

CHAPTER 11

INVENTORY MANAGEMENT

Learning Objectives

- LO1** Explain the different purposes for keeping inventory.
- LO2** Understand that the type of inventory system logic that is appropriate for an item depends on the type of demand for that item.
- LO3** Calculate the appropriate order size when a one-time purchase must be made.
- LO4** Describe what the economic order quantity is and how to calculate it.
- LO5** Summarize fixed-order quantity and fixed-time period models, including ways to determine safety stock when there is variability in demand.
- LO6** Discuss why inventory turn is directly related to order quantity and safety stock.

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DIRECT TO STORE—THE UPS VISION

Logistics visionaries have talked for years about eliminating—or at least drastically reducing—the role of inventory in modern supply chains. The most efficient, slack-free supply chains, after all, wouldn't require any inventory buffer because supply and demand would be in perfect sync. This vision certainly has its appeal: The death of inventory would mean dramatically reduced logistics costs and simplified fulfillment.

There's no need to write a eulogy for inventory just yet. Most companies haven't honed their networks and technologies well enough to eliminate the need for at least minimal inventory. Logistics managers have to perform a daily, delicate act, balancing

- Transportation costs against fulfillment speed
- Inventory costs against the cost of stock outs
- Customer satisfaction against cost to serve
- New capabilities against profitability

What's more, two accelerating business trends are making it even harder to synchronize supply chains.

First, global sourcing is forcing supply chains to stretch farther



Global



across borders. The goods people consume are increasingly made in some other part of the world, particularly in Asia. This acceleration in global sourcing changes the logistics equation. When goods cross borders, considerations such as fulfillment speed (these are the activities performed once an order is received) and inventory costs get more complicated. Second, powerful retailers and other end customers with clout are starting to push value-added supply chain responsibilities further up the supply chain. More customers are asking manufacturers or third-party logistics providers to label and prepare individual items so the products are ready to go directly to store shelves. With added responsibilities, of course, come added costs. Upstream suppliers are always looking for ways to squeeze more costs out of other areas of the supply chain, such as transportation and distribution.

The UPS Direct Approach

A growing number of companies are overcoming these barriers by taking a more direct approach to global fulfillment. This direct-to-store approach—also known as distribution center bypass or direct distribution—keeps inventory moving from manufacturer to end customer by eliminating stops at warehouses along the way. Because companies can shrink the fulfillment cycle and eliminate inventory costs, direct-to-store can offer a good balance between fulfillment speed and logistics costs.

What accounts for the emergence of the direct-to-store model?

Global sourcing and the upstream migration of value-added logistics services are certainly primary drivers. But other pieces of the puzzle have fallen into place in recent years to make direct-to-store shipments feasible.

Internet-enabled electronic links between supply chain partners have allowed better coordination and collaboration among the various supply chain segments. Meanwhile, at the front of the supply chain, increasingly sophisticated point-of-sale systems can capture product demand patterns. This information can then be fed up the supply chain to manufacturers and components suppliers. More accurate sales-forecasting tools take some of the guesswork out of production and reduce the need for large inventory safety stocks. Tracking and tracing tools are also available to follow orders across borders and through the hands of different supply partners.

In short, companies no longer need as much inventory gathering dust in warehouses because they can better synchronize production and distribution with demand. Direct-to-store lets them keep inventory in motion—across borders and around the world.

See United Parcel Service of America (UPS) Supply Chain Solutions for more information about these types of services: www.ups.com.

You should visualize inventory as stacks of money sitting on forklifts, on shelves, and in trucks and planes while in transit. That's what inventory is—money. For many businesses, inventory is the largest asset on the balance sheet at any given time, even though it is often not very liquid. It is a good idea to try to get your inventory down as far as possible.

A few years ago, Heineken, the Netherlands beer company, figured it could save a whole bunch of money on inventory-in-transit if it could just shorten the forecasting lead time. They expected two things to happen. First, they expected to reduce the need for inventory in the pipeline, therefore cutting down the amount of money devoted to inventory itself. Second, they figured that with a shorter forecasting time, forecasts would be more accurate, reducing emergencies and waste. The Heineken system, called HOPS, cut overall inventory in the system from 16 to 18 weeks down to 4 to 6 weeks—a huge drop in time, and a big gain in cash. Forecasts were more accurate, and there was another benefit, too.

Heineken found that its salespeople were suddenly more productive. That is because they were not dealing with all those calls where they had to check on inventory or solve bad forecasting problems, or change orders that were already in process. Instead, they could concentrate on good customer service and helping distributors do better. It was a “win” all the way around.

The key here involves doing things that decrease your inventory order cycle time and increase the accuracy of your forecast. Look for ways to use automated systems and electronic communication to substitute the rapid movement of electrons for the cumbersome movement of masses of atoms.

The economic benefit from inventory reduction is evident from the following statistics: The average cost of inventory in the United States is 30 to 35 percent of its value. For example, if a firm carries an inventory of \$20 million, it costs the firm more than \$6 million per year. These costs are due mainly to obsolescence, insurance, and opportunity costs. If the amount of inventory could be reduced to \$10 million, for instance, the firm would save over \$3 million, which goes directly to the bottom line; that is, the savings from reduced inventory results in increased profit.

This chapter and Chapter 9 present techniques designed to manage inventory in different supply chain settings. In this chapter, the focus is on settings where the desire is to maintain a stock of inventory that can be delivered to our customers on demand. Recall in Chapter 6 the concept of *customer order decoupling point*, which is a point where inventory is positioned to allow processes or entities in the supply chain to operate independently. For example, if a product is stocked at a retailer, the customer pulls the item from the shelf and the manufacturer never sees a customer order. In this case, inventory acts as a buffer to separate the customer from the manufacturing process. Selection of decoupling points is a strategic decision that determines customer lead times and can greatly impact inventory investment. The closer this point is to the customer, the quicker the customer can be served.

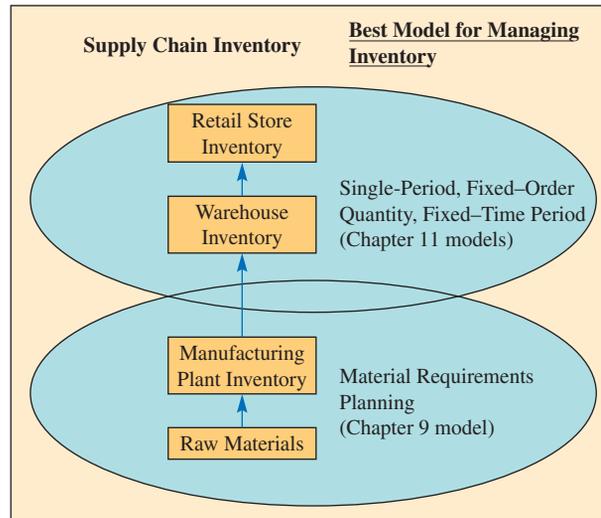
The techniques described in this chapter are suited for managing the inventory at these decoupling points. Typically, there is a trade-off where quicker response to customer demand comes at the expense of greater inventory investment. This is because finished goods inventory is more expensive than raw material inventory. In practice, the idea of a single decoupling point in a supply chain is unrealistic. There may actually be multiple points where buffering takes place.

Good examples of where the models described in this chapter are used include retail stores, grocery stores, wholesale distributors, hospital suppliers, and suppliers of repair parts needed to fix or maintain equipment quickly. Situations in which it is necessary to have the item “in-stock” are ideal candidates for the models described in this chapter. A distinction that needs to be made with the models included in this chapter is whether this is a one-time purchase, for example, for a seasonal item or for use at a special event, or whether the item will be stocked on an ongoing basis.

Exhibit 11.1 depicts different types of supply chain inventories that would exist in a make-to-stock environment, typical of items directed at the consumer. In the upper echelons of the supply chain, which are supply points closer to the customer, stock usually

exhibit 11.1

Supply Chain Inventories—Make-to-Stock Environment



is kept so that an item can be delivered quickly when a customer need occurs. Of course, there are many exceptions, but in general this is the case. The raw materials and manufacturing plant inventory held in the lower echelon potentially can be managed in a special way to take advantage of the planning and synchronization that are needed to efficiently operate this part of the supply chain. In this case, the models in this chapter are most appropriate for the upper echelon inventories (retail and warehouse), and the lower echelon should use the Material Requirements Planning (MRP) technique described in Chapter 9. The applicability of these models could be different for other environments such as when we produce directly to customer order as in the case of an aircraft manufacturer.

The techniques described here are most appropriate when demand is difficult to predict with great precision. In these models, we characterize demand by using a probability distribution and maintain stock so that the risk associated with stock out is managed. For these applications, the following three models are discussed:

1. **The single-period model.** This is used when we are making a one-time purchase of an item. An example might be purchasing T-shirts to sell at a one-time sporting event.
2. **Fixed-order quantity model.** This is used when we want to maintain an item “in-stock,” and when we resupply the item, a certain number of units must be ordered each time. Inventory for the item is monitored until it gets down to a level where the risk of stocking out is great enough that we are compelled to order.
3. **Fixed-time period model.** This is similar to the fixed-order quantity model; it is used when the item should be in-stock and ready to use. In this case, rather than monitoring the inventory level and ordering when the level gets down to a critical quantity, the item is ordered at certain intervals of time, for example, every Friday morning. This is often convenient when a group of items is ordered together. An example is the delivery of different types of bread to a grocery store. The bakery supplier may have 10 or more products stocked in a store, and rather than delivering each product individually at different times, it is much more efficient to deliver all 10 together at the same time and on the same schedule.

In this chapter, we want to show not only the mathematics associated with great inventory control but also the “art” of managing inventory. Ensuring accuracy in inventory records is essential to running an efficient inventory control process. Techniques such as ABC analysis and cycle counting are essential to the actual management of the system since they focus attention on the high-value items and ensure the quality of the transactions that affect the tracking of inventory levels.

DEFINITION OF INVENTORY

Inventory is the stock of any item or resource used in an organization. An *inventory system* is the set of policies and controls that monitor levels of inventory and determine what levels should be maintained, when stock should be replenished, and how large orders should be.

By convention, *manufacturing inventory* generally refers to items that contribute to or become part of a firm’s product output. Manufacturing inventory is typically classified into *raw materials*, *finished products*, *component parts*, *supplies*, and *work-in-process*. In distribution, inventory is classified as *in-transit*, meaning that it is being moved in the system, and *warehouse*, which is inventory in a warehouse or distribution center. Retail sites carry inventory for immediate sale to customers. In services, *inventory* generally refers to the tangible goods to be sold and the supplies necessary to administer the service.

The basic purpose of inventory analysis, whether in manufacturing, distribution, retail, or services, is to specify (1) when items should be ordered and (2) how large the order should be. Many firms are tending to enter into longer-term relationships with vendors to supply their needs for perhaps the entire year. This changes the “when” and “how many to order” to “when” and “how many to deliver.”

Inventory The stock of any item or resource used in an organization.

PURPOSES OF INVENTORY

All firms (including JIT operations) keep a supply of inventory for the following reasons:

- 1. To maintain independence of operations.** A supply of materials at a work center allows that center flexibility in operations. For example, because there are costs for making each new production setup, this inventory allows management to reduce the number of setups.
Independence of workstations is desirable on assembly lines as well. The time that it takes to do identical operations will naturally vary from one unit to the next. Therefore, it is desirable to have a cushion of several parts within the workstation so that shorter performance times can compensate for longer performance times. This way the average output can be fairly stable.
- 2. To meet variation in product demand.** If the demand for the product is known precisely, it may be possible (though not necessarily economical) to produce the product to exactly meet the demand. Usually, however, demand is not completely known, and a safety or buffer stock must be maintained to absorb variation.
- 3. To allow flexibility in production scheduling.** A stock of inventory relieves the pressure on the production system to get the goods out. This causes longer lead times, which permit production planning for smoother flow and lower-cost operation through larger lot-size production. High setup costs, for example, favor producing a larger number of units once the setup has been made.

LO1



4. **To provide a safeguard for variation in raw material delivery time.** When material is ordered from a vendor, delays can occur for a variety of reasons: a normal variation in shipping time, a shortage of material at the vendor's plant causing backlogs, an unexpected strike at the vendor's plant or at one of the shipping companies, a lost order, or a shipment of incorrect or defective material.
5. **To take advantage of economic purchase order size.** There are costs to place an order: labor, phone calls, typing, postage, and so on. Therefore, the larger each order is, the fewer the orders that need be written. Also, shipping costs favor larger orders—the larger the shipment, the lower the per-unit cost.
6. **Many other domain-specific reasons.** Depending on the situation, inventory may need to be carried. For example, in-transit inventory is material being moved from the suppliers to customers and depends on the order quantity and the transit lead time. Another example is inventory that is bought in anticipation of price changes such as fuel for jet planes or semiconductors for computers. There are many other examples.

For each of the preceding reasons (especially for items 3, 4, and 5), be aware that inventory is costly and large amounts are generally undesirable. Long cycle times are caused by large amounts of inventory and are undesirable as well.

INVENTORY COSTS

In making any decision that affects inventory size, the following costs must be considered:

1. **Holding (or carrying) costs.** This broad category includes the costs for storage facilities, handling, insurance, pilferage, breakage, obsolescence, depreciation, taxes, and the opportunity cost of capital. Obviously, high holding costs tend to favor low inventory levels and frequent replenishment.
2. **Setup (or production change) costs.** To make each different product involves obtaining the necessary materials, arranging specific equipment setups, filling out the required papers, appropriately charging time and materials, and moving out the previous stock of material.
If there were no costs or loss of time in changing from one product to another, many small lots would be produced. This would reduce inventory levels, with a resulting savings in cost. One challenge today is to try to reduce these setup costs to permit smaller lot sizes. (This is the goal of a JIT system.)
3. **Ordering costs.** These costs refer to the managerial and clerical costs to prepare the purchase or production order. Ordering costs include all the details, such as counting items and calculating order quantities. The costs associated with maintaining the system needed to track orders are also included in ordering costs.

4. **Shortage costs.** When the stock of an item is depleted, an order for that item must either wait until the stock is replenished or be canceled. When the demand is not met and the order is canceled, this is referred to as a stock out. A backorder is when the order is held and filled at a later date when the inventory for the item is replenished. There is a trade-off between carrying stock to satisfy demand and the costs resulting from stock outs and backorders. This balance is sometimes difficult to obtain because it may not be possible to estimate lost profits, the effects of lost customers, or lateness penalties. Frequently, the assumed shortage cost is little more than a guess, although it is usually possible to specify a range of such costs.

Establishing the correct quantity to order from vendors or the size of lots submitted to the firm's productive facilities involves a search for the minimum total cost resulting from the combined effects of four individual costs: holding costs, setup costs, ordering costs, and shortage costs. Of course, the timing of these orders is a critical factor that may impact inventory cost.

INDEPENDENT VERSUS DEPENDENT DEMAND

In inventory management, it is important to understand the trade-offs involved in using different types of inventory control logic. Exhibit 11.2 is a framework that shows how characteristics of demand, transaction cost, and the risk of obsolete inventory map into different types of systems. The systems in the upper left of the exhibit are described in this chapter, and those in the lower right in Chapter 9.

