Game theory and the real world

Structure of a Game

- Players
- Strategies
- Pay off

In Game theory models those are fixed elements.

In the business world?

US abolished Advertising for tobacco in 1971

And Profit boom……

example

- Market for Cigarette worth 3 billion $ (without advertising)
- Suppose there are two firms that can spend 500 million $ in advertising
- A firm increase market share (5%) if advertises. If the other doesn’t the first take 80% of the market. If both do or don’t they share the market equally.

Strategie dominanti

- For both firms advertising is a dominat strategy

	adv	No adv
adv	1,15 – 1,15	2,02 – 0,630
No adv	0,63 – 2,02	1,5 – 1,5

US Govern helped firms avoiding the possibility to deviate

truel

Sometime the best doesn’t win….

rules

3 gunmen: Adam, Bob e Charlie

- The firing order is randomly decided.
- Firing process continue till there is only one alive.
- Every shot if arrive to the target are lethal. Adam hits with 100% probability, Bob 80%, Charlie50%.
- Every player decides his own strategy.
- You can always decide to fire in the air.

question

Who has the highest chance to survive?

probabilities

- P(A hits) = 1
- P(A misses) = 0
- P(B hits) = 4/5
- P(B misses) = 1/5
- P(C hits) = 1/2
- P(C misses) = 1/2

definition

- P(S,XYZ). Probability that a player survives with this firing order.

duels

Fire order	S (A)	S (B)	S (C)	explanation
AB	P(A,AB) = 1	P(B,AB) = 0	P(C,AB) = 0	A never miss
AC	P(A,AC) = 1	P(B,AC) = 0	P(C,AC) = 0	A never miss
BA	P(A,BA) = 1/5	P(B,BA) = 4/5	P(C,BA) = 0	P(A,BA) = P(B misses) × P(A,AB) = 1/5. P(B,BA) = P(B hits) + P(B misses) × P(B,AB) = 4/5.
BC	P(A,BC) = 0	P(B,BC) = 8/9	P(C,BC) = 1/9	P(B,BC) = P(B hits) + P(B misses) × P(C misses) × P(B,BC) = 4/5 + 1/5 × 1/2 × P(B,BC), which gives P(B,BC) = 8/9. P(C,BC) = P(B misses) × P(C hits) + P(B misses) × P(C misses) × P(C,BC) = 1/5 × 1/2 + 1/5 × 1/2 × P(C,BC), which gives P(C,BC) = 1/9.
CA	P(A,CA) = 1/2	P(B,CA) = 0	P(C,CA) = 1/2	P(A,CA) = P(C misses) = 1/2. P(C,CA) = P(C hits) = 1/2.
CB	P(A,CB) = 0	P(B,CB) = 4/9	P(C,CB) = 5/9	P(B,CB) = P(C misses) × P(B hits) + P(C misses) × P(B misses) × P(B,CB) = 1/2 × 4/5 + 1/2 × 1/5 × P(B,CB), which gives P(B,CB) = 4/9. P(C,CB) = P(C hits) + P(C misses) × P(B misses) × P(C,CB) = 1/2 + 1/2 × 1/5 × P(C,CB), which gives P(C,CB) = 5/9.

Shooting order:	S when I fire A	S when I fire B	S when I Fire C	S if I miss.	Conclusion:
ABC	0	**P(A hits) × P(A,CA) + P(A misses) × P(A,BCA) = 1/2**	P(A hits) × P(A,BA) + P(A misses) × P(A,BCA) = 1/5	P(A,BCA) < 1/2 (since B will definitely shoot at A, because P(B,AB) = 0!)	So A shoots B, which means that P(A,ABC) = 1/2, P(B,ABC) = 0, and P(C,ABC) = P(C,CA) = 1/2
ACB	0	**P(A hits) × P(A,CA) + P(A misses) × P(A,BCA) = 1/2**	P(A hits) × P(A,BA) + P(A misses) × P(A,BCA) = 1/5	P(A,CBA) < 1/2 (since B will definitely shoot at A, because P(B,ABC) = 0!)	So A shoots B, which means that P(A,ACB) = 1/2, P(B,ACB) = 0, and P(C,ACB) = P(C,CA) = 1/2
BAC	**P(B hits) × P(B,CB) + P(B misses) × P(B,ACB) = 16/45**	0	P(B hits) × P(B,AB) + P(B misses) × P(B,ACB) = 0	P(B,ACB) = 0	So B shoots A, which means that P(B,BAC) = 16/45, P(A,BAC) = P(B misses) × P(A,ACB) = 1/10, and P(C,BAC) = P(B hits) × P(C,CB) + P(B misses) × P(C,ACB) = 49/90
CAB	P(C hits) × P(C,BC) + P(C misses) × P(C,ABC) = 11/36	P(C hits) × P(C,AC) + P(C misses) × P(C,ABC) = 1/4	0	**P(C,ABC) = 1/2**	So C should miss deliberately (fire "into the air"), which means that P(C,CAB) = 1/2, P(A,CAB) = P(A,ABC) = 1/2, and P(B,CAB) = P(B,ABC) = 0
BCA	**P(B hits) × P(B,CB) + P(B misses) × P(B,CAB) = 16/45**	0	P(B hits) × P(B,AB) + P(B misses) × P(B,CAB) = 0	P(B,CAB) = 0	So B shoots A, which means that P(B,BCA) = 16/45, P(A,BCA) = P(B misses) × P(A,CAB) = 1/10, and P(C,BCA) = P(B hits) × P(C,CB) + P(B misses) × P(C,CAB) = 49/90
CBA	P(C hits) × P(C,BC) + P(C misses) × P(C,BAC) = 59/180	P(C hits) × P(C,AC) + P(C misses) × P(C,BAC) = 49/180	0	**P(C,BAC) = 49/90**	So C should miss deliberately (fire "into the air"), which means that P(C,CBA) = 49/90, P(A,CBA) = P(A,BAC) = 1/10, and P(B,CBA) = P(B,BAC) = 16/45

Total probabilities

Shooting order:	Survival chance of A:	Survival chance of B:	Survival chance of C:
ABC	P(A,ABC) = 1/2	P(B,ABC) = 0	P(C,ABC) = 1/2
ACB	P(A,ACB) = 1/2	P(B,ACB) = 0	P(C,ACB) = 1/2
BAC	P(A,BAC) = 1/10	P(B,BAC) = 16/45	P(C,BAC) = 49/90
CAB	P(A,CAB) = 1/2	P(B,CAB) = 0	P(C,CAB) = 1/2
BCA	P(A,BCA) = 1/10	P(B,BCA) = 16/45	P(C,BCA) = 49/90
CBA	P(A,CBA) = 1/10	P(B,CBA) = 16/45	P(C,CBA) = 49/90
Total survival chances (sum of the probabilities divided by 6):	27/90	16/90	47/90

What can we learn?

- If you are in a weak position in a competition where the winner takes all is better to have a waiting strategy
- Sometime is better to hide yourself … if this is the case is better to pretend to be a worse shooter than you really are.

When is better to hide yourself…

- If you decide not to do a project it is better that nobody does it.…
- Follow the mob (speculative bubbles….)
- Fix impossible goals.

