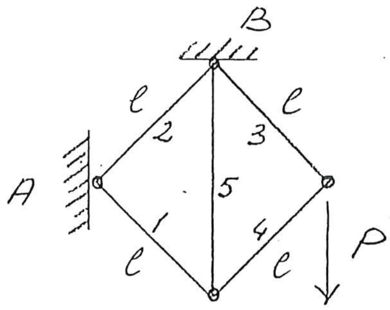
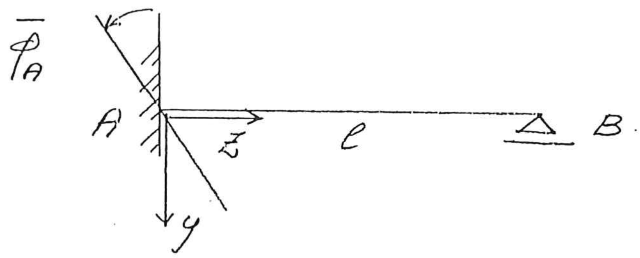
SCIENZA delle COSTRUZIONI **TEST 1** **12/10/1996**

Cognome..Nome..
Anno di Corso..................................

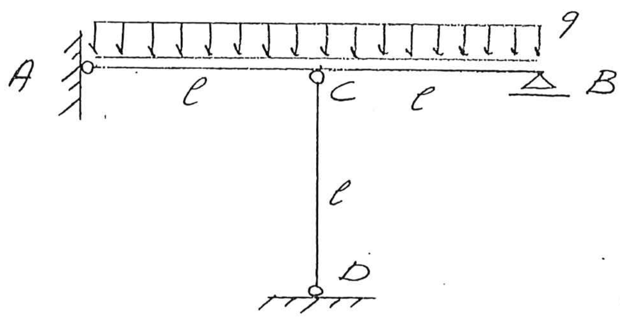
A.1 Calcolare via P.L.V. la componente orizzontale della reazione in B. Completare lo studio della reticolare con il calcolo delle rimanenti componenti di reazione e degli sforzi nelle aste.



A.2 Determinare l'equazione della linea elastica per la trave iperstatica in figura. Completare lo studio con la determinazione delle reazioni vincolari e dei diagrammi T, M.



A.3 Determinare lo sforzo nell'asta DC mediante il Metodo delle Forze.



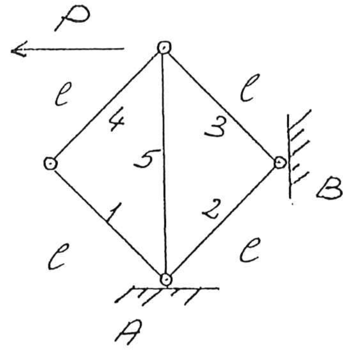
B.1 Enunciare e dimostrare il Teorema dei Lavori Virtuali per strutture reticolari deformabili.

B.2 Illustrare il modello di Eulero-Bernoulli nell'ambito della Teoria tecnica della trave e in relazione a quello di Timoshenko.

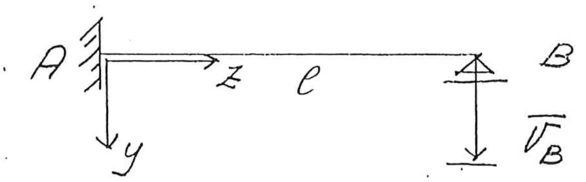
SCIENZA delle COSTRUZIONI **TEST 1** **12/10/1996**

Cognome..Nome...
Anno di Corso...................................

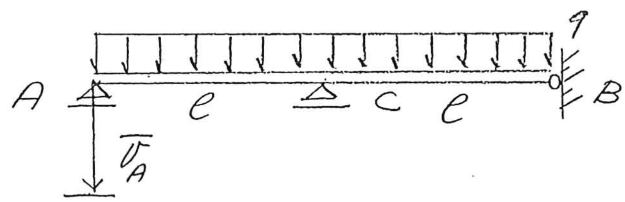
A.1 Calcolare via P.L.V. la componente orizzontale della reazione in B. Completare lo studio della reticolare con il calcolo delle rimanenti componenti di reazione e degli sforzi nelle aste.



A.2 Determinare l'equazione della linea elastica per la trave iperstatica in figura. Completare lo studio con la determinazione delle reazioni vincolari e dei diagrammi T, M.



A.3 Determinare la reazione del carrello in C mediante il Metodo delle Forze.



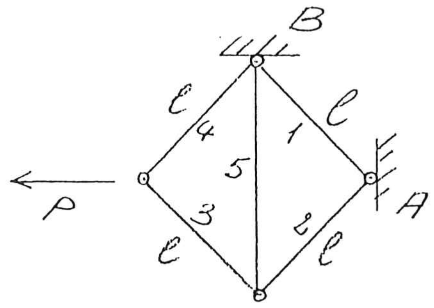
B.1 Illustrare il metodo degli spostamenti nell'ambito delle strutture reticolari iperstatiche.

B.2 Teoria tecnica della trave: ipotesi di base , caratteristiche della deformazione, equazioni indefinite di congruenza e legami costitutivi.

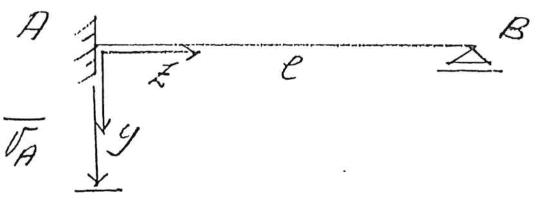
SCIENZA delle COSTRUZIONI **TEST 1** **12/10/1996**

Cognome...Nome...
Anno di Corso...................................

