dati

$$[\sigma_1]_p = \begin{bmatrix} 100 & -80 & 0 \\ -80 & 150 & 0 \\ 0 & 0 & -100 \end{bmatrix}_p \quad \text{kg/cmq}$$

$$[\sigma]_p = \begin{bmatrix} 1600 & 500 & 0 \\ 500 & 800 & 0 \\ 0 & 0 & -1000 \end{bmatrix}_p \quad \text{kg/cmq}$$

$$[\sigma]_r = \begin{bmatrix} 500 & -300 & 0 \\ -300 & -300 & 0 \\ 0 & 0 & -500 \end{bmatrix} \quad \text{kg/cmq}$$

determinare tensioni principali e direzioni principali

$\xi = 1, \quad \eta = 2, \quad \zeta = 3$

$\sigma_\xi = \sigma_1, \quad \sigma_\eta = \sigma_2, \quad \sigma_\zeta = \sigma_3$

3.A)

$$\begin{vmatrix} 100-\sigma_n & -80 & 0 \\ -80 & 150-\sigma_n & 0 \\ 0 & 0 & -100-\sigma_n \end{vmatrix} = 0$$

$$(-100-\sigma_n)\left[(100-\sigma_n)(150-\sigma_n)-80^2\right]=0$$

$\sigma_3 = \sigma_Z = -100 \ \text{kg/cmq}$ $\qquad 3 \equiv Z \equiv z$

$$15000-100\,\sigma_n-150\,\sigma_n+\sigma_n^2-80^2=0$$

$$\sigma_n^2-250\,\sigma_n+8600=0$$

$$\sigma_{1,2}=\frac{250\pm\sqrt{250^2-4\cdot 8600}}{2} \left\langle \begin{array}{l} 208{,}81 \ \text{kg/cmq} = \sigma_2 \\ 41{,}18 \ \text{kg/cmq} = \sigma_1 \end{array} \right.$$

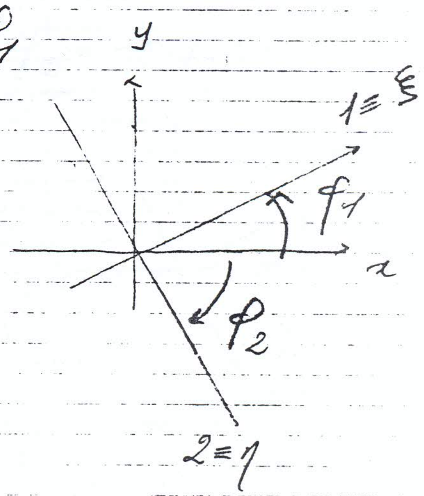
$$\sigma_x < \sigma_y \Rightarrow \sigma_1 = 41{,}18 = \sigma_\xi < \sigma_2 = 208{,}81 = \sigma_\eta$$

