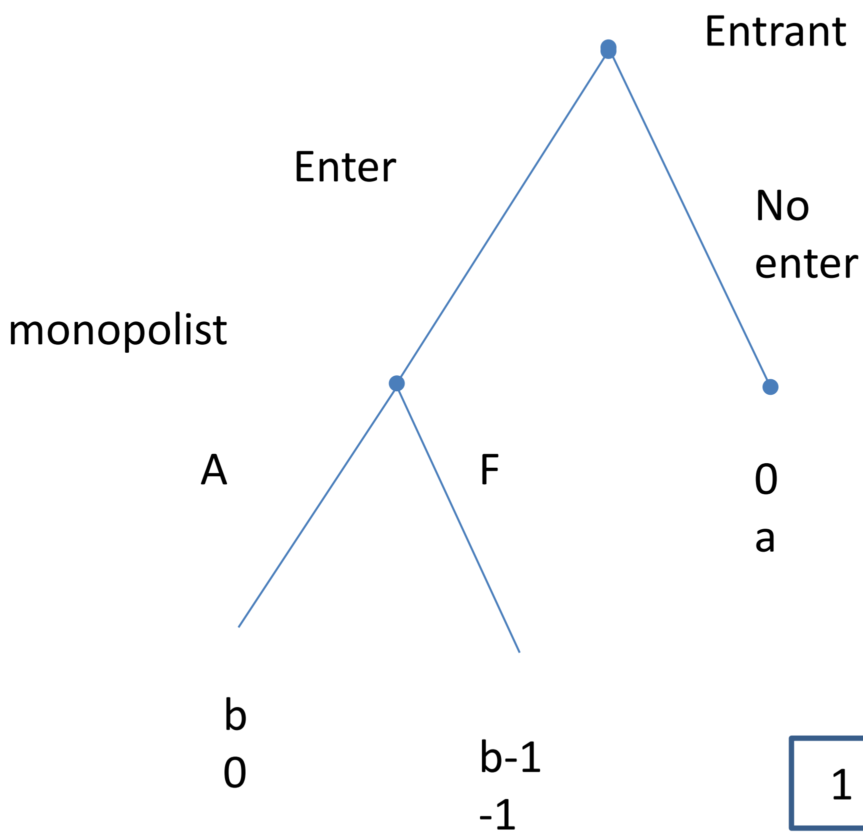
Chain store paradox

Kreps and Wilson

Recall Selten paradox

0<b<1

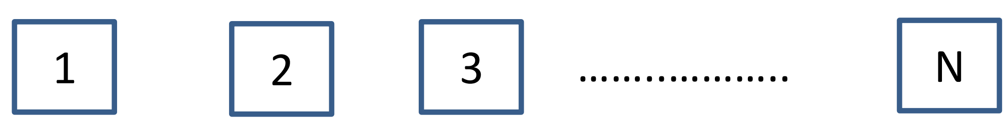
a>1



2 Nash equilibra:

(E,A) => SGP

(D,F) => No SGP



Is it a paradox?

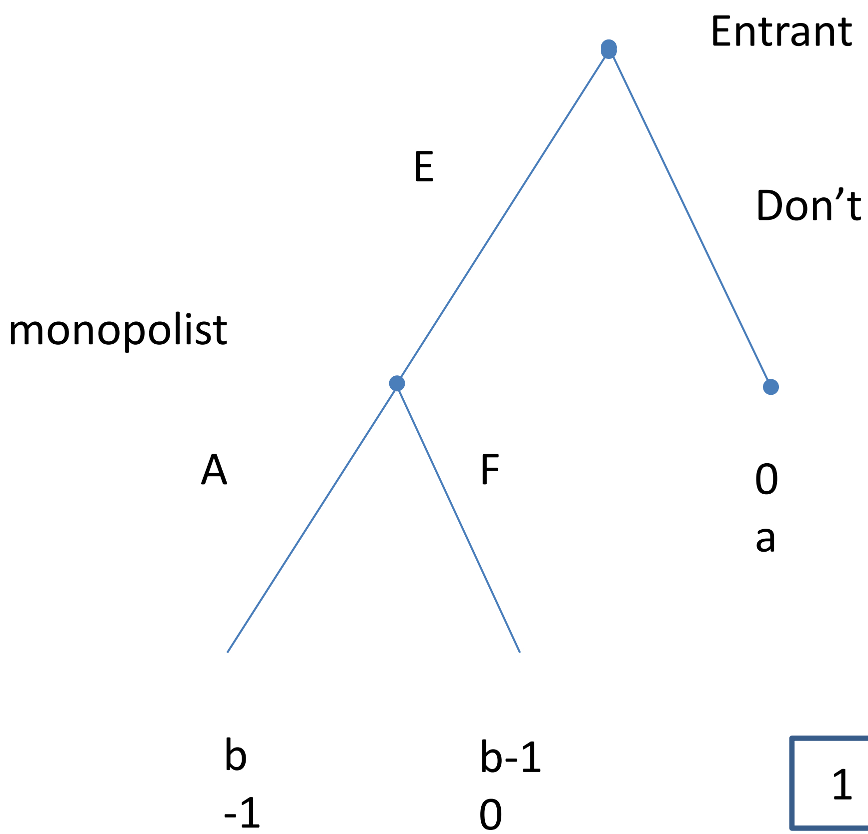
- If I go backward I have always enter and acquiesce.