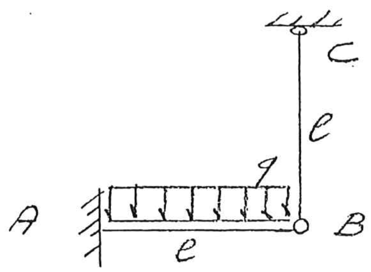
A.1 Calcolare via P.L.V. la componente orizzontale della reazione in B. Completare lo studio della reticolare con il calcolo delle rimanenti componenti di reazione e degli sforzi nelle aste.



A.2 Determinare l'equazione della linea elastica per la trave iperstatica in figura. Completare lo studio con la determinazione delle reazioni vincolari e dei diagrammi T, M.



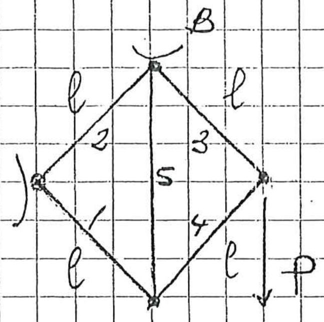
A.3 Determinare lo sforzo dell'asta BC mediante il Metodo delle Forze.

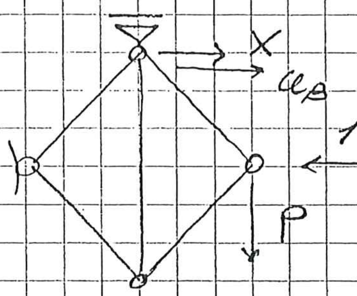


B.1 Introdurre con le opportune motivazioni le equazioni differenziali della linea elastica illustrandone brevemente le modalità di impiego.

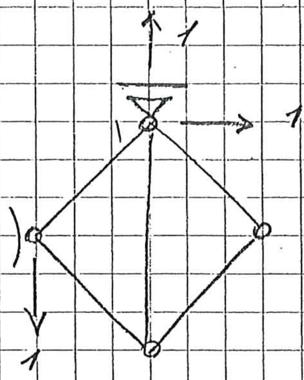
B.2 In riferimento ad una trave appoggiata soggetta a carico antisimmetrico illustrare i diagrammi dei tre parametri di spostamento. Definire inoltre questi ultimi in corrispondenza dell'asse di simmetria geometrica.

P.1)

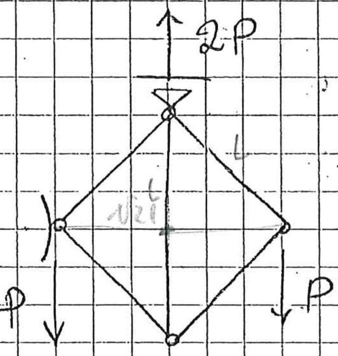




SD



FT

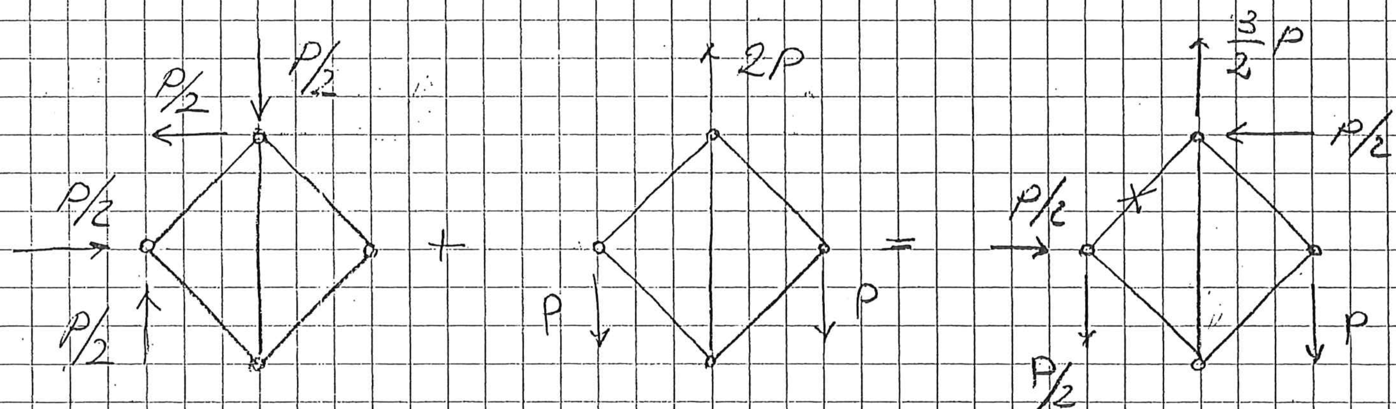


	1	2	3	4	5
l_i	l	l	l	l	$l\sqrt{2}$
N_i^1	0	$\sqrt{2}$	0	0	0
N_i^c	$-\frac{P\sqrt{2}}{2}$	$\frac{P\sqrt{2}}{2}$	$\frac{P\sqrt{2}}{2}$	$-\frac{P\sqrt{2}}{2}$	P

$$L_{ve} = 1 \cdot u_B = 0 = L_{vi} = \frac{1}{EA}\sum_{i=1}^{5} N_i^1 N_i^c l_i =$$