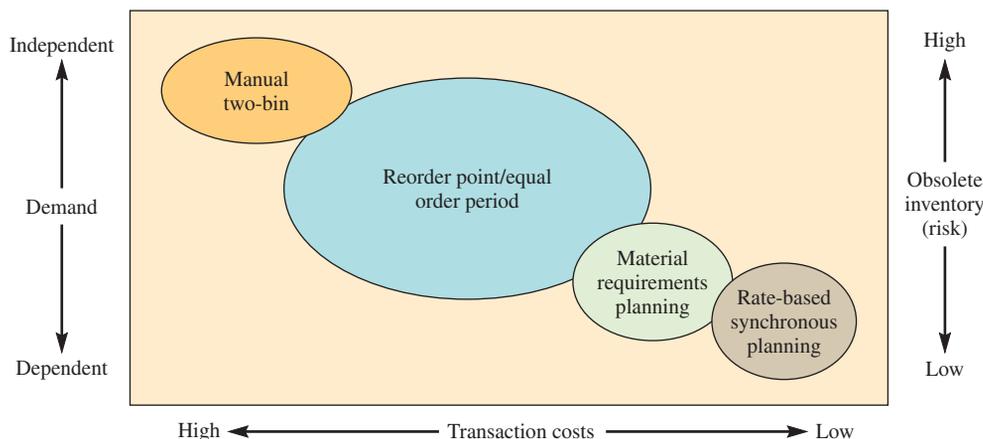
Transaction cost is dependent on the level of integration and automation incorporated in the system. Manual systems such as simple *two-bin* logic depend on human posting of the transactions to replenish inventory, which is relatively expensive compared to using a computer to automatically detect when an item needs to be ordered. Integration relates to how connected systems are. For example, it is common for orders for material to be automatically transferred to suppliers electronically and for these orders to be automatically captured by the supplier inventory control system. This type of integration greatly reduces transaction cost.

The risk of obsolescence is also an important consideration. If an item is used infrequently or only for a very specific purpose, there is considerable risk in using inventory control logic that does not track the specific source of demand for the item. Further, items

LO2

Inventory-Control-System Design Matrix: Framework Describing Inventory Control Logic

exhibit 11.2



that are sensitive to technical obsolescence, such as computer memory chips and processors, need to be managed carefully based on actual need to reduce the risk of getting stuck with inventory that is outdated.

Independent demand

The demands for various items are unrelated to each other.

Dependent demand

The need for any one item is a direct result of the need for some other item, usually an item of which it is a part.

An important characteristic of demand relates to whether demand is derived from an end item or is related to the item itself. We use the terms **independent demand** and **dependent demand** to describe this characteristic. Briefly, the distinction between independent and dependent demand is this: In independent demand, the demands for various items are unrelated to each other. For example, a workstation may produce many parts that are unrelated but that meet some external demand requirement. In dependent demand, the need for any one item is a direct result of the need for some other item, usually a higher-level item of which it is part.

In concept, dependent demand is a relatively straightforward computational problem. Needed quantities of a dependent-demand item are simply computed, based on the number needed in each higher-level item in which it is used. For example, if an automobile company plans on producing 500 cars per day, then obviously it will need 2,000 wheels and tires (plus spares). The number of wheels and tires needed is *dependent* on the production levels and is not derived separately. The demand for cars, on the other hand, is *independent*—it comes from many sources external to the automobile firm and is not a part of other products; it is unrelated to the demand for other products.

To determine the quantities of independent items that must be produced, firms usually turn to their sales and market research departments. They use a variety of techniques, including customer surveys, forecasting techniques, and economic and sociological trends, as we discussed in Chapter 3 on forecasting. Because independent demand is uncertain, extra units must be carried in inventory. This chapter presents models to determine how many units need to be ordered, and how many extra units should be carried to reduce the risk of stocking out.

INVENTORY SYSTEMS



Tutorial: Inventory

An inventory system provides the organizational structure and the operating policies for maintaining and controlling goods to be stocked. The system is responsible for ordering and receipt of goods: timing the order placement and keeping track of what has been ordered, how much, and from whom. The system also must follow up to answer such questions as: Has the supplier received the order? Has it been shipped? Are the dates correct? Are the procedures established for reordering or returning undesirable merchandise?

This section divides systems into single-period systems and multiple-period systems. The classification is based on whether the decision is just a one-time purchasing decision where the purchase is designed to cover a fixed period of time and the item will not be reordered, or the decision involves an item that will be purchased periodically where inventory should be kept in stock to be used on demand. We begin with a look at the one-time purchasing decision and the single-period inventory model.

LO3

A Single-Period Inventory Model

Certainly, an easy example to think about is the classic single-period “newsperson” problem. For example, consider the problem that the newspaper has in deciding how many newspapers to put in the sales stand outside a hotel lobby each morning. If the person does not put enough papers in the stand, some customers will not be able to purchase a paper and the newspaper will lose the profit associated with these sales. On the other hand, if too many papers are placed in the stand, the newspaper will have paid for papers that were not sold during the day, lowering profit for the day.

Actually, this is a very common type of problem. Consider the person selling T-shirts promoting a championship basketball or football game. This is especially difficult, since



the person must wait to learn what teams will be playing. The shirts can then be printed with the proper team logos. Of course, the person must estimate how many people will actually want the shirts. The shirts sold prior to the game can probably be sold at a premium price, whereas those sold after the game will need to be steeply discounted.

A simple way to think about this is to consider how much risk we are willing to take for running out of inventory. Let's consider that the newsperson selling papers in the sales stand had collected data over a few months and had found that on average each Monday 90 papers were sold with a standard deviation of 10 papers (assume that during this time the papers were purposefully overstocked in order not to run out, so they would know what "real" demand was). With these data, our newsperson could simply state a service rate that is felt to be acceptable. For example, the newsperson might want to be 80 percent sure of not running out of papers each Monday.

Recall from your study of statistics, assuming that the probability distribution associated with the sales of the paper is normal, that if we stocked exactly 90 papers each Monday morning, the risk of stocking out would be 50 percent, since 50 percent of the time we expect demand to be less than 90 papers and 50 percent of the time we expect demand to be greater than 90. To be 80 percent sure of not stocking out, we need to carry a few more papers. From the "cumulative standard normal distribution" table given in Appendix E, we see that we need approximately 0.85 standard deviation of extra papers to be 80 percent sure of not stocking out. A quick way to find the exact number of standard deviations needed for a given probability of stocking out is with the `NORMSINV(probability)` function in Microsoft Excel (`NORMSINV(0.8) = 0.84162`). Given our result from Excel, which is more accurate than what we can get from the tables, the number of extra papers would be $0.84162 \times 10 = 8.416$, or 9 papers (there is no way to sell 0.4 paper!).

To make this more useful, it would be good to actually consider the potential profit and loss associated with stocking either too many or too few papers on the stand. Let's say that our newspaper person pays \$0.20 for each paper and sells the papers for \$0.50. In this case the marginal cost associated with underestimating demand is \$0.30, the lost profit. Similarly, the marginal cost of overestimating demand is \$0.20, the cost of buying too many papers.

The optimal stocking level, using marginal analysis, occurs at the point where the expected benefits derived from carrying the next unit are less than the expected costs for that unit. Keep in mind that the specific benefits and costs depend on the problem.

In symbolic terms, define

$$\begin{aligned} C_o &= \text{Cost per unit of demand overestimated} \\ C_u &= \text{Cost per unit of demand underestimated} \end{aligned}$$

By introducing probabilities, the expected marginal cost equation becomes

$$P(C_o) \leq (1 - P)C_u$$

where P is the probability that the unit will not be sold and $1 - P$ is the probability of it being sold because one or the other must occur. (The unit is sold or is not sold.)¹

Then, solving for P , we obtain

$$P \leq \frac{C_u}{C_o + C_u} \quad [11.1]$$

This equation states that we should continue to increase the size of the order so long as the probability of selling what we order is equal to or less than the ratio $C_u/(C_o + C_u)$.

Returning to our newspaper problem, our cost of overestimating demand (C_o) is \$0.20 per paper and the cost of underestimating demand (C_u) is \$0.30. The probability therefore is $0.3/(0.2 + 0.3) = 0.6$. Now, we need to find the point on our demand distribution that corresponds to the cumulative probability of 0.6. Using the NORMSINV function to get the number of standard deviations (commonly referred to as the Z-score) of extra newspapers to carry, we get 0.253, which means that we should stock $0.253(10) = 2.53$ or 3 extra papers. The total number of papers for the stand each Monday morning, therefore, should be 93 papers.

Single-period inventory models are useful for a wide variety of service and manufacturing applications. Consider the following:

1. **Overbooking of airline flights.** It is common for customers to cancel flight reservations for a variety of reasons. Here the cost of underestimating the number of cancellations is the revenue lost due to an empty seat on a flight. The cost of overestimating cancellations is the awards, such as free flights or cash payments, that are given to customers unable to board the flight.
2. **Ordering of fashion items.** A problem for a retailer selling fashion items is that often only a single order can be placed for the entire season. This is often caused by long lead times and limited life of the merchandise. The cost of underestimating demand is the lost profit due to sales not made. The cost of overestimating demand is the cost that results when it is discounted.
3. **Any type of one-time order.** For example, ordering T-shirts for a sporting event or printing maps that become obsolete after a certain period of time.

Example 11.1: Hotel Reservations

A hotel near the university always fills up on the evening before football games. History has shown that when the hotel is fully booked, the number of last-minute cancellations has a mean of 5 and standard deviation of 3. The average room rate is \$80. When the hotel is overbooked, the policy is to find a room in a nearby hotel and to pay for the room for the customer. This usually costs the hotel approximately \$200 since rooms booked on such late notice are expensive. How many rooms should the hotel overbook?



Step by Step

SOLUTION

The cost of underestimating the number of cancellations is \$80 and the cost of overestimating cancellations is \$200.

$$P \leq \frac{C_u}{C_o + C_u} = \frac{\$80}{\$200 + \$80} = 0.2857$$

Using NORMSINV(.2857) from Excel gives a Z-score of -0.56599 . The negative value indicates that we should overbook by a value less than the average of 5. The actual value should be $-0.56599(3) = -1.69797$, or 2 reservations less than 5. The hotel should overbook three reservations on the evening prior to a football game.

Another common method for analyzing this type of problem is with a discrete probability distribution found using actual data and marginal analysis. For our hotel, consider that we have collected data and our distribution of no-shows is as follows:

| NUMBER OF NO-SHOWS | PROBABILITY | CUMULATIVE PROBABILITY |
|--------------------|-------------|------------------------|
| 0 | 0.05 | 0.05 |
| 1 | 0.08 | 0.13 |
| 2 | 0.10 | 0.23 |
| 3 | 0.15 | 0.38 |
| 4 | 0.20 | 0.58 |
| 5 | 0.15 | 0.73 |
| 6 | 0.11 | 0.84 |
| 7 | 0.06 | 0.90 |
| 8 | 0.05 | 0.95 |
| 9 | 0.04 | 0.99 |
| 10 | 0.01 | 1.00 |

Using these data, we can create a table showing the impact of overbooking. Total expected cost of each overbooking option is then calculated by multiplying each possible outcome by its probability and summing the weighted costs. The best overbooking strategy is the one with minimum cost.

| NO-SHOWS | PROBABILITY | NUMBER OF RESERVATIONS OVERBOOKED | | | | | | | | | | |
|------------|-------------|-----------------------------------|-------|-----|-------|-------|-------|-------|-------|-------|-------|-------|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | 0.05 | 0 | 200 | 400 | 600 | 800 | 1,000 | 1,200 | 1,400 | 1,600 | 1,800 | 2,000 |
| 1 | 0.08 | 80 | 0 | 200 | 400 | 600 | 800 | 1,000 | 1,200 | 1,400 | 1,600 | 1,800 |
| 2 | 0.1 | 160 | 80 | 0 | 200 | 400 | 600 | 800 | 1,000 | 1,200 | 1,400 | 1,600 |
| 3 | 0.15 | 240 | 160 | 80 | 0 | 200 | 400 | 600 | 800 | 1,000 | 1,200 | 1,400 |
| 4 | 0.2 | 320 | 240 | 160 | 80 | 0 | 200 | 400 | 600 | 800 | 1,000 | 1,200 |
| 5 | 0.15 | 400 | 320 | 240 | 160 | 80 | 0 | 200 | 400 | 600 | 800 | 1,000 |
| 6 | 0.11 | 480 | 400 | 320 | 240 | 160 | 80 | 0 | 200 | 400 | 600 | 800 |
| 7 | 0.06 | 560 | 480 | 400 | 320 | 240 | 160 | 80 | 0 | 200 | 400 | 600 |
| 8 | 0.05 | 640 | 560 | 480 | 400 | 320 | 240 | 160 | 80 | 0 | 200 | 400 |
| 9 | 0.04 | 720 | 640 | 560 | 480 | 400 | 320 | 240 | 160 | 80 | 0 | 200 |
| 10 | 0.01 | 800 | 720 | 640 | 560 | 480 | 400 | 320 | 240 | 160 | 80 | 0 |
| Total cost | | 337.6 | 271.6 | 228 | 212.4 | 238.8 | 321.2 | 445.6 | 600.8 | 772.8 | 958.8 | 1,156 |



**Excel:
Inventory
Control**

From the table, the minimum total cost is when three extra reservations are taken. This approach using discrete probability is useful when valid historic data are available. ●

LO4

Fixed-order quantity model (Q-model)

An inventory control model where the amount requisitioned is fixed and the actual ordering is triggered by inventory dropping to a specified level of inventory.

Fixed-time period model (P-model)

An inventory control model that specifies inventory is ordered at the end of a predetermined time period. The interval of time between orders is fixed and the order quantity varies.



Tutorial:
Inventory

Multiperiod Inventory Systems

There are two general types of multiperiod inventory systems: **fixed-order quantity models** (also called the *economic order quantity*, EOQ, and **Q-model**) and **fixed-time period models** (also referred to variously as the *periodic* system, *periodic review* system, *fixed-order interval* system, and **P-model**). Multiperiod inventory systems are designed to ensure that an item will be available on an ongoing basis throughout the year. Usually the item will be ordered multiple times throughout the year where the logic in the system dictates the actual quantity ordered and the timing of the order.

The basic distinction is that fixed-order quantity models are “event triggered” and fixed-time period models are “time triggered.” That is, a fixed-order quantity model initiates an order when the event of reaching a specified reorder level occurs. This event may take place at any time, depending on the demand for the items considered. In contrast, the fixed-time period model is limited to placing orders at the end of a predetermined time period; only the passage of time triggers the model.

To use the fixed-order quantity model (which places an order when the remaining inventory drops to a predetermined order point, R), the inventory remaining must be continually monitored. Thus, the fixed-order quantity model is a *perpetual* system, which requires that every time a withdrawal from inventory or an addition to inventory is made, records must be updated to reflect whether the reorder point has been reached. In a fixed-time period model, counting takes place only at the review period. (We will discuss some variations of systems that combine features of both.)