Macy Buy Out Federated Stores

Game Theory in the real world

Macy Buy Out Federated Stores

- In 1988 Macy (M) wants to buy Federated Stores (FS) that controls Bloomingdale’s shops chain
- Stock value of FS is 100$
- M offers 102$ to shareholders only if he can buy 50% of the company.
- Shareholders prefer 102$ to 100$ but if M don’t get 50% of shares they still have their own shares with value 100$

Also Robert Campeau (C) wants FS

- C makes a different offer
- C Offers 105$ till he gets 50%. The shares are set aside, and are payed on a Pro rata base (no first in first serve).
- For the shares in addition to 50% he is ready to pay only 90$.
- If C gets 50% can delist the company paying only 90$.

possibilities

1. C two-tiered offer attracts less than 50%
2. C offer attracts some amounts above 50%
3. C offer attract exactly 50%. If you tender C win, if not, lose.

1. C two-tiered offer attracts less than 50%

- In this case you get 100$ if both tenders fail or 102$ per share if the other tender succeeds
- But if you tender you get 105$

2. C offer attracts some amounts above 50%

- If you don’t tender you get 90$
- If you tender you get at worst 97,5$

(105$ *0,5+90$*0,5)

3. C offer attract exactly 50%. If you tender C win, if not, lose.

- Other people are worse off if you tender (90$ is less than 100$)
- But you are better off because you get 105$

C strategy dominate M strategy

- If M gets 50% C strategy is better because 105$ is more than 102$
- If C gets 50% the medium price is 97,5$ that is better to hold shares of the delisted company
- If C don’t get 50% it is always better to sell to him because 105$ is more than market value 100$

SO…

- C win even if his offer is on average worse than the other (97,5$ vs 102$)
- But why shareholders sell to the one who offers less?
- Because C has been able to create a sort of Prisoner Dilemma
- A conditional bid is defeated by a two-tiered bid.

What could do Macy's?

Make an unconditional bid at 102$

Why?

Shareholders understand that f C wins they get 97,5$ on average, not 105$

Advertising competition

- **"why do manufacturers and retailers prefer to offer substantial price reductions for a short period of time and then raise the price to its normal level rather than permanently reduce prices by less than the deal size?" (Blattberg, Eppen and Lieberman 1981).**

- **in case of the beverage industry, it has been reported that in any given week, either Coke or Pepsi is available on promotion and each is on promotion 26 weeks of the year (CBS 60 Minutes). A similar observation is made by Krishna (1988) over a period of 3 months where Pepsi and Coke were available on promotion in alternat-ing weeks.**

Which model?

- Prisoner's Dilemma?
- Repeated Prisoner's Dilemma (∞)?
- Price discrimination?
- Bertrand with differentiated products?
- Equilibria in mixed strategies?
- How many players?

- **Kinberg, Rao and Shakun (1974) also construct a model with two types of consumers, but with two premium brands and a private label to show that the cooperative solution between the premium brands leads to promotions by only one of the premium brands in any given period. In their analysis, quality conscious consumers always buy the most expensive brand available within the range of acceptable prices and the price conscious consumers buy the lower priced brand. Assuming that the price of the private label is fixed, they show that the cooperative solution between the premium brands is not to promote in the same period.**

hypothesis

- Some consumers are very loyal to a brand and therefore not very sensitive to price decreases of the other
- Others are more price sensitive

BERTRAND MODEL

- in Bertrand model with Homogeneous products:
  - firms start “undercutting”. This is good for consumers but bad for firms.
  - only equilibrium price:

    price = marginal cost ⟶ equilibrium is “Pereto efficient”
  - firms have zero profit.
  - there is an incentive for collusion ⟶ but this is not an equilibrium for this market

- Bertrand Model with differentiated products :
  - equilibrium price : price > marginal cost
  - firms has positive profit.
  - there is an incentive for product differentiation ⟶ “no Pareto efficient”.

Coca-Cola e Pepsi: prisoner's dilemma for one period

		STRATEGIA DI COCA-COLA	
		No advertising	**advertising**
STRATEGIA DI PEPSI	**No advertising**	$ 10.000 for each firm	C: $ 13.000 P:$ 4.000
	advertising	C: $ 4.000 P: $ 13.000	$ 8.000 for each firm

- Both advertise even if they could be more profitable without advertising.
- Why they don't simultaneously advertise?

If Bertrand (∞)?

- Go back to Repeated Bertrand

But we have to consider:

- profit is not homogeneous
- There are other unbranded market on the market with lower prices
- It is possible to demonstrate that a strategy based on alternate promotions is a Nash equilibrium

It is possible to demonstrate that a strategy based on alternate promotions is a Nash equilibrium

Why?

- If they both do promotions no one get a larger slice of the pie (a bit larger because we will attract some consumer that look for lower prices) so we simply lose money.
- If we do it not in the same period when we don't do promotion we keep our more loyal consumers.
- When we do promote alone we also keep some clients that look for lower prices.
- If there is always a branded product on sale this will limit the entry of new products.

Lets consider uncetainty in the real word

Deal or no deal

Deal or no deal

Deal or not Deal

- You are quite lucky, you are at the end of the game, with 3 boxes to be opened, your included
- Of course you don’t know what you have in your Box.
- You know the value of the 3 boxes: one has 1 bottle of low price beer, one 1€ and one 500.000€
- (to make things simple let's suppose that the value of the first 2 boxes is 0)
- The TV host open one of the two boxes you don’t have and show you that inside there is a can of beer.

Deal or no deal

- The TV host for the last time asks you if you want to change box or if you keep the one you have.
- What will you do?
- Change or not change?
- In economics / probability terms, changing box increase decrease, or doesn’t change the probability to have the box with 500.000€?

So what?

