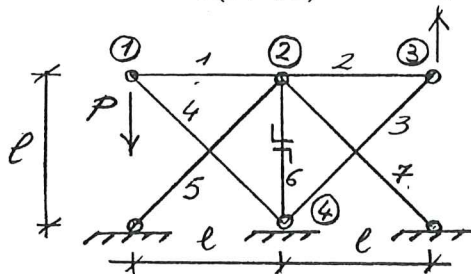


Cognome.....Nome.....
Anno di Corso.....

1 Determinare, via P.L.V., quale delle seguenti espressioni fornisce lo sforzo assiale nell'asta 7:

1: $\pm \frac{2P(1-\sqrt{2})+EA \alpha \Delta t}{\sqrt{2}(1-\sqrt{2})}$; 2: $\pm \frac{2P(1+\sqrt{2})+EA \alpha \Delta t}{2(1+\sqrt{2})}$; ~~3: $\pm \frac{2P(1+\sqrt{2})+EA \alpha \Delta t}{\sqrt{2}(1+\sqrt{2})}$~~

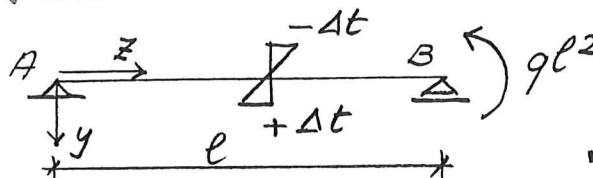


Δt

2 Determinare, via equazione differenziale del secondo ordine, quale delle seguenti espressioni fornisce lo spostamento verticale in $z = 2L/3$:

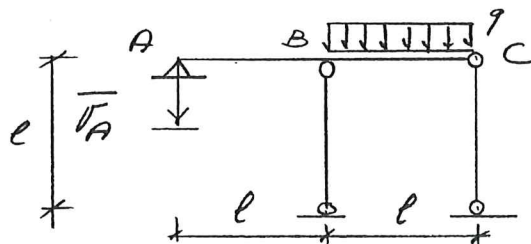
el. elastica

1: $\frac{3}{14} \frac{ql^4}{EI} + \frac{\alpha \Delta t l^2}{4h}$; ~~2: $\frac{5}{81} \frac{ql^4}{EI} + \frac{2\alpha \Delta t l^2}{9h}$~~ ; 3: $\frac{2}{25} \frac{ql^4}{EI} + \frac{2\alpha \Delta t l^2}{9h}$; 4: $\frac{3}{14} \frac{ql^4}{EI} + \frac{\alpha \Delta t l^2}{9h}$



3 Determinare con il Metodo delle Forze quale delle seguenti espressioni fornisce il Momento M_B in B.

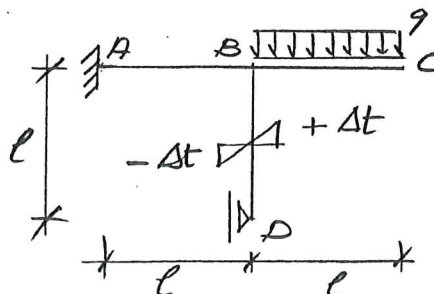
~~1: $\pm \frac{-\frac{ql}{2} \left(\frac{1}{A} - \frac{l^2}{12I} \right) + \frac{\bar{v}_A}{l} E}{\left(\frac{2l}{3I} + \frac{5}{lA} \right)}$~~ ; 2: $\pm \frac{\frac{ql}{2} \left(\frac{1}{A} - \frac{l^2}{6I} \right) + \frac{\bar{v}_A}{l} E}{\left(\frac{l}{3I} + \frac{5}{lA} \right)}$; 3: $\pm \frac{ql \left(\frac{1}{2A} + \frac{3l^2}{I} \right) - \frac{\bar{v}_A}{l} E}{\left(\frac{l}{6I} + \frac{2}{lA} \right)}$



4 Determinare con il Metodo delle Forze quale delle seguenti espressioni fornisce la reazione del carrello in D.

comp. term.

1: $\pm \frac{\frac{3ql^3}{8I} - \frac{2\alpha \Delta t l E}{h}}{\frac{2}{A} + \frac{3l^2}{2I}}$; 2: $\pm \frac{\frac{ql^3}{3I} + \frac{2\alpha \Delta t l E}{h}}{\frac{1}{A} + \frac{4l^2}{3I}}$; ~~3: $\pm \frac{\frac{ql^3}{I} + \frac{\alpha \Delta t l E}{h}}{\frac{1}{A} + \frac{4l^2}{3I}}$~~ ; 4: $\pm \frac{\frac{2ql^3}{I} - \frac{\alpha \Delta t l E}{h}}{\frac{3}{A} - \frac{3l^2}{I}}$

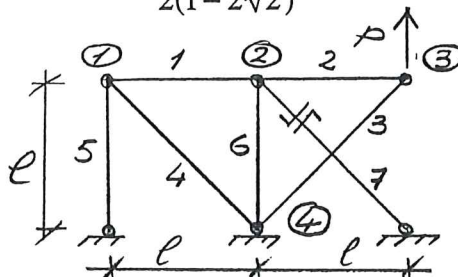


Cognome.....Nome.....

