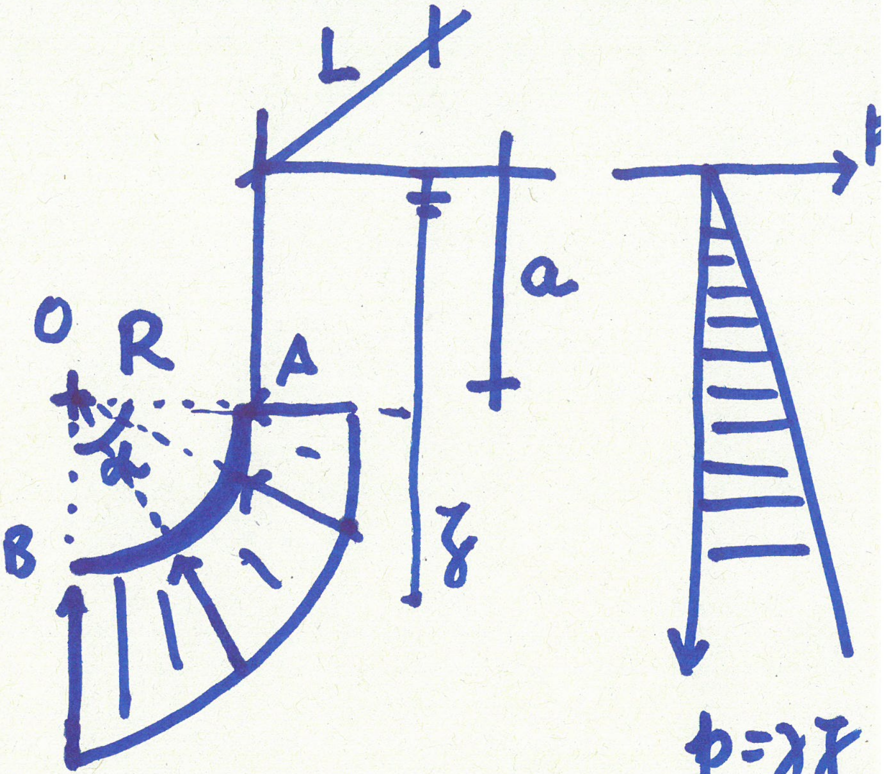


Riprova $F x_c = \frac{F}{3} x_{c1} + \frac{F}{3} x_{c2} + \frac{F}{3} \cdot x_{c3}$

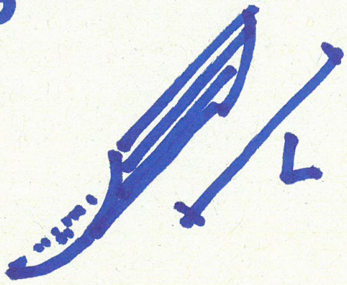
$x_c = (x_{c1} + x_{c2} + x_{c3})/3$ OK

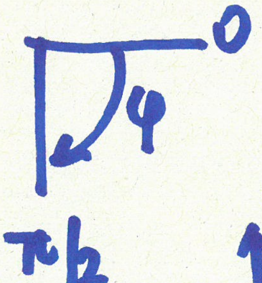
$\alpha = \pi/2$

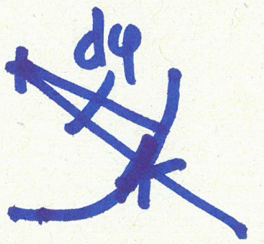




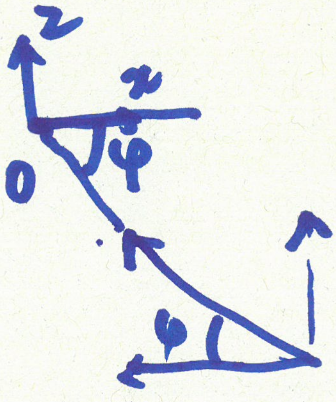
$p = \gamma z$







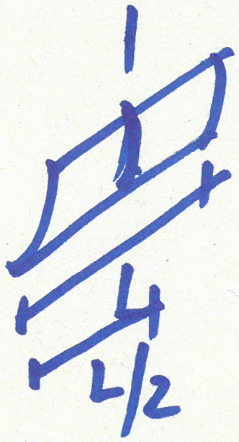
$$p\,\underbrace{dA}_{L\underbrace{R\,d\varphi}_{ds}}$$