This is the diagram when you have no information

0

0

500,000

0

O

500,000

0

0

500,000

1/3 possibilities to have the winning box with 500.00€

Here when you know where is one of the two boxes with 0 inside

0
500,000

0

0
500,000

You have 50% probability to win 500.000€

Now the TV host offers you to change box

if the real situation is

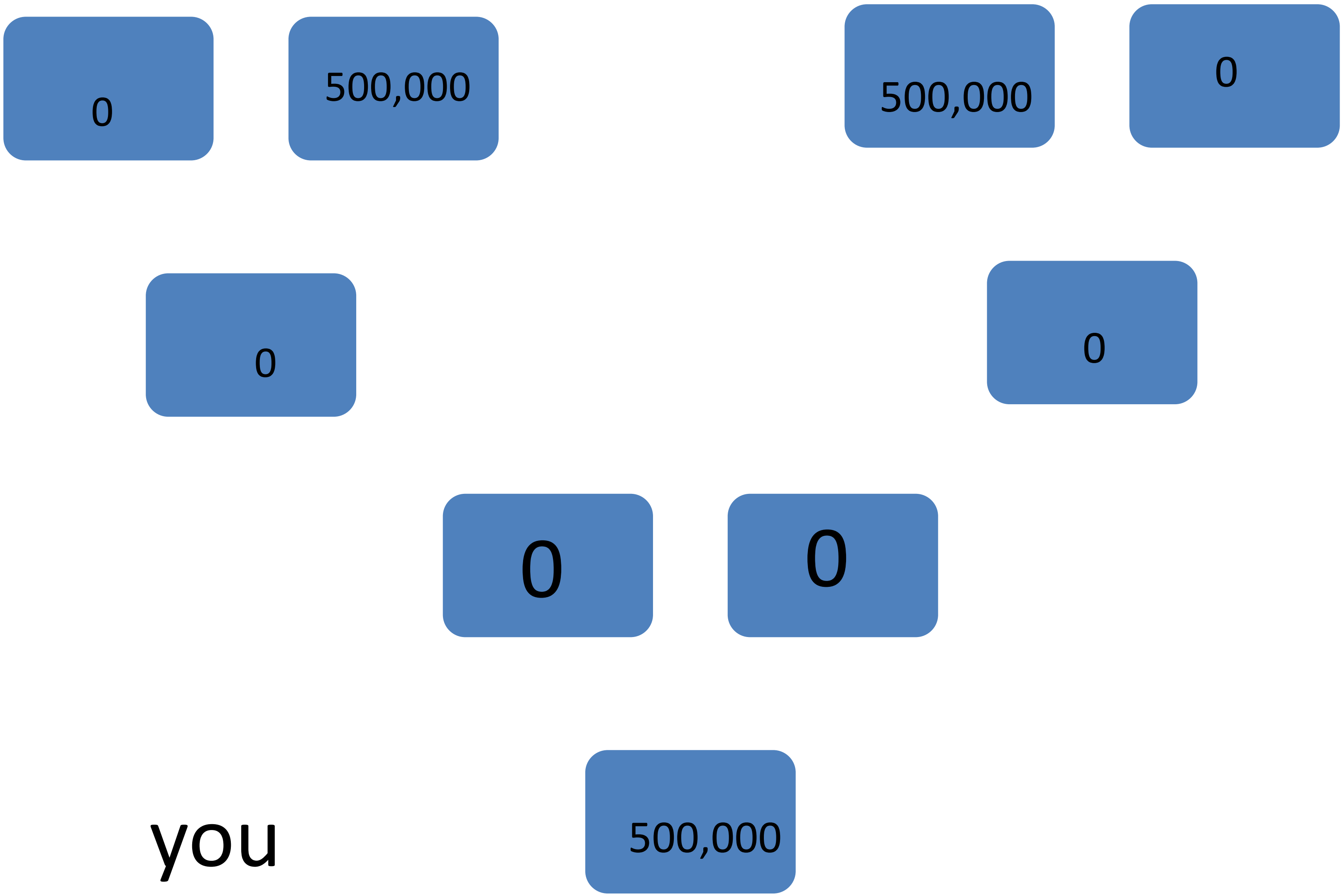
0

0

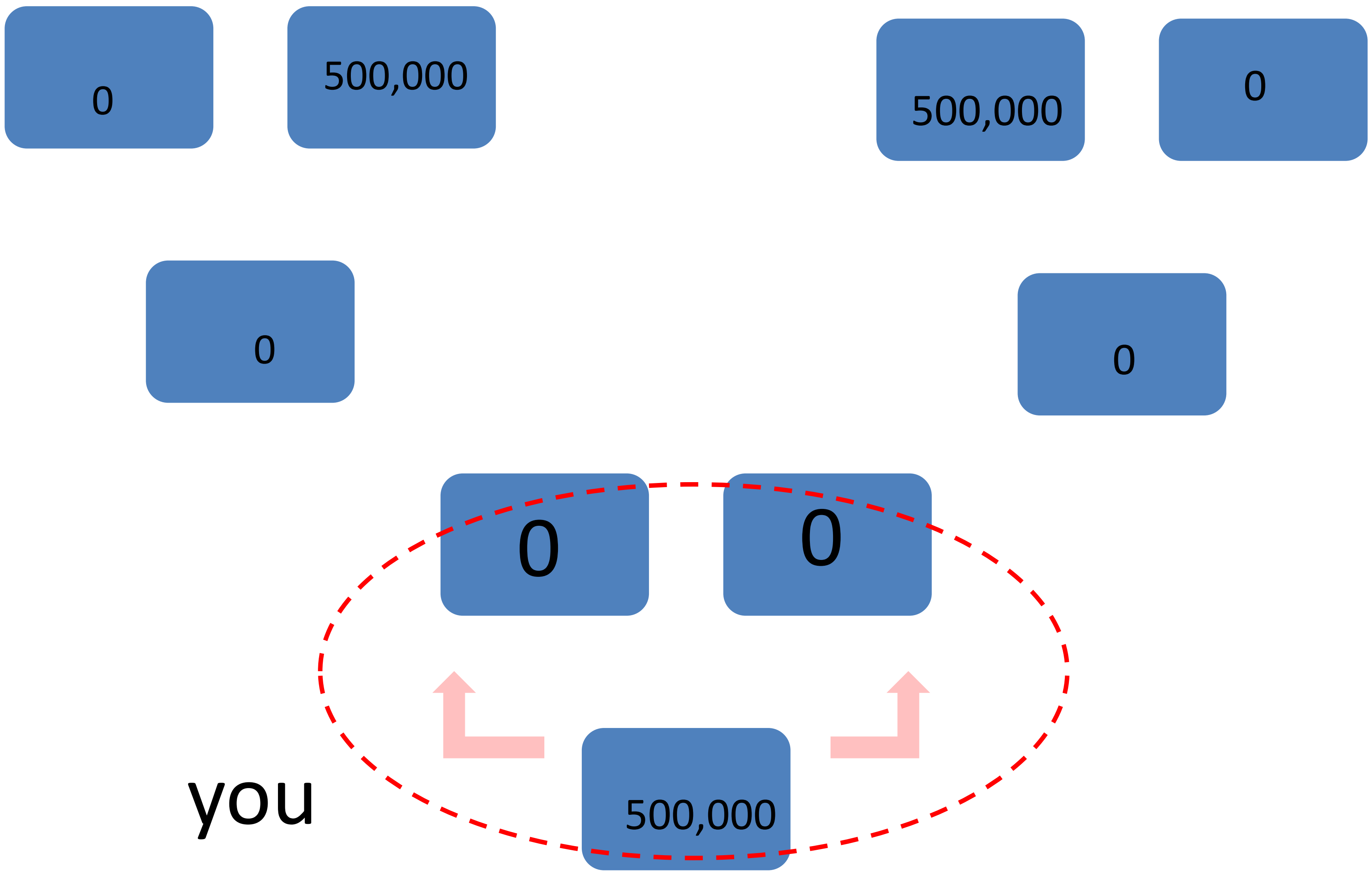
500,000

if you change you lose. With probabilities?

