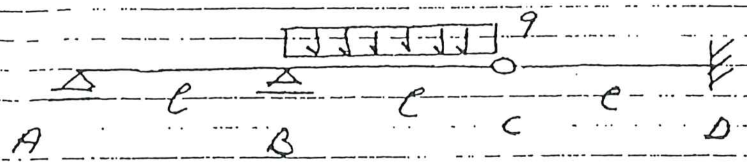
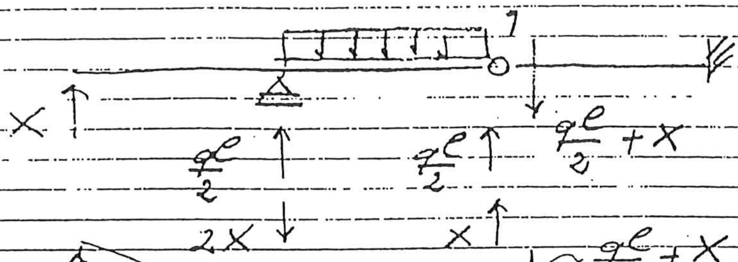
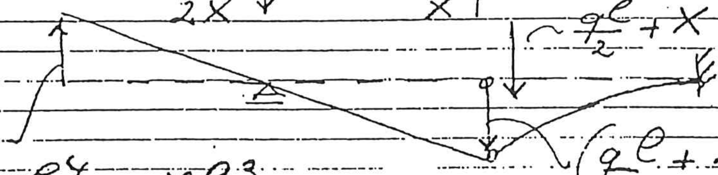
1A)

(1)

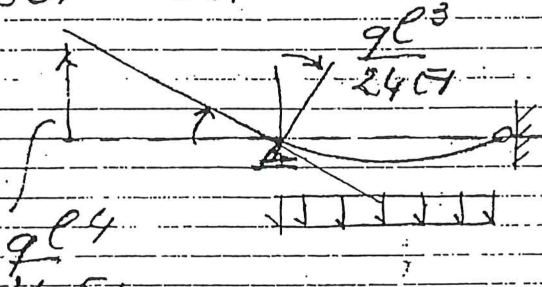






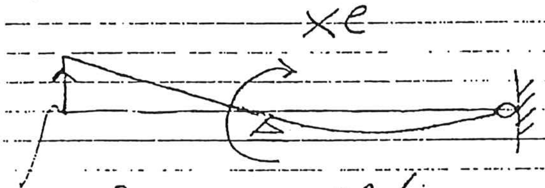
(1)

$$\frac{q\ell^4}{6EI} + \frac{X\ell^3}{3EI} \qquad \left(\frac{q\ell}{2}+X\right)\frac{\ell^3}{3EI} = \frac{q\ell^4}{6EI} + \frac{X\ell^3}{3EI}$$



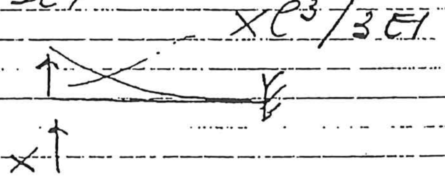
(2)

$$\frac{q\ell^4}{24EI}$$



(3)

$$\frac{X\ell^2}{3EI}\cdot\ell = X\ell^3/3EI$$



(4)

