Game Theory in Business Applications

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Contents

What is business all about? Some will say it is all about providing a service that the customers like. Some will say it is all about innovation, developing cutting-edge technology. Some will say it is all about being first to the market. Some will say it is all about reaching the right people. Some will say it is all about building value for customers and managing risk. Some will say it is all about partnerships. Some will say it is all about being ahead of your competitors. The overarching theme in all of these statements is "interactions." In any business, interactions with customers, suppliers, other business partners, and competitors, as well as interactions across people and different organizations within the firm, play an integral role in any decision and its consequences. Advances in information technology (IT) and e-commerce further enrich and broaden these interactions, by increasing the degree of connectivity between different parties involved in commerce. Thanks to globalization, now the entire world is the playground for many firms, increasing the complexity of these interactions.

Given that each firm is part of a complex web of interactions, any business decision or action taken by a firm impacts multiple entities that interact with or within that firm, and vice versa. Ignoring these interactions could lead to unexpected and potentially very undesirable outcomes. Our goal in this chapter is to provide an introduction to game theory, which is a very useful tool for studying interactive decision-making, where the outcome for each participant or "player" depends on the actions of others. Each decision maker, such as yourself, is a player in the game of business; hence, when making a decision or choosing a strategy you must take into account the potential choices of others, keeping in mind that while making their choices, other players are likely to think about and take into account your strategy as well.

Most firms certainly consider other players' actions, particularly competitors', while choosing their own. Let us look at the following example which illustrates how competitors' choices impact a firm's decisions.

> "Advanced Micro Devices (AMD) has slashed prices of its desktop and mobile Athlon processors just days after a similar move by rival Intel. 'We're going to do what it takes to stay competitive' on prices, said an AMD representative. [...] AMD's aggressive price-chopping means the company doesn't want to give up market share gains, even at the cost of losses on the bottom line, analysts said." (*CNet News.com*, May 30, 2002.) [36]

In this example, the companies compete on price in order to gain market share. Interestingly, the product under question is not a commodity, it is highly specialized, requiring a significant amount of innovation. As a result of price cuts, during the first quarter of 2002, AMD increased processor shipments from the fourth quarter of 2001, topping 8 million, but processor revenue declined by 3%. In effect, the company sold more chips for less money than in the fourth quarter. Competing companies who go into such price wars do rarely, if ever, benefit from such competition. Clearly, rather than engaging in mutual price cuts, both Intel and AMD would have done better if they kept their prices higher. Cutting prices slightly might increase the overall market potential, i.e., the "pie" might get bigger. But decreasing the prices beyond a certain limit has a diminishing impact on the market potential. Hence, eventually the size of the pie does not increase anymore and firms have to fight even harder to get a bigger portion of the pie by slashing prices, and profits. Why do firms behave this way? In this situation, and in many others, firms are caught in what is known as the "prisoner's dilemma," discussed in Section 3.

Example 1 *Prisoner's dilemma*

Two criminal accomplices are arrested and interrogated separately. Each suspect can either confess with a hope of a lighter sentence (defect) or refuse to talk (cooperate). The police does not have sufficient information to convict the suspects, unless at least one of them confesses. If they cooperate, then both will be convicted to minor offense and sentenced to a month in jail. If both defect, then both will be sentenced to jail for six months. If one confesses and the other does not, then the confessor will be released immediately but the other will be sentenced to nine months in jail. The police explains these outcomes to both suspects and tells each one that the other suspect knows the deal as well. Each suspect must choose his action without knowing what the other will do.

A close look at the outcomes of different choices available to the suspects reveals that regardless of what one suspect chooses, the other suspect is better off by choosing to defect. Hence, both suspects choose to defect and stay in jail for six months, opting for a clearly less desirable outcome than only a month in jail, which would be the case if both chose to cooperate.

There are many examples of the prisoner's dilemma in business. Given such cut-throat competition, how can companies stay in business and make money? One key concept not captured in the prisoner's dilemma example is the repetition of interactions (Section 4.1). In most business dealings, the players know that they will be in the same "game" for a long time to come, and hence they may choose to cooperate or act nice, especially if cooperation today would increase the chances of cooperation tomorrow. With their repeated actions, companies build a reputation which influences the actions of others in the future. For example, a restaurant might make a higher profit today by selling leftover food from yesterday, but the consequences of such an action could be very costly as it could mean losing many customers in the future due to bad reputation about the freshness of the food.

Companies act strategically in their relations not only with their competitors but with their supply chain partners as well. Given that each participant in a supply chain acts on self interest, the individual choices of the participants collectively do not usually lead to an "optimal" outcome for the supply chain. That is, the total profits of a typical "decentralized" supply chain which is composed of multiple, independently managed companies, is less than the total profits of the "centralized" version of the same chain, if such a chain could exist and be managed optimally by a single decision-maker to maximize the entire chain's profits. The inefficiencies in supply chains due to the participants' self-centered decision-making is generally known as "double marginalization." One possible strategy for reducing such inefficiencies is "vertical integration," where a company owns every part of its supply chain, including the raw materials, factories, and stores. An excellent example of vertical integration was Ford Motor Co. early in the 20th century. In addition to automobile factories, Henry Ford owned a steel mill, a glass factory, a rubber tree plantation, and an iron mine. Ford's focus was on "mass production," making the same car, at that time Model T, cheaper and faster. This approach worked very well at the beginning. The price of Model T fell from \$850 in 1908 to \$290 in 1924. By 1914, Ford had a 48% share of the American market, and by 1920 Ford was producing half the cars made worldwide. Vertical integration allows a company to obtain raw materials at a low cost, and exert more control over the entire supply chain, both in terms of lead times and quality. However, we do not see many examples of vertically integrated companies today. Why? Mainly because in today's fast paced economy, where customers' needs and tastes change overnight, companies which focus on core competencies and are nimble are more likely to stay ahead of competition and succeed. Hence, we see an increasing trend towards "virtual integration," where supply chains are composed of independently managed but tightly linked companies. Innovative practices, such as information sharing or vendor managed inventory (VMI), are successfully used by some companies such as Dell Corporation to get closer to virtual integration [17]. However, most companies are still reluctant to changing their supply chain practices, and in such cases it is desirable to design contracts (defining the terms of trade) or change the terms of existing contracts, to align incentives and reduce inefficiencies due to double marginalization. This is known as "supply chain coordination" and is discussed in Section 6. Similar concepts apply to independently managed divisions within a company as well.

In many business "games" the actions of some players have direct consequences for other players. For example, the performance of a company, and in turn, the value to the shareholders, depends on the actions of the managers and workers that are part of the company. In recent years, we have seen examples where top management engaged in misconduct to increase their own compensation while hurting both the shareholders and the workers.

> "While demanding that workers accept cuts, [chief executive of American Airlines] was giving retention bonuses to seven executives if they stayed through January 2005. American also made a $41 million pretax payment last October to a trust fund established to protect the pensions of 45 executives in the event of bankruptcy. This is an industry that is steadily eroding the pensions of ordinary employees. (*The Arizona Republic*, April 24, 2003) [38]

Since most shareholders are not directly involved in the operations of a company, it is important that compensation schemes align the interests of the workers and managers with those of the shareholders. This situation falls into the framework of "principal-agent" problems[1] (Section 8). The principal (shareholders) cannot directly control or monitor the actions of the agent (managers and workers), but can do so through incentives. Many business situations fall into the principal-agent framework, such as the relationships between auto manufacturers and dealers, prospective home buyers and real estate agents, franchisers and franchisees, and auctioneers and bidders.

> Airlines received billions of dollars in federal loan guarantees after the terrorist attacks. This produces what economists call moral hazard[2]. Executives realize they are

[1] Principal-agent problems fall under the more general topic of information economics, which deals with situations where there is lack of information on the part of some market participants, such as what others know, or what others are doing.

[2] Authors' note: Moral hazard occurs when the agent takes an action that affects his utility as well as the principal's in a setting where the principal can only observe the outcome, but not the action of the agent. Agent's action is not necessarily to the principal's best interest (Salanié [33]).

> ultimately using other people's money, and their accountability shrinks. The result is an industry in crisis." (*The Arizona Republic*, April 24, 2003) [38]

Game theory improves strategic decision-making by providing valuable insights into the interactions of multiple self-interested agents and therefore it is increasingly being used in business and economics.

> "Scheduling interviews for [Wharton's] 800 grads each spring has become so complex that Degnan has hired a game theory optimization expert to sort it all out." (*Business Week*, May 30, 2003) [23]

> "Even the federal government is using game theory. The Federal Communications Commission (FCC) recently turned to game theorists to help design an auction for the next generation of paging services. [...] The auction's results were better than expected." (*The Wall Street Journal*, October 12, 1994) [10]

> "When companies implement major strategies, there is always uncertainty about how competitors and potential new market entrants will react. We have developed approaches based on game theory to help clients better predict and shape these competitive dynamics. These approaches can be applied to a variety of strategic issues, including pricing, market entry and exit, capacity management, mergers and acquisitions, and R&D strategy. We sometimes use competitive 'war games' to help client executives 'live' in their competitors' shoes and develop new insights into market dynamics." (www.mckinsey.com)

In Section 1 we discuss the difference between game theory and decision analysis, another methodology which allows us to model other player's decisions. In Section 2 we provide the basic building blocks of a game, followed by static and dynamic games of complete information in Sections 3 and 4. The application of complete information games in general market settings and supply chains are discussed in Sections 5 and 6. We continue with games of incomplete information in Section 7 and their applications within the principal-agent framework in Section 8. We conclude in Section 9.

The content of this chapter is based on material from Tirole [39], Fudenberg and Tirole [21], Gibbons [22], Mas-Colell, Whinston, and Green [28], Dixit and Skeath [18], Salanié [33], Cachon [12], and Tsay, Nahmias, and Agrawal [40]. For a compact and rigorous analytical treatment of game theory, we refer the readers to Cachon and Netessine [14]. For an in-depth analysis, any of the above books on game theory would be a good start.

1 Another Perspective: Game Trees

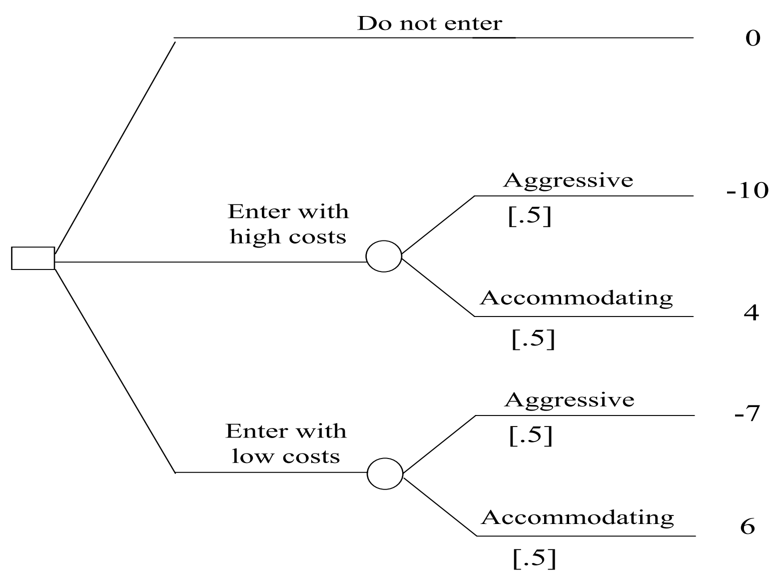
What makes game theory different than other analytical tools such as decision trees or optimization? Most analytical tools either take other parties' actions as given, or try to model or predict them. Hence, their analysis focuses on optimizing from the perspective of one player and does not endogenize the strategic behavior of other players.[3]

Example 2 *Entry and exit decisions*

The manager of a firm is considering the possibility of entering a new market, where there is only one other firm operating. The manager's decision will be based on the profitability of the market, which in turn heavily depends on how the incumbent firm will react to the entry. The incumbent firm could be accommodating and let the entrant grab his share of the market or she could respond aggressively, meeting the entrant with a cut-throat price war. Another factor that affects the revenue stream is the investment level of the entering firm. The manager of the firm may invest to the latest technology and lower his operating costs (low cost case) or he may go ahead with the existing technology and have higher operating costs (high cost case). The manager estimates that if his firm enters the market and the incumbent reacts aggressively, the total losses will be $7 million in low cost case and $10 million in high cost case. If the incumbent accommodates, however, the firm will enjoy a profit of $6 million in low cost case and $4 million in high cost case.

One possible approach for studying this problem is "decision analysis," which requires us to assess the probabilities for the incumbent being aggressive and accommodating. Assume that in this case, the manager thinks there is an equal chance of facing an aggressive and an accommodating rival. Given the estimated probabilities, we can draw the decision tree:

[3] The discussion in this section is partially based on a Harvard Business Review article by Krishna [26].



When we look at the profits, it is easy to see that if the manager chooses to enter he should invest to the latest technology. But still with a simple analysis, we see that it does not make sense to enter the new market, as in expectation, the company loses \$0.5 million. Can we conclude that the firm should not enter this market? What if the probabilities were not equal, but the probability of finding an accommodating rival were 0.55? The point is, the manager's decision is very much dependent on the probabilities that he assessed for the incumbent's behavior.

As an alternative approach, we can use game theory. The best outcome for the incumbent is when she is the only one in the market (Section 5). In this case, she would make a profit of, say \$15 million. If she chooses to be accommodating, her profits would be \$10 if the entrant enters with the existing technology, i.e., high cost case, and \$8 million if he enters with the latest technology, i.e., low cost case. If she chooses to be aggressive, her profits would be \$3 and \$1 million, respectively. Using the new information, we can draw a new tree, a *game* tree (Figure 1).

We can solve this problem by folding the tree backwards. If the firm were to enter, the best strategy for the incumbent is to accommodate. Knowing that this would be the case, entering this new market would be worthwhile for the entrant.

The idea here is simple: We assume that the incumbent is rational and interested in maximizing her profits. As Krishna [26] states "This simple example illustrates what is probably the key feature of game theory: the assumption that all actors in a strategic situation behave as rationally

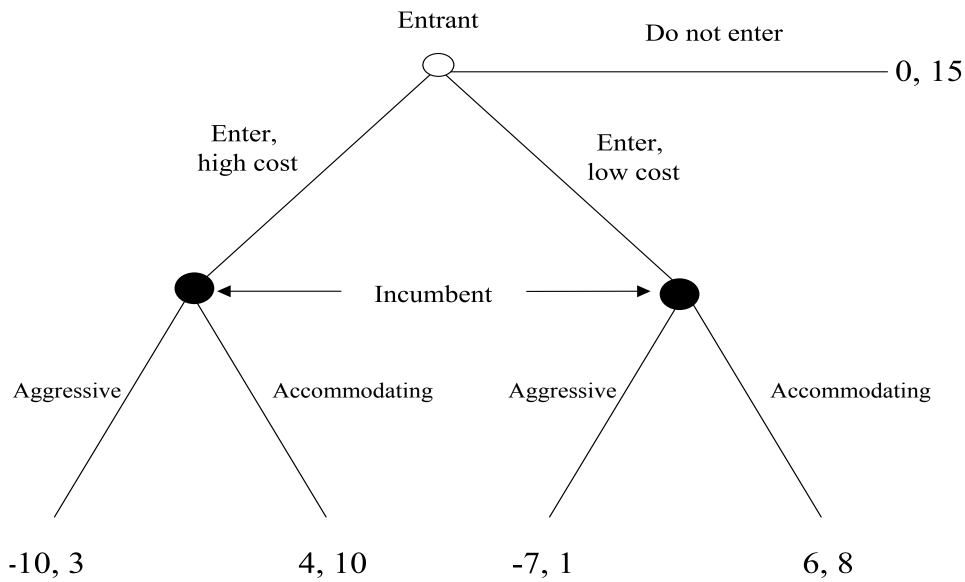


Figure 1: Game tree for the entry-exit model.

as you did. In other words, while thinking strategically, there is no reason to ascribe irrational behavior to your rivals."