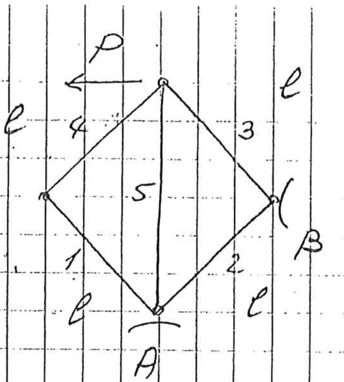
$$\frac{l}{EA}\left[\sqrt{2}\left(\frac{P\sqrt{2}}{2} + \sqrt{2}X\right)\right] = \frac{l}{EA}\left(P + 2X\right)$$

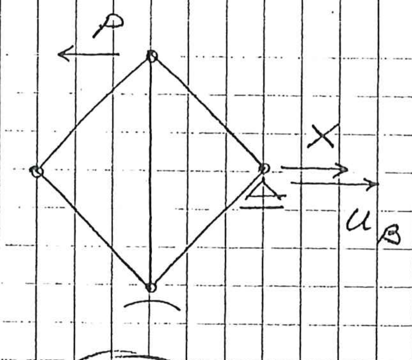
$$\boxed{X = -P/2}$$

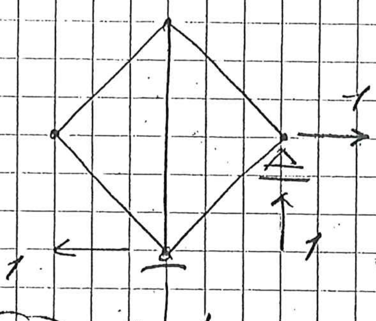


	1	2	3	4	5
N_i	$-\frac{P\sqrt{2}}{2}$	$\frac{P\sqrt{2}}{2} - \frac{P\sqrt{2}}{2} = 0$	$\frac{P\sqrt{2}}{2}$	$-\frac{P\sqrt{2}}{2}$	P

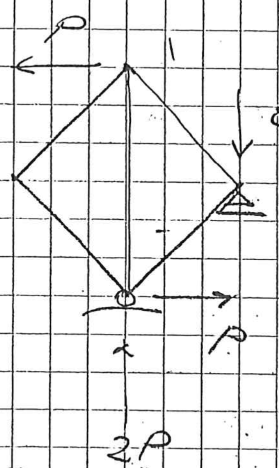
A.1)







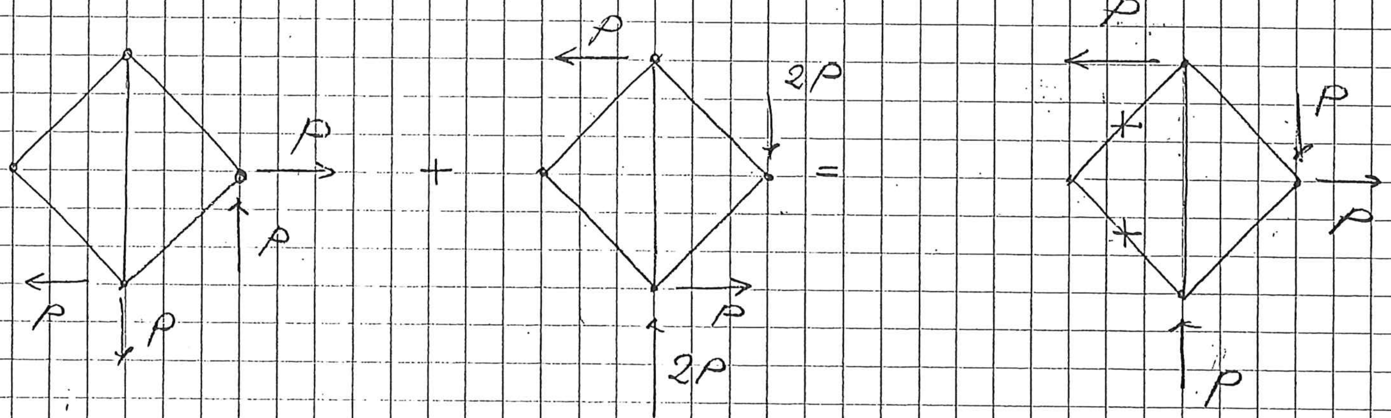
	1	2	3	4	5
l_i	l	l	l	l	$l\sqrt{2}$
N_i^1	0	$\sqrt{2}$	0	0	0
N_i^c	0	$-P\sqrt{2}$	$P\sqrt{2}$	0	$-P$



$$L_{re} = 1 \cdot u_B = 0 = L_{ri} = \frac{1}{EA}\sum_{i=1}^{5} N_i^1 N_i^c l_i =$$

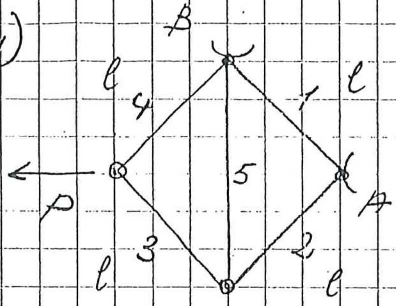
$$= \frac{l}{EA}\left[\sqrt{2}\left(-P\sqrt{2}+\sqrt{2}X\right)\right] = \frac{l}{EA}\left(-2P+2X\right)$$

