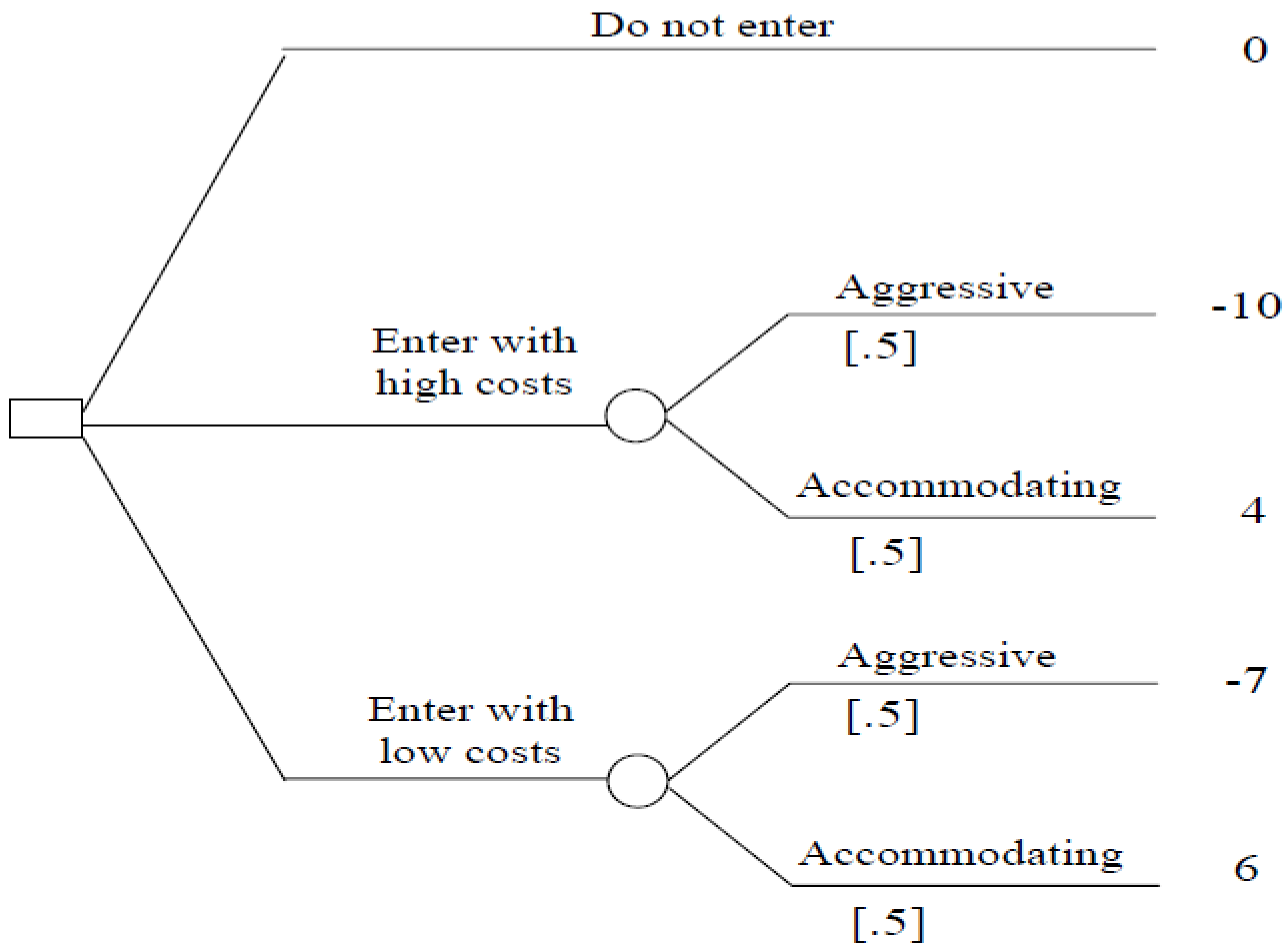
Teoria dei giochi

- Ci aiuta ad analizzare i comportamenti degli agenti economici che si trovano ad interagire tra loro.