2 Games of Strategy

We discussed various business scenarios where game theory could be useful in analyzing and understanding the involved parties' decisions and actions. But we did not formally define game theory so far. What is game theory anyway? There are several different answers to this question. Game theory is ...

- ... a collection of tools for predicting outcomes of a group of interacting agents where an action of a single agent directly affects the payoff of other participating agents.
- ... the study of multiperson decision problems. (Gibbons [22])
- ... a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact. (Osborne and Rubinstein [32])
- ... the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. (Myerson [31])

Game theory studies interactive decision-making and our objective in this chapter is to understand the impact of interactions of multiple players and the resulting dynamics in a market environment. There are two key assumptions that we will make throughout the chapter:

1. Each player in the market acts on self-interest. They pursue well-defined exogenous objectives; i.e., they are *rational*. They understand and seek to maximize their own payoff functions.

2. In choosing a plan of action (strategy), a player considers the potential responses/reactions of other players. She takes into account her knowledge or expectations of other decision makers' behavior; i.e., she reasons strategically.

These two assumptions rule out games of pure chance such as lotteries and slot machines where strategies do not matter and games without strategic interaction between players, such as Solitaire.

A game describes a strategic interaction between the players, where the outcome for each player depends upon the collective actions of all players involved. In order to describe a situation of strategic interaction, we need to know:

1. The players who are involved.

2. The rules of the game that specify the sequence of moves as well as the possible actions and information available to each player whenever they move.

3. The outcome of the game for each possible set of actions.

4. The (expected) payoffs based on the outcome.

Each player's objective is to maximize his or her own payoff function. In most games, we will assume that the players are **risk neutral**, i.e., they maximize expected payoffs. As an example, a risk neutral person is indifferent between \$25 for certain or a 25% chance of earning \$100 and a 75% chance of earning \$0.

An important issue is whether all participants have the same information about the game, and any external factors that might affect the payoffs or outcomes of the game. We assume that the rules of the game are **common knowledge**. That is, each player knows the rules of the game, say X, and that all players know X, that all players know that all players know X, that all players know that all players know that all players know X and so on, . . ., ad infinitum. In a game of **complete information** the players' payoff functions are also common knowledge. In a game of **incomplete information** at least one player is uncertain about another player's payoff function. For example, a sealed bid art auction is a game of incomplete information because a bidder does not know how much other bidders value the item on sale, i.e., the bidders' payoff functions are not common knowledge.

Some business situations do not allow for a recourse, at least in the short term. The decisions are made once and for all without a chance to observe other players' actions. Sealed-bid art auctions are classical examples of this case. Bids are submitted in sealed envelopes and the participants do not have any information about their competitors' choices while they make their own. In the game theory jargon these situations are called **static** (or **simultaneous move**) games. In contrast, **dynamic or (sequential move)** games have multiple stages, such as in chess or bargaining. Even though static games are also called simultaneous move games, this does not necessarily refer to the actual sequence of the moves but rather to the information available to the players while they make their choices. For example, in a sealed-bid auction that runs for a week, the participants may submit their bids on any day during that week, resulting in a certain sequence in terms of bid submission dates and times. However, such an auction is still a simultaneous move game because the firms do not know each others' actions when they choose their own.

Finally, in game theory, one should think of **strategy** as a complete action plan for all possible ways the game can proceed. For example, a player could give this strategy to his lawyer, or write a computer code, such that the lawyer or the computer would know exactly what to play in all possible scenarios every time the player moves.

3 Static Games of Complete Information

As in the case of art auctions, most business situations can be modelled by incomplete information games. However, to introduce basic concepts and building blocks for analyzing more complex games, we start our discussion with static games of complete information in this section.

We can represent a static game of complete information with three elements: the set of players, the set of strategies, and the payoff functions for each player for each possible outcome (for every possible combination of strategies chosen by all players playing the game). Note that in static

		Prisoner 2	
		Cooperate (C)	Defect (D)
Prisoner 1	C	-1, -1	-9, 0
	D	0, -9	-6, -6

Figure 2: Prisoner's dilemma in normal form.

games, actions and (pure, i.e., deterministic) strategies coincide as the players move once and simultaneously. We call a game represented with these three elements a game in **normal form**.

Example 1 *Prisoner's dilemma (continued)*

The prisoner's dilemma game is a static game, because the players simultaneously choose a strategy (without knowing the other player's strategy) only once, and then the game is over. There are two possible actions for each player: cooperate or defect. The payoffs for every possible combination of actions by the players is common knowledge, hence, this is a game of complete information. Figure 2 shows the normal form representation of this game.

Having defined the elements of the game, can we identify how each player will behave in this game, and the game outcome? A closer look at the payoffs reveals that regardless of the strategy of player 1, player 2 is better off (receives a higher payoff) by choosing to defect. Similarly, regardless of the strategy of player 2, player 1 is better off by choosing to defect. Therefore, no matter what his opponent chooses, "defect" maximizes a player's payoff.

Definition 3 *A strategy s_i is called a* **dominant strategy** *for player i, if no matter what the other players choose, playing s_i maximizes player i's payoff. A strategy s_i is called a* **strictly dominated strategy** *for player i, if there exists another strategy $\hat{s}_i$ such that no matter what the other players choose, playing $\hat{s}_i$ gives player i a higher payoff than playing s_i.*

In the prisoner's dilemma game, "defect" is a dominant strategy and "cooperate" is a strictly dominated strategy for both players.

Dominant or dominated strategies do not always exist in a game. However, when they do, they greatly simplify the choices of the players and the determination of the outcome. A rational player will never choose to play a strictly dominated strategy, hence, such strategies can be eliminated from consideration. In the prisoner's dilemma game, by eliminating the action "cooperate", which is a strictly dominated action, from the possible choices of both players, we find that (D, D) is the only possible outcome of this game. Alternatively, a strictly dominant strategy, if it exists for a player, will be the only choice of that player.

In the prisoner's dilemma game, the outcome (D,D) makes both players worse off than the outcome (C,C).

Definition 4 *An outcome A* **Pareto dominates** *an outcome B, if at least one player in the game is better off in A and no other player is worse off.*

One might find the outcome of the prisoner's dilemma game very strange, or counterintuitive. The players act such that the game results in a Pareto dominated outcome (and the players end up in jail for six months), whereas they would both be better off by cooperating and staying only one month in jail.

> " 'It's a prisoner's dilemma,' says Carl Steidtmann, chief retail economist with PricewaterhouseCoopers. 'It's the same problem that OPEC has. You benefit if you cheat a little bit, but if everyone cheats a little bit, everyone gets hurt.' " (*The Street.com*, November 22, 2000) [24]

Now let us modify the payoffs of the prisoner's dilemma game so that if one of the suspects confesses and the other does not, then the confessor will be released immediately but the other will be sentenced to 3 months in jail (Figure 3).

In this modified game, the players no longer have a dominated strategy. If prisoner 1 cooperates, the best response for prisoner 2 is to defect. If prisoner 1 defects, however, prisoner 2 is better off by cooperating.

Definition 5 *The strategy s_i which maximizes a player's payoff, given a set of strategies s_{-i} chosen by the other players, is called the best response of player i. Formally, in an N player*

Prisoner 1 \ Prisoner 2	Cooperate (C)	Defect (D)
C	-1, -1	-3, 0
D	0, -3	-6, -6

Figure 3: Modified prisoner's dilemma in normal form. In this modified game, there are no dominant or dominated strategies.

game, the **best response** *function of player i is such that*

$$R_i(s_{-i}) = argmax \;\; \pi_i\left(s_i, s_{-i}\right)$$

where $\pi_i\left(s_i, s_{-i}\right)$ *is the payoff function of player i for the strategy profile* $\left(s_i, s_{-i}\right)$.

In the modified prisoner's dilemma game, we cannot predict the final outcome of the game by simply eliminating the strictly dominated strategies as before (since there are no dominated strategies), hence, we need a stronger solution concept:

> "The theory constructs a notion of 'equilibrium,' to which the complex chain of thinking about thinking could converge. Then the strategies of all players would be mutually consistent in the sense that each would be choosing his or her best response to the choices of the others. For such a theory to be useful, the equilibrium it posits should exist. Nash used novel mathematical techniques to prove the existence of equilibrium in a very general class of games. This paved the way for applications." (*The Wall Street Journal*, October 12, 1994) [10]

Definition 6 *A* **Nash Equilibrium (NE)** *is a profile of strategies* $\left(s_i^*, s_{-i}^*\right)$ *such that each player's strategy is an optimal response to the other players' strategies:*

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*)$$

In a Nash equilibrium, no player will have an incentive to deviate from his strategy. Gibbons [22] calls this a *strategically stable* or *self-enforcing* outcome. He also argues that "... if a convention is to develop about how to play a given game then the strategies prescribed by the convention must be a Nash equilibrium, else at least one player will not abide by the convention." Therefore, a strategic interaction has an outcome only if a Nash equilibrium exist. That is why the following theorem is very important.

Theorem 7 *(Nash 1950) Every finite normal form game*[4] *has at least one Nash equilibrium.*

Example 8 *Matching pennies*

Consider a game where two players choose either Head or Tail simultaneously. If the choices differ, player 1 pays player 2 a dollar; if they are the same, player 2 pays player 1 a dollar. The normal form representation of this game is as follows:

		Player 2	
		Head (H)	Tail (T)
Player 1	H	1, -1	-1, 1
	T	-1, 1	1, -1

This is a **zero-sum** game where the interests of the players are diametrically opposed. When we look at the best responses of the players, it seems like the game does not have a Nash equilibrium outcome. Regardless of what actions the players choose, in the final outcome at least one player will have an incentive to "deviate" from that outcome. For example, if they play (H,H), then player 2 has an incentive to deviate and switch to T. But if they play (H,T), then player 1 has an incentive to switch to T, and so on. Theorem 7 tells us that every finite normal form game has at least one Nash equilibrium. The reason we could not find one in this game is because we

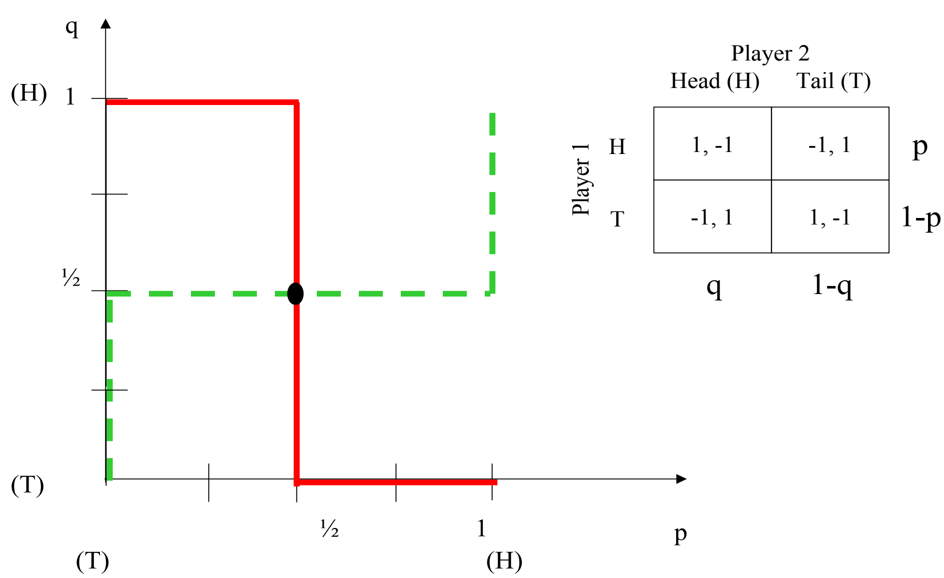
[4] A normal form game is finite if it has finite number of players and each player has a finite set of strategies.

limited ourselves to pure strategies so far, i.e., we considered only those strategies where each player chooses a single action.

The players' decisions in this game are very similar to that of a penalty kicker's and a goalkeeper's in soccer who have to choose between right and left. If the penalty kicker has a reputation of kicking to the goalkeeper's right or left (i.e., adopts a pure strategy) he will fare poorly as the opponent will anticipate and counter the action. In games like these, a player may want to consciously randomize his strategies, so that his action is unpredictable.

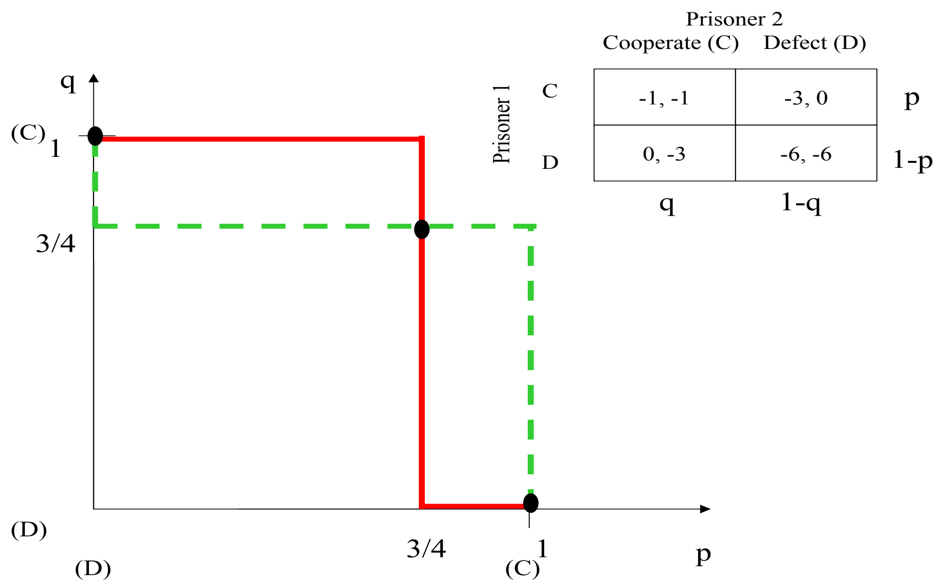
Definition 9 *A* **mixed strategy** σ *is a probability distribution over the set of pure strategies.*

Assume that player 2 plays H with probability q and T with probability $1-q$. Player 1's payoff if he plays H is $q-(1-q)=2q-1$. His payoff if he plays T is $-q+(1-q)=1-2q$. Player 1 plays H if $q>\frac{1}{2}$ and T if $q<\frac{1}{2}$. When $q=\frac{1}{2}$, he is indifferent between playing H or T. A similar argument applies to player 2. Assume that player 1 plays H with probability p and T with probability $1-p$. Player 2's payoff if she plays H is $-p+(1-p)=1-2p$. Her payoff if she plays T is $p-(1-p)=2p-1$. Player 2 plays T if $p>\frac{1}{2}$ and H if $p<\frac{1}{2}$. When $p=\frac{1}{2}$, she is indifferent between playing H or T. We can now use these best response functions to find all Nash equilibria of this game graphically. The best response function of player 1 is given by the dashed line; that of player 2 is given by the solid line. The Nash equilibrium is the point where the best response functions intersect.



Notice that in the mixed strategy Nash equilibrium, the players mix their strategies such that the opponent is indifferent between playing H and T.