Anno di Corso.....

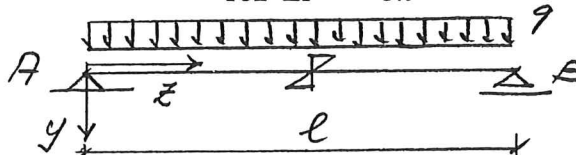
1 Determinare, via P.L.V., quale delle seguenti espressioni fornisce lo sforzo assiale nell'asta 5:

$$1: \pm \frac{P(1+2\sqrt{2})-2EA \propto \Delta t}{3+4\sqrt{2}}; \quad 2: \pm \frac{P(1-2\sqrt{2})+2EA \propto \Delta t}{2(1-2\sqrt{2})}; \quad 3: \pm \frac{2P(1+\sqrt{2})+EA \propto \Delta t}{2(1+\sqrt{2})}$$


$$15 - \Delta t$$

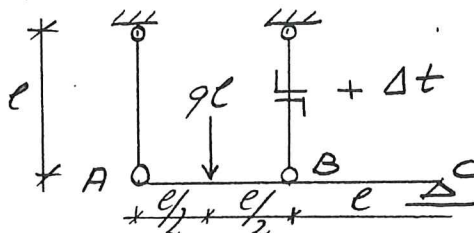
2 Determinare, via equazione differenziale del secondo ordine, quale delle seguenti espressioni fornisce la rotazione in $z = L/3$:

$$1: -\frac{11}{162} \frac{ql^3}{EI} + \frac{\alpha \Delta t l}{h}; \quad 2: -\frac{7}{162} \frac{ql^3}{EI} + \frac{2\alpha \Delta t l}{3h}; \quad \cancel{3: -\frac{13}{648} \frac{ql^3}{EI} + \frac{\alpha \Delta t l}{3h}}$$


$$\frac{\Delta z}{\Delta t} + \Delta t$$

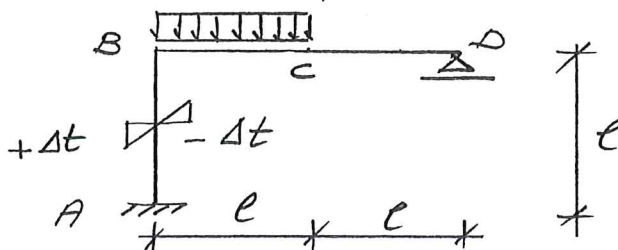
3 Determinare con il Metodo delle Forze quale delle seguenti espressioni fornisce il Momento M_B in B.

$$\begin{array}{lll} \text{1: } \pm \frac{\frac{ql}{2} \left(-\frac{1}{A} + \frac{l^2}{8I} \right) - 2\alpha\Delta t E}{\left(\frac{2l}{3I} + \frac{5}{lA} \right)}; & \text{2: } \pm \frac{\frac{ql}{2} \left(\frac{1}{A} - \frac{l^2}{6I} \right) - 2\alpha\Delta t E}{\left(\frac{l}{3I} + \frac{5}{lA} \right)}; & \text{3: } \pm \frac{\frac{ql}{2} \left(\frac{1}{2A} + \frac{3l^2}{I} \right) + \alpha\Delta t E}{\left(\frac{l}{6I} + \frac{2}{lA} \right)}; \end{array}$$



4. Determinare con il Metodo delle Forze quale delle seguenti espressioni fornisce la reazione del carrello in D.

$$1: \pm \frac{ql \left(\frac{3}{A} - \frac{7l^2}{48I} \right) - \frac{2\alpha \Delta t l E}{h}}{\frac{3}{A} + \frac{3l^2}{2I}}; \quad 2: \pm \frac{ql \left(\frac{2}{A} + \frac{13l^2}{24I} \right) + \frac{2\alpha \Delta t l E}{h}}{\frac{2}{A} + \frac{4l^2}{3I}}; \quad \cancel{3: \pm \frac{ql \left(\frac{1}{A} + \frac{19l^2}{24I} \right) + \frac{4\alpha \Delta t l E}{h}}{\frac{1}{A} + \frac{20l^2}{3I}}}$$

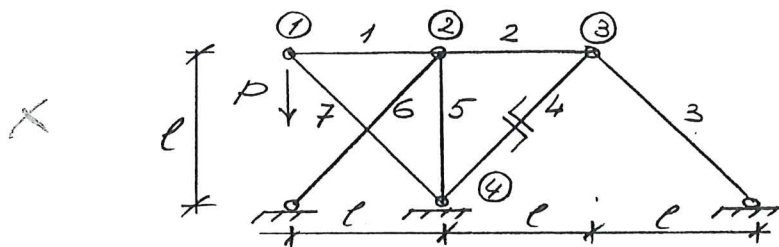


Cognome.....Nome.....

