Mathematical foundations of Econometrics

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Required mathematical and Statistical background

- Probability
- Random variables and probability distributions
- Normal probability distribution, related PDFs
- Classical statistical inference
- Estimators
- Sampling distributions from a normal population
- Interval estimation
- Confidence intervals and tests of hypotheses
- Combining independent tests

References

Mathematical and statistical foundations

- Bierens J. Herman, 2005, *Introduction to the mathematical and statistical foundations of Econometrics*, Themes in modern Econometrics, Cambridge University Press.
- White Halbert, 2000, *Asymptotic theory for Econometricians*, Academic Press.
- Pitman Jim, 1997, *Probability*, Springer texts in statistics, Springer

Econometrics

- Wooldridge M. Jeffrey, 2012, *Introductory Econometrics: A Modern Approach*, South -Western Pub
- Stock H. James & Watson W. Mark, 2010, *Introduction to Econometrics*, Pearson
- Maddala G.S. & Lahiri Kajal, *Introduction to Econometrics*, Wiley

Motivations

This course provides the statistical and probability theoretic foundations of econometrics, and will have practical value to Economics, Finance and Statistics students.

The principal goals are:

- to present some basic notions of statistical, mathematical and probability theory necessary for understanding regression
- a more in depth understanding of the underlying concepts of Econometrics

The short run goal of the course is for the student to understand the implications in Econometrics of measure theory, probability theory, mathematical expectation, modes of convergence, limit theorems and asymptotics.

Probability

The notion of probability has been widely debated. Maybe the most famous quote with direct notions in probability is "Alea iacta est ", quoted by Julius Caesar on January 10, 49 BC as he led his army across the Rubicon river in Northern Italy.

Classical view
Gerolamo Cardano (1501-1576) was the first to systematically compute probabilities, giving the following definition:

$$\text{probability} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Frequentistic view *Poisson* (1837) gave a different definition. If n is the number of trials and $n(E)$ the number of occurrences of an event E, then the probability of E is:

$$P(E) = \lim_{n\to\infty} \frac{n(E)}{n}$$

Therefore, according to the frequentistic view, probability is a limit rather than a theoretical number.

Probability

Subjective view
Subjective probability is based on personal beliefs (Bruno de Finetti). Suppose a game where someone wins q units if event A occurs and loses p units if A does not occur. If the person is willing to play the bet, his subjective probability for the occurance of A is at least equal to $\frac{p}{q+p}$. Following this notion we can find which his objective probability is.

Axiomatic definition of probability
Kolmogorov (1933) introduced a new view of probability, according to which first we define a measurable space $\{\Omega, \mathscr{F}\}$ and then we specify a probability measure P in it. The probability P must satisfy three basic axioms.

Our lesson is based on Kolmogorov's view of probability.

The Lotto game

In a LOTTO game, we draw (without replacement) six (6) numbers out of a "pool" of fifty (50) equally probable ones. The one who bets on the 6 selected numbers wins.

To find how probable it is for 6 numbers to be the winning ones, we calculate how many possible *unordered sets* exist ie.

$$\binom{50}{6} = \frac{50!}{6!(50-6)!}$$

$\frac{50!}{(50-6)!}$ represents the number of possible *ordered sets* there are, while 6! is the number of ordered sets that corresponds to just one unordered.

Simple questions

1. Do we understand what the **sample space** of the game is? Is, for example, a $\{21\}$ a possible outcome of the game?
2. Generally, which is the mathematical framework in which we assign probabilities?

Basic definitions

Definition

Experiment is any process whose result is not known in advance with certainty. **Outcome** ω is defined as a result of *one trial* in a statistical experiment.

Therefore, a (21) **is not** a possible outcome of the particular game, but the unordered set (21, 22, 23, 24, 25, 26) can.

Definition

The set Ω of possible outcomes ω of a statistical experiment is called **sample space**.

Note: The outcomes are not necessarily numerical.

Definition

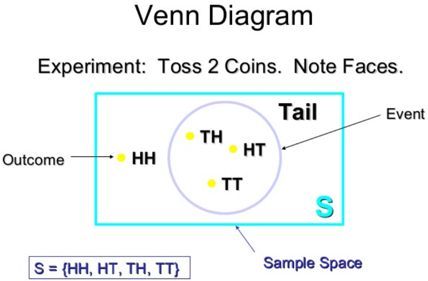
An **event** E is any subset of outcomes from the subspace Ω, ie. it is a collection of outcomes from Ω.

For example, all unordered 6-numbered sets which contain 21, is an event, i.e. $E = \{\omega \in \Omega_{Lotto} | 21 \in \omega\}$

Example on the above definitions

Experiment:
Consider the game of tossing a fair coin two times.

Questions:
What can be an outcome of the game? Can just "heads" H or HTH be a possible outcome?
Which is the sample space Ω of the game? *answer*: $\Omega = \{HH, HT, TH, TT\}$
What can be an event of this game?



Understanding sample space

Example: Survey 10 people on their employment status and count the number of unemployed.

Questions:
Which are the possible outcomes?
Which is the sample space of this "experiment"?
Tell me an event of the experiment.
Which is the event {more than 40% of the sample are unemployed }?
Can we show an outcome, an event and the sample space of this experiment in a *Venn diagram*?

Algebras and σ-algebras

Definition

A collection $\mathscr{F}$ of events of a non-empty set Ω satisfying the following conditions is called an *algebra*.

1. $\Omega \in \mathscr{F}$
2. If $\mathrm{A} \in \mathscr{F}$ then $\tilde{\mathrm{A}} \in \mathscr{F}$, where $\tilde{\mathrm{A}}$ is the *complement* of A, i.e. $\tilde{\mathrm{A}} = \Omega \backslash \mathrm{A}$
   This property is called *"closed under complements"*.
3. If $\mathrm{A}, \mathrm{B} \in \mathscr{F}$ then $\mathrm{A} \cup \mathrm{B} \in \mathscr{F}$. By induction, the condition extends to any **finite** union of sets in $\mathscr{F}$.
   This property is called *"closed under finite unions"*.

Algebras and σ-algebras

Definition

A collection $\mathscr{F}$ of subsets of a non-empty set Ω satisfying the following conditions is called a σ-*algebra*.

1. $\Omega \in \mathscr{F}$
2. If $\mathrm{A} \in \mathscr{F}$ then $\tilde{\mathrm{A}} \in \mathscr{F}$, where $\tilde{\mathrm{A}}$ is the *complement* of A, i.e. $\tilde{\mathrm{A}} = \Omega \backslash \mathrm{A}$ (*"closed under complements"*)
3. If $\mathrm{A}_j \in \mathscr{F}$ for $j = 1, 2, 3, \ldots$ then $\cup_{j=1}^{\infty} \mathrm{A}_j \in \mathscr{F}$. (*"closed under countable unions"*)

Therefore, a σ-*algebra* is also an *algebra*. Moreover, a finite *algebra* is also a σ-*algebra*.

Note: σ-*algebra* provides us the "questions" we are allowed to make in a game. We will see more of it later...