Let see what happens if We change pay off

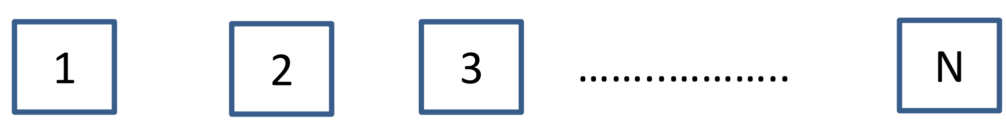
Selten modified

$0<b<1$

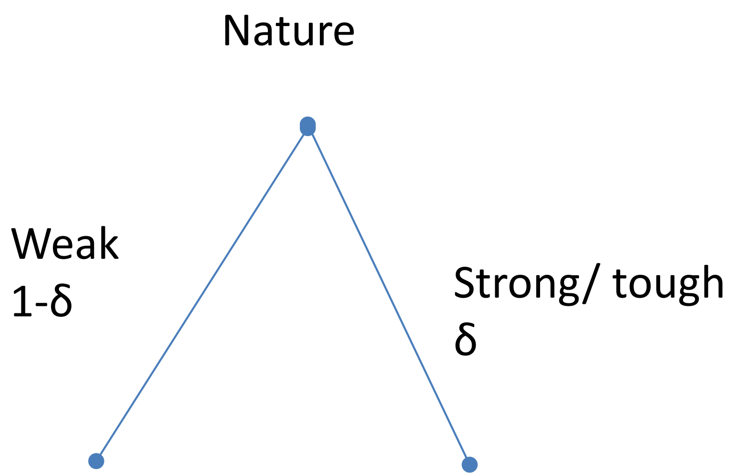
$a>1$



Suppose there is even a small probability that a Monopolist prefers fight (different pay offs, different cost structure, monopolist can be «crazy», business strategies / different incentives etc.)



Selten modified



N+1 Players
The monopolist plays the game against every entrant.
Pay off is the sum of pay offs in every game

I need to find a behavior Strategy that define what to do in every node of the Game (taking into consideration we have to deal with probabilities.

Sequential Equlibrium

a) Every time a player makes a move he/she has to take into consideration the probabilities he/she has to be in a determinate node.

b) We apply bayes rules whenever is possible

c) Given the information set we utilize optimal strategies. => given the probabilities based on past moves of the other player and Nature.

There is a small probability the Monopolist is taught.

Kreps e Wilson

- N periods or N games
- 1 monopolist and 1 entrant per game
- N+1 player

To have a sequential equilibrium I need

1. A strategy for the incumbent /monopolist
2. An Entrant's Strategy
3. Belief of the entrant (incumbent knows if he/she is strong or not while the entrant has only an idea based on probabilities $P_n(h_n)$

Rename games backward

N | N-1 | …………… | 2 | 1

A priori $p_N = \delta$ is random and exogenous

δ *grows while time pass.*

the more the incumbent fights the higher is δ. If the entrant doesn't enter, the incumbent doesn't fight and so δ doesn't grow $p_n = p_{n+1}$

if the entrant enters and the incumbent doesn't fight the probability assigned that he/she is strong goes to zero. $p_{n+1} = 0$ and cannot grow anymore.

beliefs

Rule: if there is an entrant and $p_{n+1} > 0$

$a.$ $p_n = \max(b^n, p_{n+1})$

$b.$ $p_n = 0$

If the monopolist acts as it is tough, the probability for him / her to be strong doesn't decrease