Anno di Corso.....

1 Determinare, via P.L.V., quale delle seguenti espressioni fornisce lo sforzo assiale nell'asta 3:

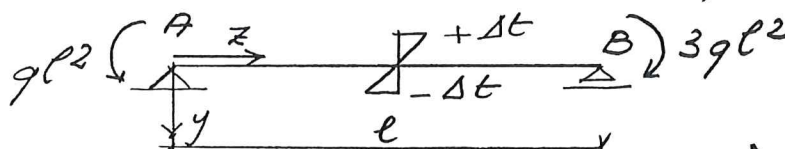
1: $\pm \frac{P(1+2\sqrt{2})+2EA \alpha \Delta t}{3+\sqrt{2}}$; ~~2: $\pm \frac{P(1+2\sqrt{2})+EA \alpha \Delta t}{2(3+\sqrt{2})}$~~ ; 3: $\pm \frac{P(1+\sqrt{2})+2EA \alpha \Delta t}{2(1+\sqrt{2})}$



]Γ + Δt

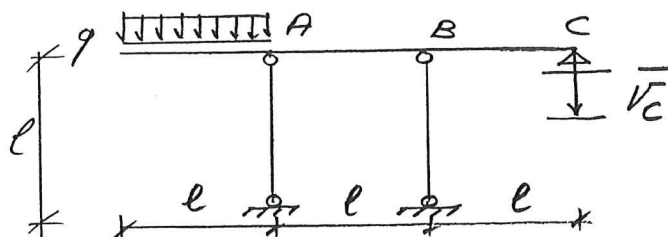
2 Determinare, via equazione differenziale del secondo ordine, quale delle seguenti espressioni fornisce la rotazione in $z = 2L/3$:

1: $-\frac{7}{24} \frac{ql^3}{EI} - \frac{2\alpha \Delta t l}{h}$; 2: $-\frac{9}{31} \frac{ql^3}{EI} - \frac{2\alpha \Delta t l}{3h}$; ~~3: $-\frac{5}{18} \frac{ql^3}{EI} - \frac{\alpha \Delta t l}{3h}$~~



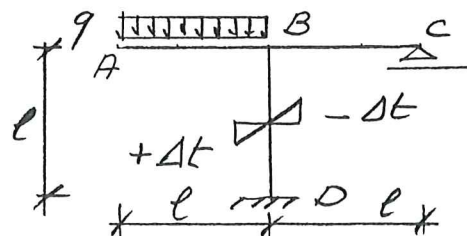
3 Determinare con il Metodo delle Forze quale delle seguenti espressioni fornisce il Momento M_B in B.

1: $\pm \frac{\frac{ql}{2} \left(-\frac{1}{A} + \frac{l^2}{3I} \right) + \bar{v}_{CE}}{\left(\frac{2l}{3I} + \frac{3}{lA} \right)}$; 2: $\pm \frac{\frac{ql}{2} \left(\frac{2}{A} - \frac{l^2}{6I} \right) - \bar{v}_{CE}}{\left(\frac{l}{3I} + \frac{5}{lA} \right)}$; ~~3: $\pm \frac{\frac{ql}{2} \left(\frac{5}{A} - \frac{l^2}{6I} \right) + \bar{v}_{CE}}{\left(\frac{2l}{3I} + \frac{5}{lA} \right)}$~~

