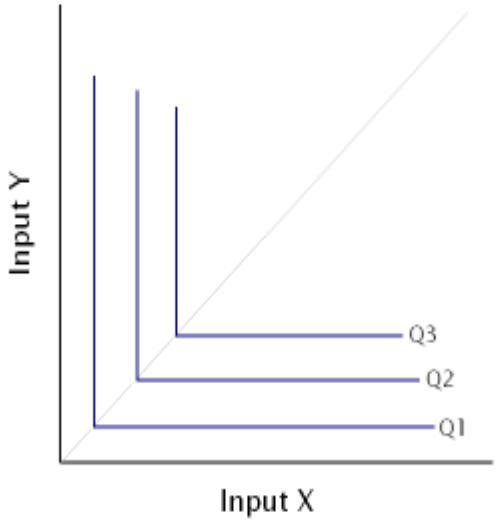
COMPLEMENTARITY

- Preliminary concept:

If the two inputs are perfect complements, the isoquant map takes the form of fig. A:



With a given level of production, input X and input Y can only be combined efficiently in the certain ratio occurring at the kink in the isoquant.

They are both necessary and must be used by the firm jointly and in fixed proportions.

Fixed-Proportions Production Functions

"Leontief production function"

CES function, with $\sigma \rightarrow 0$:

$$Q = \left[(\alpha X)^{\frac{\sigma-1}{\sigma}} + (\beta Y)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Example: chemical industry:

H_2O: one more atom of hydrogen doesn't permit to produce more molecules of water, because the two components (hydrogen and oxygen) must be used in fixed proportions.

$$TRS = \begin{cases} 0 \\ undetermined \\ \infty \end{cases}$$

LIMITS:
A pair of inputs;
analysis conducted on the same isoquant of production.

WE EXPLORE:
A broader concept of complementarity that goes beyond the static framework we have just described.

Decision variables of the firm…broader than just inputs
And that is not limited at the analysis of changes on the same isoquant of production, but rather at the analysis of the changes of the benefit function of a firm.

DEFINITION:

Activities are *Edgeworth complements* if doing more of any one of them increases the returns to doing more of the others.

In particular, with TWO activities:
the two activities are complements if doing more of one activity increases the returns to doing more of the other activity.

ANALITICALLY:

Given a set of two variables (activities of the firm or any other decision variable of the firm):

$$\{x_1, x_2\} \in X \subseteq R^2$$

And some firm's objective function $F(x)$ defined on the space R^2, that is smooth (continuous and twice differentiable)
The two variables are complements if:

$$\frac{\partial^2 F(x)}{\partial x_1 \partial x_2} \geq 0$$

increases in the objective function due to increases of variable x_1 are increasing at the increasing of the other variable too.

Complementarity goes beyond the boundaries imposed by the hypothesis of decreasing return to scales.

In fact:

- doing more of one activity does not prevent the possibility of doing more also of another activity.
- if in a pair of complementary activities one of the two is incremented, this makes more attractive to increase also the other activity.

The relationship of complementarity is symmetric:

$$\frac{\partial^2 F(x)}{\partial x_2 \partial x_1} \geq 0$$

increases in the objective function due to increases of variable x_2 are increasing at the increasing of the variable x_1.

The relationship of complementarity may involve more than two variables simultaneously

through a chain reaction that starts from a complementarity relationship between two variables and involves a complementarity relationship between one of the two variables and a third variable and so on.

Milgrom-Roberts (1995): complementarity among 12 choice variables of the firm.

Essentially what economists investigate through the analysis of complementarity is the extent to which different elements of strategy, structure and managerial processes in a firm fit one another and generate higher performances.

Complementarities in firms' innovation practices.

firms' innovation activity is a complex outcome deriving from the influence of many factors that are interrelated through complementary relationships which might give rise:

"to systemic effects, with the whole being more than the sum of its parts"

→**The firm is a system.**

What has induced the economic literature to go beyond the analytical treatment of Edgeworth complementarity?

For many of the problems one wants to address in the theory of the firm, it is unnatural to assume continuity and even divisibility of the choice variables of the firm.

The firm's activities and practices are typically investigated in discrete settings

e.g:

- the firm adopts or doesn't adopt an activity, a practice;
- the firm adopts a practice at an intensity higher or lower than the average;
- the firm decides to enter or not to enter some specific market
- etc...

