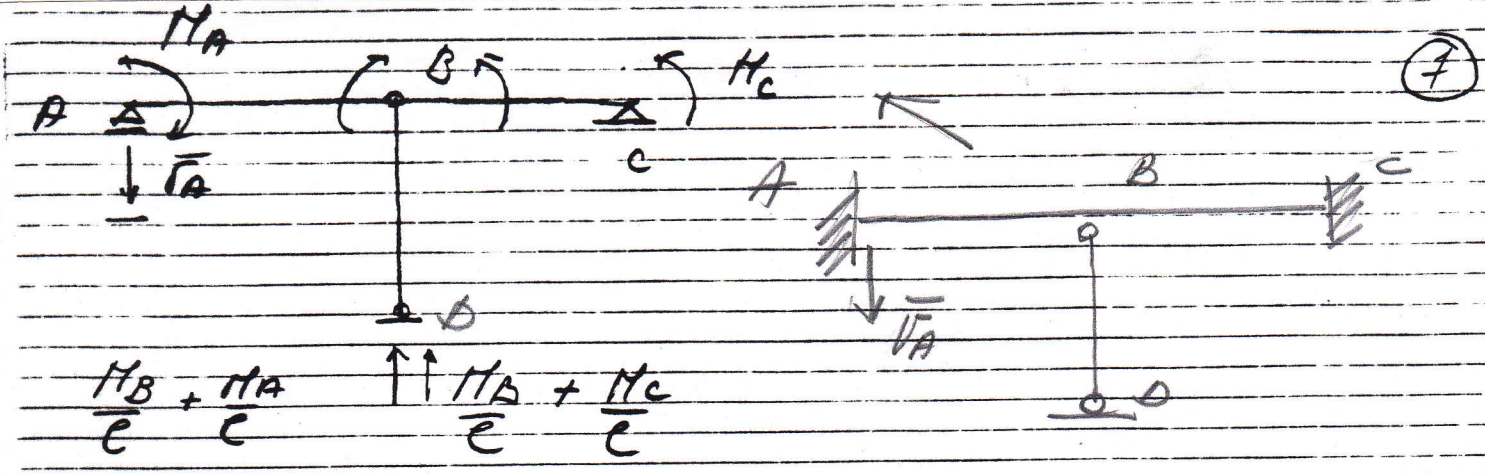
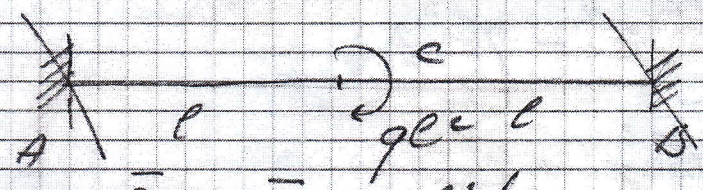


$$\varphi_A = \frac{\bar{V}_A}{e} - \left(\frac{2M_B}{e} + \frac{M_A}{e} + \frac{M_C}{e}\right)\frac{e}{EA}\cdot\frac{1}{e} + \frac{M_B e}{6EI} - \frac{M_A e}{3EI} = 0$$

$$\varphi_B^S = -\left(2M_B + M_A + M_C\right)\frac{1}{eEA} + \frac{M_A e}{6EI} - \frac{M_B e}{3EI} + \frac{\bar{V}_A}{e} =$$

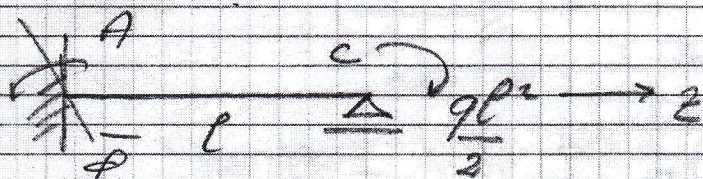
$$= \varphi_B^0 = \left(\frac{2M_B}{e} + \frac{M_A}{e} + \frac{M_C}{e}\right)\frac{1}{eEA} + \frac{M_B e}{3EI} - \frac{M_C e}{6EI} =$$

$$\varphi_C = \left(2M_B + M_A + M_C\right)\frac{1}{eEA} - \frac{M_B e}{6EI} + \frac{M_C e}{3EI} = 0$$