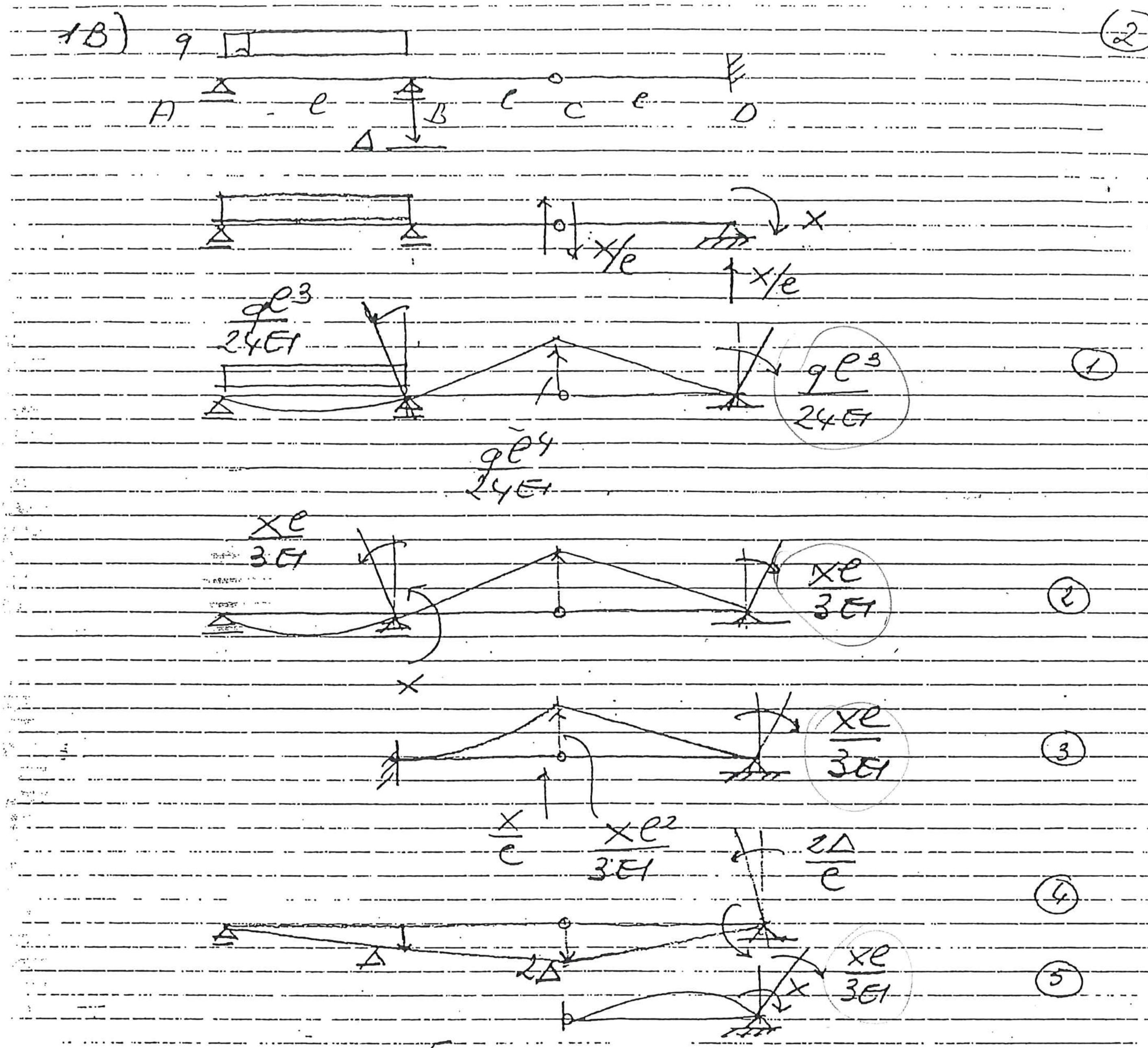
$$v_A = \sum_{i=1}^{4} v_A^i = \frac{q\ell^4}{6EI} + \frac{X\ell^3}{3EI} + \frac{q\ell^4}{24EI} + \frac{X\ell^3}{3EI} + \frac{X\ell^3}{3EI} =$$

$$= \frac{5q\ell^4}{24\cancel{EI}} + \frac{X\ell^3}{\cancel{EI}} = 0 \qquad \boxed{X = -\frac{5q\ell}{24}}$$

Risultati notevoli:

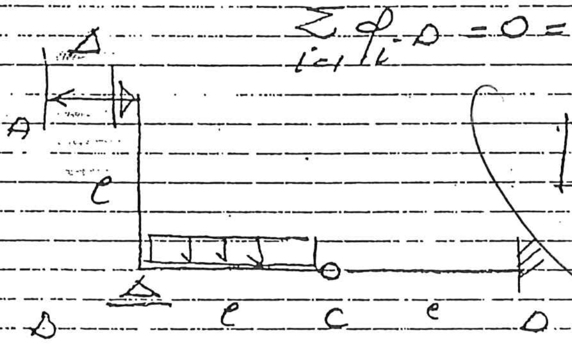
$$\frac{P\ell^3}{3EI}, \quad \frac{q\ell^3}{24EI}, \quad \frac{M\ell}{3EI}$$

1B)



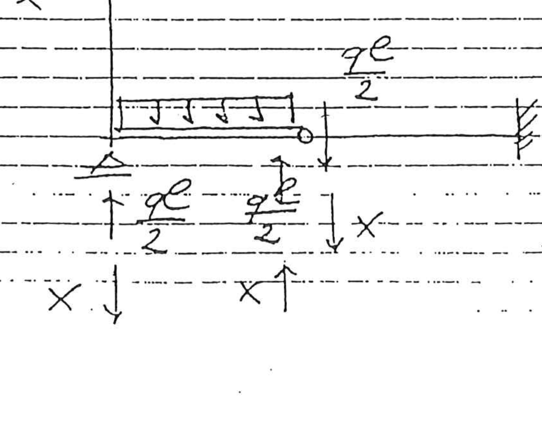
$$\sum_{i=1}^{5} \varphi_{i,D} = 0 = \frac{Xl}{EI} + \frac{ql^3}{24EI} - \frac{2\Delta}{l}$$