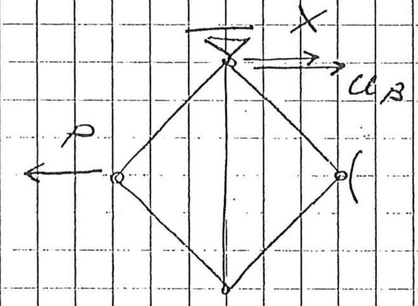
$$\boxed{X = P}$$

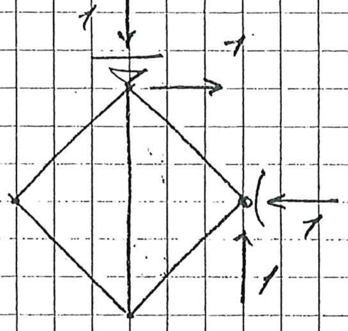


	1	2	3	4	5
N_i	0	$P\sqrt{2} - P\sqrt{2} = 0$	$P\sqrt{2}$	0	$-P$

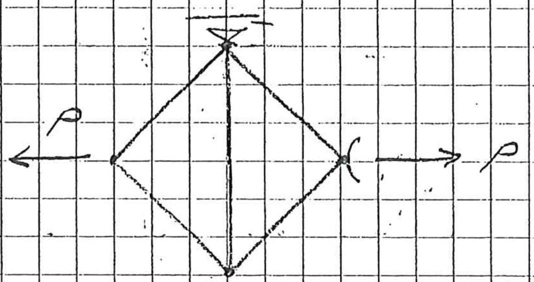
A.1)







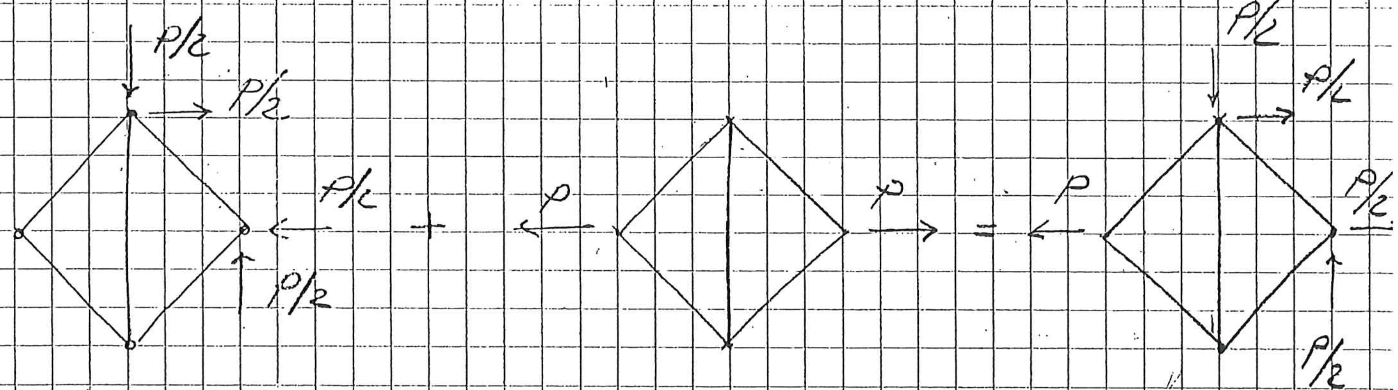
	1	2	3	4	5
l_i	l	l	l	l	$l\sqrt{2}$
N_i^f	$-\sqrt{2}$	0	0	0	0
N_i^c	$\frac{P\sqrt{2}}{2}$	$\frac{P\sqrt{2}}{2}$	$\frac{P\sqrt{2}}{2}$	$\frac{P\sqrt{2}}{2}$	$-P$



$$L_{re} = 1\cdot u_B = 0 = L_{ri} = \frac{1}{EA}\sum_i N_i^f N_i^c l_i$$

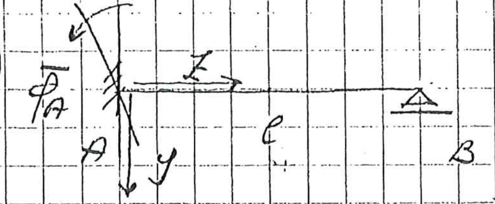
$$= \frac{l}{EA}\left[-\sqrt{2}\left(-\sqrt{2}\,X + \frac{P\sqrt{2}}{2}\right)\right] = \frac{l}{EA}\left(2X - P\right)$$

$$\boxed{X = P/2}$$



	1	2	3	4	5
N_i	$\frac{P\sqrt{2}}{2} - \frac{P\sqrt{2}}{2} = 0$	$\frac{P\sqrt{2}}{2}$	$\frac{P\sqrt{2}}{2}$	$\frac{P\sqrt{2}}{2}$	$-P$