Before we complete this section, we revisit the modified prisoner's dilemma game and find its Nash equilibria:



The game has two pure and one mixed Nash equilibria: (D, C), (C, D), and $\left(\left(\frac{3}{4}\text{C}, \frac{1}{4}\text{D}\right), \left(\frac{3}{4}\text{C}, \frac{1}{4}\text{D}\right)\right)$. Therefore, if we go back to Gibbons' convention concept "there may be games where game theory does not provide a unique solution and no convention will develop. In such games, Nash equilibrium loses much of its appeal as a prediction of play." (Gibbons [22])

4 Dynamic Games of Complete Information

In the previous section, we discussed simultaneous move games. However, most of the real-life situations do not require "simultaneous" moves. In contrast, most business games are played in multiple stages where players react to other players' earlier moves:

> "Dell Computer is known for ruthlessly driving down PC prices, but competitors are working hard this week to catch up with the worldwide market leader. Possibly sparked by a 10 percent price cut on Dell corporate desktops earlier in the week,

> Compaq and Hewlett-Packard have fought back with sizable cuts of their own. [...] HP announced Thursday that it would cut prices on corporate desktop PCs by as much as 28 percent. Compaq Computer reduced PC prices by as much as 31 percent Tuesday." (*CNet News.com News*, May 3, 2001) [35]

An extensive form game contains the information about the set of players, the order of moves, payoffs, the set of actions and the information available to the players when they move, and the probability distributions over any exogenous events.

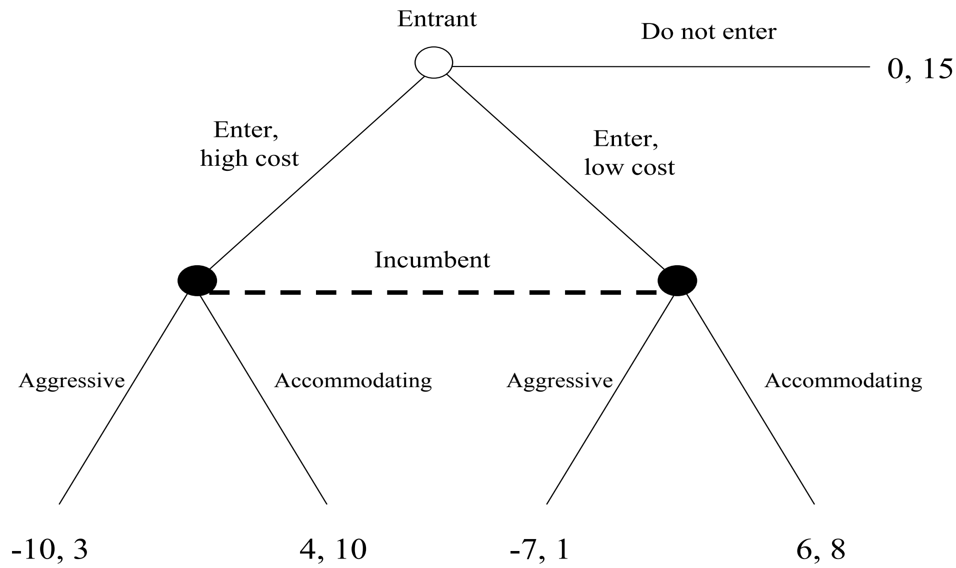
Entry-exit game revisited

The entry-exit game is a dynamic game of complete information. This is a game of complete information because the players know the payoffs of all players for every possible outcome.

There are two stages. In the first stage, the entrant has three possible actions, enter with low cost, enter with high cost, or do not enter. The action set of the incumbent is empty in the first stage, or equivalently, his only available action is "do nothing." Hence, only the entrant makes a move in the first stage. In the second stage, the action set of the incumbent contains two actions, be aggressive or accommodate. The entrant's only action in the second stage is "do nothing." In the second stage, the incumbent makes a decision fully knowing the history of the game at that stage, i.e., knowing the action chosen by the entrant in the first stage.

A multistage game has **perfect information** if (i) for every stage and history, exactly one player has a nontrivial action set, and all other players have one-element action set "do nothing" and (ii) each player knows all previous moves when making a decision. In a **finite game of perfect information**, the number of stages and the number of actions at any stage are finite. It is easy to see that the entry-exit game is a finite game of perfect information, since there are only two stages, only one player moves at each stage, and the players know the complete history of the game (previous actions of other players) at every stage of the game.

Now let us consider a slight modification of the entry-exit game. Different from the original game, suppose that the incumbent cannot observe the investment level of the entrant (if he chooses to enter), hence, the production costs. How can we represent this game with a game tree?



Notice that in the original game when it is the incumbent's turn to move, she is able to observe the entrant's moves. In the modified game, however, this is not the case; the incumbent cannot observe the investment level of the entrant. The extensive form of the game is very similar to that of the original game (Figure 1) except that we have drawn dashed lines around incumbent's decision nodes to indicate that these nodes are in a single **information set**. This means that when it is the incumbent's turn to play, she cannot tell which of these nodes she is at because she has not observed the entrant's investment level. Due to the definition of information sets, at every node within a given information set, a player must have the same set of possible actions. Note that a game is one of perfect information if each information set contains a single decision node.

It is easy to see that the outcome of the original entry-exit game is for the entrant to enter the market with the latest technology (low cost case) and the incumbent to accommodate, i.e., (Enter with low cost, Accommodate). However, this game has another pure strategy Nash equilibrium: (Do not enter, Aggressive). This profile is a Nash equilibrium because given that the entrant does not enter, the incumbent's information set is not reached and she loses nothing by being aggressive. But this equilibrium is not "credible" because it is an "empty threat" by the incumbent to play aggressive. (We know that if the entrant chooses to enter, the best response of the incumbent is to accommodate.) Credibility is a central issue in all dynamic games:

> “Coca-Cola is developing a vanilla-flavored version of its best-selling flagship cola, a report says, extending the company’s palette of flavorings from Cherry Coke and Diet Coke with lemon. [...] But don’t expect to see a vanilla-flavored Pepsi anytime soon. ‘It’s not something we’re looking at,’ said spokesman Larry Jabbonsky of Pepsi. ‘We think it’s a bit vanilla.’ ” (*USA Today*, April 01, 2002) [1]

> “PepsiCo [...] is launching Pepsi Vanilla and its diet version in stores across the country this weekend. [...] Coke came out with Vanilla Coke in May 2002 and it was a resounding success, selling 90 million cases, according to trade publication Beverage Digest. [...] ‘We’re a little surprised that Pepsi decided to enter the vanilla segment,’ said Mart Martin, spokesman for Coca-Cola. ‘When we came out with Vanilla Coke, Pepsi originally said the idea sounded ‘a bit vanilla.’ But, whatever.’ ” (*CNN/Money*, August 8, 2003) [11]

The non-credible equilibrium (Do not enter, Aggressive) of the entry-exit game suggests that we need to have a concept to deal with the non-credible “threats” in dynamic games. This concept is the subgame perfect Nash equilibrium (SPNE).

First let us define what a subgame is. A **subgame** is any part of a game that remains to be played after a series of moves and it starts at a point where both players know all the moves that have been made up to that point (Colman [15]). Formally,

Definition 10 *A* **subgame** *is an extensive form game that (a) begins at a decision node n that is a singleton information set, (b) includes all the decision and terminal nodes following n in the game, and (c) does not cut any information sets.*

According to this definition, we can make three observations. A game is a subgame of itself. Every subgame taken by itself is a game. In a finite game of perfect information, every node in the game tree starts a subgame.

The logic of subgame perfection is simple: replace any proper subgame of the tree with one of its Nash equilibrium payoffs and perform backwards induction on the reduced tree. In backwards induction we start by solving for the optimal choice (best response) of the last mover for each possible situation she might face. Once we solve for the last stage, we move to the second-to-last stage and given the best response functions at the last stage, compute the optimal choice for the player in this stage. We continue in this manner till we solve for all stages:

- Determine the optimal action(s) in the final stage K for each possible history h^K.
- For each stage $j = K-1, \ldots, 1$: Determine the optimal action(s) in stage j for each possible history h^j given the optimal actions determined for stages $j+1, \ldots, K$.

Definition 11 *A strategy profile s of an extensive form game is a* **subgame perfect Nash equilibrium** *if the restriction of s is a Nash equilibrium of G for every proper subgame of G.*

The SPNE outcome does not involve non-credible threats as at each move the players respond optimally.

In dynamic games, even if the players have similar action sets, the sequence in which the players move significantly impacts the equilibrium outcome of the game and the players' payoffs in equilibrium.

> "... the concept of 'first mover advantage.' It goes like this: The first one to establish a viable business in any new Internet category takes all the marbles. It worked for Amazon.com and eBay." (*USA Today*, February 6, 2002) [27]

Although being the first mover is advantageous in some business games, this is not always the case.

> "In the early 1990s J. Daniel Nordstrom wanted to make [...] Nordstrom department stores a powerhouse in the next big thing – shopping via interactive TV. Instead he got a front-row seat when the glitzy idea imploded. His conclusion: 'It's very expensive and time-consuming to be at the absolute leading edge. I want to be the first guy to be second. It's much more efficient for me to ride in the draft.' " (Forbes Magazine, July 24, 2000) [16]

Example 12 *Is being the first-mover always advantageous?*

Two firms are competing against each other based on product quality. Let q_j be the quality level set by firm j. The payoff functions for the firms are:

$$\begin{aligned}\Pi^1(q_1, q_2) &= 5q_1 + 2q_1q_2 - 5q_1^2 \\ \Pi^2(q_1, q_2) &= 4q_2 + 11q_1q_2 - 4q_2^2\end{aligned}$$

Assuming that firm j is the first mover, the equilibrium quality levels can be determined using backwards induction.

First mover	Follower's best response function	Outcome	Profit Firm 1	Profit Firm 2
Firm 1	$q_2(q_1) = \frac{4+11q_1}{8}$	$\left(q_1 = \frac{4}{3}, q_2 = \frac{7}{3}\right)$	4	≈ 13
Firm 2	$q_1(q_2) = \frac{5+2q_2}{10}$	$\left(q_1 = \frac{37}{36}, q_2 = \frac{95}{36}\right)$	5.38	≈ 22

Therefore, given the choice, firm 1 would actually prefer to be the follower. This is a phenomenon known as the "first-mover disadvantage."

> "You can ascribe TiVo's struggles to the business axiom known as 'first-mover disadvantage.' Technology pioneers typically get steamrollered, then look on helplessly from the sidelines as a bunch of Johnny-come-latelies make billions. First movers, the theory goes, are too smart for their own good, churning out gizmos that are too expensive or too complex for the average consumer's taste. The big boys survive their gun-jumping – think of Apple and its proto-PDA, the Newton, which might have dusted the rival PalmPilot had the company merely waited a year or two to iron out its kinks." (*MSN News*, October 9, 2002) [25]

4.1 Repeated Games

One key concept we have not captured so far is the repetition of interactions. Especially in the prisoner's dilemma type business dealings, the players know that they will be in the same "game" for a long time to come, and hence they may think twice before they defect based on short-term incentives, especially if cooperation today would increase the chances of cooperation tomorrow. In general, players choose their actions considering not only short-term interests, but rather long term relationships and profitability.

Most business interactions, such as the ones between manufacturers and suppliers, and sellers and customers, are repetitive in nature. In such situations, as long as both parties continue to benefit, the relationships are maintained. Hence, the dependency of future actions on past outcomes plays an important role in the current choices of the players. That is why many suppliers strive for consistently achieving high quality levels, or why e-tailers put a high emphasis on the speed and quality of their customers' online shopping experience.

> "Intel Corp. [...] uses its much envied supplier ranking and rating program – which tracks a supplier's total cost, availability, service and support responsiveness, and quality – to keep its top suppliers on a course for better quality. 'We reward suppliers who have the best rankings and ratings with more business,' says Keith Erickson, director of purchasing [...] As an added incentive, Intel occasionally plugs high-quality suppliers in magazine and newspaper advertisements. The company even lets its top-performing suppliers publicize their relationship with Intel. That's a major marketing coup, considering that Intel supplies 80% of chips used in PCs today and is one of the most recognized brand names in the world.
>
> Seagate Technology [...] takes a similar approach to continuous improvement. 'Quality goals are set for each component and are measured in defective parts per million,' says Sharon Jenness, HDA mechanical purchasing manager at Seagate's design center. 'Suppliers who consistently [beat their quality] goals and maintain a high quality score are rewarded with an increased share of our business.' " (*Purchasing*, January 15, 1998) [29]

> "Customer retention is the key for e-commerce firms hoping to eventually make a profit. [...] A 10% increase in repeat customers would mean a 9.5% increase in the corporate coffers, according to McKinsey's calculations." (*The Industry Standard*, March 20, 2000) [30]

In this section, we will find out when, if at all, repeating the interactions help the firms collaborate to achieve mutually beneficial outcomes.

Example 13 *Repeated prisoner's dilemma game*

Recall the prisoner's dilemma game displayed in Figure 2. The unique equilibrium outcome of this game is (D, D) with payoffs $(-6, -6)$. This outcome is Pareto dominated by (C, C), which results in payoffs $(-1, -1)$, i.e., the prisoners settle for an inferior outcome by selfishly trying to get better at the expense of the other.

Now suppose that this game is repeated T times, i.e., consider the dynamic version of this game with T stages. For simplicity, consider $T = 2$. The prisoners observe the outcomes of all preceding stages before they engage in the current stage of the game. The payoffs for the repeated game are simply the sum of the payoffs from all the stages of the game. The question is: Can repeating the same game help the players to achieve a better outcome?

In order to find the equilibrium outcome of this new game, we can use the idea of subgame perfect equilibrium and start analyzing the second stage of the game. In the second stage, regardless of the first stage, there is a unique Nash equilibrium (D, D). After solving the second stage, we can roll back and analyze the first stage of the game. We already know that regardless of the outcome in this stage we will have an equilibrium payoff of $(-6, -6)$ from the second stage; we might as well decrease the first stage payoffs by 6 for each outcome and just look at this modified single stage game instead.

		Prisoner 2	
		Cooperate (C)	Defect (D)
Prisoner 1	C	-7, -7	-15, -6
	D	-6, -15	-12, -12

This modified single stage game has a unique Nash equilibrium: (D, D). Thus, the unique SPNE of the two-stage prisoner's dilemma game is (D, D) in the first stage, followed by (D, D) in the second stage. Unfortunately, cooperation cannot be achieved in either stage of the SPNE. The result would be the same if the game was repeated for any finite number of times.

What would happen if we repeat this game infinitely many times? Adding the payoffs of all stages would not work here, because the sum of the payoffs would go to (negative) infinity no matter what the outcome is. Since future payoffs are in general worth less today, it is reasonable to look at discounted payoffs, or the net present value of payoffs. Let δ be the value today of a dollar to be received one stage later, e.g., $\delta = 1/(1+r)$ where r is the interest rate per stage. Given the discount factor δ the present value of the infinite sequence of payoffs $\pi_1, \pi_2, \pi_3, \ldots$ is $\pi_1 + \delta\pi_2 + \delta^2\pi_3 + \ldots = \sum_{t=1}^{\infty} \delta^{t-1}\pi_t$. If the payoff is the same in each stage, then we have $\pi + \delta\pi + \delta^2\pi_3 + \ldots = \sum_{t=1}^{\infty} \delta^{t-1}\pi = \frac{\pi}{1-\delta}$.