$$dF_x = p\,dA\cos\varphi$$
$$dF_z = p\,dA\sin\varphi$$

$$z = a + R\sin\varphi$$
$$dF_x = \gamma\,(a + R\sin\varphi)\cos\varphi\,LR\,d\varphi$$
$$dF_z = \gamma\,(a + R\sin\varphi)\sin\varphi\,LR\,d\varphi$$

$\bar{F}$



$\bar{F}$ nel piano medio

retta d'azione passa per O

$$F_x = \int_A dF_x = \int_A \gamma\,(a + R\sin\varphi)\cos\varphi\, LR\, d\varphi$$

$$F_x = \gamma L R \int_0^{\pi/2} (a + R\sin\varphi)\cos\varphi\, d\varphi$$

$$\int_0^{\pi/2} a\cos\varphi\, d\varphi = a\sin\varphi \Big|_0^{\pi/2} = a$$

$$\int_0^{\pi/2} R\sin\varphi\cos\varphi\, d\varphi =$$

$\sin\varphi = t$

$$= R\int_0^1 t\, dt = R\,\frac{t^2}{2}\Big|_0^1 = \frac{R}{2}$$

$$F_x = \gamma L R \left(a + \frac{R}{2}\right)$$

$$F_z = \int_A dF_z = \gamma L R \int_0^{\pi/2} (a + R\sin\varphi)\sin\varphi \, d\varphi$$

$$\int_0^{\pi/2} a \sin\varphi \, d\varphi = a \left(-\cos\varphi\right)\Big|_0^{\pi/2}$$