Decision theory

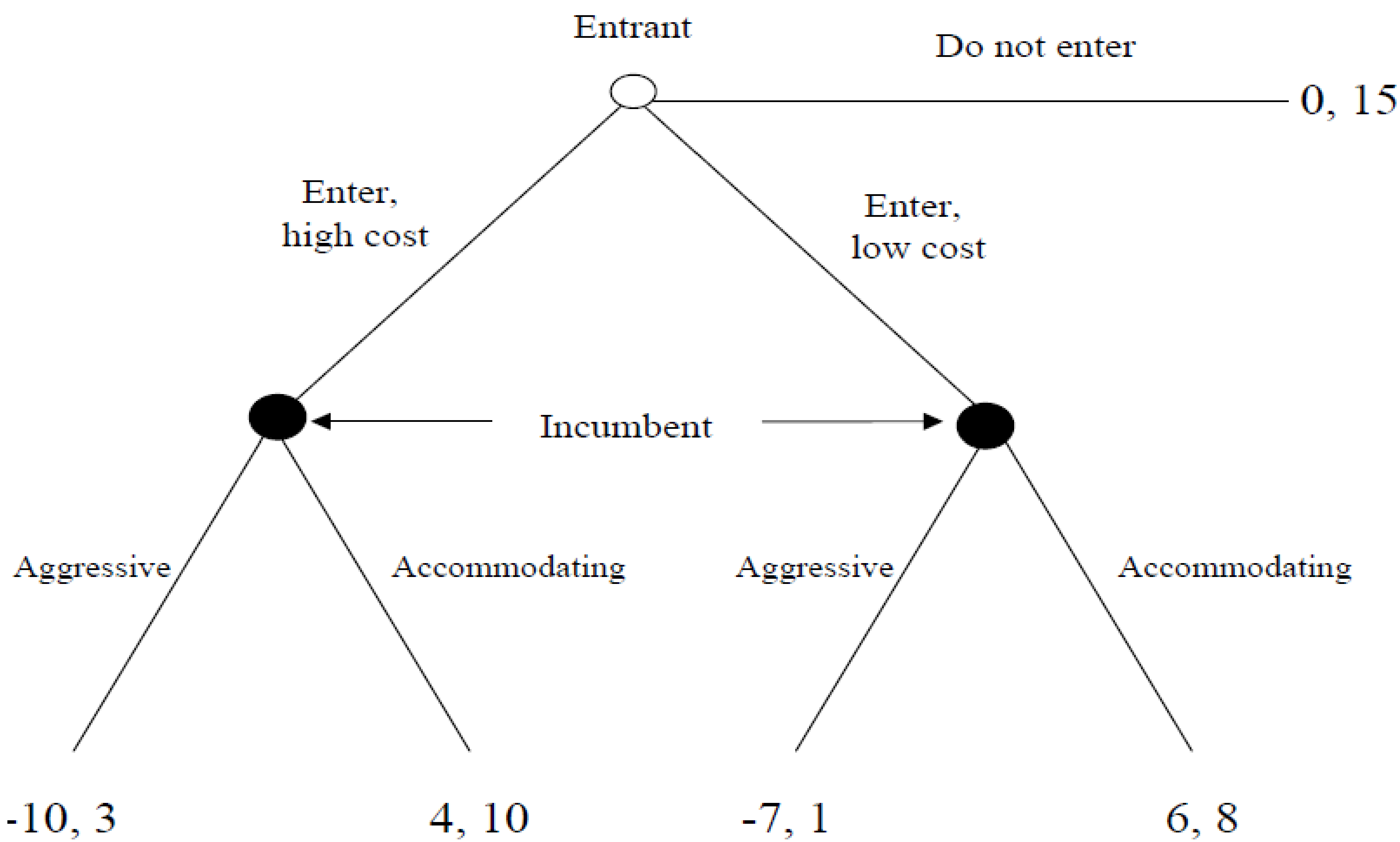


-10*0,5+4*0,5=-3 < 0

-7*0,5+6*0,5=-0,5 <0

Meglio non entrare

Game Theory



Entrant
Do not enter
0, 15
Enter,
high cost
Enter,
low cost
Incumbent
Aggressive
Accommodating
Aggressive
Accommodating
-10, 3
4, 10
-7, 1
6, 8

Game theory

- ... a collection of tools for predicting outcomes of a group of interacting agents where an action of a single agent directly affects the payoff of other participating agents.
- ... the study of multiperson decision problems. (Gibbons)
- ... a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact. (Osborne and Rubinstein)
- ... the study of mathematical models of conflict and cooperation between intelligent rational (self interested) decision-makers. (Myerson)

Game

1. The <u>players</u> who are involved.
2. The <u>rules</u> of the game that specify the sequence of moves as well as the possible actions and information available to each player whenever they move. (<u>strategies</u>)
3. The <u>outcome</u> of the game for each possible set of actions.
4. The (expected) payoffs based on the outcome.