Consider the following strategy for the infinitely repeated prisoner's dilemma game: Play C in the first stage. In the t-th stage, if the outcome of all $t-1$ preceding stages has been (C,C), then play C; otherwise, play D. Such strategies are called **trigger strategies** because a player cooperates until someone fails to cooperate, which triggers a switch to non-cooperation (punishment) forever after.

We see many examples of such trigger strategies in business. For example, a manufacturer and a supplier will continue to do business as long as the supplier meets certain quality standards and the manufacturer pays a good price and provides sufficient volume.

> "Suppliers who consistently rank poorly in quality measurements are often placed on some type of probation, which can include barring them from participating in any new business. [...] If for some reason quality improvement is not achievable with a particular supplier, PPG is free to let that supplier go." (*Purchasing*, January 15, 1998) [29]

If we assume prisoner 1 adopts this trigger strategy, what is the best response of prisoner 2 in stage $t+1$? If the outcome in stage t is any other than (C,C), then she would play D at stage $t+1$ and forever. If the outcome of stage t is (C,C), she has two alternatives – she can either continue to cooperate or she can defect. If she defects, i.e., plays D, she will receive 0 in this stage and she will switch to (D,D) forever after, leading to the following net present value:

$$V = 0 + \delta.(-6) + \delta^2.(-6) + \delta^3.(-6) + \cdots = (-6)\frac{\delta}{(1-\delta)}.$$

If she cooperates, i.e., plays C, she will receive -1 in this stage and she will face the exact same game (same choices) in the upcoming stage, leading to a net present value below:

$$V = -1 + \delta V \quad \Rightarrow \quad V = \frac{-1}{1-\delta}.$$

For prisoner 2, playing C is optimal if

$$\frac{-1}{1-\delta} \geq (-6)\frac{\delta}{(1-\delta)} \quad \Rightarrow \quad \delta \geq \frac{1}{6}.$$

Therefore, it is a Nash equilibrium for both prisoners to cooperate if $\delta \geq \frac{1}{6}$. If δ is small, then future earnings are not worth much today. Hence, players act myopically to maximize current payoffs and choose to defect rather than cooperate. However, when δ is large (in this case larger than or equal to $\frac{1}{6}$), the players value future earnings highly and hence they choose to cooperate.

Next, we will argue that such a Nash equilibrium is subgame-perfect. Recall the definition of SPNE: A Nash equilibrium is subgame-perfect if the players' strategies are a Nash equilibrium in every subgame. In this game there are two classes of subgames: Subgames where the outcomes of all previous stages have been (C,C) and subgames where the outcome of at least one earlier stage differs from (C,C). The trigger strategies are Nash equilibrium for both of these cases.

In the prisoner's dilemma game, repeating the game finitely many times does not help the prisoners achieve cooperation. However, when $\delta \geq \frac{1}{6}$, the outcome of the infinitely repeated prisoner's dilemma is to cooperate in all stages. That is, the credible threat of defect helps the players to coordinate their acts.

So the idea is simple: Repeating a game with a unique Nash equilibrium finitely many times would lead to a unique SPNE where the Nash equilibrium is played at every stage of the game. If the same game is repeated infinitely many times, however, the repeated game may have a SPNE which is not a Nash equilibrium of the original game. This result has captured some discussions among the game theorists and one such discussion can be found in Osborne and Rubinstein [32] among the authors of the book:

"[A. Rubinstein] In a situation that is objectively finite, a key criterion that determines whether we should use a model with a finite or an infinite horizon is whether the last period enters explicitly into the players' strategic considerations. For this reason, even some situations that involve a small number of repetitions are better analyzed as infinitely repeated games. For example, when laboratory subjects are instructed to play the *Prisoner's Dilemma* twenty times [...], I believe that their lines of reasoning are better modeled by an infinitely repeated game than by a 20-period repeated game, since except very close to the end of the game they are likely to ignore the existence of the final period.

[M.J. Osborne] [...] The experimental results definitely indicate that the notion of subgame perfect equilibrium in the finitely repeated *Prisoner's Dilemma* does not capture human behavior. However, this deficiency appears to have more to do with the backwards induction inherent in the notion of subgame perfect equilibrium than with the finiteness of the horizon *per se*."

Recall that repeating a game with a unique Nash equilibrium finitely many times leads us to the same outcome at every stage. Then would it ever benefit the players to repeat a game finitely many times? The answer is yes. If there are multiple Nash equilibria in a game, then finite repetition might make it possible in certain stages to achieve outcomes which are not Nash equilibria in the single-stage game.

Example 14 *Finitely repeated game with multiple static Nash equilibria*[5].

Consider the game below with two players, where each player has three possible actions.

Player 1 \ Player 2	Left	Center	Right
Top	1, 1	5, 0	0, 0
Middle	0, 5	4, 4	0, 0
Bottom	0, 0	0, 0	3, 3

[5] This example is taken from Gibbons [22].

The game has two Nash equilibria: (Top, Left) and (Bottom, Right). Note that, (Middle, Center) Pareto dominates both of these equilibria, but it is not a Nash equilibrium itself. Suppose this game is played twice, with the first stage outcome observed before the second stage begins, and the players play according to the following strategy: Play (Bottom, Right) in the second stage if the first stage outcome is (Middle, Center); otherwise, play (Top, Left) in the second stage.

We can modify the payoffs of the first stage based on this strategy. If the players play anything other than (Middle, Center) in the first stage, the second stage payoffs will be $(1, 1)$. If the first stage outcome is (Middle, Center), then the second stage payoffs will be $(3, 3)$. Therefore, we can modify the first stage by adding $(3, 3)$ to the payoffs of outcome (Middle, Center) and by adding $(1, 1)$ to the payoffs of all other outcomes.

		Player 2		
		Left	Center	Right
Player 1	Top	2, 2	6, 1	1, 1
	Middle	1, 6	7, 7	1, 1
	Bottom	1, 1	1, 1	4, 4

The modified first stage game has 3 pure-strategy Nash equilibria: (Top, Left), (Middle, Center), and (Bottom, Right). Thus, the SPNE of the two-stage game are [(Top, Left), (Top, Left)], [(Bottom, Right), (Top, Left)] and [(Middle, Center), (Bottom, Right)]. Note that (Middle, Center) is not an equilibrium in the single stage game, but it is a part of SPNE. In this example, we have seen a credible promise, which resulted in an equilibrium outcome that is good for all parties involved. If the stage game has multiple equilibria, players may devise strategies in order to achieve collaboration in a finitely repeated game.

5 Price and Quantity Competition

For most manufactures and retailers, key business decisions include what to produce/procure and sell, how much, and at what price. Hence, we commonly see competitive interactions around these decisions. Given their importance within the business world, we devote this section to well-known game theoretic models that focus on price and quantity decisions.

Quantity competition, i.e., how much competing firms produce and put into the market, can be seen in many industries. These decisions are especially important in commodity markets where there is a direct (inverse) relationship between the quantity in the market and the market price.

> "OPEC decided to slash its crude oil production by 1.5 million barrels a day (6%). The issue came to a head this autumn as the weakening world economy, together with the uncertainty caused by the Sept. 11 attacks on the United States, dragged down prices some 30 percent. [...] The cut is expected to lift OPEC's benchmark price to $22 a barrel – the group's minimum target price." (*CBS News*, December 28, 2001.) [37]

Price competition occurs in almost every market, as competing companies try to maintain or increase market share.

> "Burger King Corp. will put its flagship Whopper hamburger on sale for 99 cents [...] The move is likely to intensify and prolong the burger price wars that have been roiling the U.S. fast-food industry in recent months. [...] Burger King officials had said earlier that while they were reluctant to discount the Whopper, they had little choice given a $1 menu at archrival McDonald's Corp. that included a Whopper-like hamburger, called the Big 'N Tasty." (*Chicago Sun-Times*, January 3, 2003) [3]

> "Tesco announced plans to slash £80 million from prices of more than 1,000 products, with some prices falling by more than 30%. The cuts came as rival Asda also said it was slashing selected prices. [...] The cuts echo memories of the supermarket price wars played out in 1999 as stores fought to capture more customers and increased market share." (*Sunday Telegraph*, January 5, 2003) [19]

Next we discuss Cournot and Bertrand games, two classical models commonly used for studying and better understanding the dynamics in quantity and price competition, respectively.

Example 15 *Cournot game*

Consider a market with two competing firms selling a homogeneous good. Let q_1 and q_2 denote the quantities produced by firms 1 and 2, respectively. Suppose that the firms choose their production quantities simultaneously. This game is known as the Cournot game. It is a static game with two players, where the players' action sets are continuous (each player can produce any non-negative quantity).

Let $P(Q) = a - bQ$ be the market-clearing price when the aggregate quantity on the market is $Q = q_1 + q_2$. Assume that there are no fixed costs and the total cost of producing quantity q_i is $c_i q_i$. In order to avoid the trivial cases, in the rest of this section, we will assume that $a > 2c_i$. To illustrate this game – and the other games in this section – numerically, consider an example where $a = 130$, $b = 1$, and $c_1 = c_2 = 10$.

Before we consider the Cournot game, we will analyze the monopoly setting as a benchmark model. A monopolist sets his production quantity q_M to maximize his profit:

$$\Pi_M = \max_{0 \le q_M \le \infty} (a - bq_M)\, q_M - c_M q_M = (130 - q_M - 10)\, q_M$$

Notice that Π_M is concave in q_M, therefore we can use the first-order condition (FOC) to find the optimal monopoly quantity:

$$\frac{\partial \Pi_M}{\partial q_M} = a - 2bq_M - c_M = 0 \Rightarrow q_M^* = \frac{a - c_M}{2b} = 60.$$

The monopolist's profit is $\Pi_M = \frac{(a-c_M)^2}{4b} = 3600$.

Now, let us go back to the Cournot game. The payoff functions for players for strategy profile $s = (q_1, q_2)$ are

$$\Pi_1(s) = (a - b(q_1 + q_2))\, q_1 - c_1 q_1 = (130 - (q_1 + q_2) - 10)\, q_1 \qquad (1)$$

$$\Pi_2(s) = (a - b(q_1 + q_2))\, q_2 - c_2 q_2 = (130 - (q_1 + q_2) - 10)\, q_2 \qquad (2)$$

In the Cournot game, in order to find the best response functions of the players, we need to solve each player's optimization problem:

$$\Pi^1 = \max_{0 \le q_1 \le \infty} (a - b(q_1 + q_2) - c_1)\, q_1 = \max_{0 \le q_1 \le \infty} (120 - (q_1 + q_2))\, q_1$$

$$\Pi^2 = \max_{0 \le q_2 \le \infty} (a - b(q_1 + q_2) - c_2)\, q_2 = \max_{0 \le q_2 \le \infty} (120 - (q_1 + q_2))\, q_2$$

FOC for the optimization problems are as follows:

$$\begin{aligned}\frac{\partial \Pi_1(s)}{\partial q_1} &= a - 2bq_1 - bq_2 - c_1 = 120 - 2q_1 - q_2 = 0 \\ \frac{\partial \Pi_2(s)}{\partial q_2} &= a - 2bq_2 - bq_1 - c_2 = 120 - 2q_2 - q_1 = 0\end{aligned}$$

Notice that, the objective functions in this case are concave and therefore we can use FOC to find the best response functions:

$$\begin{aligned}R_1(q_2) &= \left(\frac{a-c_1}{2b} - \frac{q_2}{2}\right)^+ = \left(60 - \frac{q_2}{2}\right)^+ \\ R_2(q_1) &= \left(\frac{a-c_2}{2b} - \frac{q_1}{2}\right)^+ = \left(60 - \frac{q_1}{2}\right)^+\end{aligned}$$

When we solve these reaction functions together, we get the Cournot equilibrium:

$$\begin{aligned}q_1^* &= \frac{a - 2c_1 + c_2}{3b} = 40 \\ q_2^* &= \frac{a - 2c_2 + c_1}{3b} = 40\end{aligned}$$

When we compare this outcome with the monopoly result, we see that companies, when engaged in competition, end up producing less than what they would produce otherwise. However, the total quantity to the market $q_1^* + q_2^*$ would be higher than what a monopolist would produce and sell, q_M^*. As the number of firms in the market increases, each firm's profitability decreases. The firms begin to have smaller market shares and the end-customers benefit more and more from the tough competition.

Example 16 *Infinitely repeated Cournot game*

Let us consider the infinitely repeated version of the static Cournot duopoly model. Each firm has a marginal cost of c and both firms have a discount factor of δ. Assume that the firms play the following trigger strategy.

> Produce half the monopoly quantity, $q_M/2$, in the first stage. In the t^{th} stage, produce $q_M/2$ if both firms have produced $q_M/2$ in all previous stages; otherwise, produce q_C.

Each firm's profit when they both produce half the monopoly quantity, $\left(\frac{q_M}{2} = \frac{a-c}{4b} = 30\right)$, is $\frac{(a-c)^2}{8b} = 1800$. If both produce the Cournot quantity, $\left(q_C = \frac{a-c}{3b} = 40\right)$, the profit of each is $\frac{(a-c)^2}{9b} = 1600$. We want to compute the values of δ for which this trigger strategy is a SPNE.

If the outcome in stage t is other than $(q_M/2, q_M/2)$, then firm i's best response in stage $t+1$ (and forever) is to produce q_C. If the outcomes of stages $1, \cdots, t$ are $(q_M/2, q_M/2)$, however, he has two choices. He may deviate or he may continue to cooperate. If he deviates, he will produce the quantity that maximizes the current period's profit in stage $t+1$ and then produce q_C forever:

$$\max_{0 \leq q_i \leq \infty} \left(a - bq_i - \frac{a-c}{4} - c\right) q_i.$$

The quantity is $q_i = \frac{3(a-c)}{8b} = 45$ with a profit of $\frac{9(a-c)^2}{64b} = 2025$. The discounted profit is:

$$V = \frac{9(a-c)^2}{64b} + \left(\frac{(a-c)^2}{9b}\right)\left(\frac{\delta}{1-\delta}\right) = 2025 + 1600\left(\frac{\delta}{1-\delta}\right).$$

If he continues to cooperate, he will produce $q_M/2$ and receive $\frac{(a-c)^2}{8b} = 1800$ in this stage and he will face the exact same game (same choices) in stage $t+2$.

$$V = \frac{(a-c)^2}{8b} + \delta.V \quad \Rightarrow \quad V = \frac{(a-c)^2}{8b(1-\delta)} = \frac{1800}{1-\delta}.$$

Therefore, producing $q_M/2$ is optimal if

$$\frac{1800}{1-\delta} \geq 2025 + 1600\left(\frac{\delta}{1-\delta}\right) \Rightarrow \delta \geq \frac{9}{17}.$$

This is a tacit collusion to raise the prices to the jointly optimal level and it is called cartel. A cartel "is a combination of producers of any product joined together to control its production, sale, and price, so as to obtain a monopoly and restrict competition in any particular industry or commodity." (http://www.legal-database.com) As the example above shows, cartels can be quite unstable. At each stage, the players have a huge incentive to cheat, especially when they think it can go unnoticed.

> "The coffee bean cartel, the Association of Coffee Producing Countries, whose members produce 70% of the global supply, will shut down in January after failing to control international prices. [...] Mr Silva also said the failure of member countries to comply with the cartel's production levels was a reason for the closure." (*BBC News*, October 19, 2001) [8]

> "[...] But the oil market is notoriously difficult to balance – demonstrated by the rollercoaster of prices over the last few years. [...] Member states of OPEC do not

necessarily have identical interests and often find it difficult to reach consensus on strategy. Countries with relatively small oil reserves [...] are often seen as 'hawks' pushing for higher prices. Meanwhile, producers [...] with massive reserves and small populations fear that high prices will accelerate technological change and the development of new deposits, reducing the value of their oil in the ground." (*BBC News*, February 12, 2003) [2]

Example 17 *Bertrand game*

Bertrand game models situations where firms choose prices rather than quantities as in Cournot's model. Assume that two firms produce identical goods which are perfect substitutes from the consumers' perspective, i.e., consumers buy from the producer who charges the lowest price. If the firms charge the same price, they will split the market evenly. There are no fixed costs of production and the marginal costs are constant at c. As in the Cournot game, the firms act simultaneously.