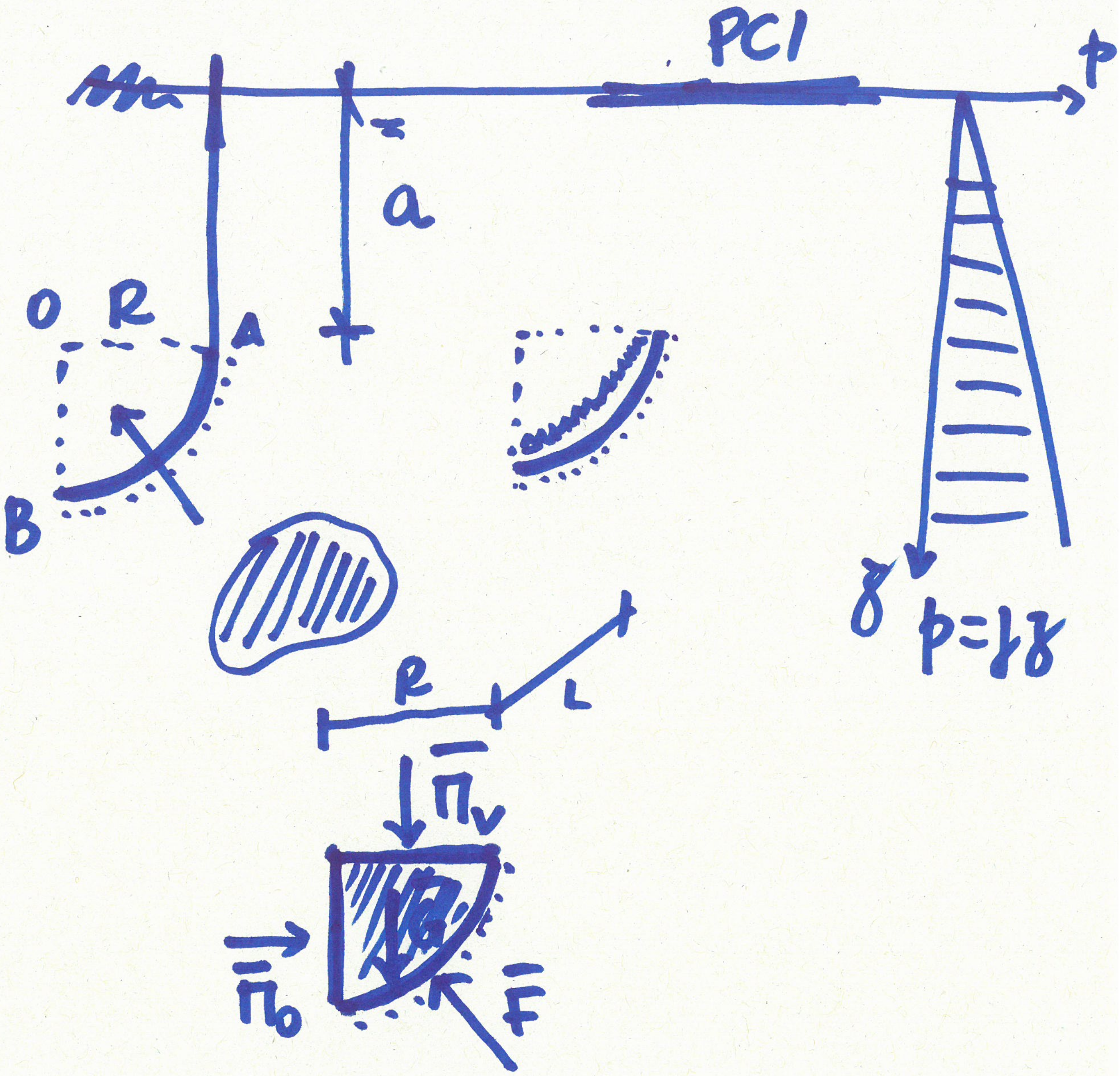
$$= a$$

$$\int_0^{\pi/2} R \sin^2\varphi \, d\varphi = \frac{\pi}{4} R$$

$$\int_0^{k} \sin^2\varphi \, d\varphi = k/2$$

$$F_z = \gamma L R \left(a + \frac{\pi}{4} R\right)$$

$$F_x = \gamma L R \left(a + \frac{R}{2}\right)$$



2) $$|\bar{\Pi}_v| + |\bar{G}| = |F_z|$$

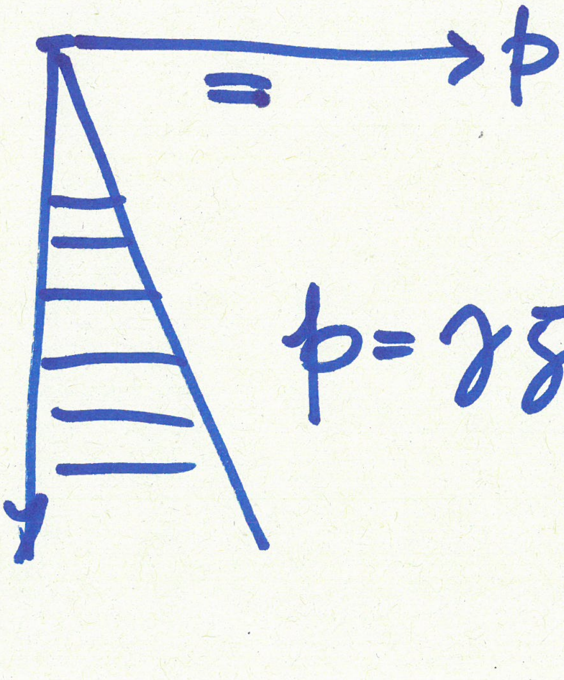
$$\gamma a R L + \gamma L \frac{\pi R^2}{4} = F_z$$

$$F_z = \gamma L R \left(a + \frac{\pi R}{4}\right)$$

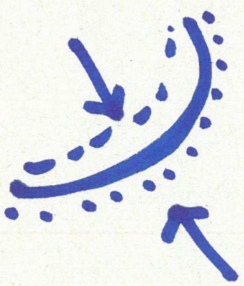
$$|\overline{\Pi}_0| = |F_x|$$

$$\gamma\left(a+\frac{R}{2}\right)LR = F_x$$

PCI



$$p = \gamma \xi$$



SUP ISOLATA DAL RESTO: COME SUP. ASTRATTA