1C)

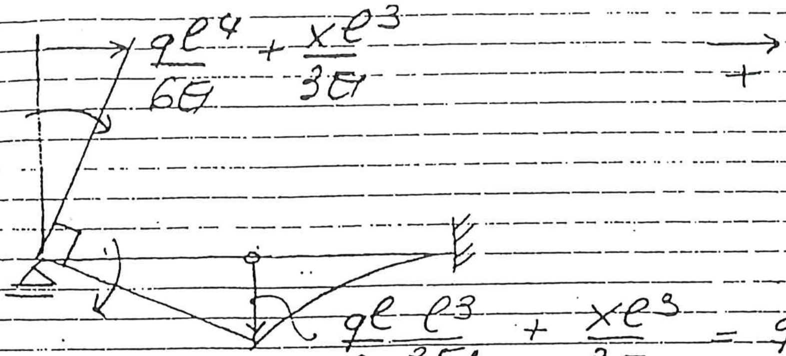


$$X = -\frac{ql^2}{24} + \frac{2\Delta EI}{l^2}$$

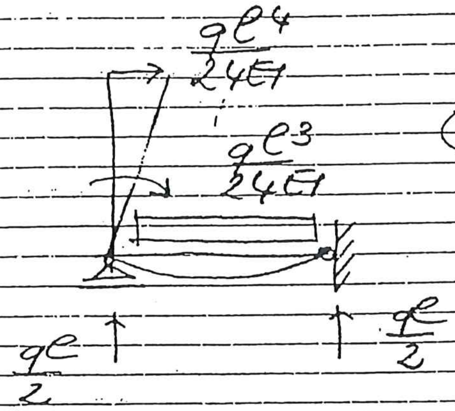
$$\frac{ql^3}{24EI} + 3\left(\frac{Xl}{3EI}\right) - \frac{2\Delta}{l} = 0$$

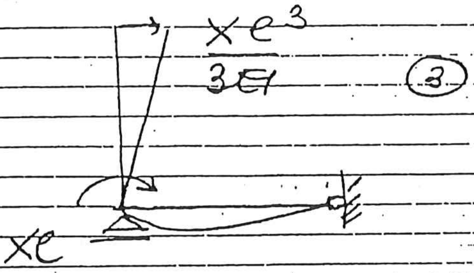


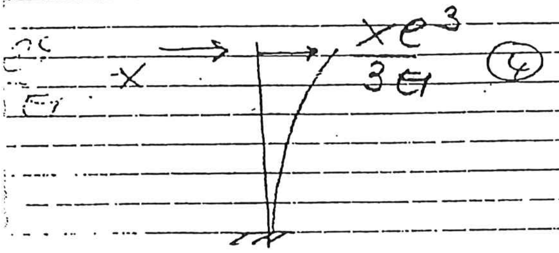
1



$$\frac{ql}{2}\frac{l^3}{3EI} + \frac{Xl^3}{3EI} = \frac{ql^4}{6EI} + \frac{Xl^3}{3EI}$$





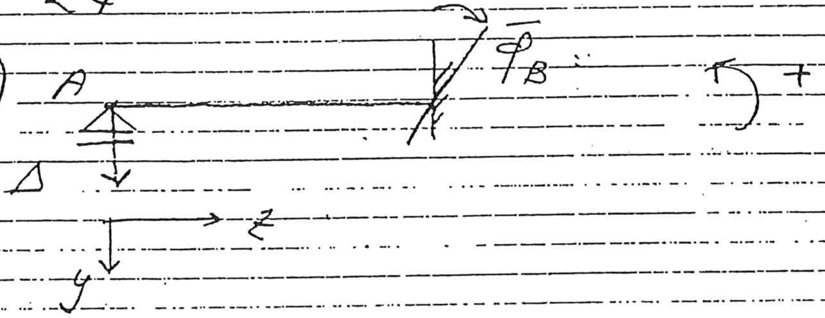


$$\sum_{i=1}^{4} v_A^i = \frac{ql^4}{6EI} + \frac{Xl^3}{3EI} + \frac{ql^4}{24EI} +$$

$$+ \frac{Xl^3}{3EI} + \frac{Xl^3}{3EI} = \frac{5ql^4}{24EI} + \frac{Xl^3}{EI} = -\Delta$$

$$\frac{5ql}{24} + X = -\frac{\Delta EI}{l^3} \qquad -\frac{5ql}{24} - \frac{\Delta EI}{l^3} = X$$