If you have 500.000 and change you get 0

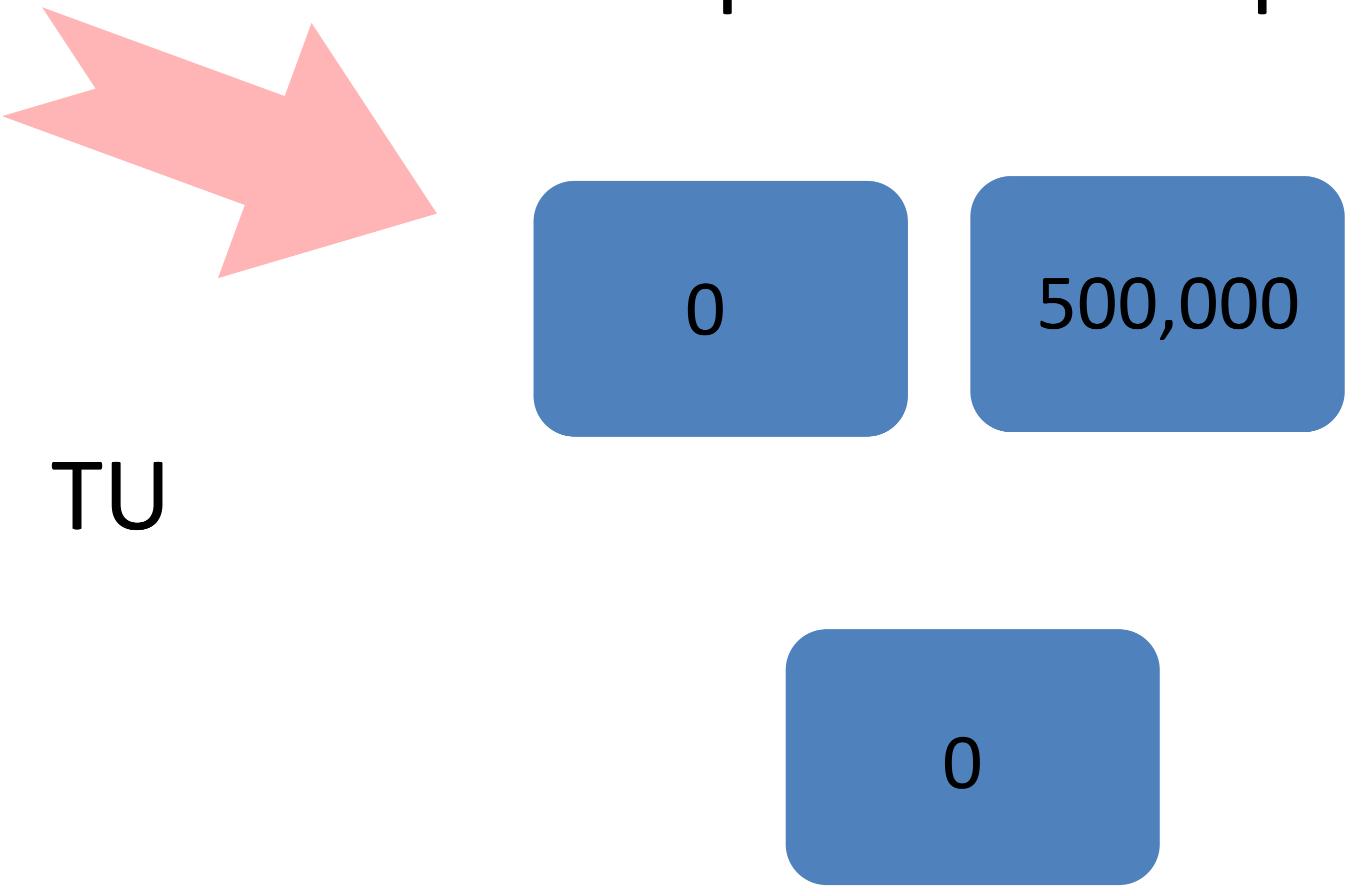


If you have the 500.000€ and change you get 0 with probability 1/3

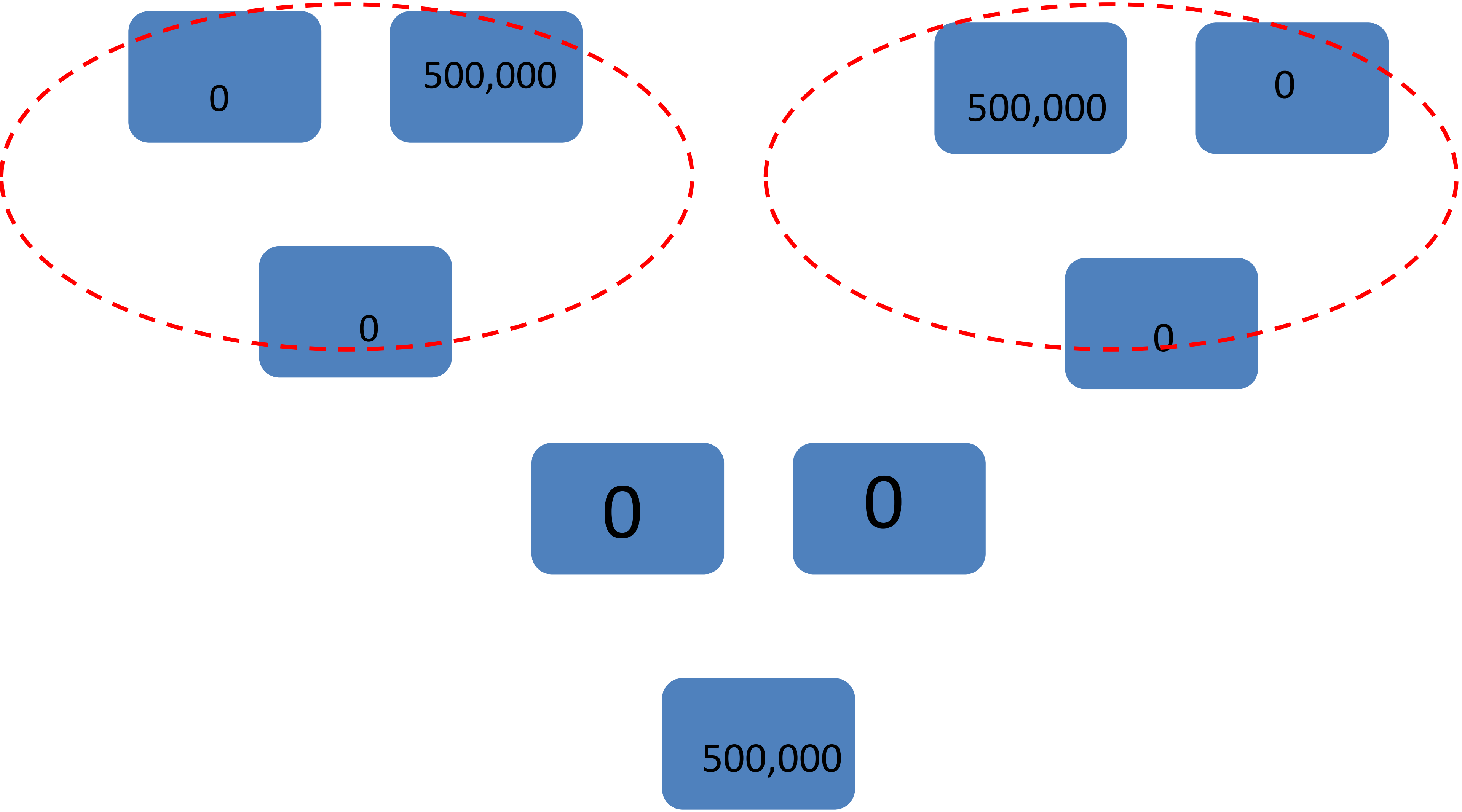


Let see if you have the 1€ box

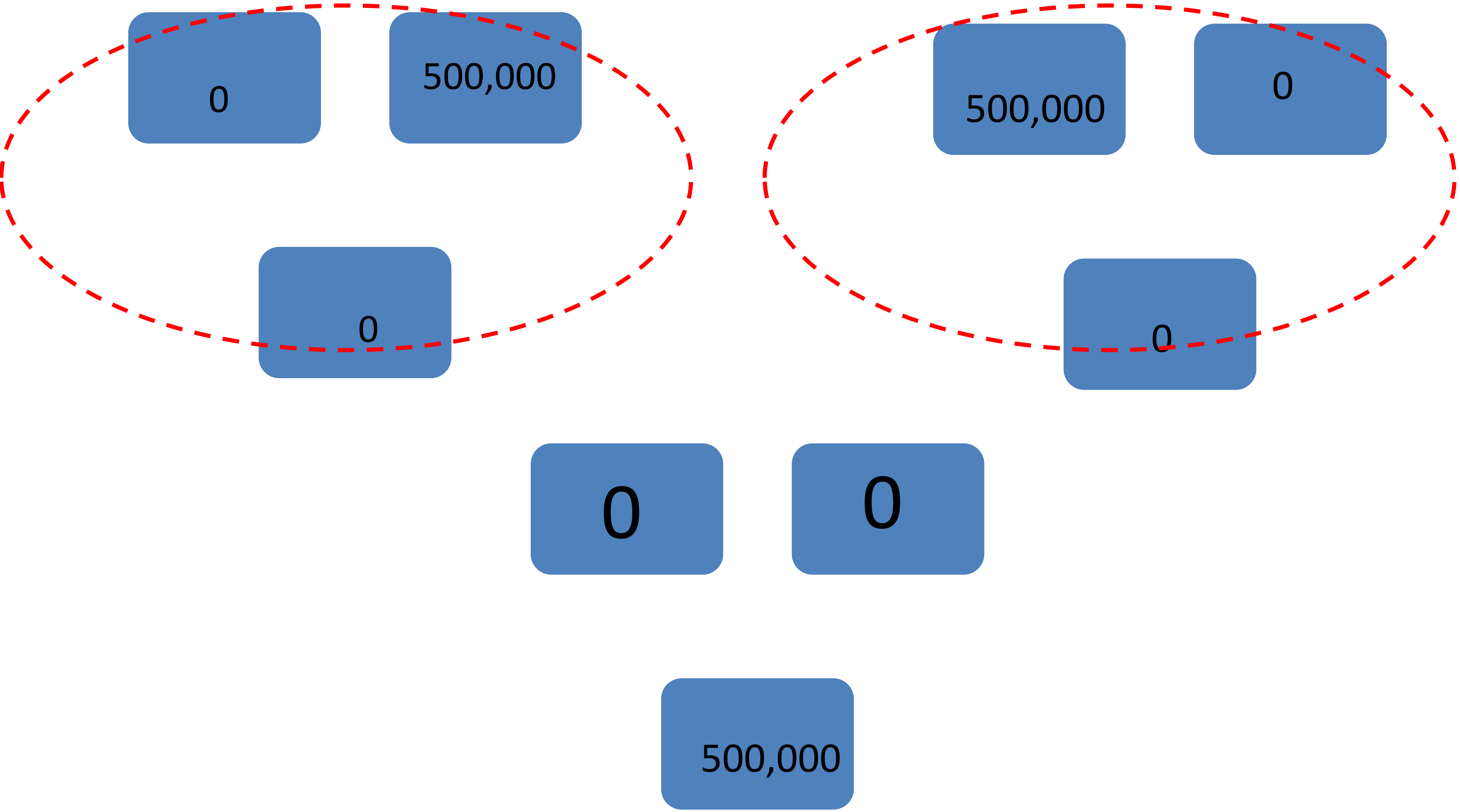
Se non hai il pacco vincente. Il conduttore è costretto ad aprire l’unica porta con 0



What is the probability to be in this situation?



Che probabilità c'è di trovarci in questa situazione: 2/3



Now we have to calculate expected pay offs

If you change

- There are 2/3 prob. to have 500.000€ =333.333€ + 1/3 to have 0
- If you don't change you have ½ to have 500.000 + ½ prob. to have 0
- So it is always better to change!!

Airbus vs Boeing in Very Large aircrafts (VLA)

There can be **only one**

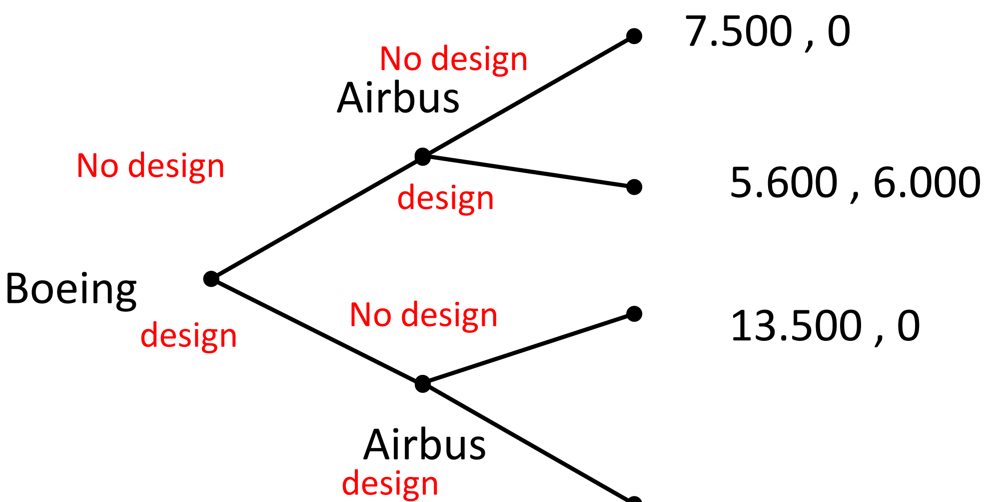
Competition in the market for super Jumbo

Airbus Vs Boeing

- Airbus e Boeing started to compete on medium, medium/long range
- Boeing had more or less monopoly power on Very Large Aircraft (VLA) thanks to Jumbo Boeing 747
- In December 2000 Airbus Officially declared in investment worth 11 bn Us$ to develop a new VLA. A «Super Jumbo» - 550 seats called A380.