Some additional differences tend to influence the choice of systems (also see Exhibit 11.3):

- The fixed-time period model has a larger average inventory because it must also protect against stock out during the review period, T ; the fixed-order quantity model has no review period.
- The fixed-order quantity model favors more expensive items because average inventory is lower.
- The fixed-order quantity model is more appropriate for important items such as critical repair parts because there is closer monitoring and therefore quicker response to potential stock out.

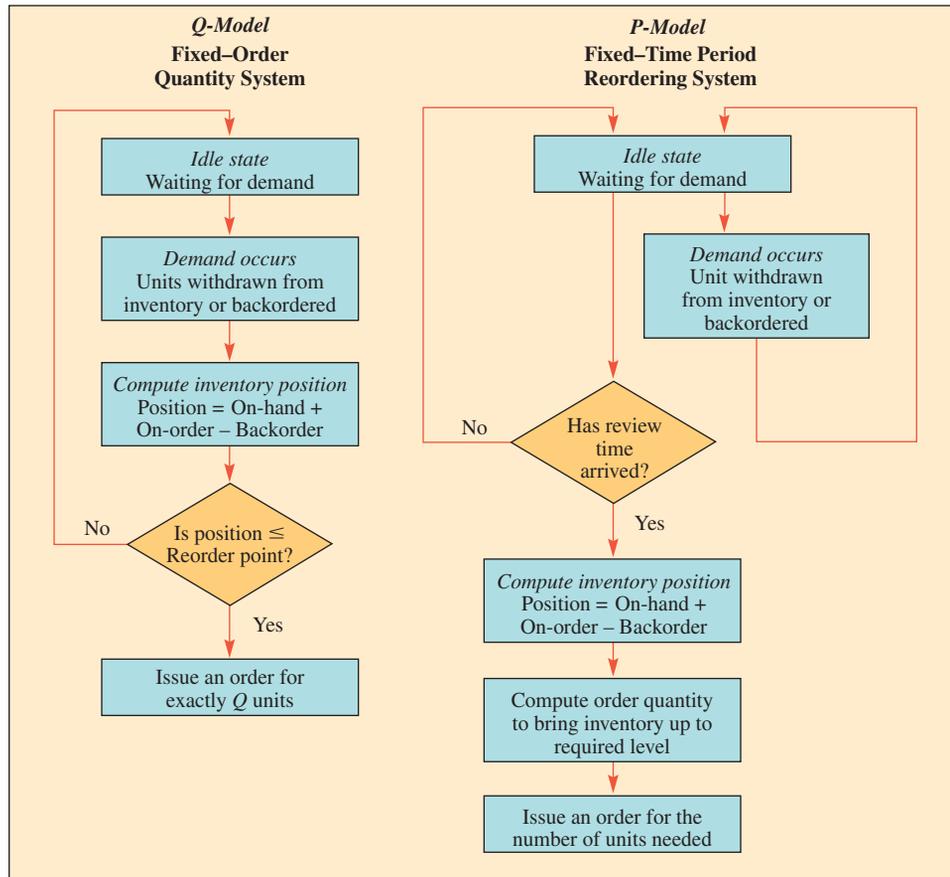
exhibit 11.3

Fixed-Order Quantity and Fixed-Time Period Differences

| FEATURE | <i>Q-MODEL</i> | <i>P-MODEL</i> |
|---------------------|---|--|
| | FIXED-ORDER QUANTITY MODEL | FIXED-TIME PERIOD MODEL |
| Order quantity | Q —constant (the same amount ordered each time) | q —variable (varies each time order is placed) |
| When to place order | R —when inventory position drops to the reorder level | T —when the review period arrives |
| Recordkeeping | Each time a withdrawal or addition is made | Counted only at review period |
| Size of inventory | Less than fixed-time period model | Larger than fixed-order quantity model |
| Time to maintain | Higher due to perpetual recordkeeping | |
| Type of items | Higher-priced, critical, or important items | |

Comparison of Fixed-Order Quantity and Fixed-Time Period Reordering Inventory Systems

exhibit 11.4



- The fixed-order quantity model requires more time to maintain because every addition or withdrawal is logged.

Exhibit 11.4 shows what occurs when each of the two models is put into use and becomes an operating system. As we can see, the fixed-order quantity system focuses on order quantities and reorder points. Procedurally, each time a unit is taken out of stock, the withdrawal is logged and the amount remaining in inventory is immediately compared to the reorder point. If it has dropped to this point, an order for Q items is placed. If it has not, the system remains in an idle state until the next withdrawal.

In the fixed-time period system, a decision to place an order is made after the stock has been counted or reviewed. Whether an order is actually placed depends on the inventory position at that time.

FIXED-ORDER QUANTITY MODELS

Fixed-order quantity models attempt to determine the specific point, R , at which an order will be placed and the size of that order, Q . The order point, R , is always a specified number of units. An order of size Q is placed when the inventory available (currently in stock and

Inventory position

The amount on-hand plus on-order minus backordered quantities. In the case where inventory has been allocated for special purposes, the inventory position is reduced by these allocated amounts.

on order) reaches the point *R*. **Inventory position** is defined as the on-hand plus on-order minus backordered quantities. The solution to a fixed-order quantity model may stipulate something like this: When the inventory position drops to 36, place an order for 57 more units.

The simplest models in this category occur when all aspects of the situation are known with certainty. If the annual demand for a product is 1,000 units, it is precisely 1,000—not 1,000 plus or minus 10 percent. The same is true for setup costs and holding costs. Although the assumption of complete certainty is rarely valid, it provides a good basis for our coverage of inventory models.

Exhibit 11.5 and the discussion about deriving the optimal order quantity are based on the following characteristics of the model. These assumptions are unrealistic, but they represent a starting point and allow us to use a simple example.

- Demand for the product is constant and uniform throughout the period.
- Lead time (time from ordering to receipt) is constant.
- Price per unit of product is constant.
- Inventory holding cost is based on average inventory.
- Ordering or setup costs are constant.
- All demands for the product will be satisfied. (No backorders are allowed.)

The “sawtooth effect” relating *Q* and *R* in Exhibit 11.5 shows that when the inventory position drops to point *R*, a reorder is placed. This order is received at the end of time period *L*, which does not vary in this model.

In constructing any inventory model, the first step is to develop a functional relationship between the variables of interest and the measure of effectiveness. In this case, because we are concerned with cost, the following equation pertains:

$$\text{Total annual cost} = \text{Annual purchase cost} + \text{Annual ordering cost} + \text{Annual holding cost}$$

or

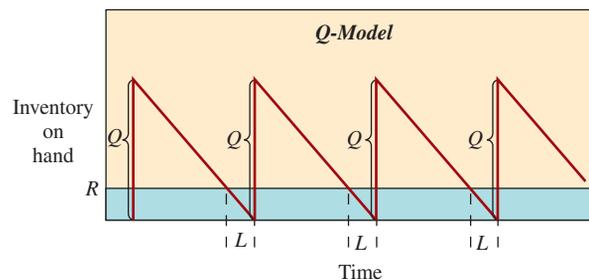
$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H \quad [11.2]$$

exhibit 11.5

Basic Fixed-Order Quantity Model



Excel:
Inventory Control



where

TC = Total annual cost

D = Demand (annual)

C = Cost per unit

Q = Quantity to be ordered (the optimal amount is termed the *economic order quantity*—EOQ—or Q_{opt})

S = Setup cost or cost of placing an order

R = Reorder point

L = Lead time

H = Annual holding and storage cost per unit of average inventory (often holding cost is taken as a percentage of the cost of the item, such as $H = iC$, where i is the percent carrying cost)

On the right side of the equation, DC is the annual purchase cost for the units, $(D/Q)S$ is the annual ordering cost (the actual number of orders placed, D/Q , times the cost of each order, S), and $(Q/2)H$ is the annual holding cost (the average inventory, $Q/2$, times the cost per unit for holding and storage, H). These cost relationships are graphed in Exhibit 11.6.

The second step in model development is to find that order quantity Q_{opt} at which total cost is a minimum. In Exhibit 11.5, the total cost is minimal at the point where the slope of the curve is zero. Using calculus, we take the derivative of total cost with respect to Q and set this equal to zero. For the basic model considered here, the calculations are

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H$$

$$\frac{dTC}{dQ} = 0 + \left(\frac{-DS}{Q^2}\right) + \frac{H}{2} = 0$$

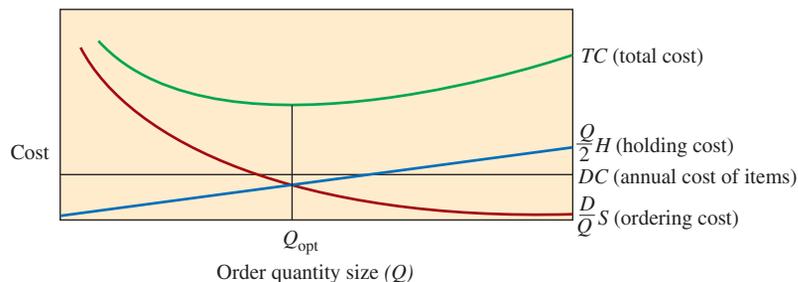
$$Q_{\text{opt}} = \sqrt{\frac{2DS}{H}} \quad [11.3]$$

Because this simple model assumes constant demand and lead time, neither safety stock nor stock-out cost is necessary, and the reorder point, R , is simply

$$R = \bar{d}L \quad [11.4]$$

Annual Product Costs, Based on Size of the Order

exhibit 11.6

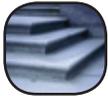


Excel:
Inventory Control

where

\bar{d} = Average daily demand (constant)

L = Lead time in days (constant)



Step by Step



Excel:
Inventory
Control

Example 11.2: Economic Order Quantity and Reorder Point

Find the economic order quantity and the reorder point, given

Annual demand (D) = 1,000 units

Average daily demand (\bar{d}) = 1,000/365

Ordering cost (S) = \$5 per order

Holding cost (H) = \$1.25 per unit per year

Lead time (L) = 5 days

Cost per unit (C) = \$12.50

What quantity should be ordered?

SOLUTION

The optimal order quantity is

$$Q_{\text{opt}} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(1,000)5}{1.25}} = \sqrt{8,000} = 89.4 \text{ units}$$

The reorder point is

$$R = \bar{d}L = \frac{1,000}{365}(5) = 13.7 \text{ units}$$

Rounding to the nearest unit, the inventory policy is as follows: When the inventory position drops to 14, place an order for 89 more.

The total annual cost will be

$$\begin{aligned} TC &= DC + \frac{D}{Q}S + \frac{Q}{2}H \\ &= 1,000(12.50) + \frac{1,000}{89}(5) + \frac{89}{2}(1.25) \\ &= \$12,611.81 \end{aligned}$$

Note that the total ordering cost $(1,000/89) \times 5 = \$56.18$ and the total carrying cost $(89/2) \times 1.25 = \$55.62$ are very close but not exactly the same due to rounding Q to 89.

Note that in this example, the purchase cost of the units was not required to determine the order quantity and the reorder point because the cost was constant and unrelated to order size. ●

Establishing Safety Stock Levels

The previous model assumed that demand was constant and known. In the majority of cases, though, demand is not constant but varies from day to day. Safety stock must therefore be maintained to provide some level of protection against stock outs. **Safety stock** can be defined as the amount of inventory carried in addition to the expected demand. In a normal distribution, this would be the mean. For example, if our average monthly demand is 100 units and we expect next month to be the same, if we carry 120 units, then we have 20 units of safety stock.

Safety stock The amount of inventory carried in addition to the expected demand.

Safety stock can be determined based on many different criteria. A common approach is for a company to simply state that a certain number of weeks of supply needs to be kept in safety stock. It is better, though, to use an approach that captures the variability in demand.

For example, an objective may be something like “set the safety stock level so that there will only be a 5 percent chance of stocking out if demand exceeds 300 units.” We call this approach to setting safety stock the probability approach.

The Probability Approach Using the probability criterion to determine safety stock is pretty simple. With the models described in this chapter, we assume that the demand over a period of time is normally distributed with a mean and a standard deviation. *Again, remember that this approach considers only the probability of running out of stock, not how many units we are short.* To determine the probability of stocking out over the time period, we can simply plot a normal distribution for the expected demand and note where the amount we have on hand lies on the curve.

Let’s take a few simple examples to illustrate this. Say we expect demand to be 100 units over the next month, and we know that the standard deviation is 20 units. If we go into the month with just 100 units, we know that our probability of stocking out is 50 percent. Half of the months we would expect demand to be greater than 100 units; half of the months we would expect it to be less than 100 units. Taking this further, if we ordered a month’s worth of inventory of 100 units at a time and received it at the beginning of the month, over the long run we would expect to run out of inventory in six months of the year.

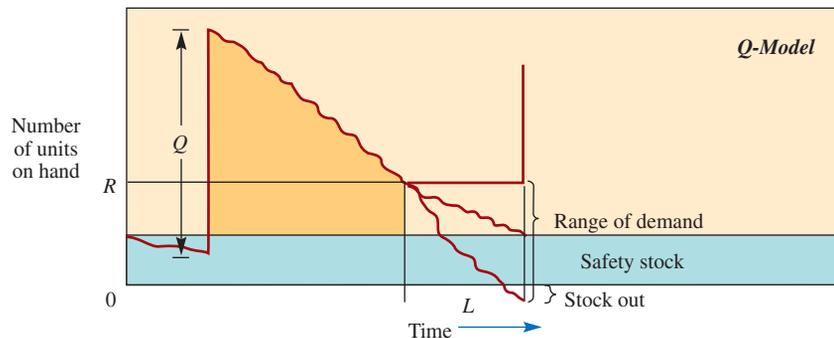
If running out this often was not acceptable, we would want to carry extra inventory to reduce this risk of stocking out. One idea might be to carry an extra 20 units of inventory for the item. In this case, we would still order a month’s worth of inventory at a time, but we would schedule delivery to arrive when we still have 20 units remaining in inventory. This would give us that little cushion of safety stock to reduce the probability of stocking out. If the standard deviation associated with our demand was 20 units, we would then be carrying one standard deviation worth of safety stock. Looking at the Cumulative Standard Normal Distribution (Appendix E), and moving one standard deviation to the right of the mean, gives a probability of 0.8413. So approximately 84 percent of the time we would not expect to stock out, and 16 percent of the time we would. Now if we order every month, we would expect to stock out approximately two months per year ($0.16 \times 12 = 1.92$). For those using Excel, given a z value, the probability can be obtained with the NORMSDIST function.

It is common for companies using this approach to set the probability of not stocking out at 95 percent. This means we would carry about 1.64 standard deviations of safety stock, or 33 units ($1.64 \times 20 = 32.8$) for our example. Once again, keep in mind that this does not mean that we would order 33 units extra each month. Rather, it means that we would still order a month’s worth each time, but we would schedule the receipt so that we could expect to have 33 units in inventory when the order arrives. In this case, we would expect to stock out approximately 0.6 month per year, or that stock outs would occur in 1 of every 20 months.