Different games

- Non cooperative
- Cooperative
- Game with complete information
- Game with incomplete information (auction/ sealed bid – you don't know how valuable is a good for other bidders)
- Game with perfect information (chess - bargaining)
- Game with imperfect information
- Zero (costant) sum game (divide a pie)
- Non zero sum game
- Static game
- Dynamic game

Nash equlibrium

- Un insieme di strategie, una per giocatore, è un Nash equilibrio se, data la strategia scelta dagli altri, nessuno ha interesse a cambiare la propria.

Definition 6 *A* **Nash Equilibrium (NE)** *is a profile of strategies* (s_i^*, s_{-i}^*) *such that each player's strategy is an optimal response to the other players' strategies:*

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*)$$

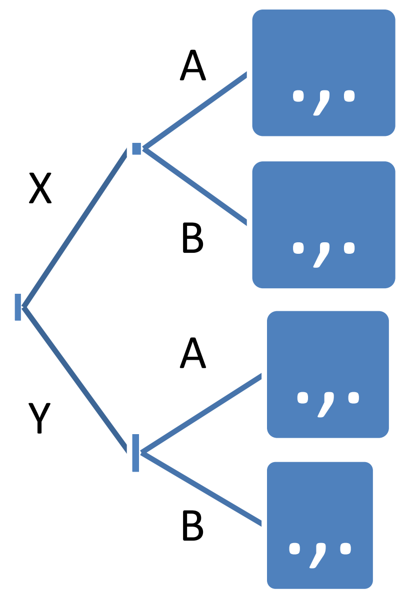
Static game – complete information (prisoner's dilemma)

		Prisoner 2	
		Cooperate (C)	Defect (D)
Prisoner 1	C	-1, -1	-9, 0
	D	0, -9	-6, -6

Normal form

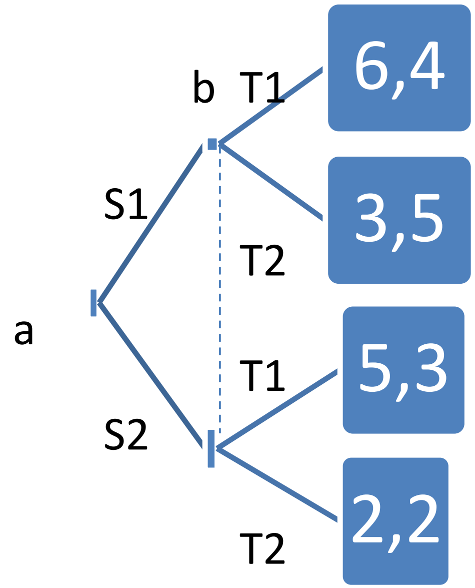
	A	B
X	.,.	.,.
Y	.,.	.,.

Game tree



Static game – complete and imperfect information

a\b	T1	T2
S1	6,4	3,5
S2	5,3	2,2



Equilibrium?

S1T1	NO given a S1	b – T2	
S1T2	NE given b T2	a – S1 , given a S1	b – T2
S2T1	NO given b T1	a – S1	
S2T2	NO given a S2	b – T1	

Dominated stategies

a\b	T1	T2	T3
S1	3,4	0,4	4,-2
S2	4,2	1,1	-1,1

Step 1 a don't have dominated strategies
Step 2 B – T3 is dominated (T1 always better)
Step 3 Without T3 for a S1 is a dominated strategy
NE S2T1