A.2)



$$\frac{d^4 v}{dz^4} = 0$$

$$\frac{d^3 v}{dz^3} = C_1 \qquad \frac{d^2 v}{dz^2} = C_1 z + C_2$$

$$\frac{dv}{dz} = \frac{C_1}{2} z^2 + C_2 z + C_3$$

$$v = \frac{C_1 z^3}{6} + \frac{C_2 z^2}{2} + C_3 z + C_4$$

$$z = 0, \quad v_A = 0 \quad \Rightarrow \quad \boxed{C_4 = 0}$$

$$z = 0, \quad \varphi_A = \bar{\varphi}_A, \quad \varphi_A = -\left.\frac{dv}{dz}\right|_{z=0} = -C_3 = \bar{\varphi}_A \quad \Rightarrow \quad \boxed{C_3 = -\bar{\varphi}_A}$$

$$z = \ell, \quad v_B = 0, \quad \frac{C_1 \ell^3}{6} + \frac{C_2 \ell^2}{2} - \bar{\varphi}_A \ell = 0$$

$$\frac{C_1}{3} \ell^2 + C_2 \ell - 2\bar{\varphi}_A = 0$$

$$z = \ell, \quad M_B = 0, \quad \frac{M(\ell)}{EI} = -\left.\frac{d^2 v}{dz^2}\right|_{z=\ell} = -C_1 \ell - C_2 = 0$$

$$C_2 = -C_1 \ell$$

$$\frac{C_1}{3} \ell^2 - C_1 \ell^2 - 2\bar{\varphi}_A = 0 \qquad -\frac{\cancel{2}}{3} C_1 \ell^2 - \cancel{2} \bar{\varphi}_A = 0$$

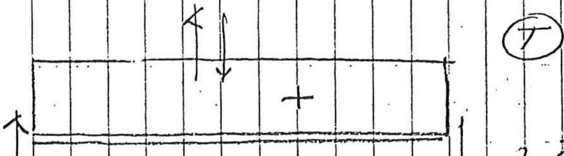
$$\boxed{C_1 = -\frac{3\bar{\varphi}_A}{\ell^2}} \qquad \boxed{C_2 = \frac{3\bar{\varphi}_A}{\ell}}$$

$$\boxed{v_{(z)} = -\frac{\bar{\varphi}_A}{2\ell^2} z^3 + \frac{3\bar{\varphi}_A}{2\ell} z^2 - \bar{\varphi}_A z} = \bar{\varphi}_A \left(-\frac{z^3}{2\ell^2} + \frac{3}{2\ell} z^2 - z \right)$$

$$\boxed{\varphi_{(z)} = -\frac{dv}{dz}} = \frac{3\bar{\varphi}_A}{2\ell^2} z^2 - \frac{3\bar{\varphi}_A}{\ell} z + \bar{\varphi}_A$$

$$\frac{d^3 v}{dz^3} = -\frac{T(z)}{EI} = C_1 = -\frac{3\bar{\varphi}_A}{\ell^2}$$

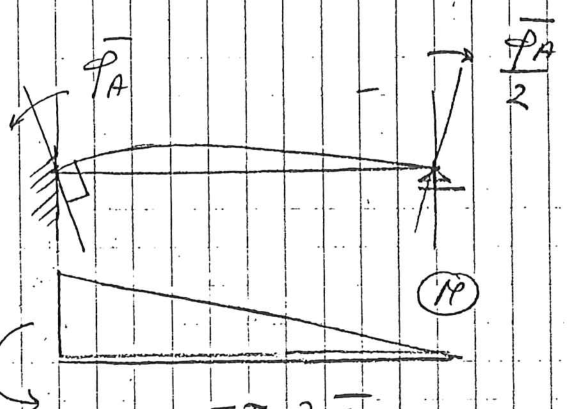
$$\boxed{T(z) = \frac{3\bar{\varphi}_A}{\ell^2} EI}$$



$$\boxed{Y_A = \frac{3\bar{\varphi}_A}{\ell^2} EI} \qquad \boxed{Y_B = -\frac{3\bar{\varphi}_A}{\ell^2} EI}$$

$$\frac{d^2 v}{dz^2} = -\frac{M(z)}{EI} = C_1 z + C_2 = -\frac{3\bar{\varphi}_A}{\ell^2} z + \frac{3\bar{\varphi}_A}{\ell}$$

$$\boxed{M(z) = \frac{EI\, 3\bar{\varphi}_A}{\ell}\left(\frac{z}{\ell} - 1\right)}$$



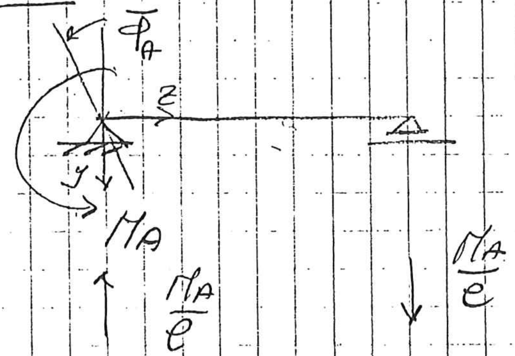
$z = 0, \quad M_A = -\frac{EI\, 3\bar{\varphi}_A}{\ell}$

$z = \ell, \quad M_B = 0$

$M_A = \frac{EI\, 3\bar{\varphi}_A}{\ell}$

in B è $\boxed{\varphi_B = -\bar{\varphi}_A/2}$

<u>oppure</u>

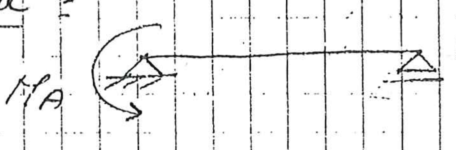


$$\frac{d^2 v}{dz^2} = \frac{1}{EI}\left(-\frac{M_A}{\ell} z + M_A\right) \quad \ldots \Rightarrow C_1, C_2, M_A$$