Fixed-Order Quantity Model with Safety Stock A fixed-order quantity system perpetually monitors the inventory level and places a new order when stock reaches some level, R . The danger of stock out in this model occurs only during the lead time, between the time an order is placed and the time it is received. As shown in Exhibit 11.7, an order is placed when the inventory position drops to the reorder point, R . During this lead time L , a range of demands is possible. This range is determined either from an analysis of past demand data or from an estimate (if past data are not available).

exhibit 11.7

Fixed-Order Quantity Model



The amount of safety stock depends on the service level desired, as previously discussed. The quantity to be ordered, Q , is calculated in the usual way considering the demand, shortage cost, ordering cost, holding cost, and so forth. A fixed-order quantity model can be used to compute Q , such as the simple Q_{opt} model previously discussed. The reorder point is then set to cover the expected demand during the lead time plus a safety stock determined by the desired service level. Thus, *the key difference between a fixed-order quantity model where demand is known and one where demand is uncertain is in computing the reorder point. The order quantity is the same in both cases. The uncertainty element is taken into account in the safety stock.*

The reorder point is

$$R = \bar{d}L + z\sigma_L \quad [11.5]$$

where

R = Reorder point in units

\bar{d} = Average daily demand

L = Lead time in days (time between placing an order and receiving the items)

z = Number of standard deviations for a specified service probability

σ_L = Standard deviation of usage during lead time

The term $z\sigma_L$ is the amount of safety stock. Note that if safety stock is positive, the effect is to place a reorder sooner. That is, R without safety stock is simply the average demand during the lead time. If lead time usage was expected to be 20, for example, and safety stock was computed to be 5 units, then the order would be placed sooner, when 25 units remained. The greater the safety stock, the sooner the order is placed.

Computing \bar{d} , σ_L , and z Demand during the replenishment lead time is really an estimate or forecast of expected use of inventory from the time an order is placed to when it is received. It may be a single number (for example, if the lead time is a month, the demand may be taken as the previous year's demand divided by 12), or it may be a summation of expected demands over the lead time (such as the sum of daily demands over a 30-day lead

time). For the daily demand situation, d can be a forecast demand using any of the models in Chapter 3 on forecasting. For example, if a 30-day period was used to calculate d , then a simple average would be

$$\begin{aligned}\bar{d} &= \frac{\sum_{i=1}^n d_i}{n} \\ &= \frac{\sum_{i=1}^{30} d_i}{30}\end{aligned}\quad [11.6]$$

where n is the number of days.

The standard deviation of the daily demand is

$$\begin{aligned}\sigma_d &= \sqrt{\frac{\sum_{i=1}^n (d_i - \bar{d})^2}{n}} \\ &= \sqrt{\frac{\sum_{i=1}^{30} (d_i - \bar{d})^2}{30}}\end{aligned}\quad [11.7]$$

Because σ_d refers to one day, if lead time extends over several days, we can use the statistical premise that the standard deviation of a series of independent occurrences is equal to the square root of the sum of the variances. That is, in general,

$$\sigma_L = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_L^2}\quad [11.8]$$

For example, suppose we computed the standard deviation of demand to be 10 units per day. If our lead time to get an order is five days, the standard deviation for the five-day period, assuming each day can be considered independent, is

$$\sigma_5 = \sqrt{(10)^2 + (10)^2 + (10)^2 + (10)^2 + (10)^2} = 22.36$$

Next we need to find z , the number of standard deviations of safety stock.

Suppose we wanted our probability of not stocking out during the lead time to be 0.95. The z value associated with a 95 percent probability of not stocking out is 1.64 (see Appendix E or use the Excel NORMSINV function). Given this, safety stock is calculated as follows:

$$\begin{aligned}SS &= z\sigma_L \\ &= 1.64 \times 22.36 \\ &= 36.67\end{aligned}\quad [11.9]$$

We now compare two examples. The difference between them is that in the first, the variation in demand is stated in terms of standard deviation over the entire lead time, while in the second, it is stated in terms of standard deviation per day.

Example 11.3: Reorder Point

Consider an economic order quantity case where annual demand $D = 1,000$ units, economic order quantity $Q = 200$ units, the desired probability of not stocking out $P = .95$, the standard deviation of demand during lead time $\sigma_L = 25$ units, and lead time $L = 15$ days. Determine the reorder point. Assume that demand is over a 250-workday year.



SOLUTION

In our example, $\bar{d} = \frac{1000}{250} = 4$, and lead time is 15 days. We use the equation

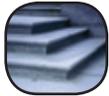
$$\begin{aligned} R &= \bar{d}L + z\sigma_L \\ &= 4(15) + z(25) \end{aligned}$$

In this case, z is 1.64.

Completing the solution for R , we have

$$R = 4(15) + 1.64(25) = 60 + 41 = 101 \text{ units}$$

This says that when the stock on hand gets down to 101 units, order 200 more. ●



Step by Step



**Excel:
Inventory
Control**

Example 11.4: Order Quantity and Reorder Point

Daily demand for a certain product is normally distributed with a mean of 60 and standard deviation of 7. The source of supply is reliable and maintains a constant lead time of six days. The cost of placing the order is \$10 and annual holding costs are \$0.50 per unit. There are no stock-out costs, and unfilled orders are filled as soon as the order arrives. Assume sales occur over the entire 365 days of the year. Find the order quantity and reorder point to satisfy a 95 percent probability of not stocking out during the lead time.

SOLUTION

In this problem we need to calculate the order quantity Q as well as the reorder point R .

$$\begin{aligned} \bar{d} &= 60 & S &= \$10 \\ \sigma_d &= 7 & H &= \$0.50 \\ D &= 60(365) & L &= 6 \end{aligned}$$

The optimal order quantity is

$$Q_{\text{opt}} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(60)365(10)}{0.50}} = \sqrt{876,000} = 936 \text{ units}$$

To compute the reorder point, we need to calculate the amount of product used during the lead time and add this to the safety stock.

The standard deviation of demand during the lead time of six days is calculated from the variance of the individual days. Because each day's demand is independent,²

$$\sigma_L = \sqrt{\sum_{i=1}^L \sigma_d^2} = \sqrt{6(7)^2} = 17.15$$

Once again, z is 1.64.

$$R = \bar{d}L + z\sigma_L = 60(6) + 1.64(17.15) = 388 \text{ units}$$

To summarize the policy derived in this example, an order for 936 units is placed whenever the number of units remaining in inventory drops to 388. ●

FIXED-TIME PERIOD MODELS



In a fixed-time period system, inventory is counted only at particular times, such as every week or every month. Counting inventory and placing orders periodically are desirable in situations such as when vendors make routine visits to customers and take orders for their complete line of products, or when buyers want to combine orders to save transportation costs. Other firms operate on a fixed time period to facilitate planning their inventory count; for example, Distributor X calls every two weeks and employees know that all Distributor X's product must be counted.

Fixed-time period models generate order quantities that vary from period to period, depending on the usage rates. These generally require a higher level of safety stock than a fixed-order quantity system. The fixed-order quantity system assumes continual tracking of inventory on hand, with an order immediately placed when the reorder point is reached. In contrast, the standard fixed-time period models assume that inventory is counted only at the time specified for review. It is possible that some large demand will draw the stock down to zero right after an order is placed. This condition could go unnoticed until the next review period. Then the new order, when placed, still takes time to arrive. Thus, it is possible to be out of stock throughout the entire review period, T , and order lead time, L . Safety stock, therefore, must protect against stock outs during the review period itself as well as during the lead time from order placement to order receipt.

Fixed-Time Period Model with Safety Stock

In a fixed-time period system, reorders are placed at the time of review (T), and the safety stock that must be reordered is

$$\text{Safety stock} = z\sigma_{T+L} \quad [11.10]$$

Exhibit 11.8 shows a fixed-time period system with a review cycle of T and a constant lead time of L . In this case, demand is randomly distributed about a mean d . The quantity to order, q , is

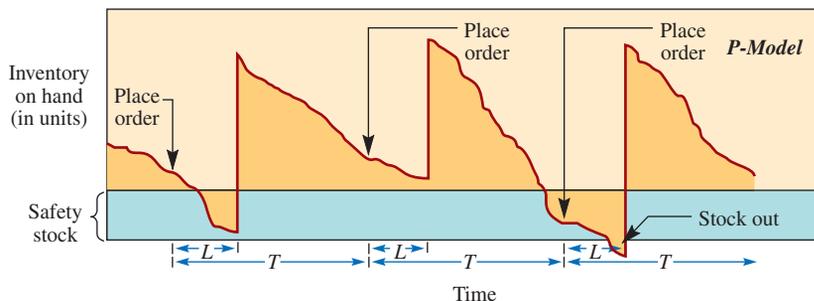
$$\begin{aligned} \text{Order quantity} &= \text{Average demand over the vulnerable period} + \text{Safety stock} - \text{Inventory currently on hand (plus on order, if any)} \\ q &= \bar{d}(T + L) + z\sigma_{T+L} - I \end{aligned} \quad [11.11]$$



Tutorial: Inventory

Fixed-Time Period Inventory Model

exhibit 11.8



where

q = Quantity to be ordered

T = The number of days between reviews

L = Lead time in days (time between placing an order and receiving it)

\bar{d} = Forecast average daily demand

z = Number of standard deviations for a specified service probability

σ_{T+L} = Standard deviation of demand over the review and lead time

I = Current inventory level (includes items on order)

Note: The demand, lead time, review period, and so forth can be any time units such as days, weeks, or years so long as they are consistent throughout the equation.

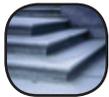
In this model, demand (\bar{d}) can be forecast and revised each review period if desired, or the yearly average may be used if appropriate. We assume that demand is normally distributed.

The value of z is dependent on the probability of stocking out and can be found using Appendix E or by using the Excel NORMSINV function.

Example 11.5: Quantity to Order

Daily demand for a product is 10 units with a standard deviation of 3 units. The review period is 30 days, and lead time is 14 days. Management has set a policy of satisfying 98 percent of demand from items in stock. At the beginning of this review period, there are 150 units in inventory.

How many units should be ordered?



Step by Step



Excel:
Inventory
Control

SOLUTION

The quantity to order is

$$\begin{aligned} q &= \bar{d}(T + L) + z\sigma_{T+L} - I \\ &= 10(30 + 14) + z\sigma_{T+L} - 150 \end{aligned}$$

Before we can complete the solution, we need to find σ_{T+L} and z . To find σ_{T+L} , we use the notion, as before, that the standard deviation of a sequence of independent random variables equals the square root of the sum of the variances. Therefore, the standard deviation during the period $T + L$ is the square root of the sum of the variances for each day:

$$\sigma_{T+L} = \sqrt{\sum_{i=1}^{T+L} \sigma_d^2} \quad [11.12]$$

Because each day is independent and σ_d is constant,

$$\sigma_{T+L} = \sqrt{(T + L)\sigma_d^2} = \sqrt{(30 + 14)(3)^2} = 19.90$$

The z value for $P = 0.98$ is 2.05.

The quantity to order, then, is

$$q = \bar{d}(T + L) + z\sigma_{T+L} - I = 10(30 + 14) + 2.05(19.90) - 150 = 331 \text{ units}$$

To ensure a 98 percent probability of not stocking out, order 331 units at this review period. ●

INVENTORY CONTROL AND SUPPLY CHAIN MANAGEMENT

It is important for managers to realize that how they run items using inventory control logic relates directly to the financial performance of the firm. A key measure that relates to company performance is inventory turn. Recall that inventory turn is calculated as follows:

LO6

$$\text{Inventory turn} = \frac{\text{Cost of goods sold}}{\text{Average inventory value}}$$

So what is the relationship between how we manage an item and the inventory turn for that item? Here, let us simplify things and consider just the inventory turn for an individual item or a group of items. First, if we look at the numerator, the cost of goods sold for an individual item relates directly to the expected yearly demand (D) for the item. Given a cost per unit (C) for the item, the cost of goods sold is just D times C . Recall this is the same as what was used in our total cost equation when calculating. Next, consider average inventory value. Recall from EOQ that the average inventory is $Q/2$, which is true if we assume that demand is constant. When we bring uncertainty into the equation, safety stock is needed to manage the risk created by demand variability. The fixed-order quantity model and fixed-time period model both have equations for calculating the safety stock required for a given probability of stocking out. In both models, we assume that when going through an order cycle, half the time we need to use the safety stock and half the time we do not. So on average, we expect the safety stock (SS) to be on hand. Given this, the average inventory is equal to the following:

$$\text{Average inventory value} = (Q/2 + SS)C \quad [11.13]$$

The inventory turn for an individual item then is

$$\text{Inventory turn} = \frac{DC}{(Q/2 + SS)C} = \frac{D}{Q/2 + SS} \quad [11.14]$$

Example 11.6: Average Inventory Calculation—Fixed-Order Quantity Model

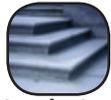
Suppose the following item is being managed using a fixed-order quantity model with safety stock.

Annual demand (D) = 1,000 units
 Order quantity (Q) = 300 units
 Safety stock (SS) = 40 units

What are the average inventory level and inventory turn for the item?

SOLUTION

$$\begin{aligned} \text{Average inventory} &= Q/2 + SS = 300/2 + 40 = 190 \text{ units} \\ \text{Inventory turn} &= \frac{D}{Q/2 + SS} = \frac{1,000}{190} = 5.263 \text{ turns per year} \bullet \end{aligned}$$



Step by Step

Example 11.7: Average Inventory Calculation—Fixed-Time Period Model

Consider the following item that is being managed using a fixed-time period model with safety stock.

Weekly demand (d) = 50 units

Review cycle (T) = 3 weeks

Safety stock (SS) = 30 units

What are the average inventory level and inventory turn for the item?

SOLUTION

Here we need to determine how many units we expect to order each cycle. If we assume that demand is fairly steady, then we would expect to order the number of units that we expect demand to be during the review cycle. This expected demand is equal to dT if we assume that there is no trend or seasonality in the demand pattern.

$$\text{Average inventory} = dT/2 + SS = 50(3)/2 + 30 = 105 \text{ units}$$

$$\text{Inventory turn} = \frac{52d}{dT/2 + SS} = \frac{52(50)}{105} = 24.8 \text{ turns per year}$$

Here we assume the firm operates 52 weeks in the year. ●

PRICE-BREAK MODELS

Price-break models deal with the fact that, generally, the selling price of an item varies with the order size. This is a discrete or step change rather than a per-unit change. For example, wood screws may cost \$0.02 each for 1 to 99 screws, \$1.60 per 100, and \$13.50 per 1,000. To determine the optimal quantity of any item to order, we simply solve for the economic order quantity for each price and at the point of price change. But not all of the economic order quantities determined by the formula are feasible. In the wood screw example, the Q_{opt} formula might tell us that the optimal decision at the price of 1.6 cents is to order 75 screws. This would be impossible, however, because 75 screws would cost 2 cents each.