The profit of firm i when it chooses the price p_i is

$$\Pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j),$$

where the demand of firm i as a function of prices is given by

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & p_i < p_j \\ \frac{1}{2}D(p_i) & p_i = p_j \\ 0 & p_i > p_j \end{cases}$$

This time the strategies available to each firm are the different prices it might charge, rather than the quantities it might produce. The Bertrand paradox states that the unique equilibrium has the firms charge the competitive price: $p_1^* = p_2^* = c$.

To see why, first observe that for each firm, any strategy which sets the price p_i lower than the marginal cost c is strictly dominated by $p_i = c$, and hence can be eliminated. Therefore, in the equilibrium $p_i \geq c$. Now assume that firm 1 charges higher than firm 2, which charges higher than the marginal cost; i.e. $p_1^* > p_2^* > c$. Then firm 1 has no demand and its profits are zero. On the other hand, if firm 1 charges $p_2^* - \epsilon$, where ϵ is positive and small, it obtains the entire market and has a positive profit margin of $p_2^* - \epsilon - c$. Therefore, $p_1^* > p_2^* > c$ cannot be an equilibrium as firm 1 is not acting in its best interest. Similar arguments can be applied to cases where $p_1^* = p_2^* > c$ and $p_1^* > p_2^* = c$ to show that neither of these cases can be an equilibrium. Hence, the only possible outcome is $p_1^* = p_2^* = c$. Therefore, when the costs and the products are identical, there exits a unique equilibrium in which all output is sold at the price which is equal to the marginal cost.

The Bertrand game suggests that when the firms compete on prices (and the costs are symmetric) we obtain a perfectly competitive market even in a duopoly situation. However, in real life, most customers do not only choose based on price, but also based on other factors such as quality and convenience of location. For example, Wendy's, the other large player in fast-food industry, decided to stay out of the Burger King-McDonald's price war and aimed at gaining market share by offering higher-quality products.

In many business situations, firms choose their actions sequentially rather than simultaneously. To be more precise, one firm chooses its action after it observes another firm's action and responds to it. The next game that we will look at considers the quantity competition when one firm reacts to another.

Example 18 *Stackelberg game*

Let us modify the Cournot model so that firms move sequentially rather than simultaneously. Firm 1 moves first and chooses her quantity to sell q_1. We call firm 1 the "Stackelberg leader." Firm 2 *observes* q_1 before choosing his own output level q_2. We call firm 2 the "Stackelberg follower." The total market demand is $Q = q_1 + q_2$. Both firms seek to maximize their profits. This game is known as Stackelberg game, and it is a dynamic game of complete and perfect information. The game has two players and two stages. In stage 1, player 1's action set is $[0, \infty)$, as player 1 can produce any non-negative quantity, whereas player 2's only available action is "do nothing." In stage 2, player 2's action set is $[0, \infty)$, and player 1's only available action is "do nothing."

The players' payoffs as a function of the moves that were made are given in Equations 1 and 2. We can find the outcome of the Stackelberg game by using backwards induction, starting at stage

2. Firm 2's best response function is

$$R_2(q_1) = \left(\frac{a - c_2}{2b} - \frac{q_1}{2}\right)^+ .$$

We can plug in this function into firm 1's optimization problem in order to find the outcome:

$$q_1 = \frac{a - 2c_1 + c_2}{2b} = 60 \qquad q_2 = \frac{a + 2c_1 - 3c_2}{4b} = 30$$

Table below summarizes the outcomes of the Cournot, the Stackelberg, and the Bertrand games when the production costs of the two firms are equal. Note that firm 2 does worse in the Stackelberg game than in the Cournot game, even though it has more "information" in the Stackelberg game while choosing its production quantity, as it observes firm 1's decision before making its own. This illustrates an important difference between single- and multi-person decision problems. In single-person decision theory, having more information can never make the decision maker worse off. In game theory, however, having more information (or, more precisely, having it known to the other players that one has more information) *can* make a player worse off.

	Monopoly	**Cournot**	**Stackelberg**	**Bertrand**
Price	$c + \frac{a-c}{2} = 70$	$c + \frac{a-c}{3} = 50$	$c + \frac{a-c}{4} = 40$	$c = 10$
Quantity	$\frac{a-c}{2b} = 60$	$\frac{2(a-c)}{3b} = 80$	$\frac{3(a-c)}{4b} = 90$ $\left(\frac{a-c}{2b} = 60, \frac{a-c}{4b} = 30\right)$	$\frac{a-c}{b} = 120$
Total Firm Profits	$\frac{(a-c)^2}{4b} = 3600$	$\frac{2(a-c)^2}{9b} = 3200$	$\frac{3(a-c)^2}{16b} = 2700$ $\left(\frac{(a-c)^2}{8b} = 1800, \frac{(a-c)^2}{16b} = 900\right)$	0

Let us look at two other measures. Total consumer surplus is consumers' willingness to pay that is not captured by the firm. It is positive if the firm sells below consumers' willingness to pay. Deadweight loss is demand that is not satisfied by the firm and is positive if the firm sells below

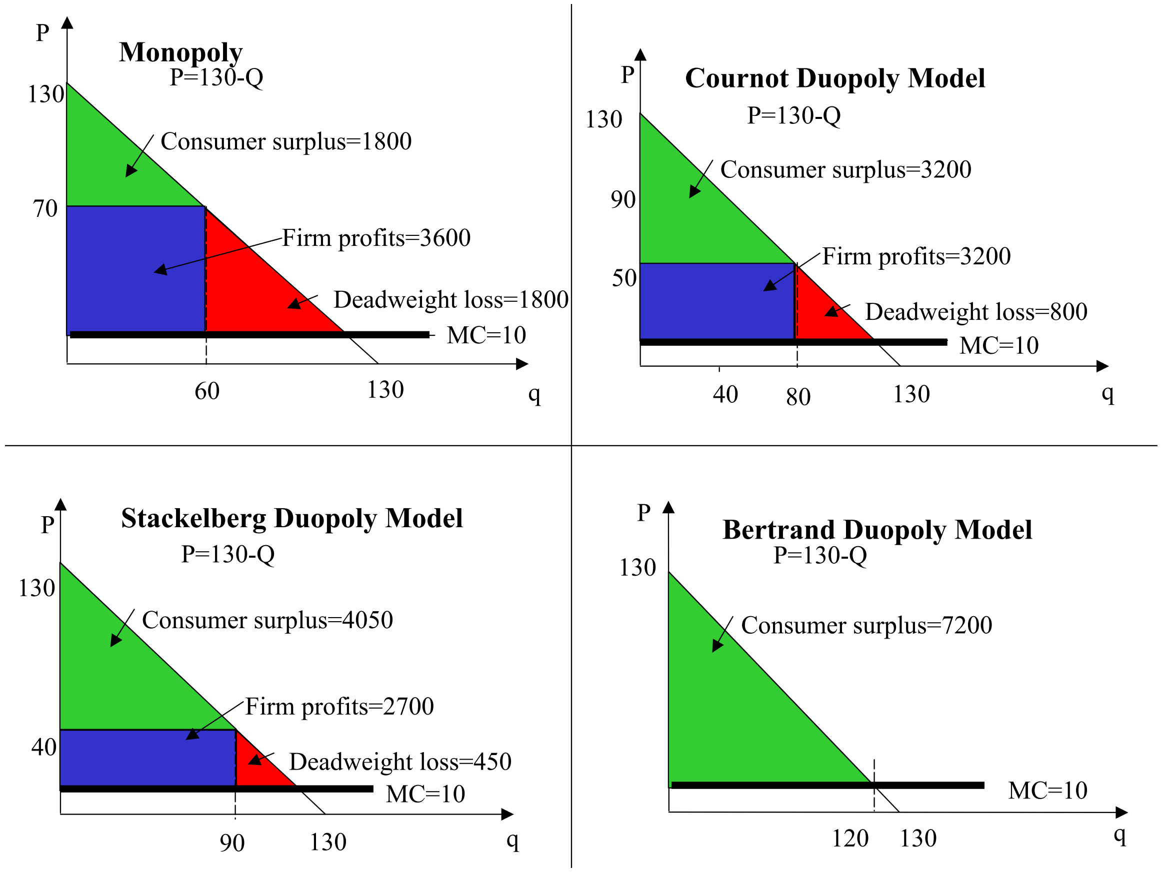


Figure 4: Division of the total potential surplus in the market among the participants.

the maximum market demand. Consumer surplus is highest in the Bertrand model ($7200) and lowest in the monopoly model ($1800). This confirms our intuition that price competition always benefits the consumer.

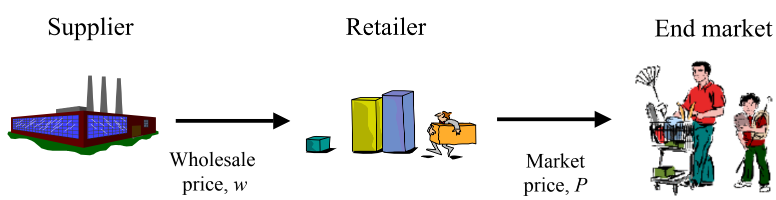
In our example, the size of the entire "pie" in the market is $7200, which is the total potential surplus. Figure 4 shows the division of the "pie" among the market participants. In the Bertrand model, the consumers get the entire pie. As we move from Bertrand to Stackelberg to Cournot to Monopoly, the portion of the pie allocated to the consumers gets smaller, as the portion of the pie allocated to the firms get bigger. The deadweight loss increases at the same time; this means that the decrease in the consumer surplus surpasses the increase in the firms' profits and the surplus obtained by the firms and the consumers together decreases. Hence, part of the pie remains unused, benefiting neither the firms nor the consumers.

6 Supply Chain Coordination

In a supply chain (SC), there are multiple firms owned and operated by different parties, and each of these firms take decisions which are in line with their own goals and objectives. As in all decentralized systems, the actions chosen by SC participants might not always lead to the "optimal" outcome if one considers the supply chain as one entity. That is, since each player acts out of self-interest, we usually see inefficiencies in the system, i.e., the results look different than if the system was managed "optimally" by a single decision-maker who could decide on behalf of these players and enforce the type of behavior dictated by this globally (or centrally) optimal solution.

In this section, we will take a look at the nature of inefficiencies that might result from the decentralized decision-making in supply chains, and if and how one can design contracts such that even though each player acts out of self interest, the decentralized solution might approach the centralized optimal solution. For excellent reviews on SC contracts and coordination we refer the reader to Tsay, Nahmias and Agrawal [40] and Cachon [12].

To better formalize these ideas, we will look at a simple stylized two-stage supply chain with one supplier and one retailer, where the retailer buys goods from the supplier and sells them in the end market:



For simplicity, assume that the supplier is uncapacitated, and has unit cost of production c. The retailer faces a market where the price is inversely related to the quantity sold. For simplicity, let us assume a linear demand curve $P = a - bq$ where P is the market price and q is the quantity sold by the retailer[6]. Assume that all of this information is common knowledge.

First let us consider the simple "wholesale price" contract where the supplier charges the retailer

[6]Note that in such a deterministic environment, the retailer will always purchase from the supplier exactly as much as he will sell in the market.

Table 1: Wholesale price contract versus CSC.

	DSC	CSC
Supplier's wholesale price (w)	$w = (a+c)/2$	w
Retailer's quantity (q)	$q = (a-c)/4b$	$q^* = (a-c)/2b$
Market Price (P)	$P = (3a+c)/4$	$P^* = (a+c)/2$
Supplier's profit (Π_S)	$\Pi_S = (a-c)^2/8b$	$\Pi_S^* = (w-c)q$
Retailer's profit (Π_R)	$\Pi_R = (a-c)^2/16b$	$\Pi_R^* = (P-w)q$
Total SC profits (Π)	$\Pi = 3(a-c)^2/16b$	$\Pi^* = (a-c)^2/4b$

w per unit. The supplier's and the retailer's profits are $\Pi_S = (w-c)q$ and $\Pi_R = (a-bq-w)q$, respectively. The supply chain's profits are $\Pi = \Pi_S + \Pi_R = (a-bq-c)q$. Note that the choice of w only indirectly affects the total SC profits, since the choice of w impacts the choice of q.

Decentralized Supply Chain (DSC): As in most real-world supply chains, suppose that the supplier and the retailer are two independently owned and managed firms, where each party is trying to maximize his/her own profits. The supplier chooses the unit wholesale price w and after observing w, the retailer chooses the order quantity q. Note that this is a dynamic game of complete information with two players, supplier and retailer, where the supplier moves first and the retailer moves second. Hence, we can solve this game using backwards induction.

Given a w, first we need to find the retailer's best response $q(w)$. The retailer will choose q to maximize $\Pi_R = (a-bq-w)q$. This is a concave function of q, and hence from FOC we get

$$\frac{\partial \Pi_R}{\partial q} = a - 2bq - w = 0 \;\Rightarrow q(w) = \frac{a-w}{2b}.$$

Next, given the retailer's best response $q(w) = (a-w)/2b$, the supplier maximizes $\Pi_S = (w-c)q = (w-c)(a-w)/2b$. This is a concave function of w and from FOC we get

$$\frac{\partial \Pi_S}{\partial w} = a - w - w + c = 0 \;\Rightarrow w = \frac{a+c}{2}.$$

The equilibrium solution for this decentralized supply chain is given in the second column of Table 1. In this contractual setting, the supplier gets two-thirds of the SC profits, the retailer gets only one-third. This is partly due to the first-mover advantage of the supplier.

Now, let us consider a centralized (integrated) supply chain (CSC) where both the retailer and the supplier are part of the same organization and managed by the same entity.

Centralized Supply Chain (CSC): In this case there is a single decision-maker who is concerned with maximizing the entire chain's profits $\Pi = (a - bq - c)q$. This is a concave function of q and from FOC we get

$$\frac{\partial \Pi}{\partial q} = a - 2bq - c = 0 \Rightarrow q^* = \frac{a-c}{2b}.$$

The solution for the CSC is given in the third column of Table 1.

From Table 1 we see that the quantity sold as well as the profits are higher and the price is lower in the CSC than in the DCS. Hence, both the supply chain and the consumers are better off in the CSC.

What about the retailer and the supplier? Are they both better off, or is one of them worse off in the CSC? What is the wholesale price? How does the choice of w affect the market price, quantity, and the supply chain profits? A closer look would reveal that w has no impact on these quantities. Any positive w would result in the same outcome for the CSC because the firm would be paying the wholesale price to itself! However, the choice of w in the CSC is still very important as it determines how the profits will be allocated between the supplier and the retailer. We can interpret w as a form of transfer payment from the retailer to the supplier. What is the minimum w that is reasonable? For positive supplier profits, we need $w \geq c$. If we set $w = c$, the supplier's profits are zero whereas the retailer captures the entire supply chain's profits. What is the w that splits the SC profits equally between the retailer and the supplier? If we set $w = (a + 3c)/4$, $w - c = P - w = (a - c)/4$ and each party's profits are $(a - c)^2/8b$. Note that this is the same as the supplier's profits in the DSC. Hence, if the supplier and the retailer split the profits equally in the CSC, the supplier is at least as good, and the retailer is strictly better off than in the DCS.