Condizioni al contorno: $v_A = 0 \qquad v_B = 0$

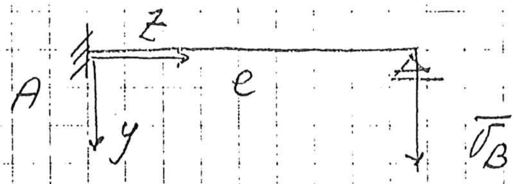
$\varphi_A = \bar{\varphi}_A$

<u>Per verifica</u>:



$$\varphi_A = \frac{M_A \ell}{3EI} = \bar{\varphi}_A \qquad \boxed{M_A = \frac{3EI}{\ell}\bar{\varphi}_A}$$

A.2)



$$\frac{d^4 v}{dz^4} = 0$$

idem come nel precedente

$$z = 0, \quad v_A = 0 \quad \Rightarrow \quad \boxed{C_4 = 0}$$

$$z = 0 \quad \varphi_A = 0 \quad \Rightarrow \quad \boxed{C_3 = 0}$$

$$z = l, \quad v_B = \bar{v}_B, \quad v_{(l)} = \frac{C_1}{6} l^3 + \frac{C_2}{2} l^2 = \bar{v}_B$$

$$\frac{C_1}{3} l^3 + C_2 l^2 = 2 \bar{v}_B$$

$$z = l, \quad M_B = 0, \quad \frac{M(l)}{EI} = - \frac{d^2 v}{dz^2}\Big|_{z=l} = - C_1 l - C_2 = 0$$

$$C_2 = - C_1 l$$

$$\frac{C_1 l^3}{3} - C_1 l^3 = 2 \bar{v}_B \qquad - \frac{\cancel{2}}{3} C_1 l^3 = \cancel{2} \bar{v}_B$$

$$\boxed{C_1 = - \frac{3 \bar{v}_B}{l^3}} \qquad \boxed{C_2 = \frac{3 \bar{v}_B}{l^2}}$$

$$\boxed{v_{(z)} = - \frac{\bar{v}_B}{2 l^3} z^3 + \frac{3}{2 l^2} \bar{v}_B z^2}$$

$$\boxed{\varphi_{(z)} = - \frac{dv}{dz} = \frac{3}{2} \frac{\bar{v}_B}{l^3} z^2 - \frac{3 \bar{v}_B}{l^2} z}$$

Infatti $\quad z = 0 \quad v_A = 0, \quad \varphi_A = 0$

$\quad z = l \quad v_B = \bar{v}_B$

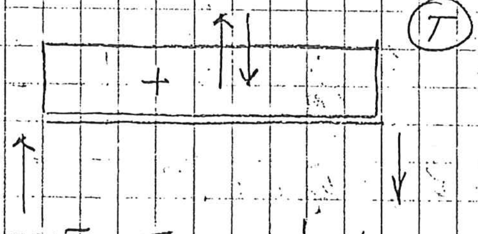
$$\boxed{\varphi_B = - \frac{3}{2} \frac{\bar{v}_B}{l}}$$

(7)

$$\varphi_B = -\frac{3}{2}\frac{\bar{v}_B}{\ell}$$

$\downarrow \bar{v}_B$

$$\frac{d^3 v}{dz^3} = -\frac{T(z)}{EI} = C_1 = -\frac{3\bar{v}_B}{\ell^3}$$



$$T(z) = \frac{3\bar{v}_B EI}{\ell^3}$$

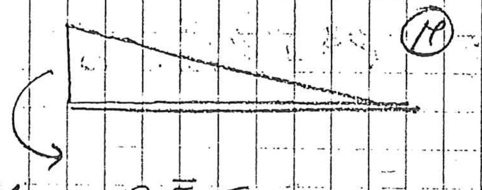
$$Y_A = \frac{3\bar{v}_B EI}{\ell^3} \qquad Y_B = -\frac{3\bar{v}_B EI}{\ell^3}$$

$$\frac{d^2 v}{dz^2} = -\frac{M(z)}{EI} = C_1 z + C_2$$

$$M(z) = EI\left(\frac{3\bar{v}_B}{\ell^3} z - \frac{3\bar{v}_B}{\ell^2}\right) = \frac{3\bar{v}_B EI}{\ell^2}\left(\frac{z}{\ell} - 1\right)$$

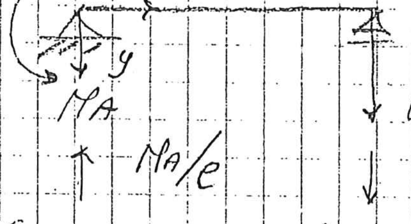
$$z = 0,\quad M_A = -\frac{3\bar{v}_B EI}{\ell^2}$$

$$z = \ell,\quad M_B = 0$$



$$M_A = \frac{3\bar{v}_B EI}{\ell^2}$$

oppure:



$$\frac{d^2 v}{dz^2} = \frac{1}{EI}\left(-\frac{M_A}{\ell} z + M_A\right) \quad \text{ecc.} \Rightarrow C_1, C_2, M_A$$

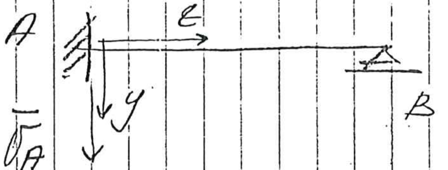
Condizione: $\quad v_A = 0 \qquad v_B = \bar{v}_B$

$\varphi_A = 0$

Per esempio:

$$\varphi_A = \frac{M_A \ell}{3EI} - \frac{\bar{v}_B}{\ell} = 0 \Rightarrow M_A = \frac{3EI\bar{v}_B}{\ell^2}$$