(A) ①

$\varphi_A = \varphi_B = ql^3/EI$



$$\frac{d^4v(z)}{dz^4} = 0 \qquad \frac{d^3v(z)}{dz^3} = C_1 = -T(z)/EI$$

$$\frac{d^2v(z)}{dz^2} = -\frac{M(z)}{EI} = C_1 z + C_2$$

$$\frac{dv(z)}{dz} = -\varphi(z) = \frac{C_1 z^2}{2} + C_2 z + C_3$$

$$v(z) = \frac{C_1 z^3}{6} + \frac{C_2 z^2}{2} + C_3 z + C_4$$

$$v_A(z=0) = 0 \rightarrow \underline{C_4 = 0}$$

$$\varphi_A(z=0) = ql^3/EI = -C_3 \qquad \underline{C_3 = -\frac{ql^3}{EI}}$$

$$v_C(z=l) = 0 = \frac{C_1 l^3}{6} + \frac{C_2 l^2}{2} - \frac{ql^3 l}{EI}$$

$$M_C(z=l) = -EI(C_1 l + C_2) = -EI C_1 l - EI C_2 = -\frac{ql^2}{2}$$

$$C_2 = \frac{ql^2}{2EI} - C_1 l$$

$$C_1 \frac{l^3}{6} + \frac{q}{4EI} l^4 - \frac{C_1 l^3}{2} - \frac{ql^4}{EI} = -C_1 \frac{l^3}{3} - \frac{3}{4}\frac{ql^4}{EI} = 0$$

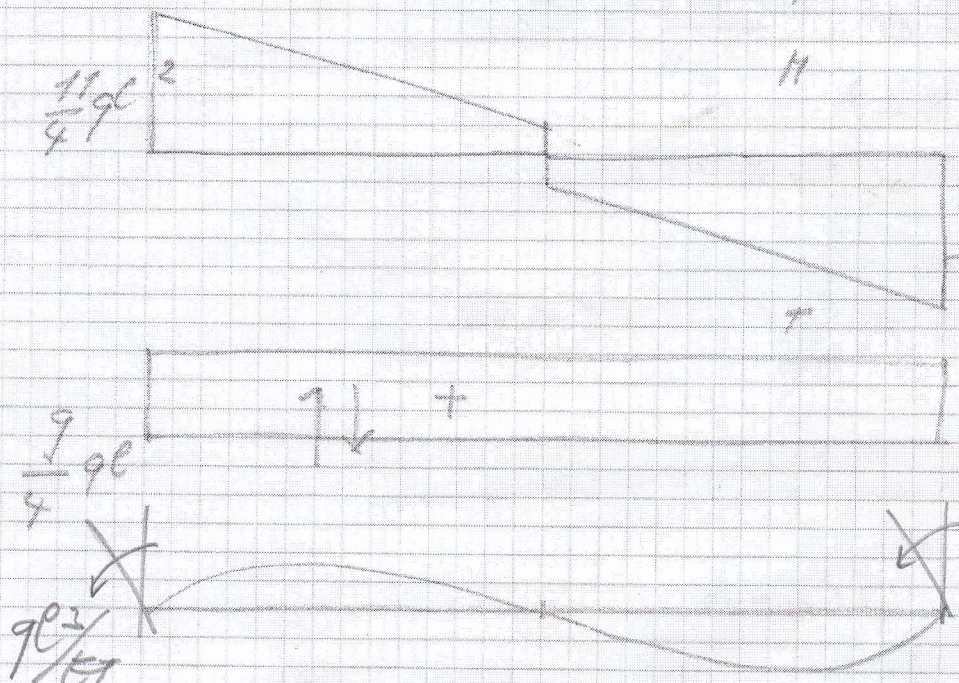
$$\underline{C_1 = -\frac{9}{4}\frac{ql}{EI}} \qquad \underline{C_2 = \frac{ql^2}{2EI} + \frac{9}{4}\frac{ql^2}{EI} = \frac{11}{4}\frac{ql^2}{EI}}$$

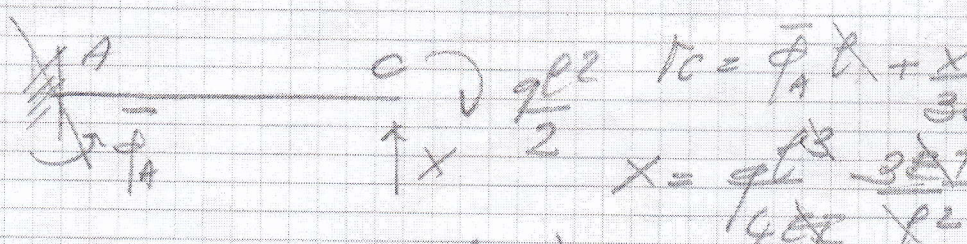
$$v(z) = -\frac{3}{24}\frac{q}{EI}z^3 + \frac{11}{8}\frac{q}{EI}z^2 - \frac{q}{EI}z$$

$$\varphi(z) = \frac{3}{8}\frac{q}{EI}z^2 - \frac{11}{4}\frac{q}{EI}z + \frac{q}{EI}$$