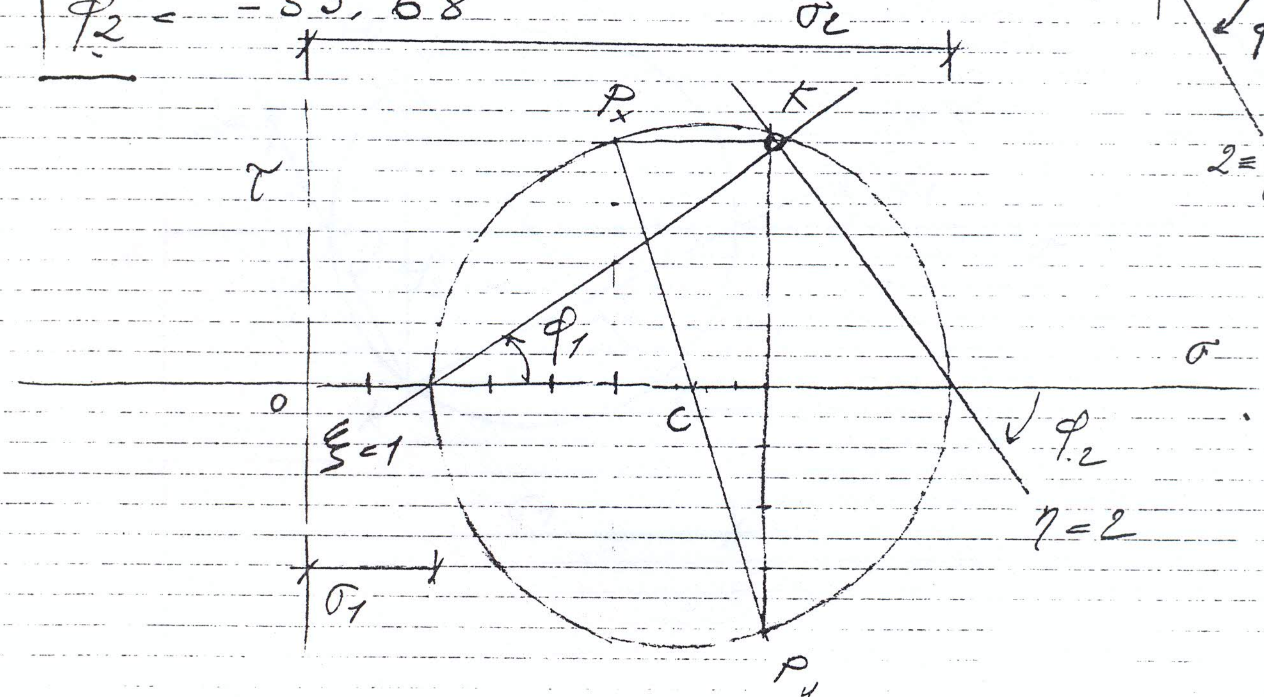
$$(100-41{,}18)\,n_x^1-80\,n_y^1=0$$

$$\frac{n_y^1}{n_x^1}=\frac{58{,}82}{80}=0{,}73525=\text{tg}\,\varphi_1$$

$\varphi_1 = \text{arctg}\ 0{,}73525 = 36{,}32^\circ$

$\varphi_2 = -53{,}68$





3.B)

$$\begin{vmatrix} 1600-\sigma_n & 500 & 0 \\ 500 & 800-\sigma_n & 0 \\ 0 & 0 & -1000-\sigma_n \end{vmatrix} = 0$$

$\sigma_3 = \sigma_\zeta = -1000$ kg/cmq $\qquad 3 \equiv \zeta \equiv z$

$(800-\sigma_n)(1600-\sigma_n) - 500^2 = 0$

$1'280\cdot000 - 800\,\sigma_n - 1600\,\sigma_n + \sigma_n^2 - 500^2 =$

$= 1030\cdot000 + \sigma_n^2 - 2400\,\sigma_n = 0$

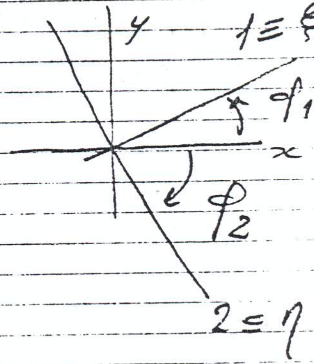
$$\sigma_{1,2} = \frac{2400 \pm \sqrt{2400^2 - 4\cdot 1030.000}}{2} = \begin{matrix} 1840{,}31 = \sigma_1 \\ 559{,}68 = \sigma_2 \end{matrix}$$