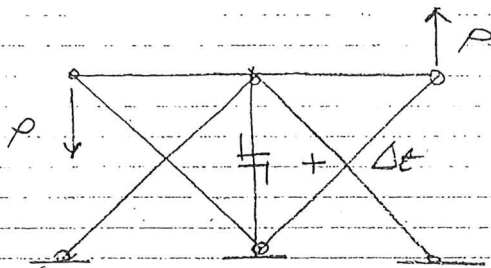


4 Determinare con il Metodo delle Forze quale delle seguenti espressioni fornisce la reazione del carrello in C.

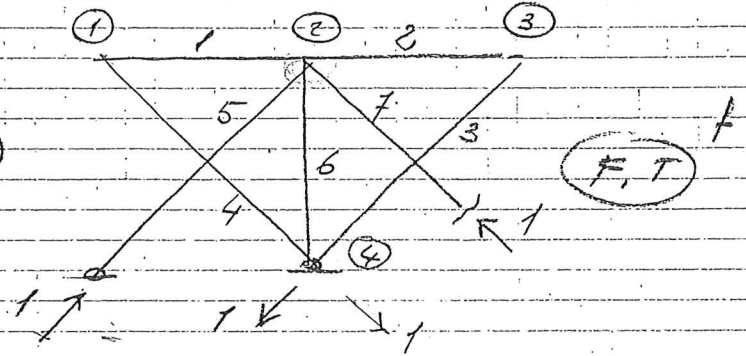
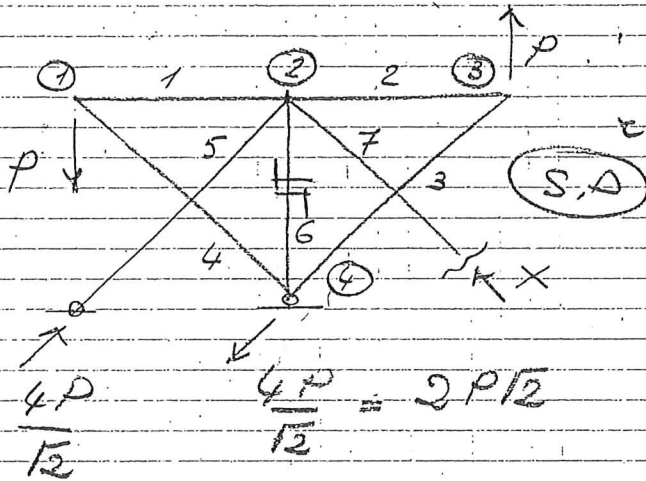
~~1: $\pm \frac{ql \left(\frac{1}{A} - \frac{l^2}{2I} \right) + \frac{2\alpha \Delta t l E}{h}}{\frac{1}{A} + \frac{4l^2}{3I}}$~~ ; 2: $\pm \frac{ql \left(\frac{2}{A} + \frac{3l^2}{2I} \right) + \frac{\alpha \Delta t l E}{h}}{\frac{2}{A} + \frac{3l^2}{4I}}$; 3: $\pm \frac{ql \left(\frac{1}{A} - \frac{3l^2}{4I} \right) + \frac{2\alpha \Delta t l E}{h}}{\frac{1}{A} - \frac{3l^2}{4I}}$



1. A)

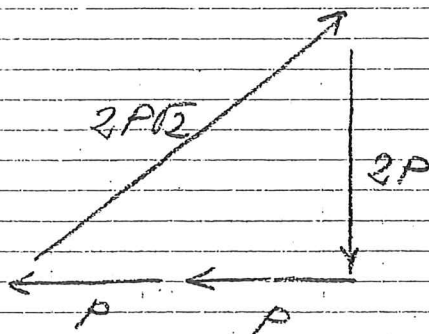


asta 7



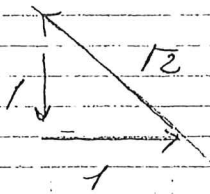
asta	1	2	3	4	5	6	7
N^t	0	0	0	0	-1	$\sqrt{2}$	-1
$N^z_{(P)}$	P	-P	$P\sqrt{2}$	$-P\sqrt{2}$	$-2P\sqrt{2}$	2P	0
Δt	0	0	0	0	0	$+\Delta t$	0
l_i	l	l	$l\sqrt{2}$	$l\sqrt{2}$	$l\sqrt{2}$	l	$l\sqrt{2}$

②

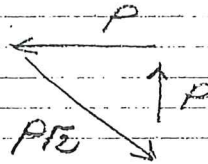


$$dwe = 1, \eta = 0 = \int \eta = \frac{1}{EA} \left[(-1)(-x - 2P\sqrt{2})l\sqrt{2} + \sqrt{2}(\sqrt{2}x + 2P)l + (-1)(-x)l\sqrt{2} \right] + \sqrt{2} \alpha l \Delta t =$$