2A)



$$\frac{d^4v}{dz^4} = 0 \qquad \frac{d^3v}{dz^3} = C_1 \qquad \frac{d^2v}{dz^2} = C_1 z + C_2 = -\frac{M(z)}{EI}$$

$$\frac{dv}{dz} = \frac{C_1 z^2}{2} + C_2 z + C_3$$

$$v = \frac{C_1 z^3}{6} + \frac{C_2 z^2}{2} + C_3 z + C_4$$

$$v_A = \Delta \ , \quad M_A = 0 \ , \quad v_B = 0 \ , \quad \varphi_B = -\bar{\varphi}_B$$

$$z = 0 \rightarrow \boxed{v_A = \Delta = C_4}$$

$$\varphi = -\frac{dv}{dz}$$

$$z = \ell \rightarrow \varphi_B = -\frac{C_1 \ell^2}{2} - C_2 \ell - C_3 = -\bar{\varphi}_B$$

$$z = \ell \rightarrow v_B = \frac{C_1 \ell^3}{6} + C_2 \frac{\ell^2}{2} + C_3 \ell + \Delta = 0$$

$$z = 0 \rightarrow M_A = 0 = -\frac{d^2 v}{dz^2} EI = -C_2 EI \qquad \boxed{C_2 = 0}$$

$$C_3 = \bar{\varphi}_B - C_1 \frac{\ell^2}{2}$$

$$C_1 \frac{\ell^3}{6} + \bar{\varphi}_B \ell - C_1 \frac{\ell^3}{2} + \Delta = 0$$

$$-\frac{1}{3} C_1 \ell^3 + \bar{\varphi}_B \ell + \Delta = 0 \qquad \boxed{C_1 = \bar{\varphi}_B \frac{3}{\ell^2} + \frac{3\Delta}{\ell^3}}$$

$$\boxed{C_3 = \bar{\varphi}_B - \bar{\varphi}_B \frac{3}{2} - \frac{3}{2} \frac{\Delta}{\ell} = -\frac{\bar{\varphi}_B}{2} - \frac{3}{2} \frac{\Delta}{\ell}}$$

$$|\bar{\varphi}_B| = 0{,}2 \text{ rad} \qquad |\Delta| = 10 \text{ cm} \qquad \ell = 300 \text{ cm}$$

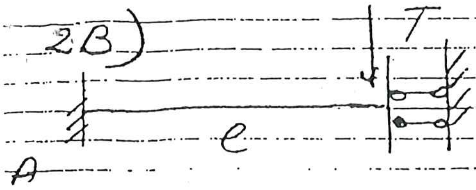
$$\boxed{C_1 = 0{,}2 \frac{3}{300^2} + \frac{3 \cdot 10}{300^3} = 7{,}7 \times 10^{-6}}$$

$$\boxed{C_3 = -\frac{0{,}2}{2} - \frac{3}{2} \frac{10}{300} = -0{,}15}$$

$$v(z) = \frac{7{,}7 \cdot 10^{-6}}{6} z^3 - 0{,}15 z + 10$$

$$z = \ell/2 \rightarrow \boxed{v(\ell/2) = -0{,}08125 \text{ m} = -8{,}125 \text{ cm}}$$

2B)



$T = 1.000\ kg$ $\qquad l = 300\ cm$

$E = 100\,000\ kg/cm^2$

$I = 50\,000\ cm^4$

$$\frac{d^4 v}{dz^4} = 0 \quad ecc.$$

$z = 0 \rightarrow v_A = 0 \rightarrow \boxed{C_4 = 0}$

$z = 0 \rightarrow \varphi_A = 0 \rightarrow \boxed{C_3 = 0}$

$$z = l \rightarrow \varphi_B = -\left.\frac{dv(z)}{dz}\right|_{z=l} = -\frac{C_1 l^2}{2} - C_2 l = 0$$

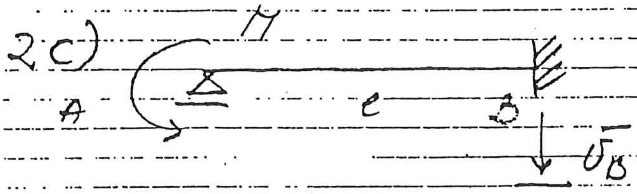
$$z = l \rightarrow T_B = -\left.\frac{d^3 v(z)}{dz^3}\right|_{z=l} EI = -C_1 EI = T$$

$$\boxed{C_1 = -T/EI = -2\times10^{-7}} \Rightarrow \boxed{C_2 = -\frac{C_1 l}{2} = 3\cdot10^{-5}}$$

$$v(z) = -\frac{2\times10^{-7}}{6} z^3 + 3\cdot10^{-5}\frac{z^2}{2} = 0.45\ cm$$

$z = l \quad \boxed{v(l) = 0.45\ cm}$

2C)