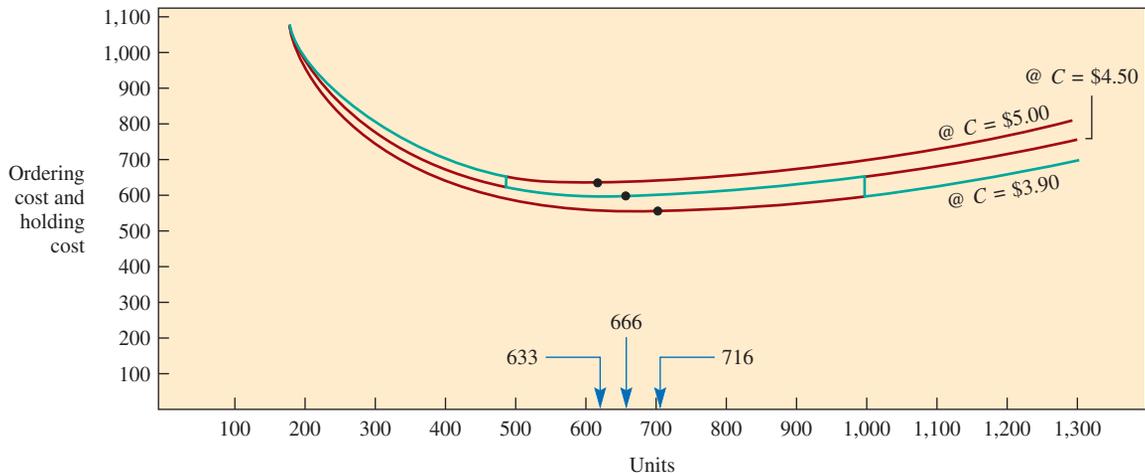
In general, to find the lowest-cost order quantity, we need to calculate the economic order quantity for each possible price and check to see whether the quantity is feasible. It is possible that the economic order quantity that is calculated is either higher or lower than the range to which the price corresponds. Any feasible quantity is a potential candidate for order quantity. We also need to calculate the cost at each of the price-break quantities, since we know that price is feasible at these points and the total cost may be lowest at one of these values.

The calculations can be simplified a little if holding cost is based on a percentage of unit price (they will be in all the examples and problems given in this book). In this case, we only need to look at a subset of the price-break quantities. The following two-step procedure can be used:

Step 1. Sort the prices from lowest to highest and then, beginning with the lowest price, calculate the economic order quantity for each price level until a feasible economic order quantity is found. By feasible, we mean that the quantity is in the correct corresponding range for that price.

Curves for Three Separate Order Quantity Models in a Three-Price-Break Situation (red line depicts feasible range of purchases)

exhibit 11.9



Step 2. If the first feasible economic order quantity is for the lowest price, this quantity is best and you are finished. Otherwise, calculate the total cost for the first feasible economic order quantity (you did these from lowest to highest price) and also calculate the total cost at each price break lower than the price associated with the first feasible economic order quantity. This is the lowest order quantity at which you can take advantage of the price break. The optimal Q is the one with the lowest cost.

Looking at Exhibit 11.9, we see that order quantities are solved from right to left, or from the lowest unit price to the highest, until a valid Q is obtained. Then the order quantity at each *price break* above this Q is used to find which order quantity has the least cost—the computed Q or the Q at one of the price breaks.

Example 11.8: Price Break

Consider the following case, where

$D = 10,000$ units (annual demand)

$S = \$20$ to place each order

$i = 20$ percent of cost (annual carrying cost, storage, interest, obsolescence, etc.)

$C =$ Cost per unit (according to the order size; orders of 0 to 499 units, \$5.00 per unit; 500 to 999, \$4.50 per unit; 1,000 and up, \$3.90 per unit)

What quantity should be ordered?



Step by Step

SOLUTION

The appropriate equations from the basic fixed-order quantity case are

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}iC$$

exhibit 11.10

Relevant Costs in a Three-Price-Break Model

| | $Q = 633$ WHERE $C = \$5$ | $Q = 667$ WHERE $C = \$4.50$ | $Q = 716$ WHERE $C = \$3.90$ | PRICE BREAK 1,000 |
|--|---------------------------------|---|------------------------------------|--|
| Holding cost $\left(\frac{Q}{2}iC\right)$ | | $\frac{667}{2}(0.20)4.50$ = \$300.15 | | $\frac{1,000}{2}(0.20)3.90$ = \$390 |
| Ordering cost $\left(\frac{D}{Q}S\right)$ | Not feasible | $\frac{10,000(20)}{667}$ = \$299.85 | Not feasible | $\frac{10,000(20)}{1,000}$ = \$200 |
| Holding and ordering cost | | \$600.00 | | \$590 |
| Item cost (DC) | | 10,000(4.50) | | 10,000(3.90) |
| Total cost | | \$45,600 | | \$39,590 |

and

$$Q = \sqrt{\frac{2DS}{iC}} \quad [11.15]$$

Solving for the economic order size, we obtain

| | | |
|---------------------|-----------------|---------------------------|
| @ $C = \$3.90$, | $Q = 716$ | Not feasible |
| @ $C = \$4.50$, | $Q = 667$ | Feasible, cost = \$45,600 |
| Check $Q = 1,000$, | Cost = \$39,590 | Optimal solution |

In Exhibit 11.10, which displays the cost relationship and order quantity range, note that most of the order quantity–cost relationships lie outside the feasible range and that only a single, continuous range results. This should be readily apparent because, for example, the first order quantity specifies buying 633 units at \$5.00 per unit. However, if 633 units are ordered, the price is \$4.50, not \$5.00. The same holds true for the third order quantity, which specifies an order of 716 units at \$3.90 each. This \$3.90 price is not available on orders of less than 1,000 units.

Exhibit 11.10 itemizes the total costs at the economic order quantities and at the price breaks. The optimal order quantity is shown to be 1,000 units. ●

One practical consideration in price-break problems is that the price reduction from volume purchases frequently makes it seemingly economical to order amounts larger than the Q_{opt} . Thus, when applying the model, we must be particularly careful to obtain a valid estimate of product obsolescence and warehousing costs.

ABC INVENTORY PLANNING

Maintaining inventory through counting, placing orders, receiving stock, and so on takes personnel time and costs money. When there are limits on these resources, the logical move is to try to use the available resources to control inventory in the best way. In other words, focus on the most important items in inventory.

In the nineteenth century Vilefredo Pareto, in a study of the distribution of wealth in Milan, found that 20 percent of the people controlled 80 percent of the wealth. This logic of the few having the greatest importance and the many having little importance has been broadened to include many situations and is termed the *Pareto principle*.³ This is true in our everyday lives (most of our decisions are relatively unimportant, but a few shape our future) and is certainly true in inventory systems (where a few items account for the bulk of our investment).

Any inventory system must specify when an order is to be placed for an item and how many units to order. Most inventory control situations involve so many items that it is not practical to model and give thorough treatment to each item. To get around this problem, the **ABC inventory classification** scheme divides inventory items into three groupings: high dollar volume (A), moderate dollar volume (B), and low dollar volume (C). Dollar volume is a measure of importance; an item low in cost but high in volume can be more important than a high-cost item with low volume.

ABC inventory classification Divides inventory into dollar volume categories that map into strategies appropriate for the category.

ABC Classification

If the annual usage of items in inventory is listed according to dollar volume, generally, the list shows that a small number of items account for a large dollar volume and that a large number of items account for a small dollar volume. Exhibit 11.11A illustrates the relationship.

The ABC approach divides this list into three groupings by value: A items constitute roughly the top 15 percent of the items, B items the next 35 percent, and C items the last 50 percent. From observation, it appears that the list in Exhibit 11.11A can be meaningfully grouped with A including 20 percent (2 of the 10), B including 30 percent, and C including 50 percent. These points show clear delineations between sections. The result of this segmentation is shown in Exhibit 11.11B and plotted in Exhibit 11.11C.

Segmentation may not always occur so neatly. The objective, though, is to try to separate the important from the unimportant. Where the lines actually break depends on the particular inventory under question and on how much personnel time is available. (With more time, a firm could define larger A or B categories.)

The purpose of classifying items into groups is to establish the appropriate degree of control over each item. On a periodic basis, for example, class A items may be more clearly controlled with weekly ordering, B items may be ordered biweekly, and C items may be ordered monthly or bimonthly. Note that the unit cost of items is not related to their classification. An A item may have a high dollar volume through a combination of either low cost and high usage or high cost and low usage. Similarly, C items may have a low dollar volume because of either low demand or low cost. In an automobile service station, gasoline would be an A item with daily or weekly replenishment; tires, batteries, oil, grease, and transmission fluid may be B items and ordered every two to four weeks; and C items would consist of valve stems, windshield wiper blades, radiator caps, hoses, fan belts, oil and gas additives, car wax, and so forth. C items may be ordered every two or three months or even be allowed to run out before reordering because the penalty for stock out is not serious.

Sometimes an item may be critical to a system if its absence creates a sizable loss. In this case, regardless of the item's classification, sufficiently large stocks should be kept on hand to prevent runoff. One way to ensure closer control is to designate this item an A or a B, forcing it into the category even if its dollar volume does not warrant such inclusion.

exhibit 11.11

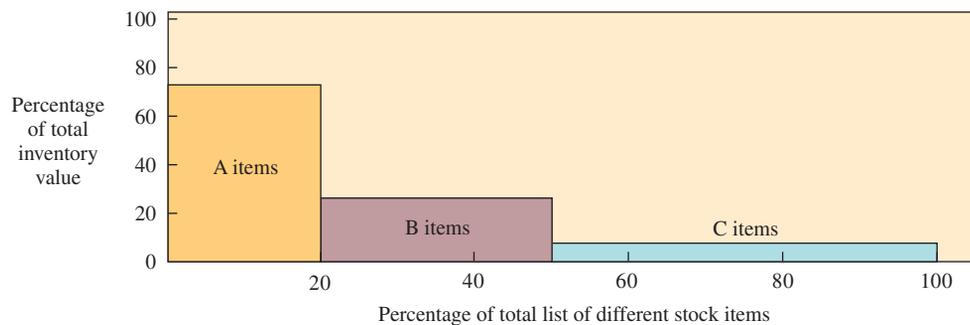
A. Annual Usage of Inventory by Value

| ITEM NUMBER | ANNUAL DOLLAR USAGE | PERCENTAGE OF TOTAL VALUE |
|-------------|---------------------|---------------------------|
| 22 | \$ 95,000 | 40.69% |
| 68 | 75,000 | 32.13 |
| 27 | 25,000 | 10.71 |
| 03 | 15,000 | 6.43 |
| 82 | 13,000 | 5.57 |
| 54 | 7,500 | 3.21 |
| 36 | 1,500 | 0.64 |
| 19 | 800 | 0.34 |
| 23 | 425 | 0.18 |
| 41 | 225 | 0.10 |
| | <u>\$233,450</u> | <u>100.00%</u> |

B. ABC Grouping of Inventory Items

| CLASSIFICATION | ITEM NUMBER | ANNUAL DOLLAR USAGE | PERCENTAGE OF TOTAL |
|----------------|--------------------|---------------------|---------------------|
| A | 22, 68 | \$170,000 | 72.9% |
| B | 27, 03, 82 | 53,000 | 22.7 |
| C | 54, 36, 19, 23, 41 | 10,450 | 4.4 |
| | | <u>\$233,450</u> | <u>100.0%</u> |

C. ABC Inventory Classification (inventory value for each group versus the group's portion of the total list)



INVENTORY ACCURACY AND CYCLE COUNTING

Inventory records usually differ from the actual physical count; inventory accuracy refers to how well the two agree. Companies such as Walmart understand the importance of inventory accuracy and expend considerable effort ensuring it. The question is, How much error is acceptable? If the record shows a balance of 683 of part X and an actual count shows 652, is this within reason? Suppose the actual count shows 750, an excess of 67 over the record; is this any better?

Every production system must have agreement, within some specified range, between what the record says is in inventory and what actually is in inventory. There are many reasons why records and inventory may not agree. For example, an open stockroom area



A SALES CLERK AT TOKYO'S MITSUKOSHI DEPARTMENT STORE READS AN RFID TAG ON JEANS TO CHECK STOCK. MITSUKOSHI AND JAPAN'S ELECTRONIC GIANT FUJITSU PARTNERED TO USE RFID TO IMPROVE STOCK CONTROL AND CUSTOMER SERVICE.

allows items to be removed for both legitimate and unauthorized purposes. The legitimate removal may have been done in a hurry and simply not recorded. Sometimes parts are misplaced, turning up months later. Parts are often stored in several locations, but records may be lost or the location recorded incorrectly. Sometimes stock replenishment orders are recorded as received, when in fact they never were. Occasionally, a group of parts is recorded as removed from inventory, but the customer order is canceled and the parts are replaced in inventory without canceling the record. To keep the production system flowing smoothly without parts shortages and efficiently without excess balances, records must be accurate.

How can a firm keep accurate, up-to-date records? Using bar codes and RFID tags is important to minimizing errors caused by inputting wrong numbers in the system. It is also important to keep the storeroom locked. If only storeroom personnel have access, and one of their measures of performance for personnel evaluation and merit increases is record accuracy, there is a strong motivation to comply. Every location of inventory storage, whether in a locked storeroom or on the production floor, should have a recordkeeping mechanism. A second way is to convey the importance of accurate records to all personnel and depend on them to assist in this effort. (This all boils down to this: Put a fence that goes all the way to the ceiling around the storage area so that workers cannot climb over to get parts; put a lock on the gate and give one person the key. Nobody can pull parts without having the transaction authorized and recorded.)

Another way to ensure accuracy is to count inventory frequently and match this against records. A widely used method is called *cycle counting*.

Cycle counting is a physical inventory-taking technique in which inventory is counted frequently rather than once or twice a year. The key to effective cycle counting and, therefore, to accurate records lies in deciding which items are to be counted, when, and by whom.

Cycle counting A physical inventory-taking technique in which inventory is counted on a frequent basis rather than once or twice a year.

Virtually all inventory systems these days are computerized. The computer can be programmed to produce a cycle count notice in the following cases:

1. When the record shows a low or zero balance on hand. (It is easier to count fewer items.)
2. When the record shows a positive balance but a backorder was written (indicating a discrepancy).
3. After some specified level of activity.
4. To signal a review based on the importance of the item (as in the ABC system) such as in the following table:

| ANNUAL DOLLAR USAGE | REVIEW PERIOD |
|---------------------|------------------|
| \$10,000 or more | 30 days or less |
| \$3,000–\$10,000 | 45 days or less |
| \$250–\$3,000 | 90 days or less |
| Less than \$250 | 180 days or less |

The easiest time for stock to be counted is when there is no activity in the stockroom or on the production floor. This means on the weekends or during the second or third shift, when the facility is less busy. If this is not possible, more careful logging and separation of items are required to count inventory while production is going on and transactions are occurring.

The counting cycle depends on the available personnel. Some firms schedule regular stockroom personnel to do the counting during lulls in the regular working day. Other companies hire private firms that come in and count inventory. Still other firms use full-time cycle counters who do nothing but count inventory and resolve differences with the records. Although this last method sounds expensive, many firms believe that it is actually less costly than the usual hectic annual inventory count generally performed during the two- or three-week annual vacation shutdown.

The question of how much error is tolerable between physical inventory and records has been much debated. Some firms strive for 100 percent accuracy, whereas others accept 1, 2, or 3 percent error. The accuracy level often recommended by experts is ± 0.2 percent for A items, ± 1 percent for B items, and ± 5 percent for C items. Regardless of the specific accuracy decided on, the important point is that the level be dependable so that safety stocks may be provided as a cushion. Accuracy is important for a smooth production process so that customer orders can be processed as scheduled and not held up because of unavailable parts.