$$T(z) = -EI\,C_1 = \frac{3}{4}ql$$

$$M(z) = -EI\,(C_1 z + C_2) = \frac{3}{4}ql\,z - \frac{11}{4}ql^2$$





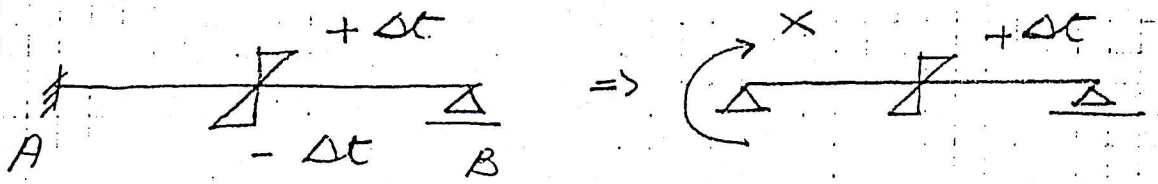
$$v_C = \varphi_A l + \frac{X l^3}{3EI} - \frac{ql^3}{4EI} = 0$$

$$X = \frac{ql^3}{4EI}\cdot\frac{3EI}{l^2} - \varphi_A\frac{3EI}{l^2} =$$

$$= \frac{3}{4}ql - \frac{ql^3}{EI}\cdot\frac{3EI}{l^2} = \frac{3}{4}ql - 3ql = -\frac{9}{4}ql$$

$$\Rightarrow \quad X = \frac{3}{4}ql$$

7.1)



$$\frac{d^2v}{dz^2} = -\chi = -\chi(x) - \chi(\Delta t)$$

$$\chi(x) = \frac{X}{EI} - \frac{X}{\ell EI} z \qquad \chi(\Delta t) = -\frac{2\alpha\Delta t}{h}$$

$$\frac{dv}{dz} = -\frac{X}{EI} + \frac{X}{\ell EI} z + \frac{2\alpha\Delta t}{h}$$

$$\frac{dv}{dz} = -\frac{X}{EI} z + \frac{X}{2\ell EI} z^2 + \frac{2\alpha\Delta t}{h} z + C_1$$

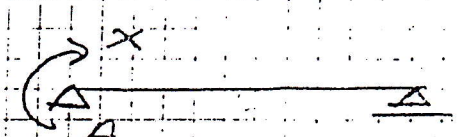
$$v = -\frac{X z^2}{2EI} + \frac{X z^3}{6\ell EI} + \frac{\alpha\Delta t}{h} z^2 + C_1 z + C_2$$

$$z = 0 \quad v = \frac{dv}{dz} = 0 \Rightarrow C_1 = C_2 = 0$$

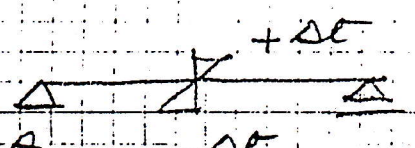
$$z = \ell \quad v = 0 \Rightarrow -\frac{X\ell^2}{2EI} + \frac{X\ell^2}{6EI} + \frac{\alpha\Delta t}{h}\ell^2 = 0$$

$$-\frac{X\ell^2}{3EI} + \frac{\alpha\Delta t \ell h}{h} = 0 \qquad \boxed{X = \frac{3\alpha\Delta t EI}{\ell}}$$

Del resto, trave compressa =



$$\varphi_A(x) = -\frac{X\ell}{3EI}$$



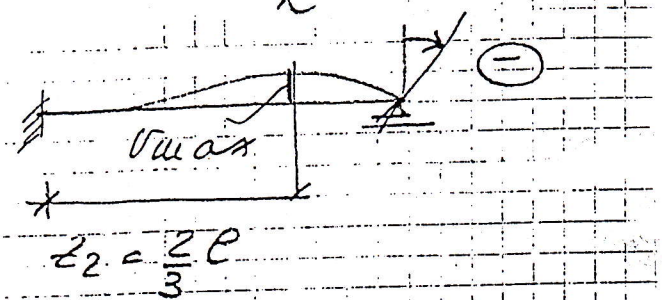
$$\varphi_A(\Delta t) = \frac{\alpha\ell\Delta t}{h}$$

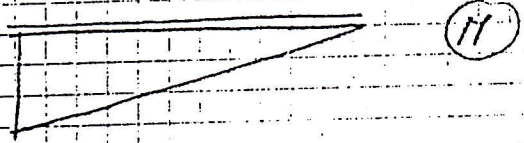
$$\varphi_A = \varphi_A(x) + \varphi_A(\Delta t) = -\frac{X\ell}{3EI} + \frac{\alpha\ell\Delta t}{h} = 0$$

$$\varphi_B = \frac{Xl}{EI} - \frac{Xl}{2EI} - \frac{2\alpha\Delta t\, l}{h} =$$

$$= \frac{Xl}{2EI} - \frac{2\alpha\Delta t\, l}{h} = \frac{3\alpha\Delta t\, l}{2h} - \frac{2\alpha\Delta t\, l}{h} =$$

$$= -\frac{\alpha l \Delta t}{2h} = -\varphi_B$$





NB il diagramma del momento è tutto fatto mentre le fibre tese sono in parte sotto e in parte sopra l'asse della trave. Ciò è dovuto al fatto che la distorsione Δt provoca deformazione (nella trave vincolata) in assenza di momento.