$|M| = 200.000\ kg\ cm$

$E = 100.000\ kg/cm^2$

$I = 50.000\ cm^4$

$l = 300\ cm$

$|\bar{v}_B| = 5\ cm$

$$\frac{d^4 v}{dz^4} = 0 \quad ecc$$

$z = 0 \rightarrow v_A = 0 \rightarrow \boxed{C_4 = 0}$

$$z = 0 \rightarrow M_A = -\left.\frac{d^2 v(z)}{dz^2}\right|_{z=0} EI = -C_2 EI = -M$$

$$\boxed{C_2 = \frac{M}{EI} = 4\cdot10^{-5}}$$

$z = l \rightarrow \varphi_B = -\frac{d\delta(z)}{dz}\Big|_{z=l} = -\frac{C_1 l^2}{2} - 4\cdot10^{-5} l - C_3 = 0$

$z = l \rightarrow \delta_B = \frac{C_1 l^3}{6} + 2\cdot10^{-5} l^2 + C_3 l = 5$

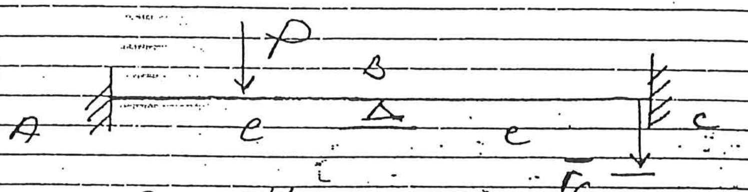
$\frac{C_1 l^3}{6} + 2\cdot10^{-5} l^2 - \frac{C_1 l^3}{2} - 4\cdot10^{-5} l^2 = -\frac{C_1 l^3}{3} - 2\cdot10^{-5} l^2 = 5$

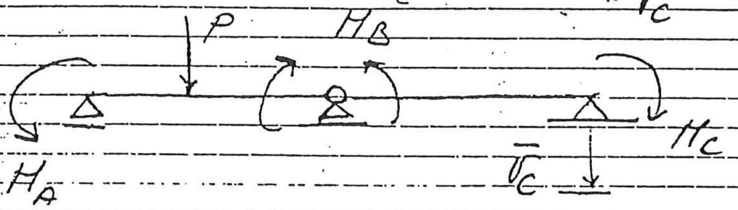
$C_1 = -\frac{6\cdot10^{-5}}{l} - \frac{15}{l^3}$ | $C_3 = 3\cdot10^{-5} l + \frac{15}{2l} - 4\cdot10^{-5} l = \frac{15}{2l} - 10^{-5} l$

$C_1 = -7{,}55\cdot10^{-7}$; $C_2 = 4\cdot10^{-5}$, $C_3 = 0{,}022$, $C_4 = 0$

$\delta\left(\frac{l}{2}\right) = -7{,}55\cdot10^{-7}\frac{l^3}{48} + 4\cdot10^{-5}\frac{l^2}{8} + 0{,}022\frac{l}{2} = 3{,}32$ cm

3A)





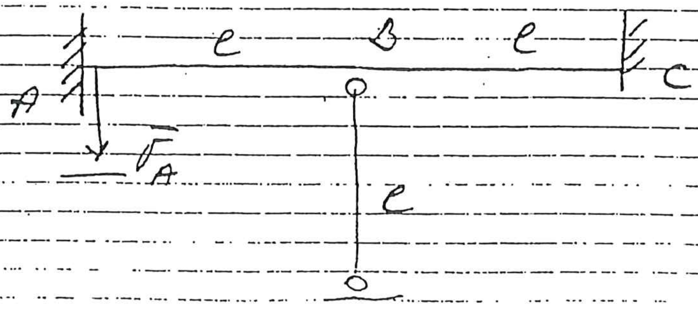
$\varphi_B^S = -\frac{M_B l}{3EI} - \frac{M_A l}{6EI} + \frac{Pl^2}{16EI} = \varphi_B^D =$

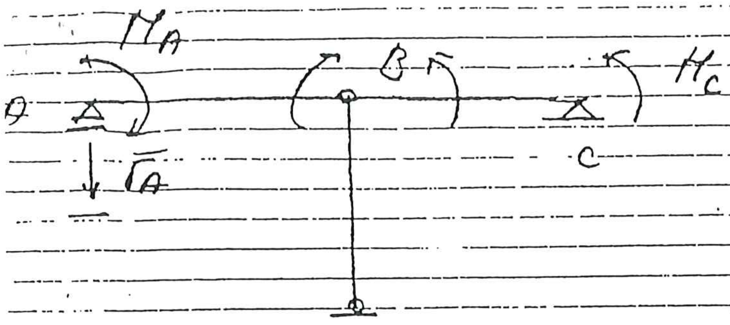
$= -\frac{\bar\delta_C}{l} + \frac{M_B l}{3EI} + \frac{M_C l}{6EI}$