②



②

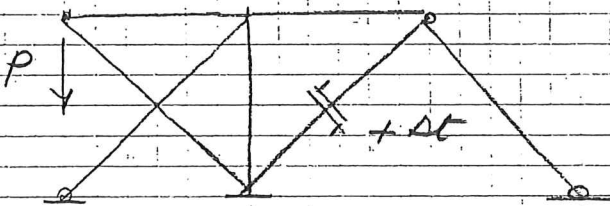


③

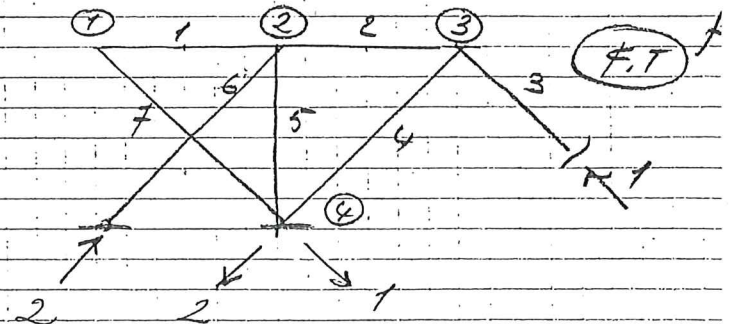
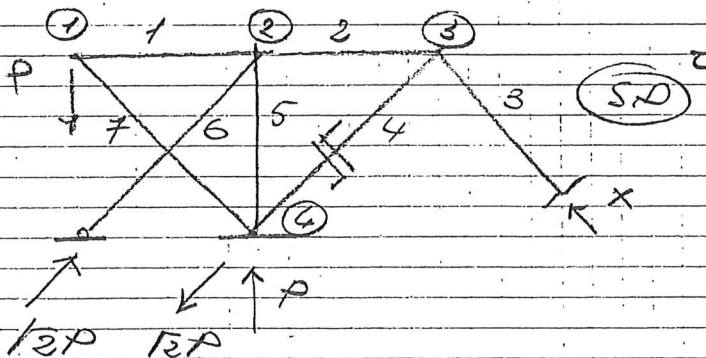
$$\begin{aligned} dwe = 1 \cdot \eta = 0 &= d\bar{w}_i = \frac{1}{EA} [(-1)(-x)l + 1/2 \cdot 1/2 \cdot x \cdot l\sqrt{2} + \\ &+ (-1)(-x)l + 1 \cdot (x - P)l + (-1/2)(-1/2 x + P/2)l\sqrt{2}] \\ &+ (-1/2)(-x \cdot l\sqrt{2} \Delta t) = \\ &= \frac{1}{EA} (xl + 2/2 xl + xl + xl - Pl + 2/2 xl - \\ &- 2/2 Pl) + 2\alpha EA \Delta t = \\ &= \frac{1}{EA} (3xl + 4/2 xl - Pl - 2/2 Pl) + 2\alpha EA \Delta t = \\ &= X(3 + 4/2) - P(1 + 2/2) + 2\alpha EA \Delta t \end{aligned}$$

$$X = \frac{P(1 + 2/2) - 2\alpha EA \Delta t}{(3 + 4/2)}$$

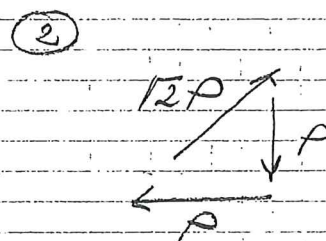
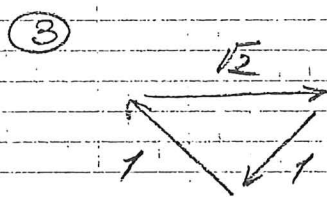
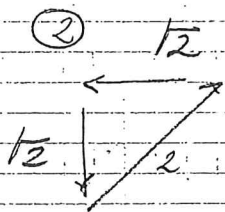
1.c)



data 3



node	1	2	3	4	5	6	7	④
u^f	0	$-\sqrt{2}$	-1	1	$\sqrt{2}$	-2	0	
$N(P)$	P	0	0	0	P	$-\sqrt{2}P$	$-\sqrt{2}P$	
Δt	0	0	0	$+\Delta t$	0	0	0	
q_i	l	l	$l\sqrt{2}$	$l\sqrt{2}$	l	$l\sqrt{2}$	$l\sqrt{2}$	



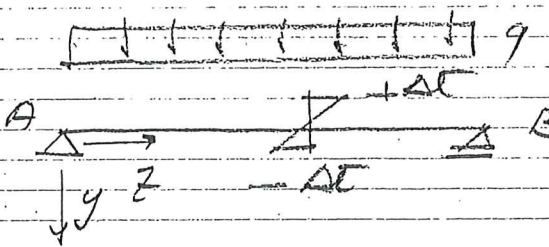
$$\begin{aligned}
 \delta w_e &= u \cdot \eta = 0 = \delta w_i = \frac{1}{EA} \left[(-\sqrt{2}) (-\sqrt{2}X) l + \right. \\
 &+ (-1) (-1) \times l\sqrt{2} + 1 \times l\sqrt{2} + \sqrt{2} (\sqrt{2}X + P) l + \\
 &+ (-2) (-2X - \sqrt{2}P) l\sqrt{2} + 1 \cdot 0 \cdot l\sqrt{2} \Delta t = \\
 &= \frac{1}{EA} \left(2Xl + 2Xl + 2Xl + 2Xl + 2Pl + \right. \\
 &+ 4\sqrt{2}Xl + 4Pl \left. \right) + 0 \cdot l\sqrt{2} \Delta t = \\
 &= 4X + 6\sqrt{2}X + 4P + 2P + 0 \cdot EA \sqrt{2} \Delta t = \\
 &= 2\sqrt{2}X(\sqrt{2} + 3) + \sqrt{2}P(2\sqrt{2} + 1) + 0 \cdot EA \sqrt{2} \Delta t
 \end{aligned}$$