$$\frac{dv}{dz} = 0 \Rightarrow$$

$$z\left(-\frac{X}{EI} + \frac{Xz}{2lEI} + \frac{2\alpha\Delta t}{h}\right) = 0$$

$$\Rightarrow \quad z_1 = 0$$

$$z_2 = \left(\frac{X}{EI} - \frac{2\alpha\Delta t}{h}\right) 2lEI = \left(\frac{3\alpha\Delta t}{h} - \frac{2\alpha\Delta t}{h}\right) \frac{2lEI}{3\alpha\Delta t}$$

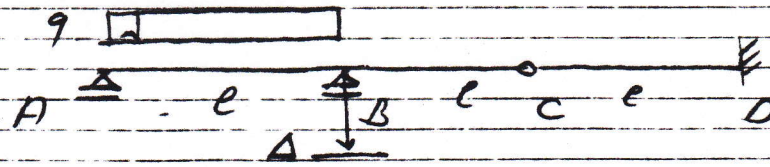
$$= \frac{\alpha\Delta t}{h} \cdot \frac{2lh}{3\alpha\Delta t} = \frac{2}{3}l$$

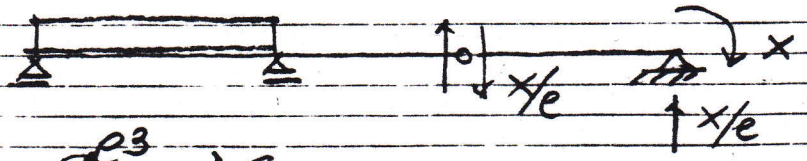
$$v_{max} = -\frac{X 4l^2}{9 \cdot 16 EI} + \frac{X 8 l^3}{81 \cdot 16 lEI} + \frac{\alpha\Delta t\, 4l^2}{9h} =$$

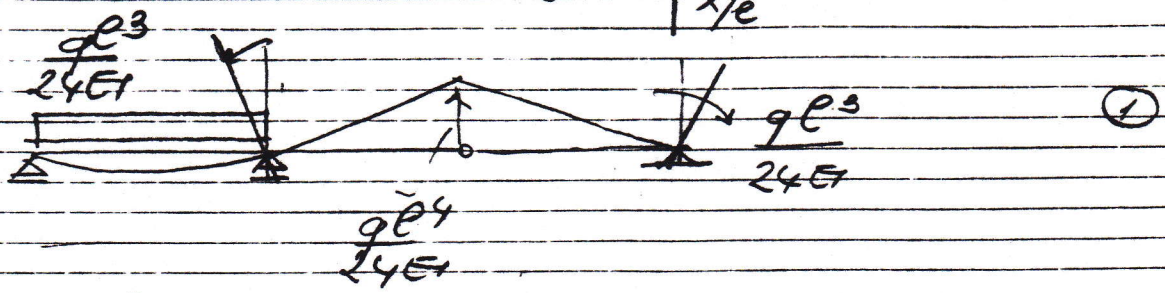
$$= -\frac{14 X l^2}{81 EI} + \frac{4\alpha\Delta t\, l^2}{9h} = -\frac{42\alpha\Delta t EI\, l^2}{81 hEI} + \frac{4\alpha\Delta t\, l^2}{9h} =$$

$$= -\frac{6\alpha\Delta t\, l^2}{81 \cdot 6}$$

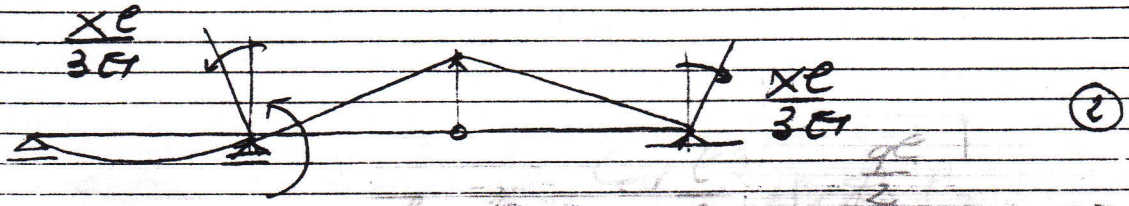
1B)