SUMMARY

This chapter introduced the two main classes of demand: (1) independent demand, referring to the external demand for a firm's end product, and (2) dependent demand, usually referring—within the firm—to the demand for items created because of the demand for more complex items of which they are a part. Most industries have items in both classes. In manufacturing, for example, independent demand is common for finished products, service and repair parts, and operating supplies; and dependent demand is common for those parts and materials needed to produce the end product. In wholesale and retail sales of consumer goods, most demand is independent—each item is an end item, with the wholesaler or retailer doing no further assembly or fabrication.

Independent demand, the focus of this chapter, is based on statistics. In the fixed-order quantity and fixed-time period models, the influence of service level was shown on safety stock and reorder point determinations. One special-purpose model—the single-period model—was also presented.

To distinguish among item categories for analysis and control, the ABC method was offered. The importance of inventory accuracy was also noted, and cycle counting was described.

In this chapter, we also pointed out that inventory reduction requires a knowledge of the operating system. It is not simply a case of selecting an inventory model off the shelf and plugging in some numbers. In the first place, a model might not even be appropriate. In the second case, the numbers might be full of errors or even based on erroneous data. Determining order quantities is often referred to as a trade-off problem; that is, trading off holding costs for setup costs. Note that companies really want to reduce both.

The simple fact is that firms have very large investments in inventory, and the cost to carry this inventory runs from 25 to 35 percent of the inventory's worth annually. Therefore, a major goal of most firms today is to reduce inventory.

A caution is in order, though. The formulas in this chapter try to minimize cost. Bear in mind that a firm's objective should be something like "making money"—so be sure that reducing inventory cost does, in fact, support this. Usually, correctly reducing inventory lowers cost, improves quality and performance, and enhances profit.

Key Terms

ABC inventory classification Divides inventory into dollar volume categories that map into strategies appropriate for the category.

Cycle counting A physical inventory-taking technique in which inventory is counted on a frequent basis rather than once or twice a year.

Dependent demand The need for any one item is a direct result of the need for some other item, usually an item of which it is a part.

Fixed-order quantity model (or Q-model) An inventory control model where the amount requisitioned is fixed and the actual ordering is triggered by inventory dropping to a specified level of inventory.

Fixed-time period model (or P-model) An inventory control model that specifies inventory is ordered at the end of

a predetermined time period. The interval of time between orders is fixed and the order quantity varies.

Independent demand The demands for various items are unrelated to each other.

Inventory position The amount on-hand plus on-order minus backordered quantities. In the case where inventory has been allocated for special purposes, the inventory position is reduced by these allocated amounts.

Inventory The stock of any item or resource used in an organization.

Safety stock The amount of inventory carried in addition to the expected demand.

Formula Review

Single-period model. Cumulative probability of not selling the last unit. Ratio of marginal cost of underestimating demand and marginal cost of overestimating demand.

$$P \leq \frac{C_u}{C_o + C_u} \quad [11.1]$$

Q -model. Total annual cost for an order Q , a per-unit cost C , setup cost S , and per-unit holding cost H .

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H \quad [11.2]$$

Q -model. Optimal (or economic) order quantity.

$$Q_{\text{opt}} = \sqrt{\frac{2DS}{H}} \quad [11.3]$$

Q -model. Reorder point R based on average daily demand \bar{d} and lead time L in days.

$$R = \bar{d}L \quad [11.4]$$

Q -model. Reorder point providing a safety stock of $z\sigma_L$.

$$R = \bar{d}L + z\sigma_L \quad [11.5]$$

Average daily demand over a period of n days.

$$\bar{d} = \frac{\sum_{i=1}^n d_i}{n} \quad [11.6]$$

Standard deviation of demand over a period of n days.

$$\sigma_d = \sqrt{\frac{\sum_{i=1}^n (d_i - \bar{d})^2}{n}} \quad [11.7]$$

Standard deviation of a series of independent demands.

$$\sigma_s = \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_i^2} \quad [11.8]$$

Q -model. Safety stock calculation.

$$SS = z\sigma_L \quad [11.9]$$

P -model. Safety stock calculation.

$$SS = z\sigma_{T+L} \quad [11.10]$$

P -model. Optimal order quantity in a fixed-period system with a review period of T days and lead time of L days.

$$q = \bar{d}(T + L) + z\sigma_{T+L} - I \quad [11.11]$$

P -model. Standard deviation of a series of independent demands over the review period T and lead time L .

$$\sigma_{T+L} = \sqrt{\sum_{i=1}^{T+L} \sigma_{d_i}^2} \quad [11.12]$$

$$\text{Average inventory value} = (Q/2 + SS)C \quad [11.13]$$

$$\text{Inventory turn} = \frac{DC}{(Q/2 + SS)C} = \frac{D}{Q/2 + SS} \quad [11.14]$$

$$Q = \sqrt{\frac{2DS}{iC}} \quad [11.15]$$

Solved Problems

SOLVED PROBLEM 1

A product is priced to sell at \$100 per unit, and its cost is constant at \$70 per unit. Each unsold unit has a salvage value of \$20. Demand is expected to range between 35 and 40 units for the period; 35 definitely can be sold and no units over 40 will be sold. The demand probabilities and the associated cumulative probability distribution (P) for this situation are shown below.

| NUMBER OF UNITS DEMANDED | PROBABILITY OF THIS DEMAND | CUMULATIVE PROBABILITY |
|--------------------------|----------------------------|------------------------|
| 35 | 0.10 | 0.10 |
| 36 | 0.15 | 0.25 |
| 37 | 0.25 | 0.50 |
| 38 | 0.25 | 0.75 |
| 39 | 0.15 | 0.90 |
| 40 | 0.10 | 1.00 |

How many units should be ordered?

Solution

The cost of underestimating demand is the loss of profit, or $C_u = \$100 - \$70 = \$30$ per unit. The cost of overestimating demand is the loss incurred when the unit must be sold at salvage value, $C_o = \$70 - \$20 = \$50$.

The optimal probability of not being sold is

$$P \leq \frac{C_u}{C_o + C_u} = \frac{30}{50 + 30} = .375$$

From the distribution data above, this corresponds to the 37th unit.

The following is a full marginal analysis for the problem. Note that the minimum cost is when 37 units are purchased.

| UNITS DEMANDED | PROBABILITY | NUMBER OF UNITS PURCHASED | | | | | |
|----------------|-------------|---------------------------|-----|-----|-----|-----|-----|
| | | 35 | 36 | 37 | 38 | 39 | 40 |
| 35 | 0.1 | 0 | 50 | 100 | 150 | 200 | 250 |
| 36 | 0.15 | 30 | 0 | 50 | 100 | 150 | 200 |
| 37 | 0.25 | 60 | 30 | 0 | 50 | 100 | 150 |
| 38 | 0.25 | 90 | 60 | 30 | 0 | 50 | 100 |
| 39 | 0.15 | 120 | 90 | 60 | 30 | 0 | 50 |
| 40 | 0.1 | 150 | 120 | 90 | 60 | 30 | 0 |
| Total cost | | 75 | 53 | 43 | 53 | 83 | 125 |

SOLVED PROBLEM 2

Items purchased from a vendor cost \$20 each, and the forecast for next year's demand is 1,000 units. If it costs \$5 every time an order is placed for more units and the storage cost is \$4 per unit per year,

- What quantity should be ordered each time?
- What is the total ordering cost for a year?
- What is the total storage cost for a year?



Excel:
Inventory
Control

Solution

- a. The quantity to be ordered each time is

$$Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(1,000)5}{4}} = 50 \text{ units}$$

- b. The total ordering cost for a year is

$$\frac{D}{Q}S = \frac{1,000}{50}(\$5) = \$100$$

- c. The storage cost for a year is

$$\frac{Q}{2}H = \frac{50}{2}(\$4) = \$100$$

SOLVED PROBLEM 3

Daily demand for a product is 120 units, with a standard deviation of 30 units. The review period is 14 days and the lead time is 7 days. At the time of review, 130 units are in stock. If only a 1 percent risk of stocking out is acceptable, how many units should be ordered?

Solution

$$\sigma_{T+L} = \sqrt{(14 + 7)(30)^2} = \sqrt{18,900} = 137.5$$

$$z = 2.33$$

$$\begin{aligned} q &= \bar{d}(T + L) + z\sigma_{T+L} - I \\ &= 120(14 + 7) + 2.33(137.5) - 130 \\ &= 2,710 \text{ units} \end{aligned}$$

SOLVED PROBLEM 4

A company currently has 200 units of a product on hand that it orders every two weeks when the salesperson visits the premises. Demand for the product averages 20 units per day with a standard deviation of 5 units. Lead time for the product to arrive is seven days. Management has a goal of a 95 percent probability of not stocking out for this product.

The salesperson is due to come in late this afternoon when 180 units are left in stock (assuming that 20 are sold today). How many units should be ordered?

Solution

Given $I = 180$, $T = 14$, $L = 7$, $d = 20$

$$\sigma_{T+L} = \sqrt{21(5)^2} = 23$$

$$z = 1.64$$

$$\begin{aligned} q &= \bar{d}(T + L) + z\sigma_{T+L} - I \\ &= 20(14 + 7) + 1.64(23) - 180 \\ q &= 278 \text{ units} \end{aligned}$$



Excel:
Inventory
Control



Excel:
Inventory
Control

Review and Discussion Questions

- 1 Distinguish between dependent and independent demand in a McDonald's restaurant, in an integrated manufacturer of personal copiers, and in a pharmaceutical supply house.
- 2 Distinguish between in-process inventory, safety stock inventory, and seasonal inventory.
- 3 Discuss the nature of the costs that affect inventory size. For example:
 - a. How does shrinkage (stolen stock) contribute to the cost of carrying inventory? How can this cost be reduced?

- b. How does obsolescence contribute to the cost of carrying inventory? How can this cost be reduced?
- 4 Under which conditions would a plant manager elect to use a fixed-order quantity model as opposed to a fixed-time period model? What are the disadvantages of using a fixed-time period ordering system?
 - 5 What two basic questions must be answered by an inventory-control decision rule?
 - 6 Discuss the assumptions that are inherent in production setup cost, ordering cost, and carrying costs. How valid are they?
 - 7 “The nice thing about inventory models is that you can pull one off the shelf and apply it so long as your cost estimates are accurate.” Comment.
 - 8 Which type of inventory system would you use in the following situations?
 - a. Supplying your kitchen with fresh food.
 - b. Obtaining a daily newspaper.
 - c. Buying gas for your car.
 To which of these items do you impute the highest stock out cost?
 - 9 What is the purpose of classifying items into groups, as the ABC classification does?

Problems

- 1 The local supermarket buys lettuce each day to ensure really fresh produce. Each morning any lettuce that is left from the previous day is sold to a dealer that resells it to farmers who use it to feed their animals. This week the supermarket can buy fresh lettuce for \$4.00 a box. The lettuce is sold for \$10.00 a box and the dealer that sells old lettuce is willing to pay \$1.50 a box. Past history says that tomorrow’s demand for lettuce averages 250 boxes with a standard deviation of 34 boxes. How many boxes of lettuce should the supermarket purchase tomorrow?
- 2 Next week, Super Discount Airlines has a flight from New York to Los Angeles that will be booked to capacity. The airline knows from past history that an average of 25 customers (with a standard deviation of 15) cancel their reservation or do not show for the flight. Revenue from a ticket on the flight is \$125. If the flight is overbooked, the airline has a policy of getting the customer on the next available flight and giving the person a free round-trip ticket on a future flight. The cost of this free round-trip ticket averages \$250. Super Discount considers the cost of flying the plane from New York to Los Angeles a sunk cost. By how many seats should Super Discount overbook the flight?
- 3 Solve the newsvendor problem. What is the optimal order quantity?

| | | | | | | |
|-------------|-----|-----|-----|-----|-----|-----|
| Probability | 0.2 | 0.1 | 0.1 | 0.2 | 0.3 | 0.1 |
| Value | 1 | 2 | 3 | 4 | 5 | 6 |

Purchase cost $c = 15$

Selling price $p = 25$

Salvage value $v = 10$

- 4 Wholemark is an Internet order business that sells one popular New Year greeting card once a year. The cost of the paper on which the card is printed is \$0.05 per card, and the cost of printing is \$0.15 per card. The company receives \$2.15 per card sold. Since the cards have the current year printed on them, unsold cards have no salvage value. Their customers are from the four areas: Los Angeles, Santa Monica, Hollywood, and Pasadena. Based on past data, the number of customers from *each* of the four regions is normally distributed with mean 2,000 and standard deviation 500. (Assume these four are independent.) What is the optimal production quantity for the card?

- 5 Lakeside Bakery bakes fresh pies every morning. The daily demand for its apple pies is a random variable with (discrete) distribution, based on past experience, given by

| | | | | | | |
|-------------|-----|-----|-----|-----|-----|----|
| Demand | 5 | 10 | 15 | 20 | 25 | 30 |
| Probability | 10% | 20% | 25% | 25% | 15% | 5% |

Each apple pie costs the bakery \$6.75 to make and is sold for \$17.99. Unsold apple pies at the end of the day are purchased by a nearby soup kitchen for 99 cents each. Assume no goodwill cost.

- If the company decided to bake 15 apple pies each day, what would be their expected profit?
 - Based on the demand distribution above, how many apple pies should the company bake each day to maximize their expected profit?
- 6 Sally's Silk Screening produces specialty T-shirts that are primarily sold at special events. She is trying to decide how many to produce for an upcoming event. During the event, Sally can sell T-shirts for \$20 apiece. However, when the event ends, any unsold T-shirts are sold for \$4 apiece. It costs Sally \$8 to make a specialty T-shirt. Sally's estimate of demand is the following:

| DEMAND | PROBABILITY |
|--------|-------------|
| 300 | .05 |
| 400 | .10 |
| 500 | .40 |
| 600 | .30 |
| 700 | .10 |
| 800 | .05 |

- What is the service rate (or optimal fractile)?
 - How many T-shirts should she produce for the upcoming event?
- 7 You are a news vendor selling *San Pedro Times* every morning. Before you get to work, you go to the printer and buy the day's paper for \$0.25 a copy. You sell a copy of *San Pedro Times* for \$1.00. Daily demand is distributed normally with mean = 250 and standard deviation = 50. At the end of each morning, any leftover copies are worthless and they go to a recycle bin.
- How many copies of *San Pedro Times* should you buy each morning?
 - Based on *a*, what is the probability that you will run out of stock?
- 8 Ray's Satellite Emporium wishes to determine the best order size for its best-selling satellite dish (model TS111). Ray has estimated the annual demand for this model at 1,000 units. His cost to carry one unit is \$100 per year per unit, and he has estimated that each order costs \$25 to place. Using the EOQ model, how many should Ray order each time?
- 9 Dunstreet's Department Store would like to develop an inventory ordering policy of a 95 percent probability of not stocking out. To illustrate your recommended procedure, use as an example the ordering policy for white percale sheets.
- Demand for white percale sheets is 5,000 per year. The store is open 365 days per year. Every two weeks (14 days) inventory is counted and a new order is placed. It takes 10 days for the sheets to be delivered. Standard deviation of demand for the sheets is five per day. There are currently 150 sheets on hand.
- How many sheets should you order?
- 10 Charlie's Pizza orders all of its pepperoni, olives, anchovies, and mozzarella cheese to be shipped directly from Italy. An American distributor stops by every four weeks to take orders. Because the orders are shipped directly from Italy, they take three weeks to arrive.
- Charlie's Pizza uses an average of 150 pounds of pepperoni each week, with a standard deviation of 30 pounds. Charlie's prides itself on offering only the best-quality ingredients and a high level of service, so it wants to ensure a 98 percent probability of not stocking out on pepperoni.
- Assume that the sales representative just walked in the door and there are currently 500 pounds of pepperoni in the walk-in cooler. How many pounds of pepperoni would you order?