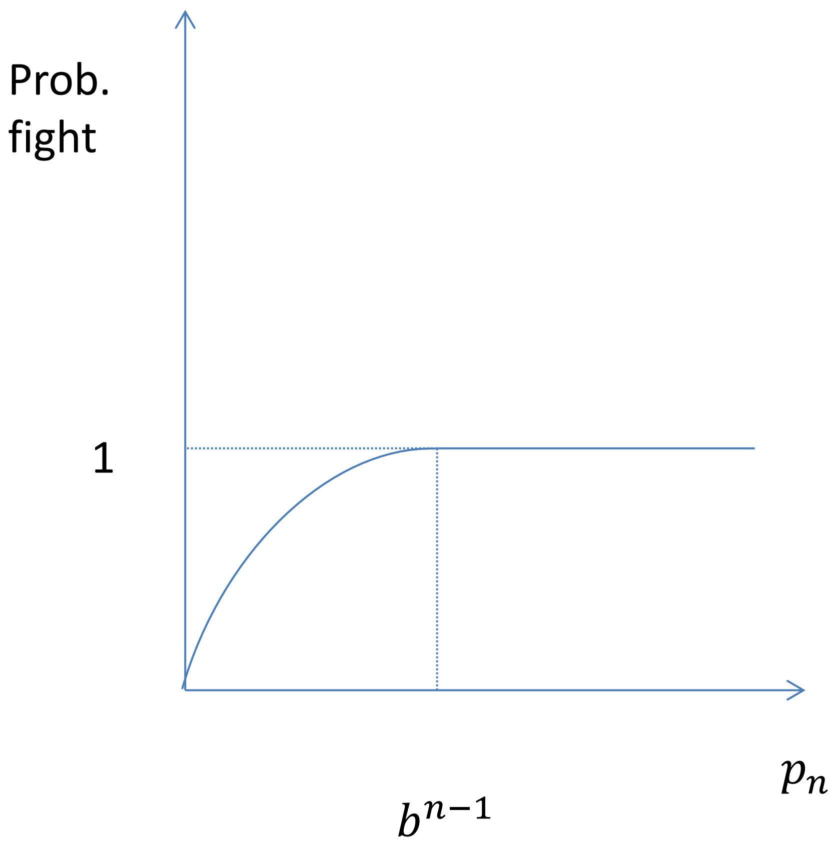
Monopolist’s Strategy

a. if tough he/she always fights

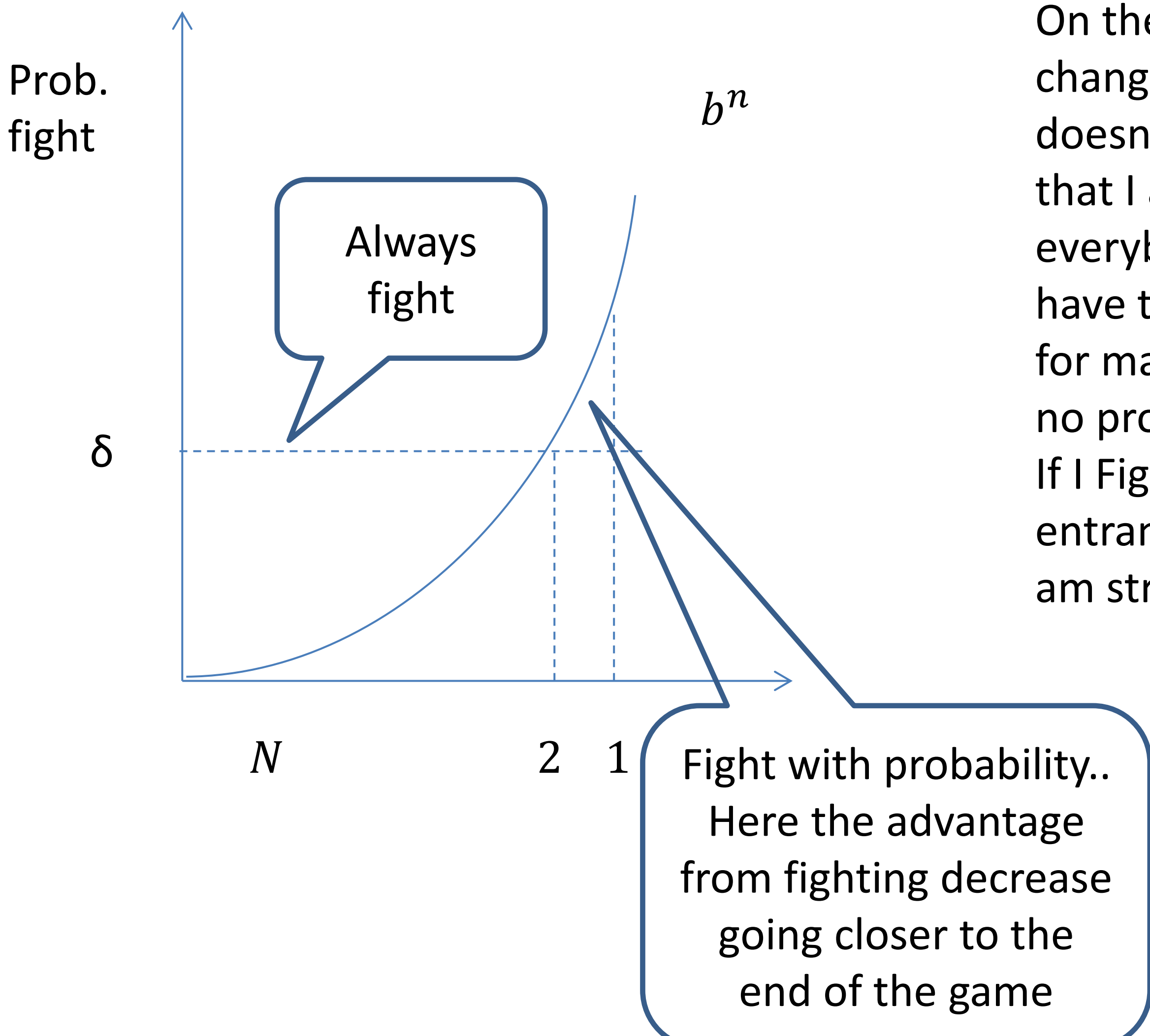
b. If he/she is Weak

- if N=1 obviously doesn’t fight
- if N>1 and $p_n \geq b^{n-1}\ fight$
- if N>1 and $p_n < b^{n-1} fight\ with\ probability$

$$\frac{\left(1-b^{n-1)}\right)p_n}{(1-p_n)b^{n-1}}$$


Prob.
fight
1
p_n
b^{n-1}

What happens to beliefs?



On the Equilibrium path things change. On the first part if I fight this doesn't change much the probability that I am perceived tough because everybody knows that if I fight I will have the possibility to have profits for many periods, if not, I will have no profits.

If I Fight at the end of the game the entrants will believe more and more I am strong.

Equilibrium strategy for the entrant

- $p_n > b^n$ $non\ entry$
- $p_n = b^n$ $entry\ with\ prob\ 1 - \frac{1}{a}$
- $p_n < b^n$ $entry$

$p_n > b^n$ nothing happens. Things are interesting at the end of the game.
Let see if this a sequential equilibrium:

a. beliefs respect Bayes law when is possible
b. The strategy is an optimal reply

$a.\quad p_{n1} = prob(I\ am\ strong|I\ fight)$

$$= \frac{prob(fight|I\ strong)\ Prob\ (strong)}{prob(fight|I\ strong)prob\ (strong + prob(fight|I\ weak)prob\ (weak)} =$$

$$= \frac{1\ p_n}{1\ p_n + \frac{(1-b^{n-1})p_n}{(1-p_n)b^{n-1}}} = b^{n-1}$$

$$p_{n-1} = b^{n-1} \ => \ p_n = b^n$$

This has to be true to respect Bayes law

we wrote $p_n = \max(b^n, p_{n+1})$ to cover the $when\ p_{n+1} = p_n = \delta > b^n\ where\ probability\ of\ fight\ is = 1$