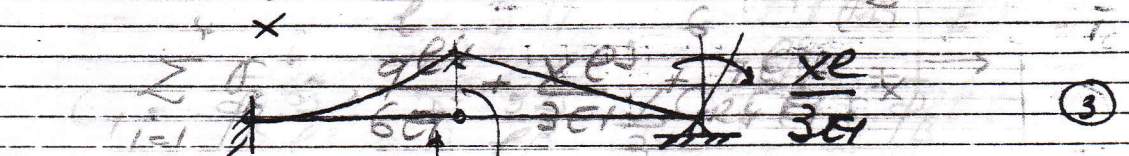


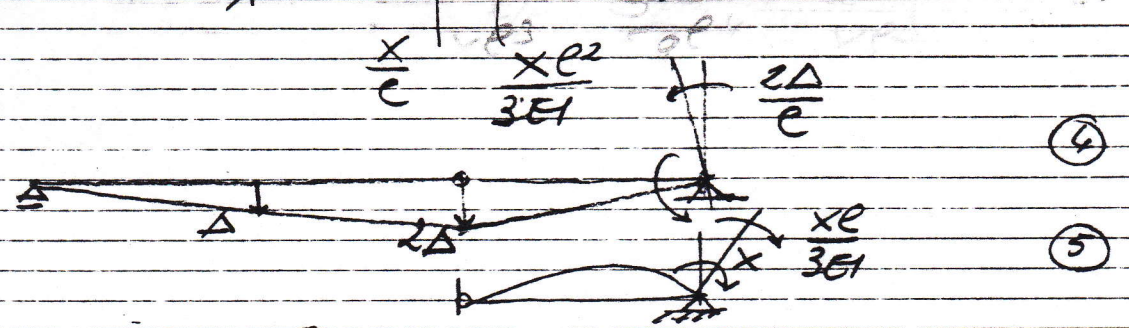




$\frac{qe^4}{24EI}$

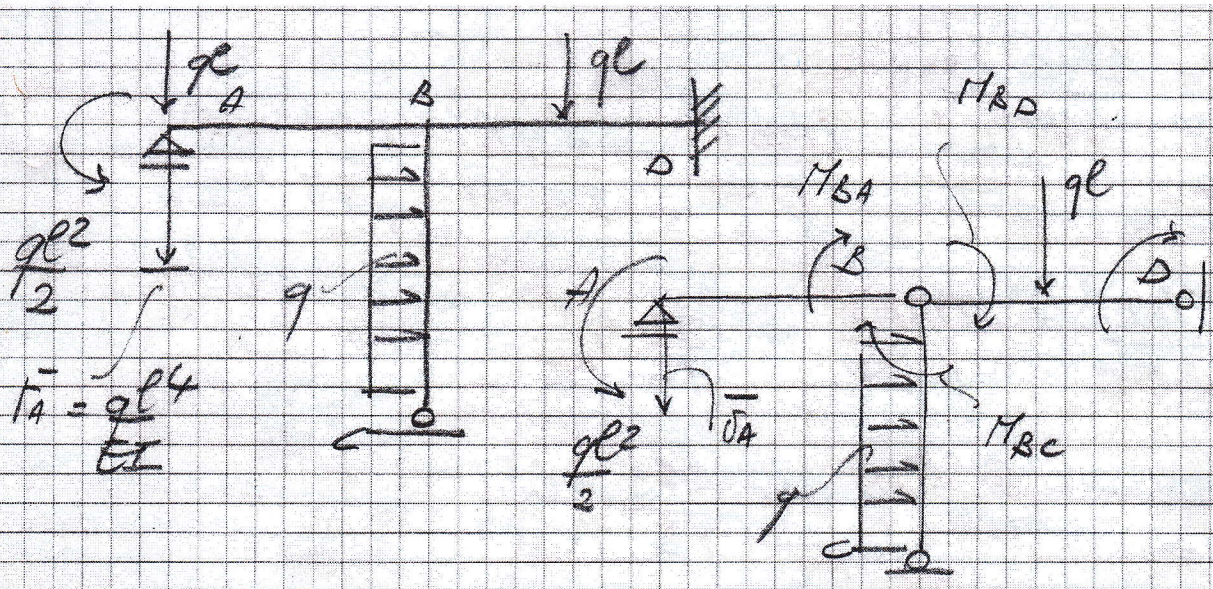






$$\sum_{i=1}^{5} \varphi_i \cdot D = 0 = \frac{Xe}{EI} + \frac{qe^3}{24EI} - \frac{2\Delta}{e}$$

$$X = -\frac{qe^2}{24} + \frac{2\Delta EI}{e^2}$$



1) $\varphi_{BA} = \varphi_{BD}$ $\quad \sum M_B = 0$ 4)

2) $\varphi_{BA} = \varphi_{BC}$ $\quad \varphi_D = 0$ 3)

1) $$-\frac{M_{BA}\,l}{3EI} - \frac{ql^3}{12EI} + \frac{ql^3}{EI} = -\frac{M_{BD}\,l}{3EI} + \frac{M_D\,l}{6EI} - \frac{ql^3}{16EI}$$

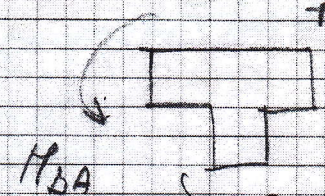
$$-\frac{M_{BA}\,l}{3EI} + \frac{M_{BD}\,l}{3EI} - \frac{M_D\,l}{6EI} + \frac{47}{48}\frac{ql^3}{EI} = 0$$

2) $$-\frac{M_{BA}\,l}{3EI} + \frac{11}{12}\frac{ql^3}{EI} = -\frac{M_{BC}\,l}{3EI} + \frac{ql^3}{24EI}$$

$$-\frac{M_{BA}\,l}{3EI} + \frac{M_{BC}\,l}{3EI} + \frac{21}{24}\frac{ql^3}{EI} = 0$$