A.2)



$\frac{d^4 v}{dz^4}=0$ soluzione come prima

$z=0$, $\varphi_A=0$, $C_3=0$

$z=0$, $v_A=\bar{v}_A$, $\boxed{C_4=\bar{v}_A}$

$z=l$; $v_B=0$, $\frac{C_1}{6}l^3+\frac{C_2}{2}l^2+\bar{v}_A=0$

$\frac{C_1}{3}l^3+C_2 l^2+2\bar{v}_A=0$

$z=l$, $M_B=0$, $\frac{M(l)}{EI}=-\frac{d^2 v}{dz^2}\Big|_{z=l}=-C_1 l-C_2=0$

$C_2=-C_1 l$

$\frac{C_1}{3}l^3-C_1 l^3+2\bar{v}_A=0$

$-\frac{\cancel{2}}{3}C_1 l^3+\cancel{2}\bar{v}_A=0$ $\boxed{C_1=\frac{3\bar{v}_A}{l^3}}$ $\boxed{C_2=-\frac{3\bar{v}_A}{l^2}}$

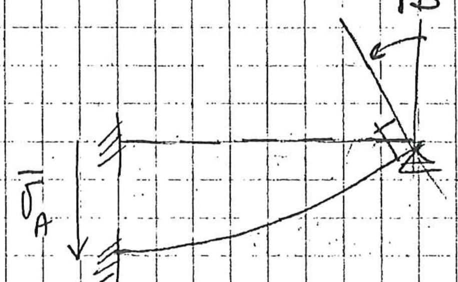
$v(z)=\frac{\bar{v}_A}{2l^3}z^3-\frac{3\bar{v}_A}{2l^2}z^2+\bar{v}_A$

$\varphi(z)=-\frac{dv}{dz}=-\frac{3\bar{v}_A}{2l^3}z^2+\frac{3\bar{v}_A}{l^2}z$

$\varphi_B=\frac{3\bar{v}_A}{2l}$

Verifica = $z=0$ $v_A=\bar{v}_A$

$\varphi_A=0$



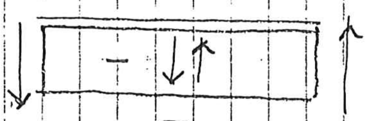
$z=l$ $v_B=0$

$\boxed{\varphi_B=\frac{3\bar{v}_A}{2l}}$

$$\frac{d^3v}{dz^3} = -\frac{T(z)}{EI} = C_1 = \frac{3\bar{v}_A}{\ell^3}$$

T

$$T(z) = -\frac{3EI\bar{v}_A}{\ell^3}$$



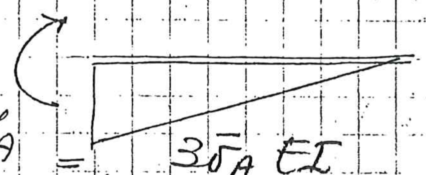
$$Y_A = -\frac{3EI\bar{v}_A}{\ell^3} \qquad Y_B = \frac{3EI\bar{v}_A}{\ell^3}$$

$$\frac{d^2v}{dz^2} = -\frac{M(z)}{EI} = C_1 z + C_2$$

$$M(z) = EI\left(-\frac{3\bar{v}_A}{\ell^3}z + \frac{3\bar{v}_A}{\ell^2}\right) = \frac{EI\,3\bar{v}_A}{\ell^2}\left(-\frac{z}{\ell}+1\right)$$

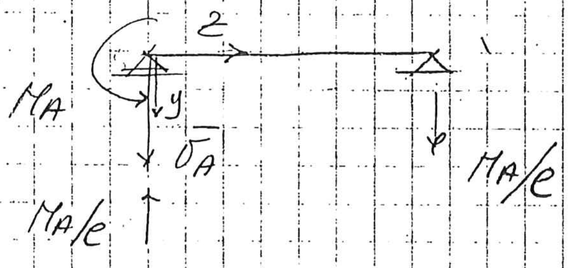
$z=0 \qquad M_A = \dfrac{3\bar{v}_A EI}{\ell^2}$

$z=\ell \qquad M_B = 0$



$$M_A = \frac{3\bar{v}_A EI}{\ell^2}$$

oppure:



$$\frac{d^2v}{dz^2} = \frac{1}{EI}\left(-\frac{M_A}{\ell}z + M_A\right) \quad \text{ecc} \implies C_1,\ C_2,\ M_A$$

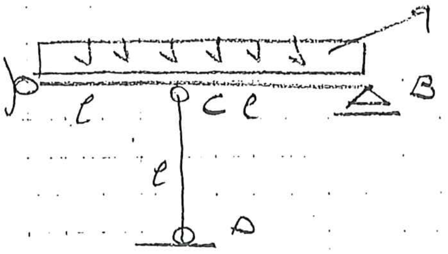
Condizioni: $v_A = \bar{v}_A$

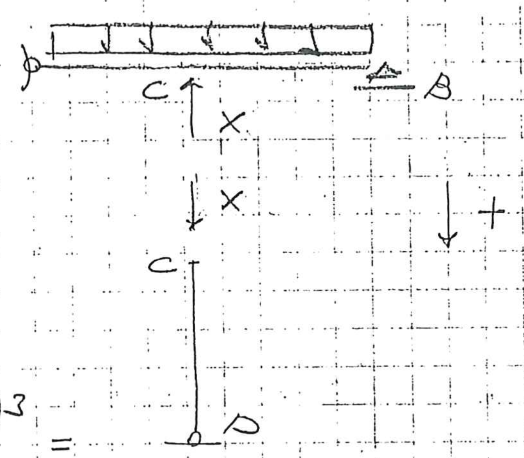
$\varphi_A = 0 \qquad v_B = 0$

Per verifica:

$$\varphi_A = \frac{M_A\ell}{3EI} + \frac{\bar{v}_A}{\ell} = 0 \qquad \boxed{M_A = -\frac{3\bar{v}_A EI}{\ell^2}}$$