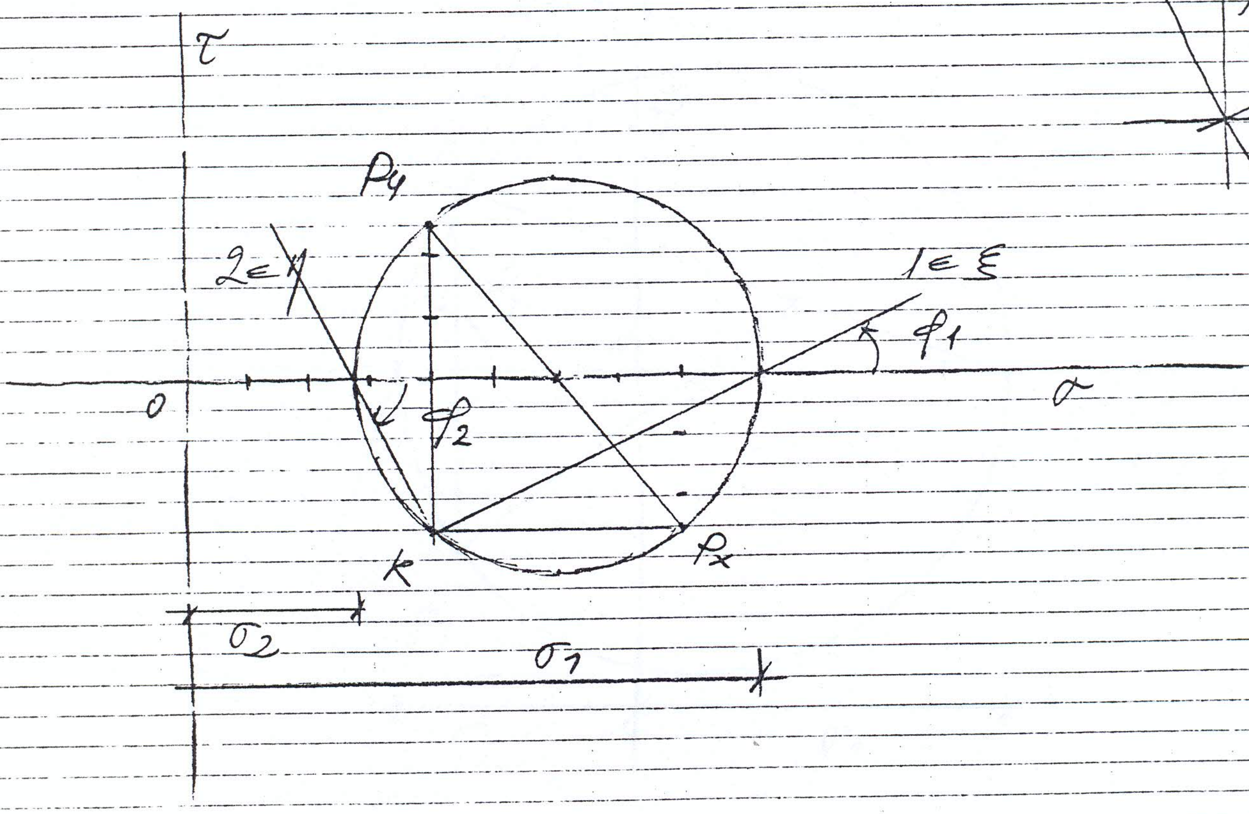
$\sigma_1 = \sigma_\xi \quad , \quad \sigma_2 = \sigma_\eta$

$\underbrace{(1600 - 1840{,}31)}_{-240{,}31} n_x' + 500\, n_y' = 0$

$$\frac{n_y'}{n_x'} = \frac{240.31}{500} = 0{,}48062 = tg\,\varphi_1$$

$\varphi_1 = \operatorname{arctg} 0{,}48062 = 25^\circ{,}67 \qquad \varphi_2 = -64^\circ{,}33$





3.c) $\begin{vmatrix} 500-\sigma_n & -300 & \\ -300 & -300-\sigma_n & \\ & & -500-\sigma_n \end{vmatrix} = 0$

$\boxed{\sigma_3 = \sigma_\zeta = \sigma_z = -500 \ kg/cm^2} \qquad 3 \equiv \zeta \equiv z$

$(500-\sigma_n)(-300-\sigma_n) - 300^2 = 0$

$-150\,000 - 500\,\sigma_n + 300\,\sigma_n + \sigma_n^2 - 300^2 =$

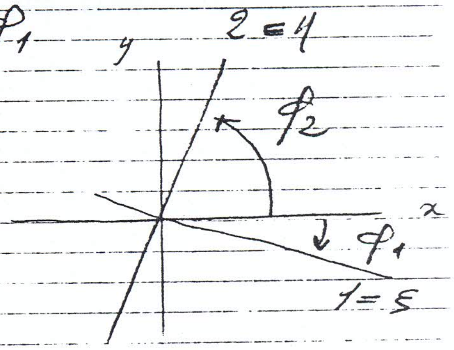
$= -240.000 + \sigma_n^2 - 200\,\sigma_n = 0$

$\boxed{\sigma_{1,2} = \dfrac{200 \pm \sqrt{200^2 + 4 \cdot 240.000}}{2}} \begin{cases} 600 \ kg/cm^2 = \sigma_1 \\ -400 \ kg/cm^2 = \sigma_2 \end{cases}$

$\sigma_1 = \sigma_\xi \qquad \sigma_2 = \sigma_\eta$

$(500 - 600)\, n_x^1 - 300\, n_y^1 = 0$

$\dfrac{n_y^1}{n_x^1} = -\dfrac{100}{300} = -0,\bar{3} = tg\,\varphi_1$



$\boxed{\varphi_1 = arctg(-0,\bar{3}) = -18,43^\circ}$

$\boxed{\varphi_2 = 71,57^\circ}$