- Airbus had orders from 50 Airlines companies plus options from 42 companies.

Airbus vs Boeing

- What should Boeing do?
- Remember that Airbus already decided to entry and has already many orders.
- Keep Airbus out of the market is not an option also because in the 90ties Boeing announced a new project for a larger VLA but didn't follow up.



A

B

C

B A	No Entry	Entry
No entry	7.500 , 0	5.600 , 6.000
Entry	13.500 , 0	4.900 , 2.900

D

Numbers are not random….

- Case A) no Super Jumbo => B remains monopolist for Jumbos. Suppose B sells 38 Jumbo per year for the next 15 years (as it was from
- Jumbo price = 165 million +2% to consider inflation
- EBITDA 20%
- Taxation 34%
- Cost of Capital 9%
- These conditions give an operating income for Boeing = 7.5 bn. US$

Numbers are not random….

- Case B) Airbus compete in super jumbos and B remains only in jumbo market. Assume that A sells 50 Super Jumbo per year for 225 mln each with operating income at 15% => 6 bn. US$ (actual value
- B continue to sell 38 aircrafts every year bot with a lower Ebitda (15%) due to A competition => 5.6 bn. US$ operating income

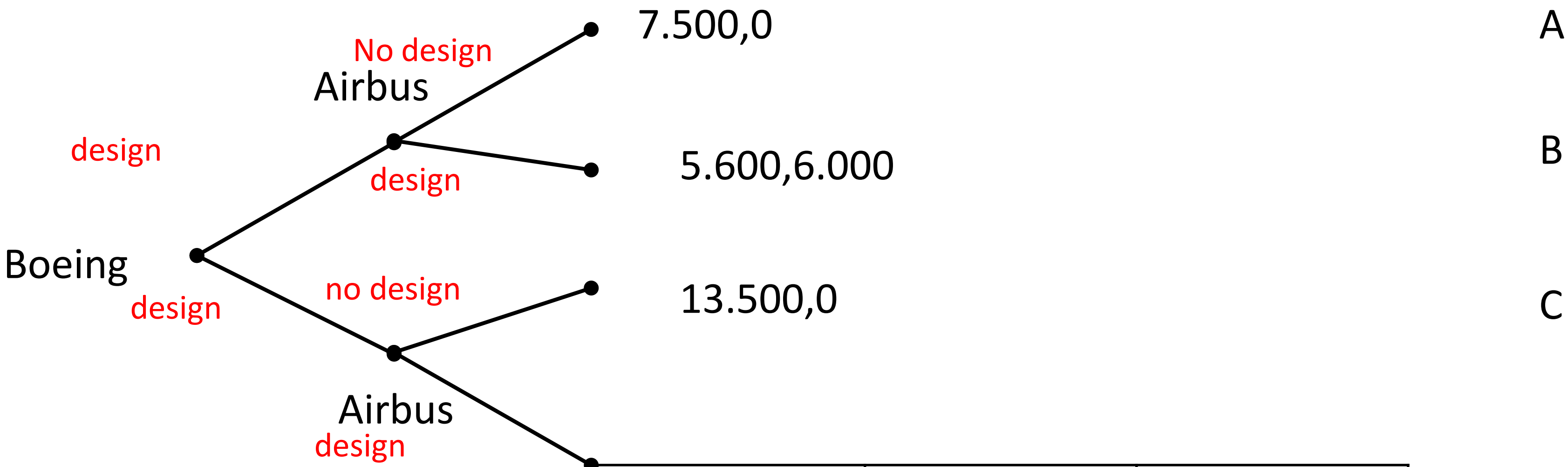
Numbers are not random….