3) $$\frac{M_{BD}\,l}{6EI} - \frac{M_D\,l}{3EI} + \frac{ql^3}{16EI} = 0$$

$$\sum M_B = M_{BA} + M_{BC} + M_{BD} = 0$$



Далее 3)

$$M_D = \frac{M_{BD}}{6EI} \cdot \frac{3EI}{l} + \frac{ql^3}{16EI} \cdot \frac{3EI}{l} =$$

$$= \frac{M_{BD}}{2} + \frac{3}{16}ql^2$$

Nella 1)

$$-\frac{M_{BA}\ell}{3EI}+\frac{M_{BD}\ell}{3EI}-\frac{M_{BD}\ell}{12EI}-\frac{3ql^3}{96EI}+\frac{7}{48}\frac{ql^3}{EI}=$$

$$=-\frac{M_{BA}\ell}{3EI}+\frac{M_{BD}\ell}{4EI}+\frac{91}{96}\frac{ql^2}{EI}=0$$

$$\boxed{M_{BA}=\frac{3}{4}M_{BD}+\frac{273}{96}ql^2}$$

Nella 4) $\boxed{M_{BC}=-M_{BA}-M_{BD}=}$

$$=-\frac{3}{4}M_{BD}-\frac{273}{96}ql^2-M_{BD}=-\frac{7}{4}M_{BD}-\frac{273}{96}ql^2$$

Nella 2)

$$-\frac{3M_{BD}\ell}{12EI}-\frac{273}{288}\frac{ql^3}{EI}-\frac{7M_{BD}\ell}{12EI}-\frac{273ql^3}{288}+$$

$$+\frac{21}{24}\frac{ql^3}{EI}=-\frac{10}{12}\frac{M_{BD}\ell}{EI}-\frac{\frac{273}{546}}{288}\frac{ql^2}{EI}=0$$

$$\boxed{M_{BD}=-\frac{91}{48}\cdot\frac{6}{5}ql^2=-\frac{91}{40}ql^2}$$

$$\boxed{M_{BC}=-\frac{7}{4}\left(-\frac{91}{40}ql^2\right)-\frac{273}{96}ql^2=\frac{637}{160}ql^2-\frac{273}{96}ql^2=}$$

$$=\frac{1911-1365}{480}ql^2=\frac{546}{480}ql^2=\frac{273}{240}ql^2=$$

$$=\frac{91}{80}ql^2$$

$$\boxed{M_{BA}=\frac{3}{4}\left(-\frac{91}{40}ql^2\right)+\frac{273}{96}ql^2=-\frac{273}{160}ql^2+\frac{273}{96}ql^2}$$

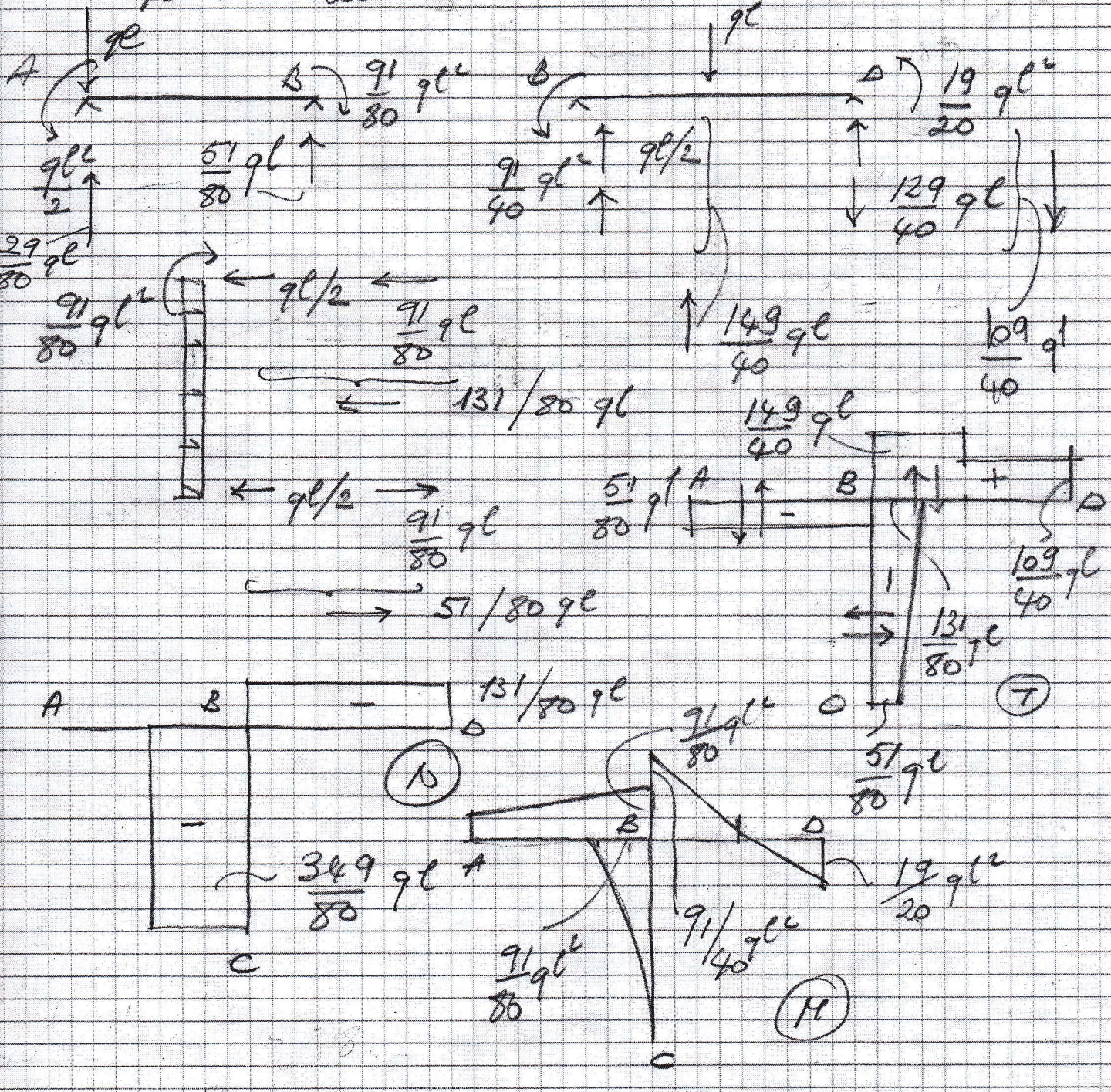
$$= -\frac{819+1365}{480}ql^2 = \frac{546}{480}ql^2 = \frac{273}{240} = \frac{91}{80}ql^2$$