$\varphi_A = \frac{M_A l}{3EI} + \frac{M_B l}{6EI} - \frac{Pl^2}{16EI} = 0$

$\varphi_C = -\frac{M_C l}{3EI} - \frac{M_B l}{6EI} - \frac{\bar\delta_C}{l} = 0$

3B)





$$\frac{M_B}{l}+\frac{M_A}{l} \qquad \frac{M_B}{l}+\frac{M_C}{l}$$

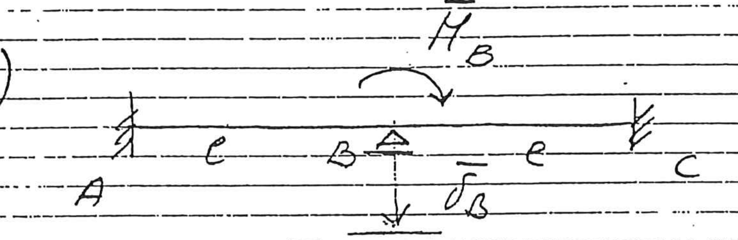
$$\varphi_A = \frac{\bar{\delta}_A}{l} - \left(\frac{2M_B}{l}+\frac{M_A}{l}+\frac{M_C}{l}\right)\frac{l}{EA}\cdot\frac{1}{l} + \frac{M_B l}{6EI} - \frac{M_A l}{3EI} = 0$$

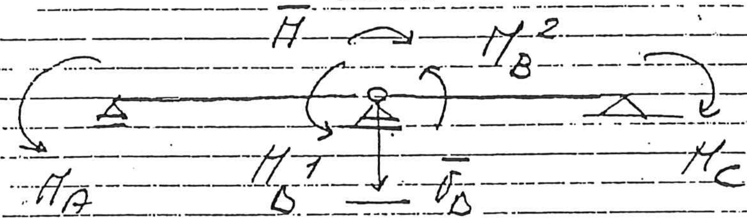
$$\varphi_B^S = -\left(2M_B+M_A+M_C\right)\frac{1}{lEA} + \frac{M_A l}{6EI} - \frac{M_B l}{3EI} + \frac{\bar{\delta}_A}{l} =$$

$$= \varphi_B^D = \left(2M_B+M_A+M_C\right)\frac{1}{lEA} + \frac{M_B l}{3EI} - \frac{M_C l}{6EI} =$$

$$\varphi_C = \left(2M_B+M_A+M_C\right)\frac{1}{lEA} - \frac{M_B l}{6EI} + \frac{M_C l}{3EI} = 0$$

3c)

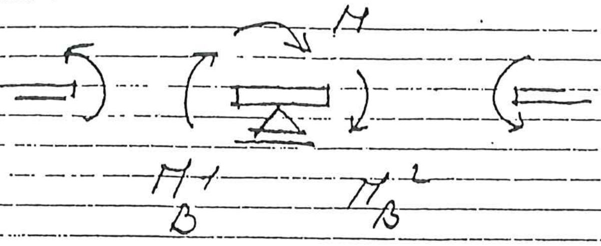




$$\varphi_A = \frac{M_A l}{3EI} - \frac{M_B^1 l}{6EI} - \frac{\bar{\delta}_B}{l} = 0$$

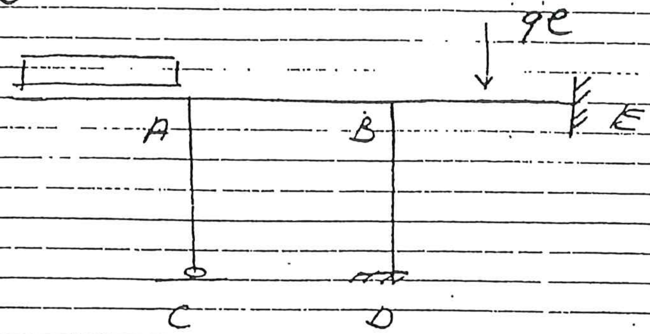
$$\varphi_B^S = \frac{M_B^1 l}{3EI} - \frac{M_A l}{6EI} - \frac{\bar{\delta}_B}{l} = \varphi_B^D = \frac{M_B^2 l}{3EI} + \frac{M_C l}{6EI} + \frac{\bar{\delta}_B}{l}$$

$$\varphi_C = -\frac{M_C l}{3EI} - \frac{M_B^2 l}{6EI} + \frac{\bar{\delta}_B}{l} = 0$$



$\Sigma M_B = M_B^1 + M_B^2 + M = 0$

4 A)



φ_E, φ_B

$M^\circ_{AC} = 0 \qquad M_{AC} = \frac{3EI}{l}\varphi_A$

$M^\circ_{AD} = 0 \qquad M_{AB} = \frac{4EI}{l}\varphi_A + \frac{2EI}{l}\varphi_B$