- Case C) Airbus out from VLA, B offers both jumbos and jumbos.
- Best Scenario: let's suppose there are not cross-effects between the two markets =>7,5 bn. (case A) + 6 bn. (case B) = 13,5 bn.
- Worst scenario: (case A) 5,6 + 6 bn. (caso B)

Numbers are not random….

- Case D) 3 outcomes have the same payoffs we already analyzed.
- One is «new» and it is the one where both companies enter the market for Super Jumbos(E,E)
- Competition erodes margins and Ebitda 15% to 10%. Sales are 35 for each producer. Operative margin 2.9 bn.
- \+ super jumbo sales decrease jumbo sales at 20 each year with a 10% margin.

Airbus vs Boeing



B A	No Entry	Entry
No entry	7.500 , 0	5.600 , 6.000
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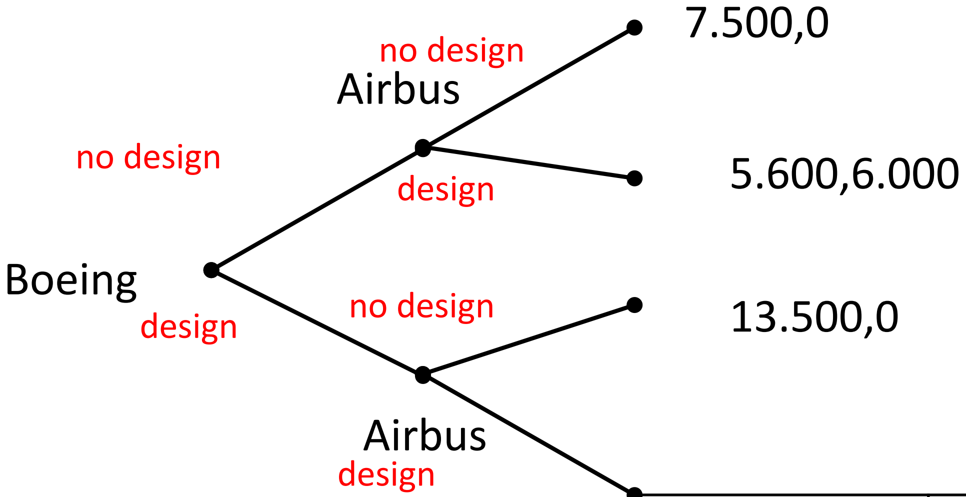
D

Airbus vs Boeing



A

B

C

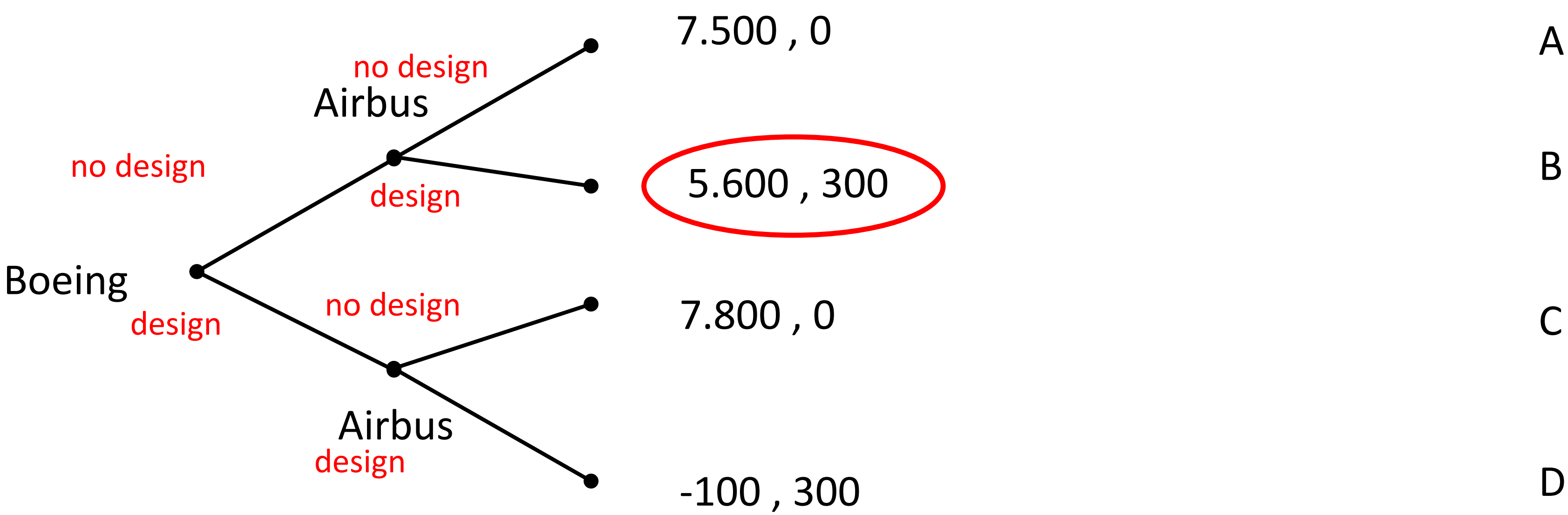
dominant strategy for Airbus

B A	No Entry	Entry
No entry	7.500 , 0	5.600 , 6.000
Entry	13.500 , 0	4.900 , 2.900