In the DSC, the outcomes are worse for all the parties involved (supplier, retailer, supply chain, and consumer) compared to the CSC, because in the DSC both the retailer and the supplier independently try to maximize their own profits, i.e., they each try to get a margin, $P - w$ and $w - c$, respectively. This effect is called "double marginalization" (DM).

> In a serial supply chain with multiple firms there is coordination failure because each firm charges a margin and neither firm considers the entire supply chain's margin when

making a decision (Spengler [34]).

In this stylized model, the profit loss in the DSC due to DM is 25% (also referred to as the DM loss). It is clearly in the firms' interest to eliminate or reduce double marginalization, especially if this can be done while allocating the additional profits to the firms such that both firms benefit. This simple model suggests that vertical integration could be one possible way of eliminating double marginalization. However, for reasons we discussed at the beginning of this chapter, vertical integration is usually not desirable, or not practical. Then the question is, can we change the terms of the trade so that independently managed companies act as if they are vertically integrated? This is the concept known as "supply chain coordination." In this stylized model, the retailer should choose $q^* = (a - c)/2b$ in any coordinating contract.

One can easily think of some very simple alternative contracts to eliminate double marginalization:

Take-it-or-leave-it-contract: The supplier offers the following contract to the retailer: Buy q^* at the wholesale price $w = (a + c)/2$, or nothing. In this case the supplier's profit is Π^*, i.e., the supplier captures 100% of the CSC profits.

Marginal pricing: The supplier sets $w = c$. In this case, the retailer's profit is Π^*, i.e., the retailer captures 100% of the CSC profits.

Note that the take-it-or-leave-it contract would require a very powerful supplier whereas the marginal pricing contract would require a very powerful retailer. In practice, neither the supplier nor the retailer is so powerful in general to dictate such contract terms. Hence, we need to consider alternative contracts that coordinate the supply chain. In general, the following aspects are important in (coordinating) contracts (Cachon [12]):

- Profitability: Achieve profits close to optimum.
- Fairness and flexibility: Allow for flexible division of profits.
- Implementability: Should be easy and low-cost to administer.

Next, we will discuss alternative contracts which have these desirable properties.

6.1 Revenue Sharing Contract

One alternative contract which has these desirable properties is revenue sharing (Cachon and Lariviere [13]) which we discuss next.

In a revenue sharing contract, the retailer shares a fraction $\alpha < 1$ of his revenues with the supplier. To study this contract, we will consider a slight variation of our earlier simple two-stage supply chain model. Let c_S and c_R denote the unit costs of the supplier and the retailer, respectively. Let $c = c_S + c_R$ be the cost of each unit sold (the supply chain's total unit cost). The retailer's revenue as a function of quantity q is given by a function $R(q)$, which is concave and increasing in q.

First, consider the centralized supply chain. In the CSC, the profits are given by $\Pi = R(q) - cq$, and from FOC we have $\frac{\partial R(q)}{\partial q} = c$. This means that in the CSC, *the marginal revenue is equal to the marginal cost* at the optimal quantity.

Next, consider the DSC under the wholesale price contract as before. The retailer's profits are $\Pi_R = R(q) - (w + c_R)q$. From FOC, $\frac{\partial R(q)}{\partial q} = w + c_R$. Note that $q < q^*$ if $w > c - c_R$, i.e., unless the supplier sells at marginal cost, the retailer orders less than the optimal CSC quantity resulting in double marginalization.

Finally, consider the revenue sharing contract. The retailer's profits in a revenue sharing contract are $\Pi_R(q) = \alpha R(q) - (w + c_R)q$. From FOC, $\alpha \frac{\partial R(q)}{\partial q} = w + c_R$ and $\frac{\partial R(q)}{\partial q} = (w + c_R)/\alpha$. Recall that we had $\frac{\partial R(q)}{\partial q} = c$ in the integrated chain. Hence, by setting the two right hand sides equal to each other, we can achieve the same quantities in the CSC and the DSC. That is, if $w + c_R = \alpha c$, i.e., $w = \alpha c - c_R$, we have marginal revenue equal to the marginal cost in the DSC as well, and $q = q^*$. In this case, the retailer's profit is $\Pi_R(q) = \alpha R(q) - (\alpha c - c_R)q - c_R q = \alpha(R(q*) - cq*) = \alpha \Pi^*$, i.e., the retailer captures α fraction of the optimal SC profits. Similarly, the supplier captures $(1 - \alpha)$ fraction of the SC profits. Note that in the revenue sharing contract, the retailer's objective function becomes an affine transformation of the supply chain's objective function, i.e., the retailer's and the supply chain's objectives are perfectly aligned.

Since the total SC profits are higher compared to the traditional wholesale price contract, partners can choose α such that both parties benefit. The choice of α depends on several factors, including the bargaining power of the supplier and the retailer.

This simple model is motivated by revenue sharing contracts implemented in practice, in particular by the agreements between Blockbuster and the movie studios. Blockbuster is a retailer, which purchases movies from the studios (suppliers) and rents them to customers. The supplier's wholesale price impacts how many videos Blockbuster orders and hence, how many units are eventually rented by customers. Before 1998, the price of purchasing a tape from the studio was very high, around $65. Given that rental fees are in the order of $3-4, Blockbuster could purchase only a limited number of videos and this resulted in lost demand; especially during the initial release period, where the demand was high (peak demand usually lasts less than 10 weeks), 20% of customers could not find what they were looking for on the shelf. Hence, the studio's high wholesale price impacted the quantity purchased by Blockbuster, and in turn, the revenues and the profitability of both firms. Seeing this problem, Blockbuster and the studios went into a revenue sharing agreement. According to this, Blockbuster pays only $8 per tape initially, but then gives a portion (somewhere around 30 to 45%) of the revenues generated from that tape back to the supplier. Since this agreement reduces Blockbuster's initial investment in movies, it orders more tapes from the studio, hence, is able to meet more demand, generate more revenues, and give back a portion of those revenues back to the supplier. Blockbuster increased its overall market share from 25% to 31% and its cash flow by 61% using this agreement. This is clearly a win-win situation. The supplier might be better off even if he sells each unit below its production cost. A similar agreement is used between studios and theaters as well. (*CNet News.com*, October 18, 2000) [7]

One of the potential drawbacks of such an agreement is the additional cost and administrative burden it creates compared to the straightforward wholesale price contract. Revenue sharing takes an organizational effort to set up the deal and follow its progress. It is worthwhile to go into such an agreement only if the increase in profits is relatively large compared to the additional cost and the administrative effort.

In our analysis of the revenue sharing contract, we considered a single supplier and a single retailer. It turns out that in case of multiple retailers, coordination is not guaranteed unless the supplier has the flexibility to offer different contracts to different retailers. Unfortunately, such "discrimination" might not always be possible due to legal considerations. Another issue is the impact of such an agreement on the behavior of a retailer who sells competing goods and also sets prices. In such a case, the retailer may have an incentive to use the goods under revenue sharing agreement as loss leaders, to drive the traffic to the store and increase overall sales. Finally, revenue sharing loses its appeal if the revenues depend on the retailer's sales effort. For a retailer who is taking only a fraction of the revenues he generates, the incentive to improve sales goes down. While revenue sharing helps to ensure that the retailers buy and sell the "right" quantity, it hurts their sales effort. This is especially true in retail industries such as automobile sales, where a retailer's sales effort makes a big difference in the overall sales rate.

6.2 Buyback Contract

Revenue sharing is just one of many different alternative contract types to coordinate a supply chain. We observed that in the decentralized chain under wholesale price contract, the retailer orders less than the optimal quantity. This was mainly due to double marginalization. An additional reason for ordering less than the optimal quantity could be the risk of excess inventory. For example, consider a newsvendor type model, where there is a single selling opportunity and the seller needs to procure/produce before knowing the demand. This is the case for most fashion retailers, where products need to be ordered months in advance, due to long lead times, way before the demand is known and the actual selling season begins. Whatever is not sold at the end of the selling season is salvaged, and the retailer bears the cost of having excess inventory.

Buyback contracts allocate the inventory risk between the supplier and the retailer. Retailer can return the goods to the supplier and get some money back. Specifically, in the buyback contract the supplier purchases leftover units at the end of the selling season for \$$b$ per unit, where $b < w$. The retailer's revenue is $R(q) = pS(q)$, where p is the unit selling price (given, i.e., not a decision variable) and $S(q)$ is the total amount of sales made by the retailer. Note that $S(q) \leq q$, i.e., the retailer cannot sell more than what he orders. As before, the total unit cost of the supply chain is $c = c_S + c_R$.

In the DSC, the retailer's profit is

$$\begin{aligned}\Pi_R &= \text{revenue} + \text{returns} - \text{cost} - \text{purchase cost} \\ &= R(q) + b\left(q - \frac{R(q)}{p}\right) - (c_R + w)q = R(q)\left(1 - \frac{b}{p}\right) - (c_R + w - b)\,q.\end{aligned}$$

Recall that in revenue sharing contracts we have $\Pi_R = \alpha\Pi^* = \alpha\left(R(q^*) - cq^*\right)$, i.e., the retailer's profit is an affine transformation of the centralized supply chain's profits. Hence, the retailer's optimal quantity is the same as the centralized supply chain's optimal quantity.

For a similar situation to hold in buyback contracts, the retailer's revenue should be equal $\alpha\Pi^*$, i.e., $R(q)\,(1-b/p)-(c_R+w-b)q=\alpha\Pi^*$ and we need to have $(c_R+w-b)=\alpha c$ and $(1-b/p)=\alpha$:

$$\left(1-\frac{b}{p}\right)=\alpha \quad\Rightarrow\quad b=p(1-a)$$
$$c_R+w-b=\alpha c \quad\Rightarrow\quad w_b=\alpha c+b-c_R=p(1-\alpha)+\alpha c-c_R.$$

Hence, if the supplier chooses w and b in this fashion, the retailer will get a fraction of the CSC's profits.

In revenue sharing contracts, we had $w=\alpha c-c_R$. In buyback contracts, we have $w_b=p(1-\alpha)+\alpha c-c_R$ and $b=p(1-\alpha)$. That is, $w_b=b+w$. Hence, in the buyback contract the supplier charges a little more for the wholesale price and in return guarantees to give back b for any unit that is not sold.

In revenue sharing contracts the wholesale price did not depend on the selling price. But in a buyback contract, the wholesale price and the return price depend on the selling price. Hence, in a buyback contract the supplier should set w and b as functions of the selling price p.

Is the buyback contract flexible in terms of how the profits are shared between the supplier and the retailer? The answer is yes. Actually, our analysis so far tells us that for every revenue sharing contract, there is an equivalent buyback contract and vice versa.

Does buyback make sense in the video rental setting? The retailer always has q units remaining at the end of the selling season, hence, is guaranteed to get back \$$b$ for every unit he purchases. Hence, the buyback contract reduces to a revenue sharing contract.

6.3 Two-Part Tariff

In a two-part tariff, the supplier charges a fixed fee F and a wholesale price w per unit. The following is an example of two-part tariffs from fractional aircraft ownership:

> "For travellers who value flexibility and the increased security of knowing everyone on the flight, there is a compelling incentive for opting for fractional ownership. [...] Under NetJets' scheme, a one-sixteenth share of a small Cessna Encore, which seats seven passengers, costs $487,500 plus a monthly management fee of $6,350 and an occupied hourly fee of $1,390 for each of the allotted 50 hours." (Financial Times, December 12, 2001) [20]

In this section, we will answer the following questions: Does a two-part tariff contract coordinate the supply chain? If yes, how should the supplier choose F and w? How are the profits allocated between the supplier and the retailer?

Recall that in the CSC, the optimality condition is $\frac{\partial R(q)}{\partial q} = c = c_R + c_S$, i.e., marginal revenue is equal to marginal cost.

In the DSC, under two-part tariff the retailer's profit is $\Pi_R = R(q) - (w + c_R)q - F$. From the FOC

$$\frac{\partial \Pi_R}{\partial q} = \frac{\partial R(q)}{\partial q} - w - c_R = 0 \Rightarrow \frac{\partial R(q)}{\partial q} = w + c_R.$$

In order to have the CSC solution, we need $w = c_S$, i.e., supplier must sell at cost. In this case the retailer gets $\Pi_R = R(q^*) - (w + c_R)q^* - F = R(q^*) - (c_S + c_R)q^* - F = \Pi^* - F$. Supplier's profit is $\Pi_S = F + (w - c_S)q^* = F$.

Notice that F determines the allocation of profits between the supplier and the retailer. This is a flexible contract as any allocation is possible.

6.4 Quantity Discount Contract

So far we assumed that w is fixed per unit. However, in many applications suppliers offer quantity discounts.

> "We offer a quantity discount for orders of 10 pieces and more of the same products." (www.decor24.com)

> "Server quantity discounts start at 10 units, with further breaks at 25, 50, 100, 250 and above." (www.greyware.com)

> "Quantity discounts on miscellaneous accessories: 0 - 4 = 0%; 5 - 9 = 5%; 10 - 24 = 10%; 25 - 49 = 15%; 50 - up = 20%" (www.frye.com)

In quantity discount contracts, if you purchase more units, the unit price goes down. Therefore, the supplier charges $w(q)$ where w is a decreasing function of q. In general, $w(q)$ is a step function as seen in the examples above, but for simplicity, we will assume it is continuous and differentiable.

The retailer's profit in the DSC is $\Pi_R = R(q) - (w(q) + c_R)q$. We observed earlier that the DSC has the same optimal quantity as the CSC, if Π_R is an affine transformation of Π. Hence, we need

$$\Pi_R = R(q^*) - (w(q^*) + c_R)q^* = \alpha(R(q^*) - cq^*) \Rightarrow w(q^*) = (1-\alpha)\left(\frac{R(q^*)}{q^*}\right) - c_R + \alpha c$$

If the supplier sets the wholesale price as $w(q) = (1-\alpha)\left(\frac{R(q)}{q}\right) - c_R + \alpha c$, there is a tradeoff for the retailer: If he chooses $q < q^*$, he will pay too much per unit, increasing his marginal cost. If he chooses $q > q*$, then the unit price will decrease but the marginal revenue will decrease more, making the extra unit unprofitable. Hence, the optimal quantity choice for the retailer is q^*.

The supplier's profit in this case is

$$\Pi_S = (w - c_S)q^* = \left(\frac{(1-\alpha)R(q*)}{q^*} - c_R + \alpha c - c_S\right)q^* = (1-\alpha)R(q^*) - (1-\alpha)q^*c = (1-\alpha)\Pi$$

How is the quantity discount contract related to the revenue sharing contract? In both contracts the retailer's (and the supplier's) revenue is proportional to the centralized supply chain's profit. If there is demand uncertainty, in a quantity discount contract the retailer bears the risk. The retailer orders the quantity that maximizes the expected supply chain profits, hence the supplier always gets $1-\alpha$ fraction of the expected profits. But depending on the demand realization, the retailer may earn more or less than an α fraction of the expected profits. In a revenue sharing contract, however, the retailer shares revenues after the sales are made. Since sharing occurs on realized, not expected, revenues, in case of uncertainty, both the supplier and the retailer bear the risk. Recall that revenue sharing does not coordinate the chain if the effort level is a decision variable. One can show that the quantity discount contract coordinates the chain with an effort- and quantity-dependent wholesale price $w(q) = \frac{(1-\alpha)R(q^*,e^*)}{q} - c_R + \alpha c + \frac{(1-\alpha)g(e^*)}{q}$, where e^* denotes the optimal effort level and $g(e^*)$ denotes the cost of exerting effort e^* ($g(e)$ is an increasing, differentiable, convex function of e). Hence, coordination is possible under a quantity discount contract when both the effort level and the quantity are decision variables.