$M>0$

$\Sigma M_A = \frac{ql^2}{2} - \frac{3EI}{l}\varphi_A - \frac{4EI}{l}\varphi_A - \frac{2EI}{l}\varphi_B =$

$= \frac{ql^2}{2} - \frac{7EI}{l}\varphi_A - \frac{2EI}{l}\varphi_B = 0 \quad *$

$M^\circ_{BA} = 0 \qquad M_{BA} = \frac{4EI}{l}\varphi_B + \frac{2EI}{l}\varphi_A$

$M^\circ_{BE} = \frac{ql^2}{8} \qquad M_{BE} = \frac{4EI}{l}\varphi_B$

$M>0$

$M^\circ_{BD} = 0 \qquad M_{BD} = \frac{4EI}{l}\varphi_B$

$\Sigma M_B = -\frac{ql^2}{8} - \frac{12EI}{l}\varphi_B - \frac{2EI}{l}\varphi_A = 0 \quad *$

dalla II^a $\quad \varphi_A = -\left(\frac{ql^2}{8} + \frac{12EI}{l}\varphi_B\right)\frac{l}{2EI}$

in I^a $\quad \frac{ql^2}{2} + \frac{7EI}{l}\left(\frac{ql^2}{8} + \frac{12EI}{l}\varphi_B\right)\frac{l}{2EI} - \frac{2EI}{l}\varphi_B =$

$= \frac{ql^2}{2} + \frac{7ql^2}{16} + \frac{\overset{42}{\cancel{84}}}{\cancel{2}}\frac{EI}{l}\varphi_B - \frac{2EI}{l}\varphi_B =$

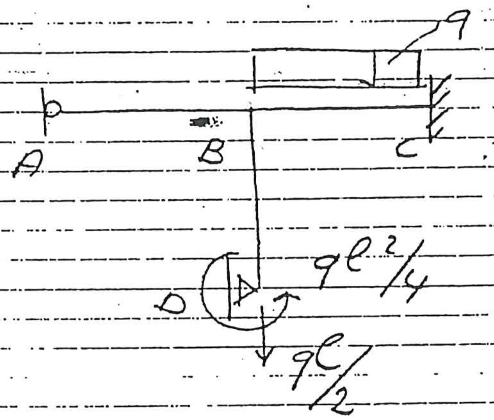
$= \frac{15ql^2}{16} + 40\frac{EI}{l}\varphi_B = 0 \qquad \varphi_B = -\frac{15ql^2}{16}\frac{l}{40EI} =$

$$= -\frac{\cancel{15}\,3\; q\ell^3}{\cancel{640}\;128\; EI} = -0{,}02343\;\frac{q\ell^3}{EI} = \varphi_B$$

$$\boxed{\varphi_A} = -\left(\frac{q\ell^2}{8} - \frac{12\,EI}{8}\,\frac{3q\ell^3}{128\,EI}\,\ell^2\right)\frac{\ell}{2EI} =$$

$$= \frac{20}{128}\,q\ell^2 \cdot \frac{\ell}{2EI} = \frac{10}{128}\,\frac{q\ell^3}{EI} = \frac{5}{64}\,\frac{q\ell^3}{EI} = 0{,}07812\,\frac{q\ell^3}{EI}$$

4B)



$$M_{BA}^{0} = 0 \qquad T_{BA}^{0} = 0$$

$$M_{BA} = \frac{3EI}{\ell}\varphi_B + \frac{3EI}{\ell^2}v_B$$

$$T_{BA} = \frac{3EI}{\ell^2}\varphi_B + \frac{3EI}{\ell^3}v_B$$

$$M_{BC}^{0} = \frac{q\ell^2}{12} \qquad T_{BC}^{0} = -\frac{q\ell}{2}$$

$$M_{BC} = \frac{4EI}{\ell}\varphi_B - \frac{6EI}{\ell^2}v_B$$

$$T_{BC} = -\frac{6EI}{\ell^2}\varphi_B + \frac{12EI}{\ell^3}v_B$$

$$M_{BD}^{0} = \frac{q\ell^2}{8} \qquad M_{BD} = \frac{3EI}{\ell}\varphi_B$$

$$\Sigma M_B = -\frac{3EI}{l}\varphi_B - \frac{3EI}{l^2}v_B - \frac{ql^2}{12} - \frac{4EI}{l}\varphi_B + \frac{6EI}{l^2}v_B - \frac{ql^2}{8} - \frac{3EI}{l}\varphi_B =$$