$$\boxed{X = \frac{P(2\sqrt{2} + 1) + 0 \cdot EA \Delta t}{2(\sqrt{2} + 3)}}$$

⑤



$$= \frac{10}{162} \frac{q l^4}{EI} + \frac{2}{9} \frac{\alpha \Delta t l^2}{h} = \frac{5}{81} \frac{q l^4}{EI} + \frac{2 \alpha \Delta t l^2}{9 h}$$

2B)  (7)

$\chi(\Delta t) = -\frac{2\alpha \Delta t}{h}$

$$\frac{d^2 v(z)}{dz^2} = -\chi(q) - \chi(\Delta t) = -\frac{17(qz)}{EI} + \frac{2\alpha \Delta t}{h} =$$

$$= -\frac{q l z}{3 EI} + \frac{q z^2}{2 EI} + \frac{2\alpha \Delta t}{h}$$

$$\frac{dv(z)}{dz} = -\frac{q l z^2}{4 EI} + \frac{q z^3}{6 EI} + \frac{2\alpha \Delta t z}{h} + C_1$$

$$v(z) = -\frac{q l z^3}{12 EI} + \frac{q z^4}{24 EI} + \frac{2\alpha \Delta t z^2}{2h} + C_1 z + C_2$$

$$z=0 \quad v_A=0 \rightarrow \underline{C_2=0}$$

$$z=l \quad v_B=0 \rightarrow -\frac{q l^4}{12 EI} + \frac{q l^4}{24 EI} + \frac{\alpha \Delta t l^2}{h} + C_1 l = 0$$

$$-\frac{q l^4}{24 EI} + \frac{\alpha \Delta t l^2}{h} + C_1 l = 0$$

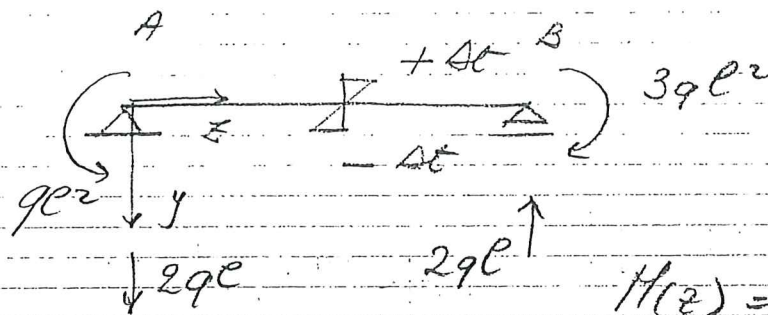
$$\underline{C_1 = \frac{q l^3}{24 EI} - \frac{\alpha \Delta t l}{h}}$$

$$\phi(z) = \frac{q l z^2}{4 EI} - \frac{q z^3}{6 EI} - \frac{2\alpha \Delta t z}{h} - \frac{q l^3}{24 EI} + \frac{\alpha \Delta t l}{h}$$

$$\left[\phi(z=l) = \frac{q l^3}{36 EI} - \frac{q l^3}{162 EI} - \frac{2\alpha \Delta t l}{3h} - \frac{q l^3}{24 EI} + \frac{\alpha \Delta t l}{h} = \right.$$

$$\left. = -\frac{13}{648} \frac{q l^3}{EI} + \frac{\alpha \Delta t l}{3h} \right]$$

2c)



$$X(\Delta t) = -\frac{2\alpha \Delta t}{L}$$

$$H(z) = -2qlz - ql^2$$

$$\frac{d^2v(z)}{dz^2} = -X(\overline{H}) - X(\Delta t) = \frac{2qlz}{EI} + \frac{ql^2}{EI} + \frac{2\alpha \Delta t}{L}$$

$$\frac{dv(z)}{dz} = \frac{2qlz^2}{2EI} + \frac{ql^2z}{EI} + \frac{2\alpha \Delta t z}{L} + C_1$$

$$v(z) = \frac{qlz^3}{3EI} + \frac{ql^2z^2}{2EI} + \frac{2\alpha \Delta t z^2}{2L} + C_1z + C_2$$

$$z=0 \quad v_A = 0 \rightarrow C_2 = 0$$

$$z=l \quad v_B = 0 \rightarrow \frac{ql^4}{3EI} + \frac{ql^4}{2EI} + \frac{\alpha \Delta t l^2}{L} + C_1 l = 0$$