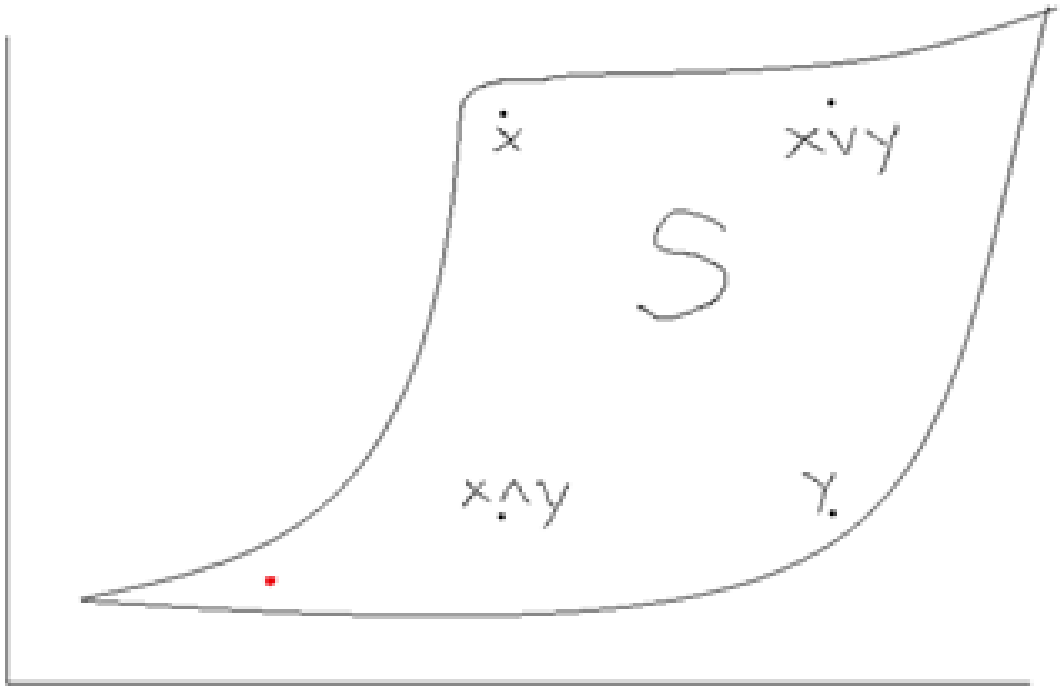
Topkis (1995, 1998), Milgrom and Roberts (1990, 1995), Milgrom and Shannon (1994)

Complementarity between different forms of actions through the properties of supermodular functions.

Technical issues.

Edgeworth complementarity is a matter of order: "doing **more** of any one activity increases the returns to doing **more** of the another"

The importance of order has led to consider for the analytical treatment of complementarity the branch of mathematics known as *lattice theory*.



x has bigger ordinate than y but smaller abscissa

y has bigger abscissa than x but smaller ordinate

$x \vee y$ the smallest element larger than both x and y has the two biggest coordinates

$x \wedge y$ the largest element smaller than both x and y has the two smallest coordinates

The red dot is smaller than the minimum element but it is not the LARGEST element smaller than both x and y

We state that two variables x and y in a *lattice* X are complements if a real-valued function $F(x,y)$ on the *lattice* X is supermodular in its arguments.

That is, if and only if:

$$(1) \quad F(x \vee y) + F(x \wedge y) \geq F(x) + F(y) \quad \forall x, y \in X$$

Or, expressed differently:

$$(2) \quad F(x \vee y) - F(x) \geq F(y) - F(x \wedge y) \quad \forall x, y \in X$$

the change in F from element x (or y) to the maximum element $(x \vee y)$

is greater than the change in F from the minimum element $x \wedge y$

to element y (or x):

raising one of the variables raises the value of increase in the second variable as well.

Or again:

$$(3)\quad F(x \vee y) - F(x \wedge y) \geq [F(x) - F(x \wedge y)] + [F(y) - F(x \wedge y)]$$

changes in the function F when both the elements are increased together are more than the changes resulting from the sum of the separate increases of the two elements.

Supermodularity gives an analytical structure to the idea that “increasing the value of some variables never prevents one from increasing the others as well”

This technical approach has the benefit of focusing on a purely economic analysis, without the need to dwell on more mathematical issues,

We have not made any particular assumption on the function F that ensures the existence of interior optima.

No divisibility or concavity assumptions are needed, so that increasing returns are easily encompassed.

Example from Antonioli-Mancinelli-Mazzanti (2013)

Aim of our work was to give an answer to the following questions:

- Do firms' actions in **organisation** and **training** foster the adoption of **environmental innovation**?
- Are **environmental strategies integrated** with **organisational changes** aimed at increasing firms' performances?