Equilibrium

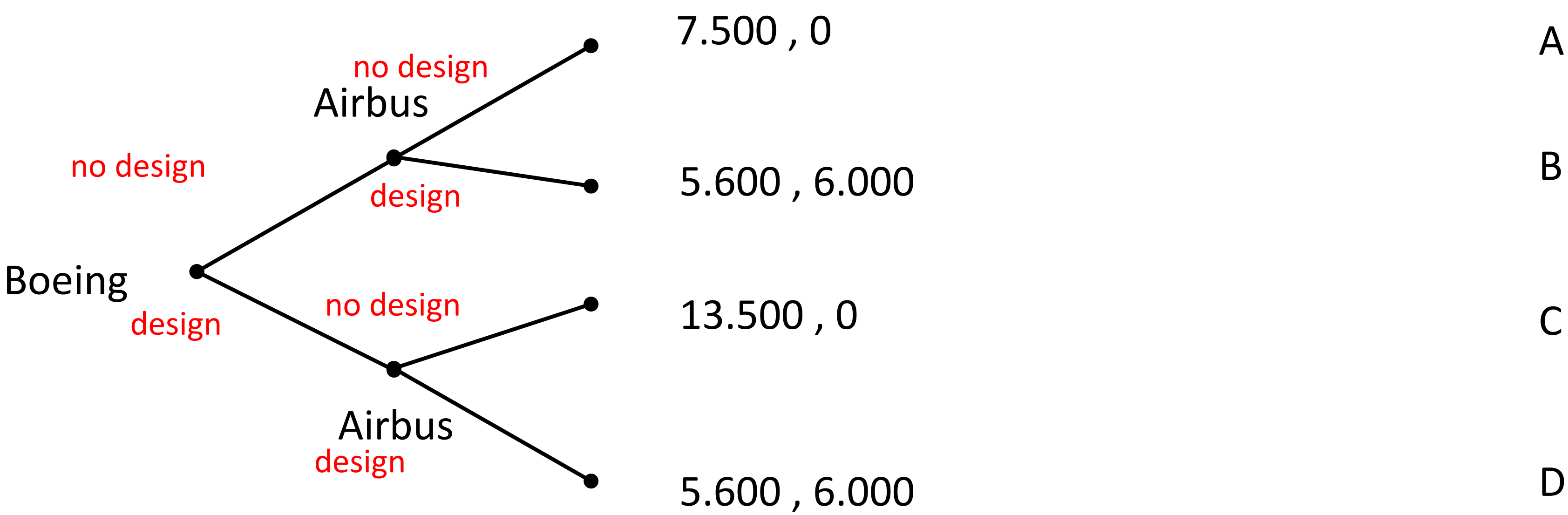
Airbus vs Boeing

- Ricordiamoci che i cosati fissi (e per la gran parte Sunk) per lo design ammontavano a 5.700



Airbus vs Boeing

- Remember that fix costs for design are very high 5.7 bn.



Note that even if the cost of designing a super jumbo were 0 for Boeing the best strategy for Boeing is still not to develop it.

Is there anything that Boeing can do?

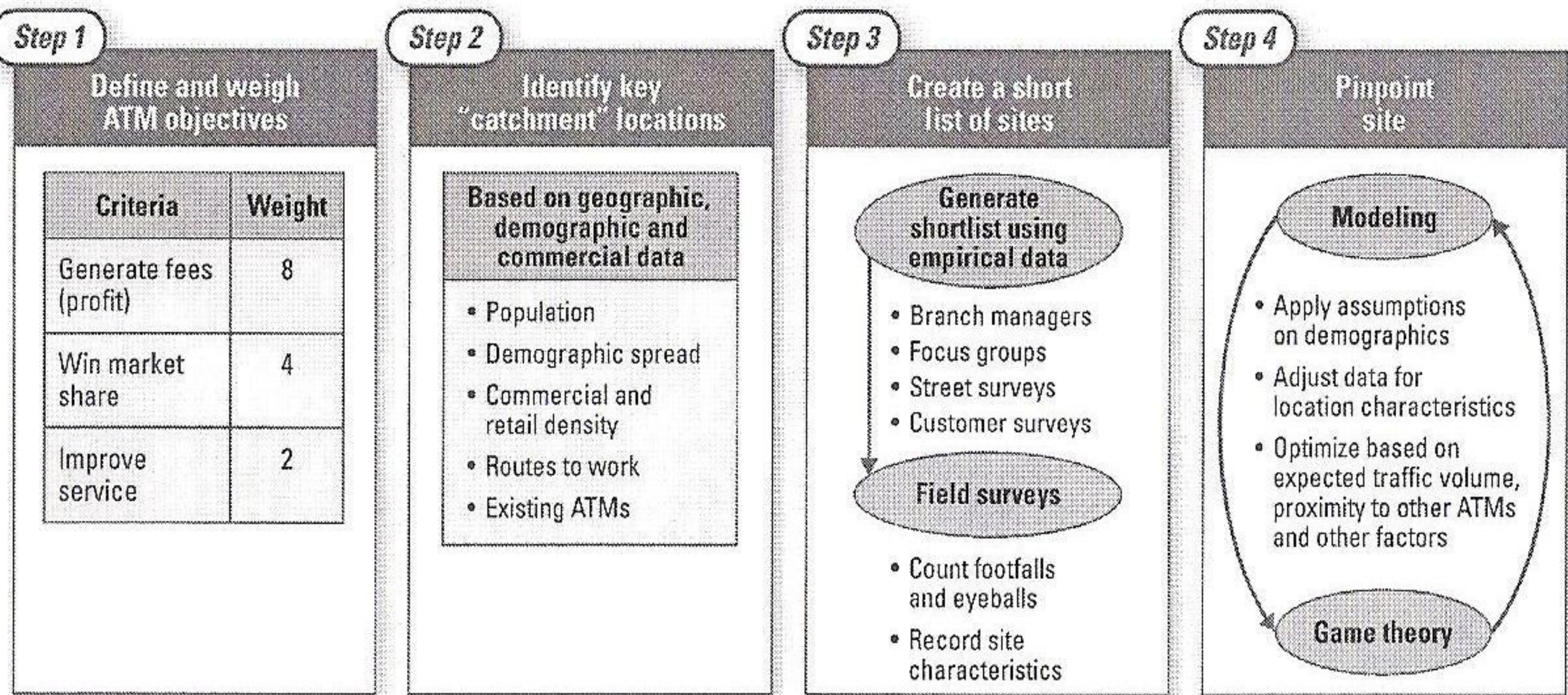
- Delay Airbus Market entry (every year of delay are 135 mln. Extra for Boeing): pretend to cooperate for the development of a shared project. Try to make agreement with Airbus suppliers to slow down Airbus project.
- Declare the project of a new VLA aircraft (double deck Jumbo but lack of credibility)

What we learnt?

- Build strategies can be complex
- Maintaining leadership is very difficult
- It is also difficult to keep competitors out to lucrative markets even if fix cost are very high
- Credible threats are difficult to implement: be careful you can end up with less money and a new, unpleased, competitor

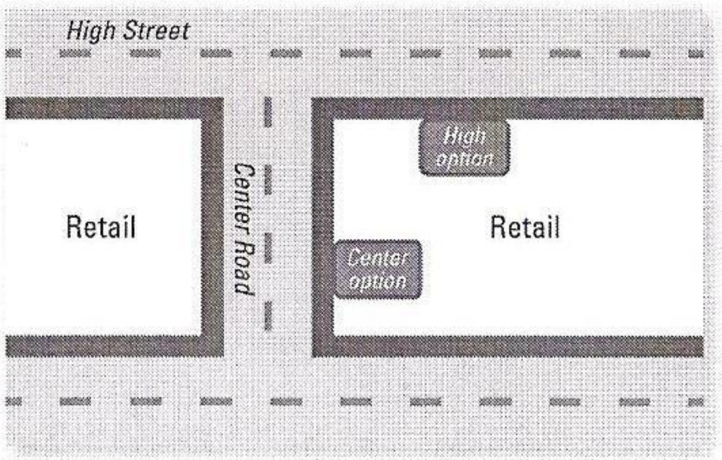
ATM location

Four steps to determining ATM locations



Source: A.T. Kearney analysis

Without considering competition, the potential revenues from new ATMs are significant *Illustrative*



ATM location	Additional revenues (US$ thousands) Beta bank	Alpha bank
High Street *(own customers)*	70	50
Center Road *(own customers)*	40	30
High Street *(foreign customers)*	20	28
Center Road *(foreign customers)*	12	16
Cost per ATM	-40	-40

Source: A.T. Kearney analysis

Figure 3

Game theory allows comparison of potential net revenue at each ATM location *Illustrative*

Figure 3

Game theory allows comparison of potential net revenue at each ATM location *Illustrative*

		Beta Bank net revenues (US$ thousands)			
	ATM placement	Center Road only	High Street only	Both locations	Neither location
Alpha Bank net revenues (US$ thousands)	Neither location	59 -22	60 -22	62 -32	0 0
	High Street only	38 19	38 16	42 B 4	-31 46
	Center Road only	37 16	42 C 18	36 7	-31 47
	Both locations	37 0	35 16	30 A 0	-56 44

A Because of the competitive situation, Alpha makes no money with ATMs at both locations.