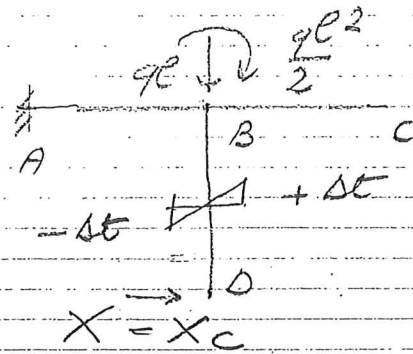
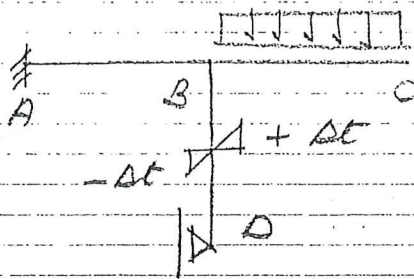
$$C_1 = -\frac{5ql^3}{6EI} - \frac{\alpha \Delta t l}{L}$$

$$\left| \phi\left(z = \frac{2}{3}l\right) = -\frac{ql}{EI} \frac{4l^2}{9} - \frac{ql^2}{EI} \frac{2l}{3} - \frac{2\alpha \Delta t \cdot \frac{2l}{3}}{L} + \right.$$

$$\left. + \frac{5}{6} \frac{ql^3}{EI} + \frac{\alpha \Delta t l}{L} = \right.$$

$$= -\frac{5ql^3}{18EI} - \frac{\alpha \Delta t l}{3L}$$

4.A)



(10)

$$U_D = \frac{Xl}{EA} + \frac{Xl}{EI} \cdot l \cdot l + \frac{Xl^3}{3EI} - \frac{ql \cdot l^2}{2EI} \cdot l - \frac{ql^2}{2} \cdot \frac{l \cdot l}{EI}$$

$$- \frac{\alpha \Delta t l^2}{h} =$$

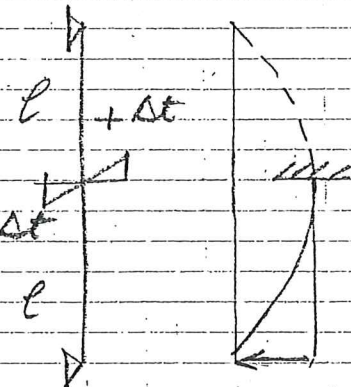
$$= \frac{Xl}{EA} + \frac{4Xl^3}{3EI} - \frac{ql^4}{EI} - \frac{\alpha \Delta t l^2}{h} =$$

$$= Xl \left(\frac{1}{A} + \frac{4l^2}{3I} \right) - \frac{ql^4}{EI} -$$

$$- \frac{\alpha \Delta t l^2}{h} = 0$$

$$\frac{ql^3}{I} + \frac{\alpha \Delta t l^2}{h}$$

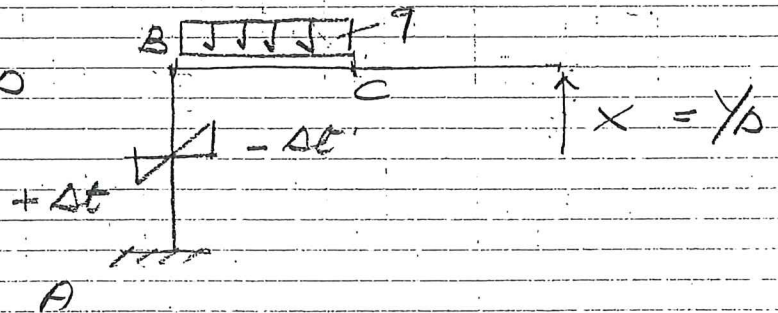
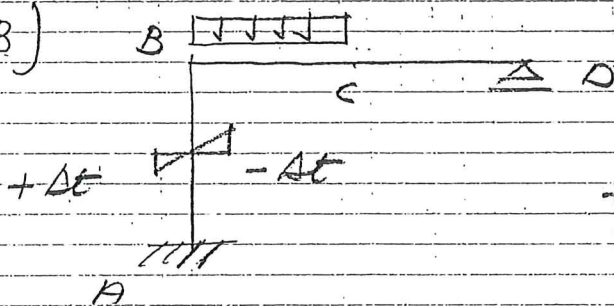
$$X = \pm \frac{\frac{ql^3}{I} + \frac{\alpha \Delta t l^2}{h}}{\frac{1}{A} + \frac{4l^2}{3I}}$$



$$\frac{\alpha \Delta t l^2}{h} =$$

$$= \frac{\alpha \Delta t (2l)^2}{4h}$$

4.B)



$$U_D = - \frac{Xl}{EA} - X \cdot 2l \cdot \frac{l}{EI} \cdot 2l - \frac{X \cdot 8l^3}{3EI} +$$