So far we assumed that the price is exogenously determined, and given a price, revenues depend only on quantity. What if price is also a decision variable? Would these contracts still coordinate? In revenue sharing, quantity discount, and two-part tariff, since the retailer's profit is an affine transformation of the centralized supply chain's profits, the retailer will choose the optimal price-quantity pair. However, in the buyback contract, the coordinating wholesale price depends on the selling price. Unless the supplier can quote such price-dependent contract parameters, when the selling price is a decision variable, it has been shown that buyback contracts coordinate only if the supplier earns zero profit. In quantity discount contract, price, effort level, and quantity can all be decision variables, and the contract will still coordinate.

7 Games of Incomplete Information

Most business interactions do not take place under complete information, i.e., the players' payoff functions are not necessarily common knowledge. For example, unlike the Cournot models that we have analyzed, the firms may not know each other's costs for sure. Similarly, in a sealed-bid auction, each bidder knows his/her own valuation but does not know any other bidder's valuation. Situations like these can be analyzed by using games of incomplete information, where at least one player is uncertain about another player's payoff function.

Example 19 *Cournot duopoly model under asymmetric information*

Let us modify the Cournot duopoly model such that there is an information asymmetry. The marginal cost of producing each unit of the good is c_1 and c_2 for firm 1 and 2, respectively. Firm 1's cost c_1 is common knowledge, however c_2 is known only by firm 2. Firm 1 believes that c_2 is "high" c_H with probability p and "low" c_L with probability $(1-p)$. Firm 1's belief about firm 2's cost is common knowledge. What is the outcome of this game?

First note that firm 2's strategy must specify its actions for each of its "types," i.e., c_H and c_L. That is, even though firm 2 knows its own cost, it still needs to devise an action plan for the other cost as well. Let q_2^H and q_2^L denote firm 2's quantity choices for each of its types, and q_1 denote firm 1's single quantity choice. The payoff functions of the players for strategy profile $s = (q_1, q_2^H, q_2^L)$ are

$$\begin{aligned}
\Pi_1(s) &= (P - c_1)q_1 = p[a - (q_1 + q_2^H)]q_1 + (1-p)[a - (q_1 + q_2^L)]q_1 - c_1 q_1 \\
\Pi_2^t(s) &= \left(a - \left(q_1 + q_2^t\right)\right) q_2^t - c_t q_2^t \qquad t \in \{H, L\}
\end{aligned}$$

Note that firm 1's payoff function is an expectation over firm 2's possible types. From the FOC of the maximization problems of firm 1 and firm 2, we get the following best-response functions[7]:

$$\begin{aligned} q_1 &= p\left(\frac{a-c_1}{2}-\frac{q_2^H}{2}\right)^+ + (1-p)\left(\frac{a-c_1}{2}-\frac{q_2^L}{2}\right)^+ \\ q_2^H &= \left(\frac{a-c_H}{2}-\frac{q_1}{2}\right)^+ \\ q_2^L &= \left(\frac{a-c_L}{2}-\frac{q_1}{2}\right)^+ \end{aligned}$$

We can solve these equations together to find the outcome:

$$\begin{aligned} q_1 &= \frac{a-2c_1+pc_H+(1-p)c_L}{3} \\ q_2^H &= \frac{a-2c_H+c_1}{3}+\frac{1-p}{6}(c_H-c_L) \\ q_2^L &= \frac{a-2c_L+c_1}{3}-\frac{p}{6}(c_H-c_L) \end{aligned}$$

Let us compare these results to the Cournot equilibrium under complete information with costs c_1 and c_2. In this case, the equilibrium quantities are $q_i^* = \frac{a-2c_i+c_j}{3}$. In the incomplete information case, $q_2^H > q_2^*$ and $q_2^L < q_2^*$. Firm 2 not only tailors its quantity to its cost but also responds to the fact that firm 1 cannot do so.

Cournot duopoly model under asymmetric information is a static game of incomplete information. Harsanyi proposed that the way to model and understand the games of this kind is to introduce a prior move by nature that determines player 2's "type," in the Cournot example her cost. Then, we can analyze the game with the following alternative timing: First, nature draws the type vector $t = (t_1, \cdots, t_n)$, where t_i is drawn from the set of possible types T_i, and reveals t_i to player i but not to any other player. Players do not know each other's types, but given her own type, each player has a "belief" about the other players' types. The players simultaneously choose actions and payoffs are received. Once this common-prior assumption is imposed, we have a standard game, which we can use the concept of Nash equilibrium. The **Bayesian equilibrium** (or Bayesian Nash equilibrium) is precisely the Nash equilibrium of a static incomplete information game.

We can represent a static game of incomplete information with five elements: the set of players, the type spaces of players, the set of strategies, the belief of each player on other players' types, and the payoff functions for each player. Each player i's type t_i is a member of the set of possible

[7]For simplicity assume that $a > 2max\{c_H, c_1\}$.

types T_i, is privately known by the player i, and determines the player's payoff function. Player i's belief $p_i(t_{-i}|t_i)$ describes i's uncertainty about the other players' possible types t_{-i}, given i's own type t_i.

We need to modify our definition of strategy as well. In a static Bayesian game, a **strategy** for player i is a function $s_i(t_i)$, where for each type t_i in T_i, $s_i(t_i)$ specifies the action from the feasible set A_i that type i would choose if drawn by nature.

Definition 20 *In the static Bayesian game, the strategies $s^* = (s_1^*, \cdots, s_n^*)$ are a (pure strategy)* **Bayesian Nash equilibrium** *if for each player i and for each of i's types t_i on T_i, $s_i^*(t_i)$ solves*

$$\max_{a_i \in A_i} \sum_{t_{-i} \in T_{-i}} u_i(s_1^*(t_1), \cdots, s_{i-1}^*(t_{i-1}), a_i, s_{i+1}^*(t_{i+1}), \cdots, s_n^*(t_n); t) p_i(t_{-i}|t_i)$$

That is, no player wants to change his/her strategy, even if the change involves only one action by one type.

Example 21 *First-price sealed-bid auctions*

Consider a case where two bidders are bidding for one good. Bidder i's valuation for the good is v_i and is known only by bidder i. Let us assume that the valuations are independently and uniformly distributed on [0,1]. Each bidder i submits a nonnegative bid b_i. The higher bidder wins and pays his bid. The other bidder pays and receives nothing. In case of a tie, the winner is determined by a coin flip. Bidder i's payoff, if wins and pays p, is $v_i - p$. Bidders are risk-neutral and all of this information is common knowledge.

First, let us specify the action and type spaces, beliefs, and expected payoff functions for each player. Each player can bid any non-negative quantity, therefore, the action spaces are $A_1 = A_2 = [0, \infty)$. The type spaces for the players are given as $T_1 = T_2 = \text{Uniform}[0, 1]$. The valuations are independently distributed, so the beliefs are $p_1(t_2|t_1) = p_1(t_2)$ and $p_2(t_1|t_2) = p_2(t_1)$. Finally, player i's (expected) payoff function is

$$\pi_1(b_1, b_2; v_1, v_2) = \begin{cases} v_i - b_i & \text{if } b_i > b_j \\ \frac{v_i - b_i}{2} & \text{if } b_i = b_j \\ 0 & \text{if } b_i < b_j \end{cases}$$

Before searching for an equilibrium, we need to construct the players' strategy spaces as well. Recall that a strategy is a function from types to actions. Strategy for player i specifies the bid (action) of player i for each of player i's types, $b_i(v_i)$. Recall that in a Bayesian Nash equilibrium, each player's strategy is a best response to the other players' strategies. Therefore, strategies $(b_1(v_1), b_2(v_2))$ are a Bayesian Nash equilibrium if for each v_i in [0,1], $b_i(v_i)$ solves[8]

$$\max \ (v_i - b_i)Prob\,(b_i > b_j(v_j)) + \frac{(v_i - b_i)Prob\,(b_i = b_j(v_j))}{2}$$

Function $b_i(v_i)$ could take any form. The higher a player bids, the higher the chances of winning, but the lower the payoff. Given that valuations are uniformly distributed, we might expect the rate of increase in the probability of winning to be proportional to the increase in the bid. Hence, let us try bids that are linear functions of the valuations:

$$b_i(v_i) = a_i + c_i v_i$$

Assuming player j adopts the strategy $b_j(v_j) = a_j + c_j v_j$, player i's best response is:

$$\max \ (v_i - b_i)Prob\,(b_i > b_j(v_j)) = (v_i - b_i)Prob\,(b_i > a_j + c_j v_j)$$

Since bidder i would never bid above his own valuation, the problem can be rewritten as follows:

$$\begin{aligned} \max \quad & (v_i - b_i)\left(\frac{b_i - a_j}{c_j}\right) \\ s.t. \quad & b_i \le v_i \end{aligned}$$

From FOC, we get

$$b_i = \begin{cases} v_i & \text{if } v_i \le a_j \\ \frac{v_i + a_j}{2} & \text{otherwise} \end{cases}$$

What are the possible values for a_j such that this strategy would make sense? If a_j is between 0 and 1, there are some values of a_j such that $v_i \le a_j$, which means that the response function is flat at first and slopes later, which violates the linearity assumption. Similarly, a_j cannot be greater than 1, as in this case $b_j(v_j) = a_j + c_j v_j > 1$ which violates the constraint that a bidder would not bid higher than his own valuation. Therefore, a_j is non-positive and the bidding strategy is

$$b_i(v_i) = a_i + c_i v_i = \frac{v_i + a_j}{2}$$

[8]Note that $Prob(b_i = b_j(v_j)) = 0$ since the valuations are uniformly distributed.

It is easy to see that $a_i = \frac{a_j}{2}$ and $c_i = \frac{1}{2}$. Similarly, $a_j = \frac{a_i}{2}$ and $c_j = \frac{1}{2}$. Therefore, we get $a_i = a_j = 0$, $c_i = c_j = \frac{1}{2}$, and $b_i(v_i) = \frac{v_i}{2}$. Note that, player i would never bid above player j's maximum bid, which is $a_j + c_j = \frac{1}{2}$. Each player bids half his/her valuation to balance the amount of payment and the chances of winning. This is a "symmetric" Bayesian equilibrium since each player has the same strategy.

Most business games are dynamic in nature, where the game has multiple stages or is repetitive in nature and the players devise their strategies under incomplete information. In this section we focused on static games of incomplete information whereas earlier in Section 4 we studied dynamic games of complete information. Building on these earlier discussions, next we turn to dynamic games of incomplete information.

Recall that an important concept in dynamic games is the credibility of the threats and promises. In dynamic games of complete information, we addressed this issue using the concepts of subgame and subgame perfect Nash equilibrium (SPNE). In dynamic games of incomplete information, the **continuation game** is similar to a subgame, but it can begin at any complete information set (whether singleton or not), rather than only at a singleton information set. Similar to SPNE, the players' strategies in a dynamic game of incomplete information induce a **perfect Bayesian Nash equilibrium (PBE)**, if they constitute a Bayesian Nash equilibrium in every continuation game as well as the entire game.

At each information set, the player with the move must have a belief about which node in the information set has been reached by the play of the game. For a nonsingleton information set, a belief is a probability distribution over the nodes in the information set. Therefore, for a singleton information set, the player's belief puts probability one on the single decision node. PBE differentiates between the information sets that are on the equilibrium path and off the equilibrium path. An information set is on (off) the equilibrium path if it will be (if it is not to be) reached with positive probability if the game is played according to the equilibrium strategies.

Given their beliefs, the players' strategies must be **sequentially rational**. That is, at each information set the action taken by the player with the move (and the player's subsequent strategy) must be optimal given the player's belief at that information set and the other players' subsequent information strategies.

In the following section, we discuss applications of dynamic games of incomplete information within the principal-agent framework.

8 Principal-Agent Problem

Principal-agent problems fall under the more general topic of information economics, which deals with situations where there is lack of information on the part of some market participants, such as what others know ("hidden information"), or what others are doing ("hidden actions"). According to Salanié [33], it is a simplifying device that models the bargaining situation that is inherent in the bilateral monopolies by allocating the power to one of the parties. Salanié classifies this general set of problems into three families:

- The models where the principal faces hidden information, i.e., she is imperfectly informed of the characteristics of the agent and she moves first. These type of models are known as **adverse selection** models.
- The models where the principal faces hidden information and the agent moves first. These type of models are known as **signaling** models.
- The models where the principal faces hidden action, i.e., she is imperfectly informed of the actions of the agent and she moves first. These type of models are known as **moral hazard** models.

Next, we provide examples for each of these families and discuss how the principal may choose the right contract to offer under each case.

Example 22 *Moral hazard: Contract between a firm and a salesman*

Consider a small firm selling specialized medical equipment via a sales force, which currently consists of a single salesman. The salesman (agent) represents the firm owner (principal) to the clients. The total amount of sales, and hence, the firm's revenues, depend on the efforts of the salesman. If the salesman does not work hard, the sales volumes from the new contracts are low, or potential customers are lost to competitors. Thus, the firm owner would like to design a contract and offer it to the salesman with the goal of providing an incentive to the salesman to work hard, such that both parties will mutually benefit. This situation is an example of the **principal-agent problem**: *The principal designs the terms of the contract and offers it to the agent. The agent decides whether or not to accept the contract. Should the agent accept the* *contract, he or she decides on the level of effort to exert. The firm's revenue is observed, and the principal pays the agent based on the terms agreed upon in the contract.*

Suppose one of the following three possible outcomes are realized based on the effort level of the salesman: (i) The client places no order. (ii) The client places a "small" order, resulting in \$100 revenues. (iii) The client places a "large" order, resulting in \$400 revenues. The probabilities for these different outcomes under each effort level chosen by the salesman is given as follows:

	No order \$0	Small order \$100	Large order \$400	Expected revenues
High effort	0.1	0.3	0.6	\$270
Low effort	0.6	0.3	0.1	\$70

The agent estimates his disutility (cost) for exerting high effort (working hard) as \$25. He incurs no cost or disutility for exerting low effort. If the agent would work somewhere else, rather than working for this firm, he estimates that he would get at least \$81 (for an amount of work that would correspond to his engagement with a potential new client); this is the reservation utility of the agent.

What kind of contract should the principal (firm owner) offer to the agent (salesman)? The principal's revenues (on expectation) are \$270 if the agent works hard, and \$70, otherwise. Therefore, employing this agent is worthwhile to the principal only if the agent works hard; otherwise, the principal gets, on expectation, at most \$70-\$81=-\$11, i.e., loses money. Hence, the principal should design a contract that provides an incentive to the agent to work hard.

First note that the agent will not accept the offer unless his utility from the wage offered exceeds his reservation utility. Let $U(w, e)$ denote the agent's utility for receiving wage w and exerting effort e. In this example, suppose the agent's utility is linear in the wage, i.e., $U(w, e) = w - e$. For the agent to work hard, his utility from working hard should exceed his reservation utility. We have $w - \$25 \geq \$81 \Rightarrow w \geq \$106$. This is called the **individual rationality** or **participation** constraint of the agent, i.e., the agent will not accept the offer unless this offer makes him better off than (or at least as good as) his other alternative, whose value is measured by the reservation utility[9].

[9]For simplicity, we will assume that the agent will accept a contract even if his (expected) payoff is equal to, i.e., does not strictly exceed, his reservation utility.