(3)

$$\frac{91}{80}ql^2 \quad \frac{91}{40}ql^2 \quad \frac{91}{80}ql^2 \qquad \Sigma M_B = \left(\frac{91}{80}\cdot\frac{1}{40} - \frac{91}{40}\right)ql^2 = 0$$

$$M_D = -\frac{91}{80}ql^2 + \frac{3}{16}ql^2 = \left(\frac{-91+15}{80}\right)ql^2 = -\frac{76}{80}ql^2 =$$

$$= -\frac{38}{40}ql^2 = -\frac{19}{20}ql^2$$



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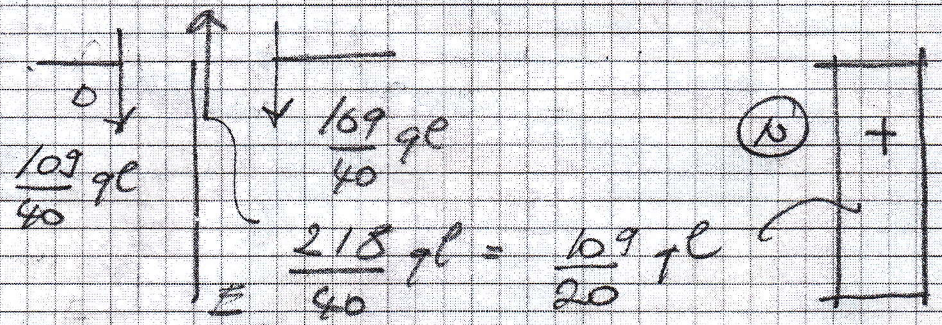
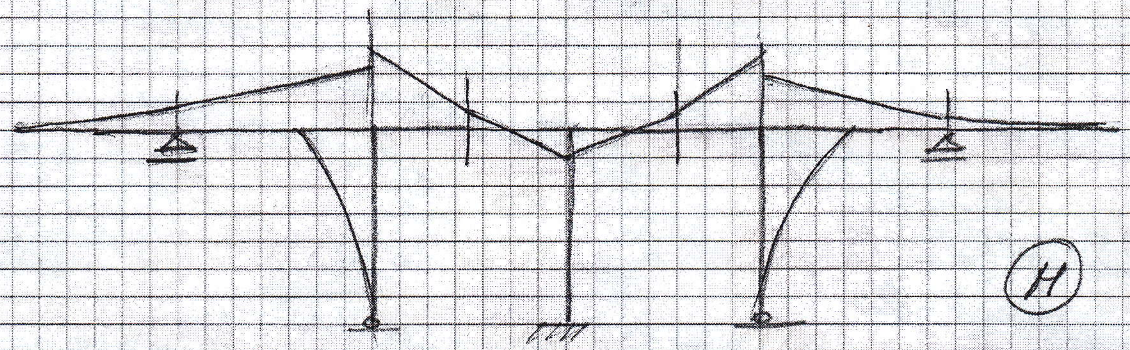
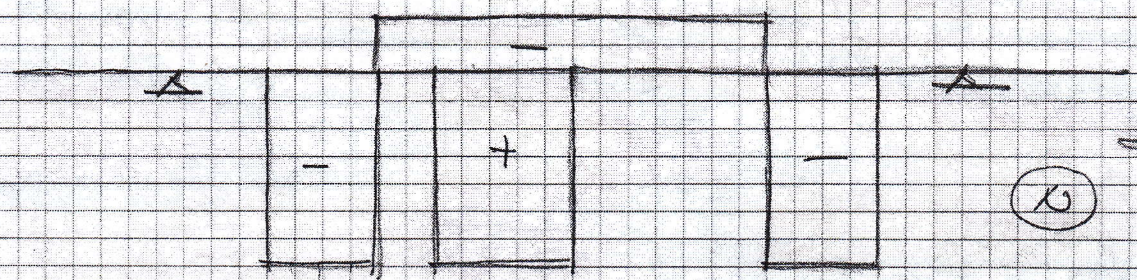
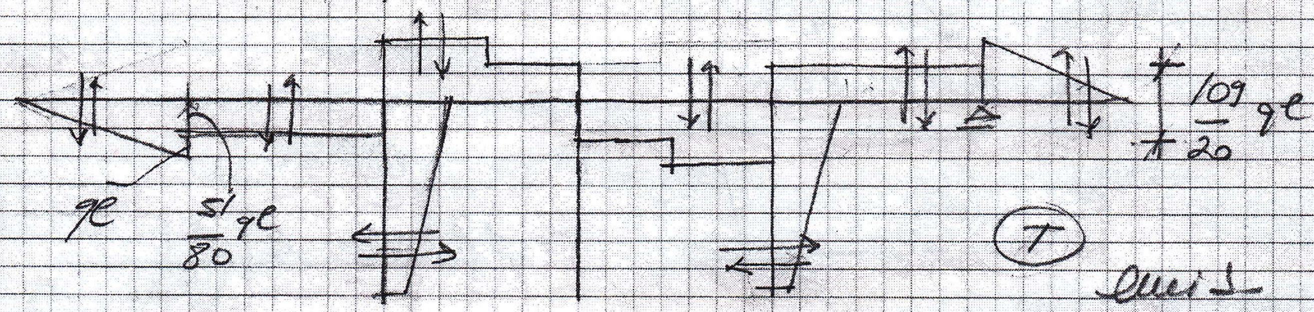