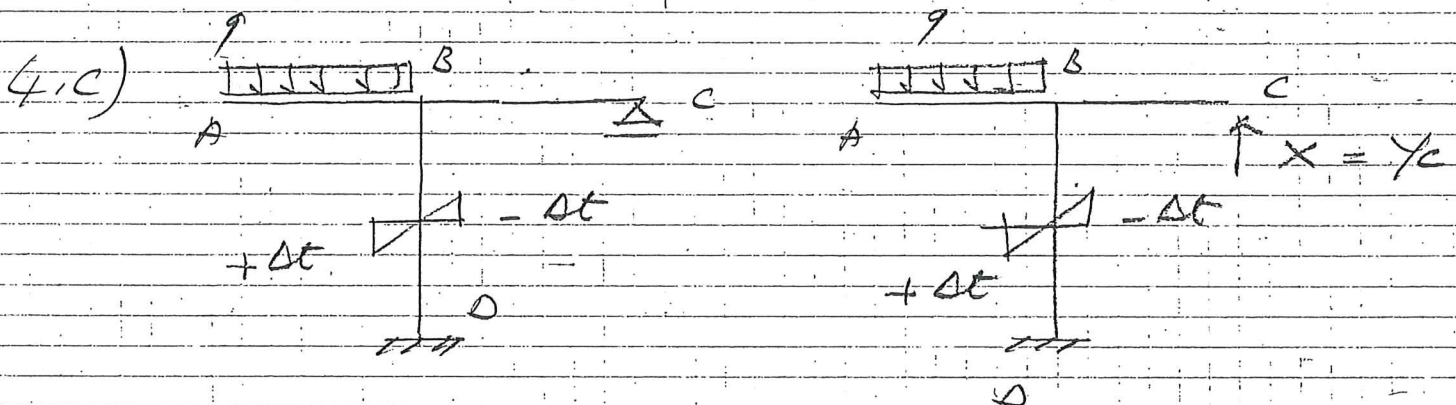
$$+ \frac{ql \cdot l}{EA} + \frac{ql^2}{2} \cdot \frac{l}{EI} \cdot l + \frac{ql^4}{8EI} + \frac{ql^3}{6EI} \cdot l +$$

$$+ \frac{2\alpha \Delta t l \cdot 2l}{h} =$$

$$= -\frac{Xl}{EA} - \frac{20 X l^3}{3 EI} + \frac{q l^3}{6 EA} + \frac{19 q l^4}{24 EI} + \frac{4 \alpha \Delta t l^2}{h} = 0$$

$$X \left(\frac{1}{A} + \frac{20 l^2}{3 I} \right) = q l \left(\frac{1}{A} + \frac{19 l^2}{24 I} \right) + \frac{4 \alpha \Delta t l^2}{h} \quad (B)$$

$$\downarrow X = \frac{q l \left(\frac{1}{A} + \frac{19 l^2}{24 I} \right) + \frac{4 \alpha \Delta t l^2}{h}}{\left(\frac{1}{A} + \frac{20 l^2}{3 I} \right)} \quad (\pm)$$



$$\bar{U} = -\frac{Xl}{EA} - Xl \cdot \frac{l}{EI} - \frac{Xl^3}{3EI} + ql \cdot \frac{l}{EA}$$

$$- \frac{ql^2}{2} \cdot \frac{l}{EI} + \frac{2 \alpha \Delta t l \cdot l}{h} = 0$$

$$-\frac{Xl}{EA} + \frac{4 X l^3}{3 EI} = \frac{ql^2}{EA} - \frac{ql^4}{2 EI} + \frac{2 \alpha \Delta t l^2}{h}$$

$$X \left(\frac{1}{A} + \frac{4 l^2}{3 I} \right) = q l \left(\frac{1}{A} - \frac{l^2}{2 I} \right) + \frac{2 \alpha \Delta t l^2}{h}$$

$$\downarrow X = \frac{q l \left(\frac{1}{A} - \frac{l^2}{2 I} \right) + \frac{2 \alpha \Delta t l^2}{h}}{\left(\frac{1}{A} + \frac{4 l^2}{3 I} \right)} \quad (\pm)$$