Battle of sexs

M\F	S	O
S	5,2	1,1
O	0,0	2,5

We have 2 NE – we need another criterium to decide

Mixed strategies

a\b	L	R
A	0,0	0,-1
B	1,0	-1,3

No NE in pure strategies

a\b	(β) L	(1-β) R
(α) A	0,0	0,-1
(1-α)B	1,0	-1,3

NE in Mixed strategies

$E\pi a(\alpha^*,\beta^*) \geq E\pi a(\alpha,\beta^*)$

$E\pi b(\alpha^*,\beta^*) \geq E\pi b(\alpha^*,\beta)$

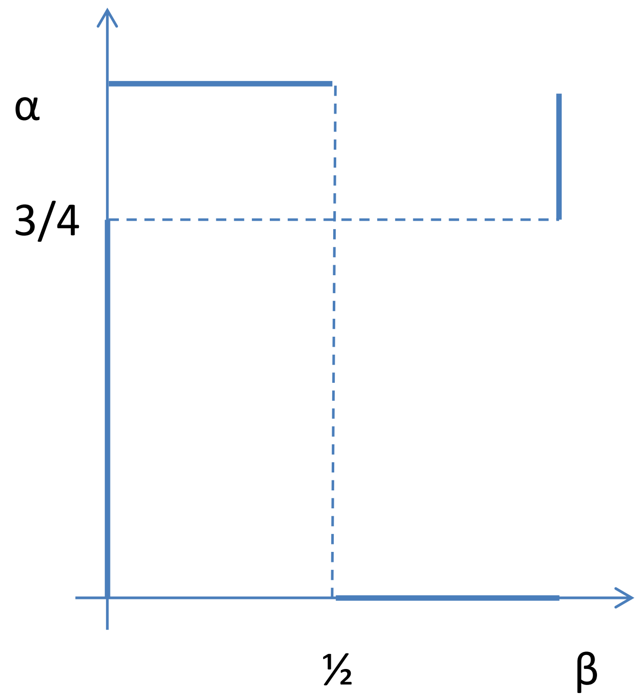
solution

a – suppose b plays L prob. β and R prob (1- β)

Eπa(A) = 0x β + 0 (1- β) = 0
Eπa(B) = 1x β + (-1) (1- β) = 2β -1
When is a indifferent?
Eπa(A)= Eπa(B) => 0=2β -1 => β=1/2
If β>1/2 a plays B if β<1/2 a plays A

The same for B
b – suppose a plays A prob. α and B prob. (1- α)

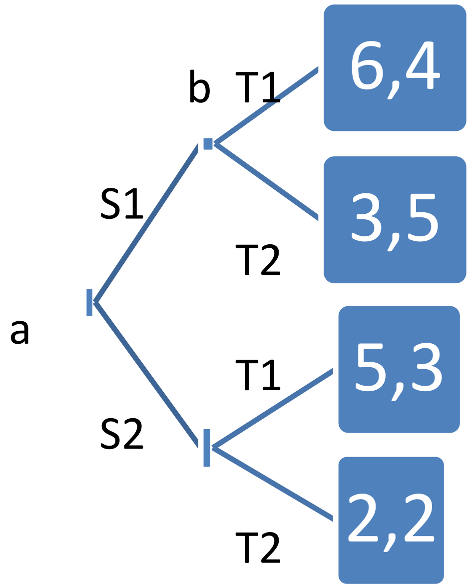
Eπb(L) = 0x α + 0 (1- α) = 0
Eπb(R) = (-1)x α + 3(1- α) = 3-4 α
When is a indifferent?
Eπb(L)= Eπb(R) => 0= 3-4 α => α=3/4
If α >3/4 b plays L if α <3/4 a plays R



NE in mix strategies
a => (A prob ¾, B prob ¼)
b=> (L prob ½, R prob ½)