b. Optimal strategy of the entrant

- Is indifferent if enter or not if prob fight =b
  Because b(1-b) + (b-1)b=0
- if $p_n > b^n \, n \, stay \, out$
- if $p_n < b^n$ n enter
- if $p_n = b^n$ enter with probability 1-$\frac{1}{a}$

(this a strategy that we define)

- The path is defined by prob(strong)=b^n

Prob (fight)= P(strong) 1 + P(weak) (fight Iweak)

$$=p_n + (1 - p_n)\frac{(1-b^{n-1})p_n}{(1-p_n)b^{n-1}}$$

We fix $p_n = b^n$ (has to be verified for beliefs). We want to see what happens on the path above δ

Prob (fight) = $b^n + \frac{(1-b^n)(1-b^{n-1})b^n}{(1-b^n)b^{n-1}}$ =b

Strategy of the weak monopolist

- Last period doesn't fight

If I have maintained my reputation till this point of the game, Pay off =1 because $p_n = b^n$ =b $[n = 1]$ =>

Probability to face an entrant (1- $\frac{1}{a}$)

monopolist doesn't fight: 0(1- $\frac{1}{a}$) + a($\frac{1}{a}$) $= 1$

let see stage 2 (second-last)

Stage 2

- Let see when the monopolist is indifferent between fight and don’t fight.

If he loses reputation, in the last period his expected pay off will 0.

If he takes his reputation (fight) his expected pay off will be 1 => if he fight today takes -1 and 1 tomorrow. If he doesn’t fight today he take 0 today and 0 tomorrow

If in 2 there is no entry get «a» but in the last game $p_n = p_1$ *becomes* $p_2 = b^2$ *and being* $b^2 < b^1$ *there will be entry at* 100%

Pay offs are $\left(1 - \frac{1}{a}\right)(0 + 0) + \frac{1}{a}(a + 0) = 1$

And so on…..