$$= -\frac{10EI}{l}\varphi_B + \frac{3EI}{l^2}v_B - \frac{5ql^2}{24} = 0$$

$$\Sigma Y_B = -\frac{3EI}{l^2}\varphi_B - \frac{3EI}{l^3}v_B + \frac{ql}{2} + \frac{6EI}{l^2}\varphi_B - \frac{12EI}{l^3}v_B + \frac{ql}{2} =$$

$$= \frac{3EI}{l^2}\varphi_B - \frac{15EI}{l^3}v_B + ql = 0$$

$$\varphi_B = \frac{l}{10EI}\left(\frac{3EI}{l^2}v_B - \frac{5}{24}ql^2\right) = \frac{3}{10}\frac{v_B}{l} - \frac{5}{240}\frac{ql^3}{EI}$$

$$\frac{3EI}{l^2}\cdot\frac{3v_B}{10l} - \frac{3\cancel{EI}}{l^2}\frac{5}{240}\frac{ql^3}{\cancel{EI}} - \frac{15EI}{l^3}v_B + ql =$$

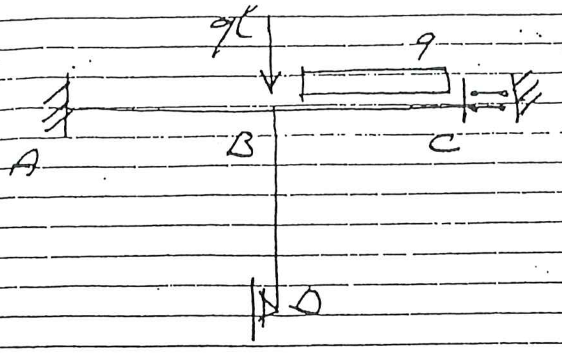
$$= \frac{9EI}{10l^3}v_B - \frac{15}{240}ql - \frac{15EI}{l^3}v_B + ql =$$

$$= -\frac{141}{10}\frac{EI}{l^3}v_B + \frac{225}{240}ql = 0$$

$$v_B = \frac{225}{240}\frac{10l^3}{141EI}ql = 0{,}06649\frac{ql^4}{EI}$$

$$\varphi_B = 0{,}3\cdot 0{,}06649\frac{ql^3}{EI} - \frac{5}{240}\frac{ql^3}{EI} = -8{,}86\times10^{-4}\frac{ql^3}{EI}$$

4c)



$$M^{0}_{BA} = 0 \qquad T^{0}_{BA} = 0$$

$$M_{BA} = \frac{4EI}{l}\varphi_B + \frac{6EI}{l^2}\delta_B$$

$$T_{BA} = \frac{6EI}{l^2}\varphi_B + \frac{12EI}{l^3}\delta_B$$

$$M^{0}_{BC} = \frac{q(2l)^2}{12} = \frac{ql^2}{3} \qquad T^{0}_{BC} = -ql$$

$$M_{BC} = \frac{EI}{l}\varphi_B \qquad T_{BC} = 0$$

$$M^{0}_{BD} = 0 \qquad M_{BD} = \frac{3EI}{l}\varphi_B$$

$$\Sigma M_B = -\frac{4EI}{l}\varphi_B - \frac{6EI}{l^2}\delta_B - \frac{ql^2}{3} - \frac{EI}{l}\varphi_B - \frac{3EI}{l}\varphi_B =$$

$$= -\frac{8EI}{l}\varphi_B - \frac{6EI}{l^2}\delta_B - \frac{ql^2}{3} = 0$$

$$\Sigma Y^{BD} = -\frac{6EI}{l^2}\varphi_B - \frac{12EI}{l^3}\delta_B + 2ql = 0$$

$$\varphi_B = -\frac{\cancel{l^2}}{\cancel{6EI}} \cdot \frac{\overset{2}{\cancel{12}}EI}{l^{\cancel{3}}}\delta_B + \frac{l^2}{6EI} \cdot 2ql =$$

$$= -\frac{2}{l}\delta_B + \frac{ql^3}{3EI}$$

$$\frac{16EI}{l^2}\delta_B - \frac{8ql^2}{3} - \frac{6EI}{l^2}\delta_B - \frac{ql^2}{3} =$$

$$= \frac{10EI}{l^2}\delta_B - 3ql^2 = 0$$

$$v_B = \frac{3}{10 EI} q l^4 = 0,3 \frac{q l^4}{EI}$$

$$\varphi_B = -\frac{2}{\cancel{l}} \cdot \frac{3}{10} q \frac{l^{\cancel{4}\,3}}{EI} + \frac{q l^3}{3EI} = (-18 + 10)\frac{q l^3}{30\, EI} = -\frac{\cancel{8}^{\,4}}{\cancel{30}_{\,15}} \frac{q l^3}{EI} = -0,2\bar{6}\, \frac{q l^3}{EI}$$