dynamic game – complete and perfect information

a\b	T1	T2
S1	6,4	3,5
S2	5,3	2,2

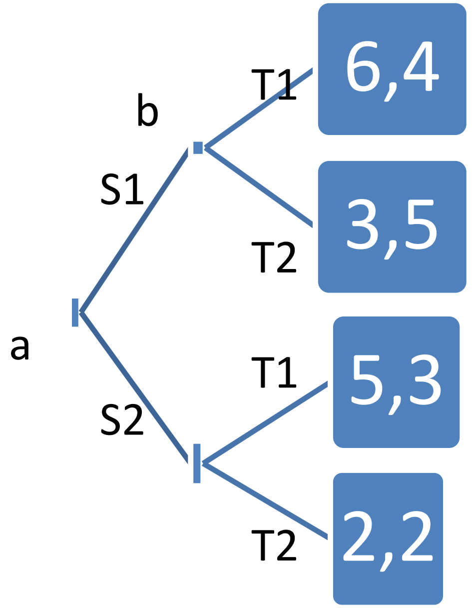


Dynamic game with complete and perfect information

a first move

b has 4 strategies

a\b	T1T1	T1T2	T2T1	T2T2
S1	6,4	6,4	3,5	3,5
S2	5,3	2,2	5,3	2,2



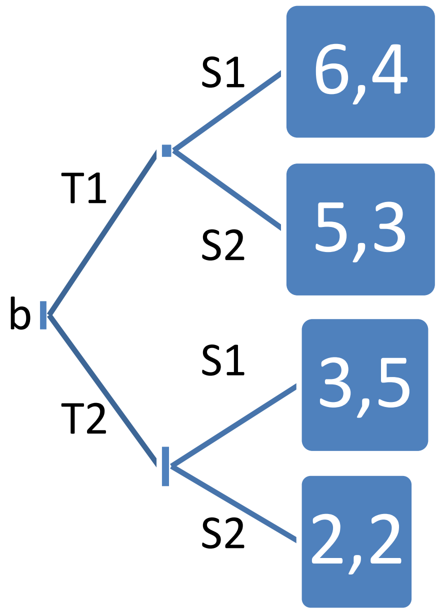
(S1, T2T2) NE no sub game perfect

(S2, T2T1) NE SGP

If B first

a\b	T1	T2
S1,S1	6,4	3,5
S1,S2	6,4	2,2
S2,S1	5,3	3,5
S2,S2	5,3	2,2

NE
(T2, S1S1) SGP
(T1, S1S2) no SGP
(T2, S2S1) no SGP



Game with incomplete information (static)

A new CEO has been hired. He can be good or bad

There is CFO close to retirement and is tired, he prefers not to work hard

But if CEO detect him he doesn't get the annual bonus

CEO good means higher profits and lower cost effort to control CFO

CEO = A CFO = B

we have 2 games

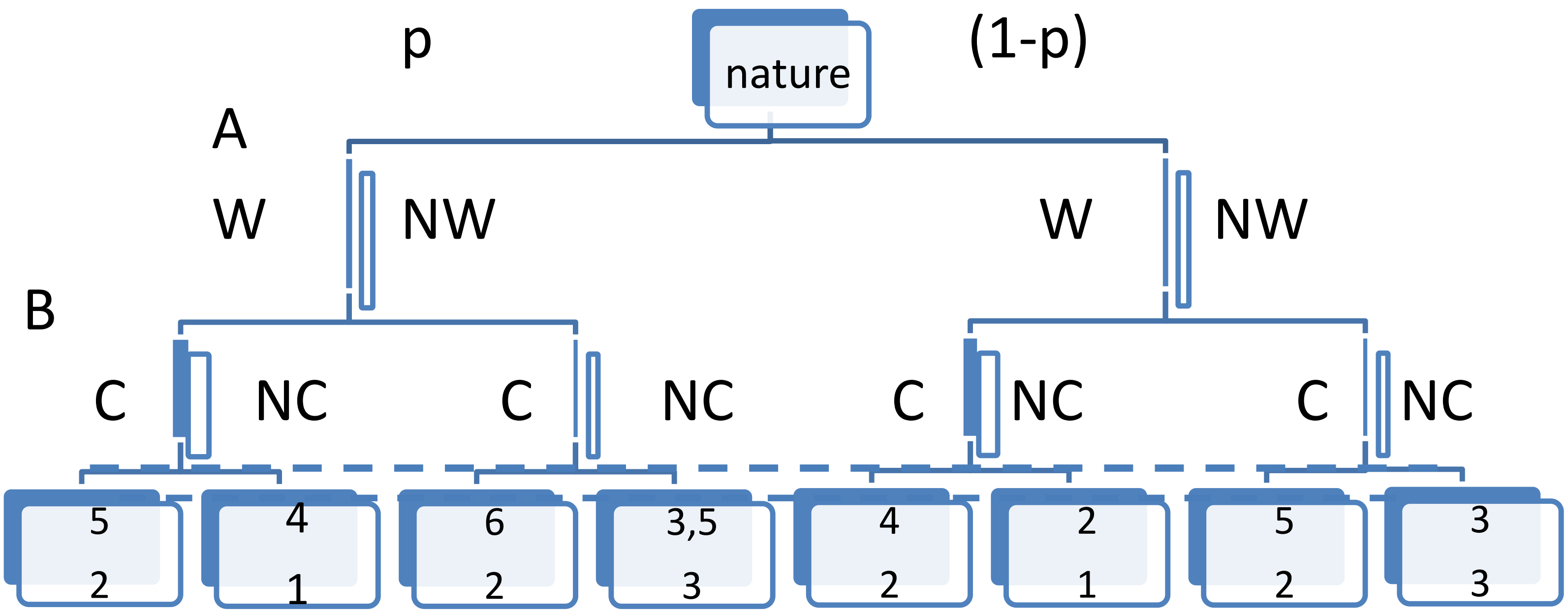
Good

	W	NW
C	5;2	4;1
NC	6;2	3,5;3

Bad

	W	NW
C	4,2	2,1
NC	5,2	3,3

- How can I find an Equlibrium?
- I change the Game in one with complete but imperfect information