DEFINITIONS:

- *Environmental innovation (EI)*: the production, assimilation or exploitation of a product, production process, service or management or business method that is novel to the organisation (developing or adopting it) and which results, throughout its life cycle, in a reduction of environmental risk, pollution and other negative impacts of resources use (including energy use) compared to relevant alternatives
- *Firms' actions in organisation and training*:

- *High Performance Workplace Practices (HPWP*): a set of organisational changes, that are related to changes in production organisation (e.g autonomous or semi-autonomous teams, quality circles) and labour organisation (e.g. job rotation, multitasking, increased workers' responsibility),
- *Human Resource Management (HRM) practices*: practices which are linked to the training activity sphere.

The human capital embodied in employees becomes a fundamental resource in the innovating activity of a firm.

When a firm undergoes organizational changes such as the introduction of HPWP, the employees can be asked to learn how to manage and how to behave in a new organizational environment. The importance of training activities that help generate and accumulate skills and competencies complementary to HPWP is clear.

The implementation of complementary HPWP/HRM practices, may be functional to the creation of an environment that smoothly absorbs and exploits even more complex types of innovation, as the Environmental innovation.

We want to scrutinize whether firms' HPWP and HRM integrated strategies can foster the adoption of EIs.

Our main research focus is:

to examine if a relationship of complementarity exists among HPWP and HRM practices when the adoption of EIs is the objective.

Complementarities in firms' innovation practices.

Proprieties of supermodularity (eq. (3)):
changes in the function F when both the elements are increased together are more than the changes resulting from the sum of the separate increases of the two elements.

- Which is firm's objective function? (of which to study supermoduarity?)
- Which are the firm's practices of which we want to study if a relationship of complementary exists?

The 'Environmental Innovation function' of firm j (*EIj*) is the firm's objective function

and we focus on two HPWP/HRM practices that can affect the firm's EI function, h' and h'' :

$$(4) \qquad EI_j = EI_j(h', h'', \theta_j) \qquad \forall j$$

The problem of firm j is to choose a combination of HPWP/HRM practices, $(h', h'') \in H$ which maximize its EI function.

What is θ_j

θ_j represents the firm's exogenous parameters.

A firm operates in an environment which is characterized by exogenous parameters
(the product market, specific sector technologies, sector-specific environmental policy)

and one could be interested in how different values of the parameter θ may imply different instances of the firms' decisional problems and hence different firms' optimal choices concerning EI.

More specifically, complementarity between the two different practices of HPWP/HRM may be analysed by testing whether:

$$EI_j = EI_j(h', h'', \theta_j) \text{ is supermodular in } h' \text{ and } h''$$

Since each firm is characterized by specific exogenous parameters (θj) even if the maximization problem is the same for all the firms, the *EI* function may result supermodular in h' and in h'' for some firms, but not for others.

- If in its *EI* maximizing problem a firm chooses to adopt neither of the two practices: $h' = 0, h'' = 0$

  the element of the set H is: $h' \wedge h'' = \{00\}.$

- If a firm chooses to adopt both practices: $h' = 1, h'' = 1$

  and the element of the set H is: $h' \vee h'' = \{1\,1\}.$

  Including the mixed cases as well, we have four elements in the set H :

$$H = \{\{00\}, \{01\}, \{10\}, \{1\,1\}\}$$

From the above we can assert that h' and h'' are complements and hence that the function is supermodular, if and only if:

$$(5) \quad EI_j(11,\theta_j) + EI_j(00,\theta_j) \geq EI_j(10,\theta_j) + EI_j(01,\theta_j),$$

or:

$$(6) \quad EI_j(11,\theta_j) - EI_j(00,\theta_j) \geq \left[EI_j(10,\theta_j) - EI_j(00,\theta_j)\right] + \left[EI_j(01,\theta_j) - EI_j(00,\theta_j)\right]$$

changes in the firm's environmental innovation processes when both forms of HPWP/HRM practices are increased <u>together</u> are <u>more</u> than the changes resulting from the sum of the <u>separate increases</u> of the two kinds of practice.

Complementarity between the two decision variables (h' and h'') exists if the EIj function is shown to be supermodular in these two variables and this happens when either inequality (5) or inequality (6) or other derived inequalities are satisfied.

Milgrom and Roberts (1995) show that The adoption of both complementary practices by a firm may be an optimal choice in some circumstances but not in others even if its behaviour is maximizing in both cases.