A.2 bis)





$$v_C^s = v_C^i$$

$$v_C^s = \frac{5}{384} q \frac{(2l)^4}{EI} - \frac{X(2l)^3}{48EI} =$$

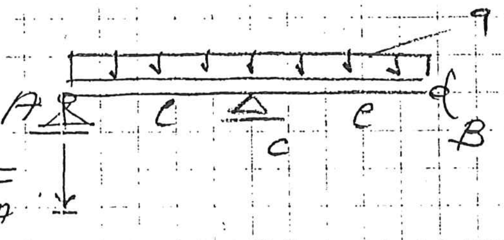
$$= v_C^i = \frac{Xl}{EA}$$

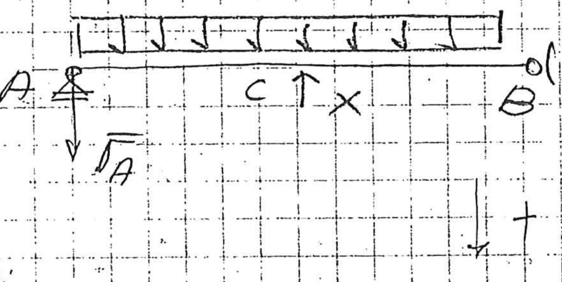
$$\frac{5}{24}\frac{q l^{\cancel{4}\,3}}{\cancel{E}I} - \frac{X l^{\cancel{3}\,2}}{6\cancel{E}I} = \frac{X\cancel{l}}{\cancel{E}A}$$

$$\frac{X}{A} + \frac{Xl^2}{6I} = \frac{5}{24}\frac{ql^3}{I}$$

$$X = \frac{5}{\cancel{24}\,4}\frac{ql^3}{\cancel{I}}\left(\frac{\cancel{6}\cancel{I}A}{6I+Al^2}\right) = \frac{5}{4}ql^3\left(\frac{A}{6I+Al^2}\right)$$

A.2 bis)

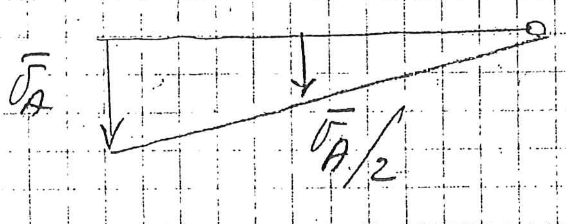




$$v_C = 0$$

$$v_C = \frac{5}{384} q \frac{(2l)^4}{EI} - \frac{X(2l)^3}{48EI} + \frac{\bar{v}_A}{2} = 0$$

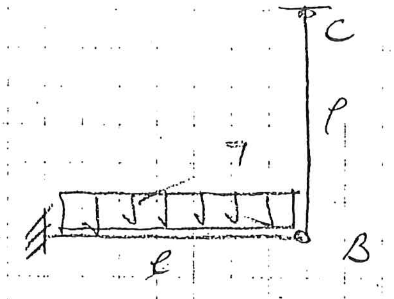
Infatti

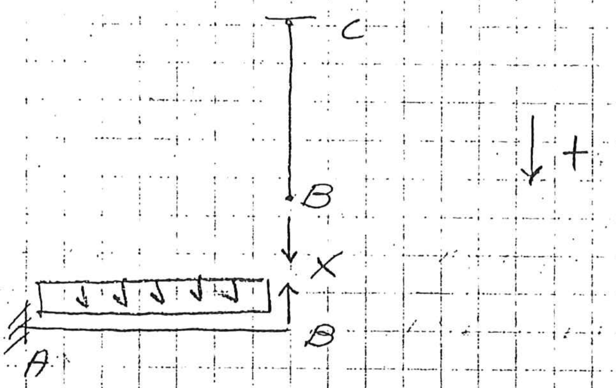


$$\frac{5}{24}\frac{q\ell^4}{EI} - \frac{X\ell^3}{6EI} + \frac{v_A}{2} = 0$$

$$X = \left(\frac{5}{24}\frac{q\ell^4}{EI} + \frac{v_A}{2}\right)\frac{6EI}{\ell^3}$$

A.2 bis)





$$v_B^s = v_B^i$$

$$v_B^s = \frac{X\ell}{EA} = -\frac{X\ell^3}{3EI} + \frac{q\ell^4}{8EI} = v_B^i$$

$$\frac{X\ell}{E}\left(\frac{1}{A} + \frac{\ell^2}{3I}\right) = \frac{q\ell^4}{8EI}$$

$$\frac{X\cancel{\ell}}{3\cancel{E}A\cancel{I}}\left(3I + \ell^2 A\right) = \frac{q\ell^{\cancel{4}\,3}}{8\cancel{E}\cancel{I}}$$

$$X = \frac{3Aq\ell^3}{8\left(3I + \ell^2 A\right)}$$