E payoff B (W,p) = p(2) + (1-p)2=2

E payoff B(NW,p) =(1-p)3 + p(1)= 3-2p

2=3-2p => p=1\2 if p<1/2 low probability to be detected =>[NW,(C,NC)] B doesn't work and A control if is Good

if P>1/2 [W,(NC,NC)] B work and A doesn't control (Bayesian NE)

Repeated Prisoner's dilemma

a\b	Lb	Hb
La	10;10	1;11
Ha	11;1	3;3

NE (Ha;Hb)
Max profit (La,Lb)
How to increase profit?
Change game

a\b	Lb	Hb	R
La	10;10	1;11	0,0
Ha	11;1	3;3	1,0
R	0,0	0,1	0,0

R is a dominated strategy
Play 2 times
First NE (H both games)
Other: first period play L and second H if you in the first L otherwise R)
Pay off if no deviations (10+3)
If deviation (11+1) non convinient
But not SGP, not credible

What if…

- We repeat the game T times?
- Nothing Change because in period T we will end up

a\b	Lb	Hb	R
La	10;10	1;11	0,0
Ha	11;1	3;3	1,0
R	0,0	0,1	0,0

- Where Ha,Hb is the only equilibrium will be in T-1
- It is not a credible threat

What if…

- We play ∞ times?
- No last period, no backward!
- Suppose we set a strategy: we play La if you play Lb but if you one play Hb we will punish you with R
- So

a 10,10,10, 1,0,0,0,0,0

b 10,10,10,11,1,1,1,1,1

To have an equilibrium you have compare what you gain from deviating today and what you loose from tomorrow onward.

Is it to “hard” ? It could be also Ha so both get b that has an outcome of 3 that is lower than 10

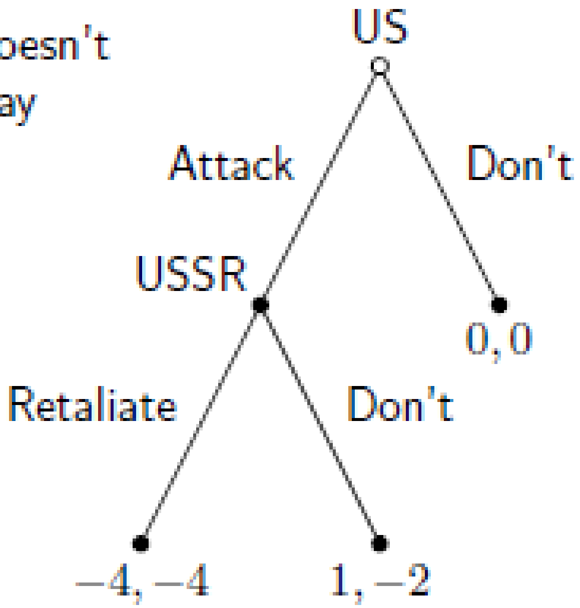
But it is not renegotiation prof.