B Alpha's economic contribution increases only marginally, while Beta does much better.

C Game theory helps both banks avoid the traps of scenarios A and B and maximize their revenues.

Source: A.T. Kearney analysis

Why?

- Look for NE
- High street is more profitable but if both go there lose money.

?

Do you want them for and Adv campaign?

Snooki

- This lady (?) is fashion addicted (with her own style, of course…)
- She loves partying and drinking (a lot),
- So she puke (often)
- Sometime in her fashionable handbag
- If you were Luis Vuitton would you be happy to see your handbag in the hands of this lady?

What to do?

- You cannot forbid someone to buy your handbag. (it is not impossible but is very complicated)
- …evil genius…. We will give to Snooky a lot of bags….the ones of our competitors!

Let see how Game theory can help us in this contest

Suppose there are two fashion firms

- Every firm can do 3 things: give to Snooky on of its bags, give to her one of the competitor, don't give anything. Give to her one of its bags is a stupid things or if we want to be polite is a dominated strategy

	No Bags	Competitor Bag	Own Bag
No Bags	-1,-1	-10,0	0,-10
Competitor Bag	0,-10	-5,-5	0,-15
Own Bag	-10,0	-15,0	-5,-5

Suppose there are two fashion firms

- Nash Equilibrium?

	No Bags	Competitor Bag	Own Bag
No Bags	-1,-1	-10,0	0,-10
Competitor Bag	0,-10	-5,-5	0,-15
Own Bag	-10,0	-15,0	-5,-5

Snooky wins!!!!!

Lets be serious…

(but still more games….)

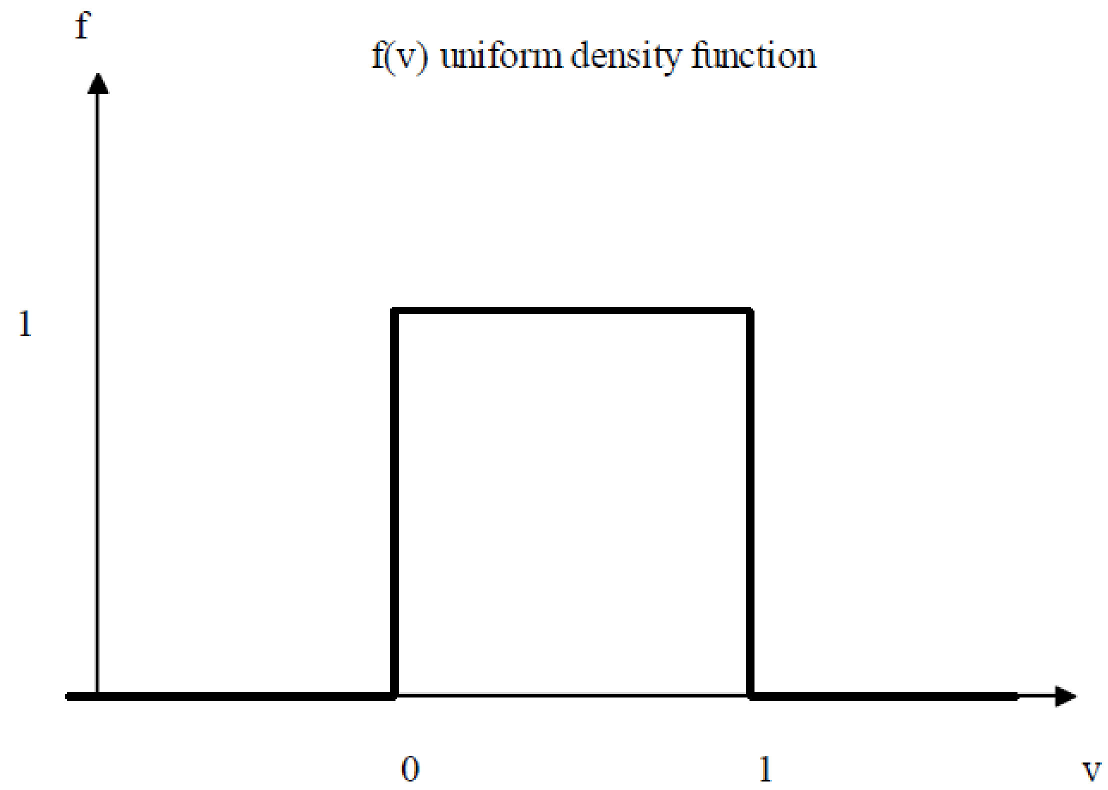
Auctions

- You have to bid for the development of a new Oil field
- You know that the value of the Oil field is between 0 and 100 (uniformely distribuited)
- One of you know the exact value

How much do you bid?

hipotesys

- Common values: Ol field Value v is the same for everybody
- Two firms 2 make a bid i = I;U
- I know the exact value of the Oil field
- U knows v is uniformelly distribuited [0; 1]


f
f(v) uniform density function
1
0
1
v

Who win?

- Who bid higher win
- If both firms bid the same value you flip a coin

Payoff

- if i Wins, get $v - b_i$
- if i looses, get 0

Do we have a Nash Equilibrium?

- NO
- Lets suppose U has a pure strategy b_U
- How should I reply?

1. if $v > b_U$?

 $b_I = b_U$ +"Ɛ"

2. if $v < b_U$?

Bid less than bU ($b_I < b_U$)

so

- So U will always loose money whatever she bid. At most it get 0
- So U has to «hide» his strategy
- U needs a mix strategy

more

- Do we have a NE with mix strategies?

NE:

- $b_I = v/2$
- b_U uniformely distributed [0; 1/2]

- How can I prove it?

U firm

$b_I = v/2$

- U wins $b_U > b_I$ or, if you prefere,v $< 2\ b_U$
- What is the expected value fof U?
- $E[v|U\ Win] = b_U$
- Because v is Uniformely distributed [0;1]

Pay off atteso per U?

1. for $b_U > 1/2$, U loses ($\pi U < 0$)
2. for $0 \leq b_U \leq 1/2$:

- $\pi U = E\,[v|U \text{ vince} - b_U] \cdot Pr(b_U > b_I)$

Payoff probability U win

$= [b_U - b_U]\ \ Pr(b_U > b_I) = 0$

Firm I

- What is the probability for I to Win the Auction?

- $Pr(b_I > b_U)$ $\begin{cases} 0 \text{ if } b_I < 0 \\ 2b_I \text{ se } 0 \leq b_I \leq 1 \\ 1 \text{ se } b_I > 1/2 \end{cases}$

Given tha b_U is uniformely distributed [0; 1/2]

Payoff?

- $\pi I = [v - b_I] \cdot Pr(b_I > b_U)$

Payoff if I wins   Probability I wins

- First order condition $0 \le b_I \le ½$

$(v - b_I)2 - 2b_I = 0$

$b_I = v/2$

Expected Payoff

- πU= 0
- ΠI= v $\frac{v}{2} > 0$

Hendricks and Porter

- The data indicate that firms owning neighbor tracts have an informational advantage over non-neighbors in offshore drainage lease auctions. They exploit this advantage by shading their bids substantially below their expectation of the value of the tract. This translates into significantly higher returns, expressed as a percentage of discounted social value, than on wildcat tracts, where the distribution of information is relatively symmetric. The non-neighbors also account for their disadvantage, by bidding conservatively. As a consequence, they do not suffer from the winner's curse, but rather break even on average.