The "first-best" alternative for the principal would be to offer \$106 (or slightly above it) to the agent since this would maximize the principal's expected profits. However, how can the principal ensure that the agent will work hard? Should she just "trust" the agent and hope that he will work hard? Given that the principal offers him a wage of \$106, what is the action that maximizes the agent's utility? The answer is simple: not working hard. No matter what wage the principal offers the agent, as long as the wage is fixed, the best response of the agent is to exert low effort.

This situation is known as the **moral hazard**: The agent takes a decision or action that affects his or her utility as well as the principal's. The principal only observes the "outcome" (as an imperfect signal of the action taken), and the agent does not necessarily choose the action in the interest of the principal.

One issue with the fixed wage contract is that once the wage is offered, the agent does not have an incentive to work hard. One alternative is to offer a contract where the wage depends on the effort level. For example, the principal could offer two wage rates: w^H if the agent exerts high effort, and w^L if the agent exerts low effort. How should the principal choose w^H and w^L such that accepting the offer and working hard is desirable for the agent? We know that from the individual rationality constraint $w^H - 25 \geq 81$. In addition, once the agent chooses to participate, his utility from working hard should exceed his utility from not working hard. The agent's utility if he chooses not to work hard is $w^L - 0$, i.e., we need $w^H - 25 \geq w^L$ for the agent to prefer working hard. This is known as the **incentive** (**incentive compatibility** or **truth-telling**) constraint. From the individual rationality and incentive constraints, we find that $w^H \geq \max\{106, w^L + 25\}$. Hence, choosing w^H a little above \$106 and $w^L = 0$ (or any number below \$81) would maximize the principal's profits.

Unfortunately, this contract would not be "implementable" in most business situations, because the effort level is either not directly observable, or even if it can be observed, it is very difficult, if not impossible, to enforce. For example, a sales level of \$400 indicates that high effort is more likely, but is not conclusive. Similarly, no sales indicates a high probability of low effort, but again, it is not conclusive. The principal may hire someone else to monitor the agent, but then this means she has another agent to pay and manage, and those two agents might collude against the principal, resulting in further complications and losses in revenue.

While the principal cannot observe the effort level, she can observe the final outcome of the effort, i.e., the output. In this example, the output is the type of order (if any) placed by the client, which can be observed and measured easily. Hence, a contract that conditions the wages on the output, rather than effort level, can provide an incentive to the agent to work hard, and would be easy to implement and enforce.

Consider the following contract offered by the principal to the agent[10]. If the client does not place an order, the wage is -\$164, i.e., the agent pays the principal \$164. If the client places a small order, the wage is -\$64. If the client places a large order, the wage is \$236.

First, let us look at whether the agent would accept this contract, and how he would behave in case of accepting. If the agent rejects the contract, he is left with his reservation utility of \$81. If he accepts the contract and does not work hard, his expected payoff is:

$$(0.1)(236) + (0.3)(-64) + (0.6)(-164) - 0 = -\$94$$

If the agent accepts the contract and works hard, his expected payoff is:

$$(0.6)(236) + (0.3)(-64) + (0.1)(-164) - 25 = \$81$$

Hence, accepting the contract and working hard is the best choice for the agent, resulting in a payoff equal to his reservation utility.

What about the principal's payoffs? In case of no order, the principal gets \$0-(\$164)=\$164. In case of a small order, she gets \$100-(\$64)=\$164. Finally, in case of a large order, she gets \$400-\$236=\$164. Hence, the principal always gets \$164 no matter what the outcome. Note that the principal's expected revenue under high effort is \$270 and the minimum wage we computed in the first-best solution was \$106. Hence, the principal's maximum (expected) profit is \$270-\$106=\$164 under any contract, which is actually achieved in this contract. In general, it is desirable to design implementable contracts that achieve the first-best solution.

In practice, it is unusual to have contracts with negative wages, where workers pay employers under certain outcomes. Hence, let us consider an alternative contract where the wages are always nonnegative regardless of the outcome. In general, how do we find the wages for different output levels to maximize the principal's profits?

[10] This is an unusual contract in which the agent pays the principal under certain outcomes. It could be modified such that the agent's wage is always positive. Nevertheless we will first use this version of the contract to better illustrate our point.

Let w^0, w^S and w^L denote the wages corresponding to no order, small order and large order outcomes. The principal's objective is:

$$\max \quad 0.6(400 - w^L) + 0.3(100 - w^S) + 0.1(0 - w^0).$$

Equivalently, the principal wants to minimize $0.6w^L + 0.3w^S + 0.1w^0$. The solution needs to satisfy the following constraints:

$$0.6w^L + 0.3w^S + 0.1w^0 - 25 \geq 81 \qquad \text{Participation constraint}$$

$$0.6w^L + 0.3w^S + 0.1w^0 - 25 \geq 0.6w^0 + 0.3w^S + 0.1w^L \qquad \text{Incentive constraint}$$

$$w^0, w^S, w^L \geq 0 \qquad \text{Nonnegativity constraints}$$

This is a linear program and can be solved easily by known methods or a software, such as Cplex or Lindo. This particular example has multiple alternative optimal solutions, e.g., $w^L = 118$, $w^S = 117$, and $w^0 = 1$. Note that the high wage is higher than the first-best wage, \$106.

Our simple example about the firm owner and the salesman illustrates why fixed wages sometimes lead to low productivity, and why firms offer bonus payments and other incentives tying the wages to the output levels.

> "In most cases, good customer service comes down to managing people. Some airlines run employee incentive schemes that rely on input from customers [...] Incentives also work well when the employee himself can capture part of the profit. In shops, for example, an employee who is on commission is more likely to force a smile from the most demanding customer than one who is not." (*The Economist*, July 12, 1997) [4]

Examples of the principal-agent framework exists in other business interactions as well. For example, a manufacturer (principal) would prefer its supplier (agent) to exert effort, e.g., by investing in new technologies, to improve the quality of the products.

> "Many purchasing organizations have gone a step further, allowing top-performing suppliers to reap a portion of the benefits gained through quality and other improvements. 'When suppliers increase quality and drive down the total cost of a product,

> [PPG Industries Inc.] will split the benefits of those efforts with them, 50-50,' says Ludlow [vice president of purchasing and distribution]." (*Purchasing*, January 15, 1998) [29]

It is interesting to note the common theme in the above examples of the principal-agent problem and in supply chain coordination. In both cases, the design of the contracts aligns the incentives of the participants with the overall goals of the system.

Example 23 *Adverse selection: Contract between a monopolist and his customer*

Consider a local monopolist wine seller who can produce wine of any quality $q \in (0, \infty)$[11]. The cost of producing a bottle of wine with quality q is $C(q)$, where C is twice differentiable, strictly convex, $C^{'}(0) = 0$, and $C^{'}(\infty) = \infty$. The price she charges per bottle is t dollars, therefore, her profit is $t - C(q)$.

A customer is planning to buy a bottle of wine and visits the local monopolist. His utility U from a bottle of wine of quality q is a function of his taste for quality θ and the price of the wine, i.e., $U = \theta q - t$, where θ is a positive parameter. If he decides not to buy any wine, his utility is 0. The consumer can either be a sophisticated customer (which we index by "1") or he may be a coarse customer (which we index by "2"). The values of θ are such that $\theta_1 < \theta_2$ and the prior probability that the consumer is of type 1 is p.

This is a dynamic game of incomplete information where the seller moves first and offers a contract (possibly a collection of (price, quality) pairs). The seller does not know an arriving customer's type, and hence needs to make a decision based on her beliefs about the customer's type. The customer moves next and chooses the (price, quality) pair that maximizes his utility.

Ideally, the seller would like to offer a (price, quality) pair to each consumer depending on the consumer's type, to achieve maximum profitability. Such a "first-best" solution could be achieved if the seller had complete information about each customer's type, and this solution would provide an upper bound to the seller's profits for the more interesting incomplete information case.

Complete Information: If the seller knows the type θ_i of the consumer, she will solve the following optimization problem:

$$\max_{q_i, t_i} \quad (t_i - C(q_i))$$

[11] Based on an example from Salanié [33].

$$s.t. \quad \theta_i q_i - t_i \geq 0$$

The Lagrangean for this problem is $\mathcal{L}_i = (t_i - C(q_i)) + \lambda_i (\theta_i q_i - t_i)$:

$$\begin{aligned} \frac{\partial \mathcal{L}_i}{\partial q_i} &= -C'(q_i) + \lambda_i \theta_i = 0 \\ \frac{\partial \mathcal{L}_i}{\partial \lambda} &= \theta_i q_i - t_i = 0 \end{aligned}$$

Therefore, the monopolist will offer a quality level $q_i = q_i^*$ such that $C^{'}(q_i^*) = \lambda_i \theta_i$ and $\theta_i q_i^* = t_i^*$. At optimality, the customer's utility will be equal to 0 and the monopolist will extract all the surplus. This is called **first-degree price discrimination** and is analogous to the "first-best" solution in Example 22.

Incomplete Information: Now, let us consider the original problem where the seller does not know the actual type of the customer but knows that the proportion of coarse consumers is p. Let us assume that she proposes the first-best contracts (q_1^*, t_1^*) and (q_2^*, t_2^*) and lets the consumers choose whichever contract they prefer. In this case a sophisticated consumer will not choose (q_2^*, t_2^*) since he gets a higher utility by buying the low quality wine:

$$\theta_2 q_1^* - t_1^* = (\theta_2 - \theta_1) q_1^* > 0 = \theta_2 q_2^* - t_2^*$$

Therefore, both types of customers will choose to buy the low quality wine and the seller will not be able to separate the two types. He has to reformulate his optimization problem and make sure that he can separate the customer types. The principal's objective is:

$$\max_{q_1, t_1, q_2, t_2} \quad p\,(t_1 - C(q_1)) + (1-p)\,(t_2 - C(q_2))\,.$$

The solution needs to satisfy the following constraints:

$$\begin{array}{cll} \theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2 & (IC_1) & \text{Incentive constraint for type 1} \\ \theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 & (IC_2) & \text{Incentive constraint for type 2} \\ \theta_1 q_1 - t_1 \geq 0 & (IR_1) & \text{Participation constraint for type 1} \\ \theta_2 q_2 - t_2 \geq 0 & (IR_2) & \text{Participation constraint for type 2} \\ q_1, t_1, q_2, t_2 \geq 0 & & \text{Nonnegativity constraints} \end{array}$$

In the optimal contract (IR_1) is active, i.e., $t_1 = \theta_1 q_1$, (IC_2) is active, i.e., $t_2 - t_1 = \theta_2(q_2 - q_1)$, and $q_2 \geq q_1$. We can neglect (IC_1) and (IR_2). Furthermore, sophisticated consumers buy the efficient quality, i.e., $q_2 = q_2^*$. Therefore, at optimality $t_1 = \theta_1 q_1$ and $t_2 = \theta_1 q_1 + \theta_2(q_2^* - q_1)$. We can substitute these values in the expression of the monopolist's profit and solve

$$\max_{q_1 \geq 0} \quad \{p\,(\theta_1 q_1 - C(q_1)) + (1-p)\,(\theta_2 - \theta_1)\,q_1\}$$

From FOC, we get

$$C^{'}(q_1) = \theta_1 - \left(\frac{1-p}{p}\right)(\theta_2 - \theta_1).$$

As $\theta_2 > \theta_1$, we get $C^{'}(q_1) < \theta_1$. Therefore, the quality sold to the coarse consumer is inefficient.

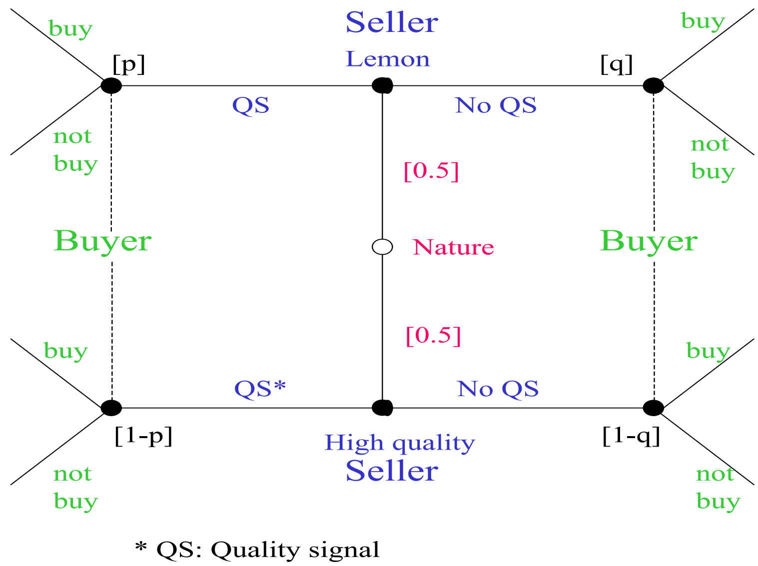
Example 24 *Signaling: Used car market*

Assume that you are selling your used car through an internet site. The used cars can be of two types, good-quality and lemon, and your's is good-quality. From your preliminary market analysis, you know that the average buyer would be willing to pay \$5,000 for a good-quality car and \$3,000 for a lemon. As for you, the value of your car is \$4500.

As the potential buyers will not be able to observe the quality of your car, their valuation of your car will not be \$5000. Assume that they believe there is a 50% chance that any used car on this site is a lemon. Therefore, the buyers will not offer more than $\frac{5000+3000}{2} = \$4000$. This amount is below your valuation, and therefore, as a seller of a good-quality car, you will end up not selling your product, but a lemon owner will be able to sell his.

> "Signalling is used in many markets, wherever a person, company or government wants to provide information about its intentions or strengths indirectly. Taking on debt might signal that a company is confident about future profits. Brands send valuable signals to consumers precisely because they are costly to create, and thus will not be lightly abused by their creators." (*The Economists*, October 11, 2001) [6]

Now let us go back to our example and assume that as a signal of the quality of his car, the seller may post an evaluation of his car by a well-known mechanics chain. We can represent this game with the following extensive form:



In a **pooling strategy** the agent would play the same strategy regardless of his type. In our example, if the seller sends the quality signal when he has a quality car as well as a lemon, then this would be a pooling strategy. In a **separating strategy** the agent would choose different actions for different types. In our example, the seller may choose to send the quality signal when he has a quality car, but not when he has a lemon and this would be a separating strategy.

9 Concluding Remarks

Game theory helps us model, analyze, and understand the behavior of multiple self-interested agents who interact while making their decisions. In particular, it is a powerful tool for analyzing situations where the agents strive to maximize their (expected) payoffs while choosing their strategies, and each agent's final payoffs depend on the profile of strategies chosen by all agents. Most business situations can be modelled by a "game," since in any business interaction involving two or more participants the payoffs of each participant depend on the other participants' actions.

The general methods and concepts offered by game theory, such as strategies and equilibrium, can be applied and provide useful insights in many applications of business and social sciences. In this chapter we provided an introductory overview of basic concepts in game theory and gave examples of their applications in supply chain management and other business settings.

> "Having game theory in your corporate 'bag of tricks' can mean the difference between success and failure." (*Investor's Business Daily*, January 25, 1996) [9]

> "Managers have much to learn from game theory – provided they use it to clarify their thinking, not as a substitute for business experience." (*The Economist*, June 15, 1996) [5]

There are many powerful tools, such as simulation, optimization, and decision analysis, which could aid managers in their decisions. We believe the use of game theory along with these other tools and the managers' business experience could significantly improve the managers' understanding of the dynamics in business interactions and lead to higher quality and more informed decisions.

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