Commitment

Doomsday

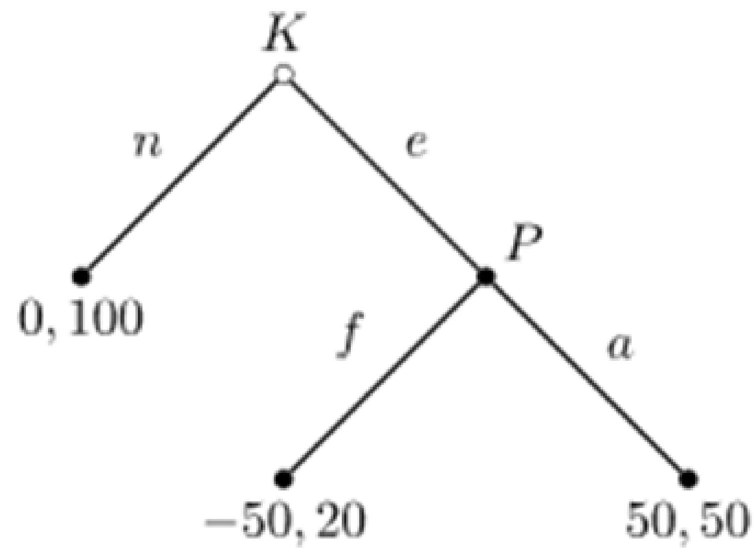
Dr. Strangelove

- What is the outcome if the U.S. doesn't know the existence of the doomsday device?
- What is the outcome if it does?
- Commitment must be observable
- What if USSR can un-trigger the device?
- Commitment must be irreversible

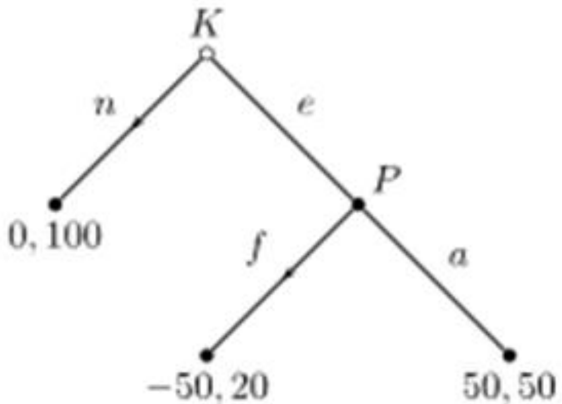


Entry Game

Consider the following entry game



Commitment



- Backward induction equilibrium (e, a)
  - Polaroid's payoff is 50
- Suppose Polaroid commits to fight (f) if entry occurs.
- What would Kodak do?

- Kodak would not enter and Polaroid would be better off
- Is this commitment credible?

Credible Commitments: Burning Bridges

- In non-strategic environments having more options is never worse
- Not so in strategic environments
- You can change your opponent's actions by removing some of your options
- 1066: William the Conqueror ordered his soldiers to burn their ships after landing to prevent his men from retreating
- 1519: Hernan Cortes sank his ships after landing in Mexico for the same reason

Sun-tzu in *The Art of War*, 400 BC

At the critical moment, the leader of an army acts like one who has climbed up a height, and then kicks away the ladder behind him.

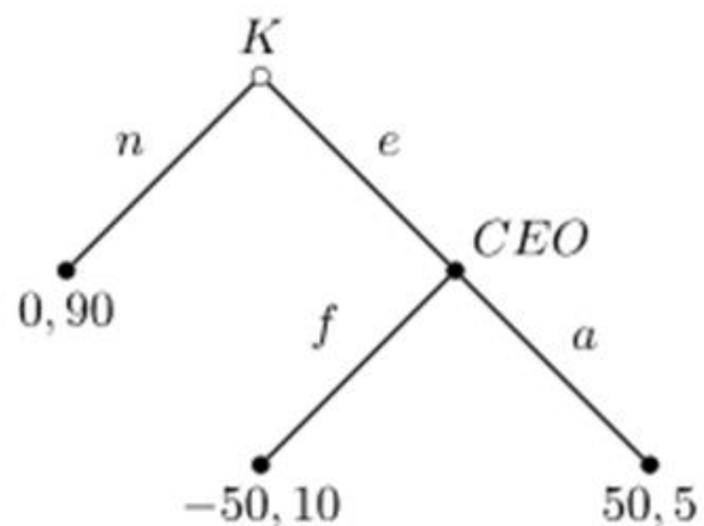
Commitment

How can you achieve credible commitment in the entry game?

- Change the game
  - Change payoffs
  - Remove accommodate as an option
- Change others' beliefs about your payoffs or options
  - Build reputation for toughness
  - Delegate to a tough CEO

Delegation

- Delegate the decision to a CEO who is known to be aggressive
- Or tie his compensation to market share
  - Pay 10 if market share is above 50%
  - Otherwise pay 5



- What is the BIE?
- Contract must be observable, non-renegotiable

Primo assignment

Portate un esempio in ambito economic/business di:

1. Dilemma del prigioniero
2. Battle of sexs
3. Game with incomplete information
4. Entry game

Consegna via email entro 23:59 del 6 ottobre