Mathematical foundations of Econometrics

March 23, 2016

Exercise 1

Start with this simple question, addressed by **Tversky and Kahneman** (Nobel prize laureate 2002) to 260 students who had not studied probability.

A six-sided die has four green and two red faces and is balanced so that each face is equally likely to come up. The die will be rolled several times. You must choose one of the following three sequences of colours; you will win $25 if the first rolls of the die give the sequence that you have chosen.

$$\{R, G, R, R, R\}$$

$$\{R, G, R, R, R, G\}$$

$$\{R, G, R, R, R, R\}$$

Without making calculations, explain which sequence you choose.

Note: 63% of the students chose the second

Exercise 2

You play draughts against an opponent who is your equal. Which of the following is more likely: (a) winning three games out of four or winning five out of eight; (b) winning at least three out of four or at least five out of eight?

Hint: Recall the Binomial distribution to count the probability of exactly k successes from n trials.

Exercise 3

An examination consists of multiple-choice questions, each having five possible answers. Suppose you are a student taking the exam and you reckon you have probability 0.75 of knowing the answer to any question that may be asked and that, if you do not know, you intend to guess an answer with probability $1/5$ of being correct. What is the probability you will give the correct answer to a question?

Hint: use law of total probability partitioning in knowing or not-knowing the correct answer.

The Monty Hall problem

In Monty Hall's successful TV show "Let's make a deal" (1963-1976), the participant was asked to choose among three closed doors. Behind one door was a car and behind the other two was nothing. After an initial choice of door, Monty Hall always opens one that does not have the car and then poses the question: Will you change your choice?

Our question: Which is the best strategy, changing our initial choice of door or not?

Exercise 4

I have in my pocket ten coins. Nine of them are ordinary coins with equal chances of coming up head and tail when tossed and the tenth has two heads.

- If I take one of the coins at random from my pocket, what is the probability that it is the coin with two heads ?
- If I toss the coin and it comes up heads, what is the probability that it is the coin with two heads ?
- If I toss the coin one further time and it comes up heads, what is the probability that it is one of the nine ordinary coins ?

Hint: use of Bayes' rule

Exercise 5

Show the following:

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $(A \cup B)^c = A^c \cap B^c$ (De Morgan's law)
- $(A \cap B)^c = A^c \cup B^c$ (De Morgan's law)

Hint: Use of Venn diagrams

Exercise 6

A certain person considers that he can drink and drive: usually he believes he has a negligible chance of being involved in an accident, whereas he believes that if he drinks two pints of beer, his chance of being involved in an accident on the way home is only one in five hundred. Assuming that he drives home from the same pub every Friday and Saturday, having drunk two pints of beer, what is the chance that he is involved in at least one accident in one year? Are there any assumptions that you make in answering the question?

Hint: Use complementarity of events and the assumption of independence between days

Exercise 7

Suppose there are 10 cards from 1 to 10 and we pick one. If A is the event that "the result is odds" and B is the event that "the result was less than 4", compute the following probabilities:

- $P(A|B)$
- $P(B|A)$ Are the two events independent? Disjoint?
- If odds **or** less than 4 appeared, what is the probability that odds occurred?
- If odds **and** less than 4 appeared, what is the probability that odds occurred?
- What is the probability of both events occur, given that at least one of them occurred?

Exercise 8

An urn contains r red balls and b blue balls, $r \geq 1$, $b \geq 3$. Three balls are selected, without replacement, from the urn. Using the notion of conditional probability to simplify the problem, find the probability of the sequence Blue, Red, Blue.

Hint: use of the **chain rule** as follows for $n = 3$.

$$P(\bigcap_{k=1}^{n} A_k) = \prod_{k=1}^{n} P(A_k | \bigcap_{j=1}^{k-1} A_j)$$

From where it stems?

Exercise 9

In a certain town, 30% of the people are Conservatives; 50% Socialists; and 20% Liberals. In this town at the last election, 65% of Conservatives voted, as did 82% of the Socialists and 50% of the Liberals. A person from the town is selected at random, and states that she voted at the last election. What is the probability that she is a Socialist?

Hint: Define the relevant events and then apply Baye's rule. Otherwise, recall the definition of conditional probability along with the law of total probability.

Exercise 10

A student is giving two exams X and Y. The probabilities of passing each exam are $P(A) = 0.8$ and $P(B) = 0.9$ respectively. The probability of passing both is $P(A \cap B) = 0.75$.

Compute the probabilities of the events $A \cup B$, $A^c \cup B^c$, $A \cup B^c$, $A^c \cap B^c$, $(A \cup B)^c$. Describe each one of these.

Hint: Use of Venn diagram to simplify the form of the events

Exercise 11

In a sample space Ω two events A and B were defined so that $P(A) = 0.7$ and $P(A \cup B) = 0.8$. Compute $P(B)$ if:

- A and B are disjoint
- A and B are independent
- $P(A|B) = 0.6$

Exercise 12

In a village 75% of the women get married for the first time before the age of 30. The probability of getting married for the first time between the age of 30 and 40 diminishes by 50%. Also, the probability of getting married for the first time above the age of 40 is a quarter of the probability of the previous group.

What is the probability of a woman NEVER getting married?