The parameter θj embodies the different environments that the different firms may face.

Our crucial question is if the joint implementation of HPWP/HRM strategies can foster the adoption of EIs especially in situations of more stringent environmental regulations, namely for firms belonging to more polluting sectors

Different HPWP/HRM strategies may result complements for some values of θ but not for others.

In our specific analysis firms operating in sectors less exposed to environmental regulations are less stimulated to adopt *EIs*, and could find it more convenient to externalise the management of workforce training.

What have we learnt?

→ complementarity gives rise to systemic effects, with the whole which is greater than the sum of its single parts!

When a relationship of complementarity is found between two activities of a firm, this implies that if one of the two activities is increased, it is more attractive for the firm to increase also the other complementary activity.

Consequences?

Obvious implications on the firm's strategic decisions: the firm's change of some choice variable may have little effect if other choice variables remain unchanged.

EVEN MORE:
The increase of just ONE variable (keeping fixed the other complementary variables) may **even worsen** the economic performance of the firm!
Whereas a joined increase could improve the firm's results!!!

Milgrom and Roberts (1995, p. 194):

"General Motors, once the most successful of mass producers, spent some $80 billion during the 1980s on robotics and other capital equipment normally associated with the new methods. It did not, however, make any serious adjustments in

- its human resources policies,
- its decision systems,
- its product development processes, on even in
- its basic manufacturing procedures.

Either it failed to see the importance of making these complementary changes or else, it was unable to make the changes that were required on these dimensions. The result was that those billion dollars were largely wasted."

And General Motors suffered a loss ever suffered before!

Several are the variables among which a relationship of compementarity has been studied by the economic literature.

Milgrom-Roberts (1995) in their analysis about modern manufacturing industry identify **12 variables** among which a relationship of complementarity exists.

They identify a relationship of complementarity among:

- Technological innovation
- Policies for staff training
- New organisational strategies.

Product and/or process innovations are fostered by trained employees, that result able to manage the complexity of innovations and to solve new problems when they arise, which implies new organizational strategies which consider more involvement of employees in the decisional processes of the firm.

In the presence of complementarities, the firm's management must be more careful (General Motors case).

It is necessary:

- to identify and coordinate the relationships of complementarity,
- to exploit the systemic effect of complementarity and
- to avoid the possible wastefulness that may occur when only one of the complementary variables is considered.

COORDINATION IS ESSENTIAL:

"When complementarities are present, *fit* is important, that is, even mistaken variations from a plan are less costly when they are coordinated than when they are made independently" (Milgrom -Roberts, 1995, p. 186).

If the different managers of the different activities of a firm wouldn't coordinate among them, but would act concentrating only on the variables relevant to their choices, considering given all the other variables, hence their answers to the changes of the economic environment would always be under-sized.

This does not imply that the firm's management must be centralized, but that all the components of the "system" (firm) must continuously and intensively communicate and cooperate with each other.

COORDINATION is also essential for the choices of economic policies, if a relationship of complementarity exists between the variable considered by the policy and other variables.

In our work, environmental policies may result ineffective, if they are not coordinate with labour policies.

Complementarity is a great resource, since it allows to encompass the boundaries of decreasing return to scale, but it must be handled with great care!

References

- Antonioli, D., Mancinelli, S., Mazzanti, M., (2013), Is Environmental Innovation Embedded within High-Performance Organisational Changes? The role of human resource management and complementarity in green business strategies, Research Policy, forthcoming
- Mancinelli, S., Mazzanti, M., 2009. Innovation, networking activities and complementarity Empirical evidence on SME performances for a local economic system in Northern Italy. Annals of Regional Sciences 43, 567-97.
- Milgrom, P., Roberts, J., 1990. The Economics of Modern Manufacturing: Technology, Strategy and Organization. American Economic Review 80, 511-528.
- Milgrom, P., Roberts, J., 1995. Complementarities and Fit Strategy, Structure and Organizational Change in Manufacturing. Journal of Accounting Economics 19, 179-208.
- Milgrom, P., Shannon, C., 1994. Monotone Comparative Statics. Econometrica 62, 157-180.
- Mohnen, P., Roller, L.H., 2005. Complementarities in Innovation Policy. European Economic Review 49, 1431-1450.
- Topkis, D. M., 1998. Super Modularity and Complementarity. Princeton University Press, Princeton, NJ.
- Topkis, D.M., 1995. Comparative statics of the firm. Journal of Economic Theory 67, 370–401.