- 11 Given the following information, formulate an inventory management system. The item is demanded 50 weeks a year.

| | | | |
|-------------------------|------------------|-------------------------------------|-------------|
| Item cost | \$10.00 | Standard deviation of weekly demand | 25 per week |
| Order cost | \$250.00 | Lead time | 1 week |
| Annual holding cost (%) | 33% of item cost | Service probability | 95% |
| Annual demand | 25,750 | | |
| Average demand | 515 per week | | |

- State the order quantity and reorder point.
 - Determine the annual holding and order costs.
 - If a price break of \$50 per order was offered for purchase quantities of over 2,000, would you take advantage of it? How much would you save annually?
- 12 Lieutenant Commander Data is planning to make his monthly (every 30 days) trek to Gamma Hydra City to pick up a supply of isolinear chips. The trip will take Data about two days. Before he leaves, he calls in the order to the GHC Supply Store. He uses chips at an average rate of five per day (seven days per week) with a standard deviation of demand of one per day. He needs a 98 percent service probability. If he currently has 35 chips in inventory, how many should he order? What is the most he will ever have to order?
- 13 Jill's Job Shop buys two parts (Tegdiws and Widgets) for use in its production system from two different suppliers. The parts are needed throughout the entire 52-week year. Tegdiws are used at a relatively constant rate and are ordered whenever the remaining quantity drops to the reorder level. Widgets are ordered from a supplier who stops by every three weeks. Data for both products are as follows:

| ITEM | TEGDIW | WIDGET |
|-------------------------------|----------|---------|
| Annual demand | 10,000 | 5,000 |
| Holding cost (% of item cost) | 20% | 20% |
| Setup or order cost | \$150.00 | \$25.00 |
| Lead time | 4 weeks | 1 week |
| Safety stock | 55 units | 5 units |
| Item cost | \$10.00 | \$2.00 |

- What is the inventory control system for Tegdiws? That is, what is the reorder quantity and what is the reorder point?
 - What is the inventory control system for Widgets?
- 14 Demand for an item is 1,000 units per year. Each order placed costs \$10; the annual cost to carry items in inventory is \$2 each. In what quantities should the item be ordered?
- 15 The annual demand for a product is 15,600 units. The weekly demand is 300 units with a standard deviation of 90 units. The cost to place an order is \$31.20, and the time from ordering to receipt is four weeks. The annual inventory carrying cost is \$0.10 per unit. Find the reorder point necessary to provide a 98 percent service probability.
- Suppose the production manager is asked to reduce the safety stock of this item by 50 percent. If she does so, what will the new service probability be?
- 16 Daily demand for a product is 100 units, with a standard deviation of 25 units. The review period is 10 days and the lead time is 6 days. At the time of review there are 50 units in stock. If 98 percent service probability is desired, how many units should be ordered?
- 17 Item X is a standard item stocked in a company's inventory of component parts. Each year the firm, on a random basis, uses about 2,000 of item X, which costs \$25 each. Storage costs, which include insurance and cost of capital, amount to \$5 per unit of average inventory. Every time an order is placed for more of item X, it costs \$10.
- Whenever item X is ordered, what should the order size be?
 - What is the annual cost for ordering item X?
 - What is the annual cost for storing item X?

- 18 Annual demand for a product is 13,000 units; weekly demand is 250 units with a standard deviation of 40 units. The cost of placing an order is \$100, and the time from ordering to receipt is four weeks. The annual inventory carrying cost is \$0.65 per unit. To provide a 98 percent service probability, what must the reorder point be?

Suppose the production manager is told to reduce the safety stock of this item by 100 units. If this is done, what will the new service probability be?

- 19 In the past, Taylor Industries has used a fixed-time period inventory system that involved taking a complete inventory count of all items each month. However, increasing labor costs are forcing Taylor Industries to examine alternative ways to reduce the amount of labor involved in inventory stockrooms, yet without increasing other costs, such as shortage costs. Here is a random sample of 20 of Taylor's items.

| ITEM NUMBER | ANNUAL USAGE | ITEM NUMBER | ANNUAL USAGE |
|-------------|--------------|-------------|--------------|
| 1 | \$ 1,500 | 11 | \$13,000 |
| 2 | 12,000 | 12 | 600 |
| 3 | 2,200 | 13 | 42,000 |
| 4 | 50,000 | 14 | 9,900 |
| 5 | 9,600 | 15 | 1,200 |
| 6 | 750 | 16 | 10,200 |
| 7 | 2,000 | 17 | 4,000 |
| 8 | 11,000 | 18 | 61,000 |
| 9 | 800 | 19 | 3,500 |
| 10 | 15,000 | 20 | 2,900 |

- a. What would you recommend Taylor do to cut back its labor cost? (Illustrate using an ABC plan.)
- b. Item 15 is critical to continued operations. How would you recommend it be classified?
- 20 Gentle Ben's Bar and Restaurant uses 5,000 quart bottles of an imported wine each year. The effervescent wine costs \$3 per bottle and is served only in whole bottles because it loses its bubbles quickly. Ben figures that it costs \$10 each time an order is placed, and holding costs are 20 percent of the purchase price. It takes three weeks for an order to arrive. Weekly demand is 100 bottles (closed two weeks per year) with a standard deviation of 30 bottles.
- Ben would like to use an inventory system that minimizes inventory cost and will provide a 95 percent service probability.
- a. What is the economic quantity for Ben to order?
- b. At what inventory level should he place an order?
- 21 Retailers Warehouse (RW) is an independent supplier of household items to department stores. RW attempts to stock enough items for a 98 percent service probability.
- A stainless steel knife set is one item it stocks. Demand (2,400 sets per year) is relatively stable over the entire year. Whenever new stock is ordered, a buyer must assure that numbers are correct for stock on hand and then phone in a new order. The total cost involved to place an order is about \$5. RW figures that holding inventory in stock and paying for interest on borrowed capital, insurance, and so on, add up to about \$4 holding cost per unit per year.
- Analysis of the past data shows that the standard deviation of demand from retailers is about four units per day for a 365-day year. Lead time to get the order is seven days.
- a. What is the economic order quantity?
- b. What is the reorder point?
- 22 Daily demand for a product is 60 units with a standard deviation of 10 units. The review period is 10 days, and lead time is 2 days. At the time of review there are 100 units in stock. If 98 percent service probability is desired, how many units should be ordered?

- 23 University Drug Pharmaceuticals orders its antibiotics every two weeks (14 days) when a salesperson visits from one of the pharmaceutical companies. Tetracycline is one of its most prescribed antibiotics, with average daily demand of 2,000 capsules. The standard deviation of daily demand was derived from examining prescriptions filled over the past three months and was found to be 800 capsules. It takes five days for the order to arrive. University Drug would like to satisfy 99 percent of the prescriptions. The salesperson just arrived, and there are currently 25,000 capsules in stock.

How many capsules should be ordered?

- 24 Famous Albert prides himself on being the Cookie King of the West. Small, freshly baked cookies are the specialty of his shop. Famous Albert has asked for help to determine the number of cookies he should make each day. From an analysis of past demand, he estimates demand for cookies as

| DEMAND | PROBABILITY OF DEMAND |
|-------------|-----------------------|
| 1,800 dozen | 0.05 |
| 2,000 | 0.10 |
| 2,200 | 0.20 |
| 2,400 | 0.30 |
| 2,600 | 0.20 |
| 2,800 | 0.10 |
| 3,000 | 0.05 |

Each dozen sells for \$0.69 and costs \$0.49, which includes handling and transportation. Cookies that are not sold at the end of the day are reduced to \$0.29 and sold the following day as day-old merchandise.

- Construct a table showing the profits or losses for each possible quantity.
 - What is the optimal number of cookies to make?
 - Solve this problem by using marginal analysis.
- 25 Sarah's Muffler Shop has one standard muffler that fits a large variety of cars. Sarah wishes to establish a reorder point system to manage inventory of this standard muffler. Use the following information to determine the best order size and the reorder point:

| | | | |
|------------------------------------|----------------------------|---------------------|----------------|
| Annual demand | 3,500 mufflers | Ordering cost | \$50 per order |
| Standard deviation of daily demand | 6 mufflers per working day | Service probability | 90% |
| Item cost | \$30 per muffler | Lead time | 2 working days |
| Annual holding cost | 25% of item value | Working days | 300 per year |

- 26 Alpha Products, Inc., is having a problem trying to control inventory. There is insufficient time to devote to all its items equally. Here is a sample of some items stocked, along with the annual usage of each item expressed in dollar volume.

| ITEM | ANNUAL DOLLAR USAGE | ITEM | ANNUAL DOLLAR USAGE |
|------|---------------------|------|---------------------|
| a | \$ 7,000 | k | \$80,000 |
| b | 1,000 | l | 400 |
| c | 14,000 | m | 1,100 |
| d | 2,000 | n | 30,000 |
| e | 24,000 | o | 1,900 |
| f | 68,000 | p | 800 |
| g | 17,000 | q | 90,000 |
| h | 900 | r | 12,000 |
| i | 1,700 | s | 3,000 |
| j | 2,300 | t | 32,000 |

- a. Can you suggest a system for allocating control time?
 b. Specify where each item from the list would be placed.
- 27 After graduation, you decide to go into a partnership in an office supply store that has existed for a number of years. Walking through the store and stockrooms, you find a great discrepancy in service levels. Some spaces and bins for items are completely empty; others have supplies that are covered with dust and have obviously been there a long time. You decide to take on the project of establishing consistent levels of inventory to meet customer demands. Most of your supplies are purchased from just a few distributors that call on your store once every two weeks.

You choose, as your first item for study, computer printer paper. You examine the sales records and purchase orders and find that demand for the past 12 months was 5,000 boxes. Using your calculator you sample some days' demands and estimate that the standard deviation of daily demand is 10 boxes. You also search out these figures:

Cost per box of paper: \$11.
 Desired service probability: 98 percent.
 Store is open every day.
 Salesperson visits every two weeks.
 Delivery time following visit is three days.

- Using your procedure, how many boxes of paper would be ordered if, on the day the salesperson calls, 60 boxes are on hand?
- 28 A distributor of large appliances needs to determine the order quantities and reorder points for the various products it carries. The following data refer to a specific refrigerator in its product line:

| | |
|---|-------------------------------------|
| Cost to place an order | \$100 |
| Holding cost | 20 percent of product cost per year |
| Cost of refrigerator | \$500 each |
| Annual demand | 500 refrigerators |
| Standard deviation of demand during lead time | 10 refrigerators |
| Lead time | 7 days |

Consider an even daily demand and a 365-day year.

- a. What is the economic order quantity?
 b. If the distributor wants a 97 percent service probability, what reorder point, R , should be used?
- 29 It is your responsibility, as the new head of the automotive section of Nichols Department Store, to ensure that reorder quantities for the various items have been correctly established. You decide to test one item and choose Michelin tires, XW size 185×14 BSW. A perpetual inventory system has been used, so you examine this as well as other records and come up with the following data:

| | |
|------------------------------------|----------------------------------|
| Cost per tire | \$35 each |
| Holding cost | 20 percent of tire cost per year |
| Demand | 1,000 per year |
| Ordering cost | \$20 per order |
| Standard deviation of daily demand | 3 tires |
| Delivery lead time | 4 days |

Because customers generally do not wait for tires but go elsewhere, you decide on a service probability of 98 percent. Assume the demand occurs 365 days per year.

a. Determine the order quantity.

b. Determine the reorder point.

- 30 UA Hamburger Hamlet (UAHH) places a daily order for its high-volume items (hamburger patties, buns, milk, and so on). UAHH counts its current inventory on hand once per day and phones in its order for delivery 24 hours later. Determine the number of hamburgers UAHH should order for the following conditions:

| | |
|------------------------------|-----|
| Average daily demand | 600 |
| Standard deviation of demand | 100 |
| Desired service probability | 99% |
| Hamburger inventory | 800 |

- 31 DAT, Inc., produces digital audiotapes to be used in the consumer audio division. DAT lacks sufficient personnel in its inventory supply section to closely control each item stocked, so it has asked you to determine an ABC classification. Here is a sample from the inventory records:

| ITEM | AVERAGE MONTHLY DEMAND | PRICE PER UNIT | ITEM | AVERAGE MONTHLY DEMAND | PRICE PER UNIT |
|------|------------------------|----------------|------|------------------------|----------------|
| 1 | 700 | \$6.00 | 6 | 100 | \$10.00 |
| 2 | 200 | 4.00 | 7 | 3,000 | 2.00 |
| 3 | 2,000 | 12.00 | 8 | 2,500 | 1.00 |
| 4 | 1,100 | 20.00 | 9 | 500 | 10.00 |
| 5 | 4,000 | 21.00 | 10 | 1,000 | 2.00 |

Develop an ABC classification for these 10 items.

- 32 A local service station is open 7 days per week, 365 days per year. Sales of 10W40 grade premium oil average 20 cans per day. Inventory holding costs are \$0.50 per can per year. Ordering costs are \$10 per order. Lead time is two weeks. Backorders are not practical—the motorist drives away.
- a. Based on these data, choose the appropriate inventory model and calculate the economic order quantity and reorder point. Describe in a sentence how the plan would work. Hint: Assume demand is deterministic.
- b. The boss is concerned about this model because demand really varies. The standard deviation of demand was determined from a data sample to be 6.15 cans per day. The manager wants a 99.5 percent service probability. Determine a new inventory plan based on this information and the data in a. Use Q_{opt} from a.
- 33 Dave's Auto Supply custom mixes paint for its customers. The shop performs a weekly inventory count of the main colors that are used for mixing paint. Determine the amount of white paint that should be ordered using the following information:

| | |
|------------------------------|----------------|
| Average weekly demand | 20 gallons |
| Standard deviation of demand | 5 gallons/week |
| Desired service probability | 98% |
| Current inventory | 25 gallons |
| Lead time | 1 week |

- 34 A particular raw material is available to a company at three different prices, depending on the size of the order:

| | |
|----------------------------|----------------|
| Less than 100 pounds | \$20 per pound |
| 100 pounds to 1,000 pounds | \$19 per pound |
| More than 1,000 pounds | \$18 per pound |

The cost to place an order is \$40. Annual demand is 3,000 units. Holding (or carrying) cost is 25 percent of the material price.

What is the economic order quantity to buy each time?

- 35 CU, Incorporated, (CUI) produces copper contacts that it uses in switches and relays. CUI needs to determine the order quantity, Q , to meet the annual demand at the lowest cost. The price of copper depends on the quantity ordered. Here are price-break and other data for the problem:

| | |
|-----------------|--|
| Price of copper | \$0.82 per pound up to 2,499 pounds |
| | \$0.81 per pound for orders between 2,500 and 5,000 pounds |
| | \$0.80 per pound for orders greater than 5,000 pounds |
| Annual demand | 50,000 pounds per year |
| Holding cost | 20 percent per unit per year of the price of the copper |
| Ordering cost | \$30 |

Which quantity should be ordered?

- 36 SY Manufacturers (SYM) is producing T-shirts in three colors: red, blue, and white. The monthly demand for each color is 3,000 units. Each shirt requires 0.5 pound of raw cotton that is imported from the Luft-Geshfet-Textile (LGT) Company in Brazil. The purchasing price per pound is \$2.50 (paid only when the cotton arrives at SYM's facilities) and transportation cost by sea is \$0.20 per pound. The traveling time from LGT's facility in Brazil to the SYM facility in the United States is two weeks. The cost of placing a cotton order, by SYM, is \$100 and the annual interest rate that SYM is facing is 20 percent.
- What is the optimal order quantity of cotton?
 - How frequently should the company order cotton?
 - Assuming that the first order is needed on April 1, when should SYM place the order?
 - How many orders will SYM place during the next year?
 - What is the resulting annual holding cost?
 - What is the resulting annual ordering cost?
 - If the annual interest cost is only 5 percent, how will it affect the annual number of orders, the optimal batch size, and the average inventory? (You are not expected to provide a numerical answer to this question. Just describe the direction of the change and explain your answer.)
- 37 Demand for a book at Amazon.com is 250 units per week. The product is supplied to the retailer from a factory. The factory pays \$10 per unit, while the total cost of a shipment from the factory to the retailer when the shipment size is Q is given by

$$\text{Shipment cost} = \$50 + 2Q$$

Assume the annual inventory carrying cost is 20 percent.

- What is the cost per shipment and annual holding cost per book?
- What is the optimal shipment size?
- What is the average throughput time?

- 38 Palin's Muffler Shop has one standard muffler that fits a large variety of cars. The shop wishes to establish a *periodic review* system to manage inventory of this standard muffler. Use the information in the following table to determine the optimal inventory target level (or order-up-to level).

| | | | |
|------------------------------------|----------------------------|---------------------|----------------|
| Annual demand | 3,000 mufflers | Ordering cost | \$50 per order |
| Standard deviation of daily demand | 6 mufflers per working day | Service probability | 90% |
| Item cost | \$30 per muffler | Lead time | 2 working days |
| Annual holding cost | 25% of item value | Working days | 300 per year |
| Review period | 15 working days | | |

- a. What is the optimal target level (order-up-to level)?
 - b. If the service probability requirement is 95 percent, the optimal target level (your answer in a) will (select one):
 - I. Increase.
 - II. Decrease.
 - III. Stay the same.
- 39 Daily demand for a certain product is normally distributed with a mean of 100 and a standard deviation of 15. The supplier is reliable and maintains a constant lead time of 5 days. The cost of placing an order is \$10 and the cost of holding inventory is \$0.50 per unit per year. There are no stock-out costs, and unfilled orders are filled as soon as the order arrives. Assume sales occur over 360 days of the year. Your goal here is to find the order quantity and reorder point to satisfy a 90 percent probability of not stocking out during the lead time.
- a. To manage inventory, the company is using (select one)
 - b. Find the order quantity.
 - c. Find the reorder point.

Analytics Exercise: Inventory Management at Big10Sweaters.com

Big10Sweaters.com is a new company started last year by two recent college graduates. The idea behind the company was simple. They will sell premium logo sweaters for Big Ten colleges with one major, unique feature. This unique feature is a special large monogram that has the customer's name, major, and year of graduation. The sweater is the perfect gift for graduating students and alumni, particularly avid football fans who want to show support during the football season. The company is off to a great start and had a successful first year while selling to only a few schools. This year they plan to expand to a few more schools and target the entire Big Ten Conference within three years.

You have been hired by Big10Sweaters.com and need to make a good impression by making good supply chain decisions. This is your big opportunity with a startup. There are only two people in the firm and you were hired with the prospect of possibly becoming a principal in the future. You majored in supply chain (operations) management in school and had a great internship at a big retailer that was getting

into Internet sales. The experience was great, but now you are on your own and have none of the great support that the big company had. You need to find and analyze your own data and make some big decisions. Of course, Rhonda and Steve, the partners who started the company, are knowledgeable about this venture and they are going to help along the way.

Rhonda had the idea to start the company two years ago and talked her friend from business school, Steve, to join her. Rhonda is into web marketing, has a degree in computer science, and has been working on completing an online MBA. She is as much an artist as a techie. She can really make the website sing.

Steve majored in accounting and likes to pump the numbers. He has done a great job of keeping the books and selling the company to some small venture capital people in the area. Last year, he was successful in getting them to invest \$2,000,000 in the company (a onetime investment). There were some significant strings attached to this investment in

that it stipulated that only \$100,000 per year could go toward paying the salary of the two principals. The rest had to be spent on the website, advertising, and inventory. In addition, the venture capital company gets 25 percent of the company profits, before taxes, during the first four years of operation, assuming the company makes a profit.

Your first job is to focus on the firm's inventory. The company is centered on selling the premium sweaters to college football fans through a website. Your analysis is important since a significant portion of the company's assets is the inventory that it carries.

The business is cyclic, and sales are concentrated during the period leading up to the college football season, which runs between late August and the end of each year. For the upcoming season, the firm wants to sell sweaters to only a few of the largest schools in the Midwest region of the United States. In particular, they are targeting The Ohio State University (OSU), the University of Michigan (UM), Michigan State University (MSU), Purdue University (PU), and Indiana University (IU). These five schools have major football programs and a loyal fan base.

The firm has considered the idea of making the sweaters in their own factory, but for now they purchase them from a supplier in China. The prices are great, but service is a problem since the supplier has a 20-week lead time for each order and the minimum order size is 5,000 sweaters. The order can consist of a mix of the different logos, such as 2,000 for OSU, 1,500 for UM, 750 for MSU, 500 for PU, and 250 for IU. Within each logo sub lot, sizes are allocated based on percentages and the supplier suggests 20 percent X-large, 50 percent large, 20 percent medium, and 10 percent small based on their historical data.

Once an order is received, a local subcontractor applies the monograms and ships the sweaters to the customer. They store the inventory of sweaters for the company in a small warehouse area located at the subcontractor.

This is the company's second year of operation. Last year they only sold sweaters for three of the schools, OSU, MU, and PU. They ordered the minimum 5,000 sweaters and sold all of them, but the experience was painful since they had too many MU sweaters and not enough for OSU fans. Last year they ordered 2,300 OSU, 1,800 MU, and 900 PU sweaters. Of the 5,000 sweaters, 342 had to be sold at a steep discount on eBay after the season. They were hoping not to do this again.

For the next year, you have collected some data relevant to the decision. Exhibit 11.12 shows cost information for the product when purchased from the supplier in China. Here we see that the cost for each sweater, delivered to the warehouse of our monogramming subcontractor, is \$60.88. This price is valid for any quantity that we order above 5,000 sweaters. This order can be a mix of sweaters for each of the five schools we are targeting. The supplier needs 20 weeks to process the order, so the order needs to be placed around April 1 for the upcoming football season.

Our monogramming subcontractor gets \$13 for each sweater. Shipping cost is paid by the customer when the order is placed.

In addition to the cost data, you also have some demand information, as shown in Exhibit 11.13. The exact sales numbers for last year are given. The exhibit indicates the retail or "full price" sales for the sweaters. Sweaters that we had at the end of the season were sold through eBay for \$50 each and were not monogrammed. Keep in mind that the retail sales

exhibit 11.12

Cost Information for the Big Ten Sweaters

| | | |
|-----------------------------|---------|---------|
| China supplier cost | | |
| Material | \$32.00 | |
| Labor | 10.50 | |
| Overhead | 1.25 | |
| Transportation within China | 1.00 | |
| Supplier profit | 8.95 | |
| Agent's fee | 2.68 | |
| Freight (ocean carrier) | 1.50 | |
| Duty, insurance, etc. | 3.00 | |
| Total supplier cost | | \$60.88 |
| Domestic subcontractor cost | | |
| Monogram Material | \$5.00 | |
| Labor | 8.00 | |
| Total subcontractor cost | | \$13.00 |
| Total (per sweater) | | \$73.88 |

Forecast Data for the Big Ten Sweaters

exhibit 11.13

| | AVERAGE FOOTBALL GAME ATTENDANCE | LAST YEAR'S ACTUAL SALES (FULL PRICE) | RHONDA'S FORECAST FOR NEXT YEAR | STEVE'S FORECAST FOR NEXT YEAR | MARKET RESEARCH FORECAST FOR NEXT YEAR | AVERAGE FORECAST | STANDARD DEVIATION |
|----------------|--|---|---------------------------------------|--------------------------------------|---|---------------------|-----------------------|
| Ohio State | 105,261 | 2,300 | 2,500 | 2,200 | 2,800 | 2,500 | 300 |
| Michigan | 108,933 | 1,468 | 1,800 | 1,500 | 2,000 | 1,767 | 252 |
| Purdue | 50,457 | 890 | 1,000 | 900 | 1,100 | 1,000 | 100 |
| Michigan State | 74,741 | — | 1,750 | 1,500 | 1,600 | 1,617 | 126 |
| Indiana | 41,833 | — | 600 | 500 | 450 | 517 | 76 |
| Penn State | 107,008 | | | | | | |
| Wisconsin | 80,109 | | | | | | |
| Iowa | 70,214 | | | | | | |
| Illinois | 59,545 | | | | | | |
| Minnesota | 50,805 | | | | | | |
| Northwestern | 24,190 | | | | | | |
| Nebraska | 85,071 | | | | | | |
| Total | | 4,658* | 7,650 | 6,600 | 7,950 | 7,400 | 430** |

*342 sweaters were sold through eBay for \$50 each (the customer pays shipping on all orders).

**Calculated assuming the demand at each school is independent = $\sqrt{\sum_{i=1}^N \sigma_i^2}$

numbers do not accurately reflect actual demand since they stocked out of the OSU sweaters toward the end of the season.

As for advertising the sweaters for next season, Rhonda is committed to using the same approach used last year. The firm placed ads in the football program sold at each game. These worked very well for reaching those attending the games, but she realized there may be ways to advertise that may open sales to more alumni. She has hired a market research firm to help identify other advertising outlets but has decided to wait at least another year to try something different.

Forecasting demand is a major problem for the company. You have asked Rhonda and Steve to predict what they think sales might be next year. You have also asked the market research firm to apply their forecasting tools. Data on these forecasts are given in Exhibit 11.13. To generate some statistics you have averaged the forecasts and calculated the standard deviation for each school and in total.

Based on advice from the market research firm, you have decided to use the aggregate demand forecast and standard deviation for the aggregate demand. The aggregate demand was calculated by adding the average forecast for each item. The aggregate standard deviation was calculated by squaring the standard deviation for each item (this is the variance), summing the variance for each item, and then taking the square root of this sum. This assumes that the demand for each school is independent, meaning that the demand for Ohio State is totally unrelated to the demand at Michigan and the other schools.

You will allocate your aggregate order to the individual schools based on their expected percentage of total demand. You discussed your analysis with Rhonda and Steve and

they are OK with your analysis. They would like to see what the order quantities would be if each school was considered individually.

You have a spreadsheet set up with all the data from the exhibits called Big10Sweater.xls and you are ready to do some calculations.

Questions

- 1 You are curious as to how much Rhonda and Steve made in their business last year. You do not have all the data, but you know that most of their expenses relate to buying the sweaters and having them monogrammed. You know they paid themselves \$50,000 each and you know the rent, utilities, insurance, and a benefit package for the business was about \$20,000. About how much do you think they made “before taxes” last year? If they must make their payment to the venture capital firm, and then pay 50% in taxes, what was their increase in cash last year?
- 2 What was your reasoning behind using the aggregate demand forecast when determining the size of your order rather than the individual school forecasts? Should you rethink this or is there a sound basis for doing it this way?
- 3 How many sweaters should you order this year? Break down your order by individual school. Document your calculations in your spreadsheet. Calculate this based on the aggregate forecast and also the forecast by individual school.

- 4 What do you think they could make this year? They are paying you \$40,000 and you expect your benefit package addition would be about \$1,000 per year. Assume that they order based on the aggregate forecast.
- 5 How should the business be developed in the future? Be specific and consider changes related to your supplier, the monogramming subcontractor, target customers, and products.

Super Quiz

- 1 Model most appropriate for making a one-time purchase of an item.
- 2 Model most appropriate when inventory is replenished only in fixed intervals of time, for example, on the first Monday of each month.
- 3 Model most appropriate when a fixed amount must be purchased each time an order is placed.
- 4 Based on an EOQ-type ordering criterion, what cost must be taken to zero if the desire is to have an order quantity of a single unit?
- 5 Term used to describe demand that can be accurately calculated to meet the need of a production schedule, for example.
- 6 Term used to describe demand that is uncertain and needs to be forecast.
- 7 We are ordering T-shirts for the spring party and are selling them for twice what we paid for them. We expect to sell 100 shirts and the standard deviation associated with our forecast is 10 shirts. How many shirts should we order?
- 8 We have an item that we stock in our store that has fairly steady demand. Our supplier insists that we buy 1,200 units at a time. The lead time is very short on the item, since the supplier is only a few blocks away and we can pick up another 1,200 units when we run out. How many units do you expect to have in inventory on average?
- 9 For the item described in question 8, if we expect to sell approximately 15,600 units next year, how many trips will we need to make to the supplier over the year?
- 10 If we decide to carry 10 units of safety stock for the item described in questions 8 and 9, and we implemented this by going to our supplier when we had 10 units left, how much inventory would you expect to have on average now?
- 11 We are being evaluated based on the percentage of total demand met in a year (not the probability of stocking out as used in the chapter). Consider an item that we are managing using a fixed-order quantity model with safety stock. We decide to double the order quantity but leave the reorder point the same. Would you expect the percent of total demand met next year to go up or down? Why?
- 12 Consider an item for which we have 120 units currently in inventory. The average demand for the item is 60 units per week. The lead time for the item is exactly 2 weeks and we carry 16 units for safety stock. What is the probability of running out of the item if we order right now?
- 13 If we take advantage of a quantity discount, would you expect your average inventory to go up or down? Assume that the probability of stocking out criterion stays the same.
- 14 This is an inventory auditing technique where inventory levels are checked more frequently than one time a year.

1. Single-period model 2. Fixed-time period model 3. Fixed-order quantity model 4. Setup or ordering cost 5. Dependent demand 6. Independent demand 7. 100 shirts 8. 600 units 9. 13 times 10. 610 units 11. Go up (we are taking fewer chances of running out) 12. 50 percent 13. Will probably go up if the probability of stocking out stays the same 14. Cycle counting

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Footnotes

- ¹ P is actually a cumulative probability because the sale of the n th unit depends not only on exactly n being demanded but also on the demand for any number greater than n .
- ² As previously discussed, the standard deviation of a sum of independent variables equals the square root of the sum of the variances.
- ³ The Pareto principle is also widely applied in quality problems through the use of Pareto charts. (See Chapter 10.)