Flex.

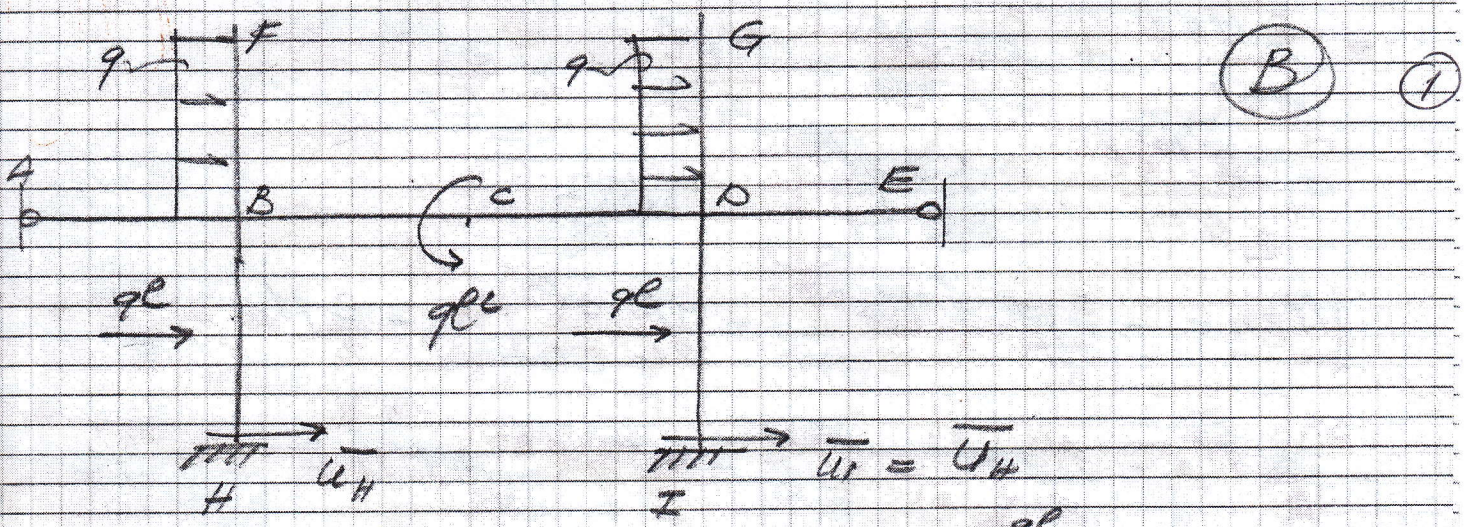


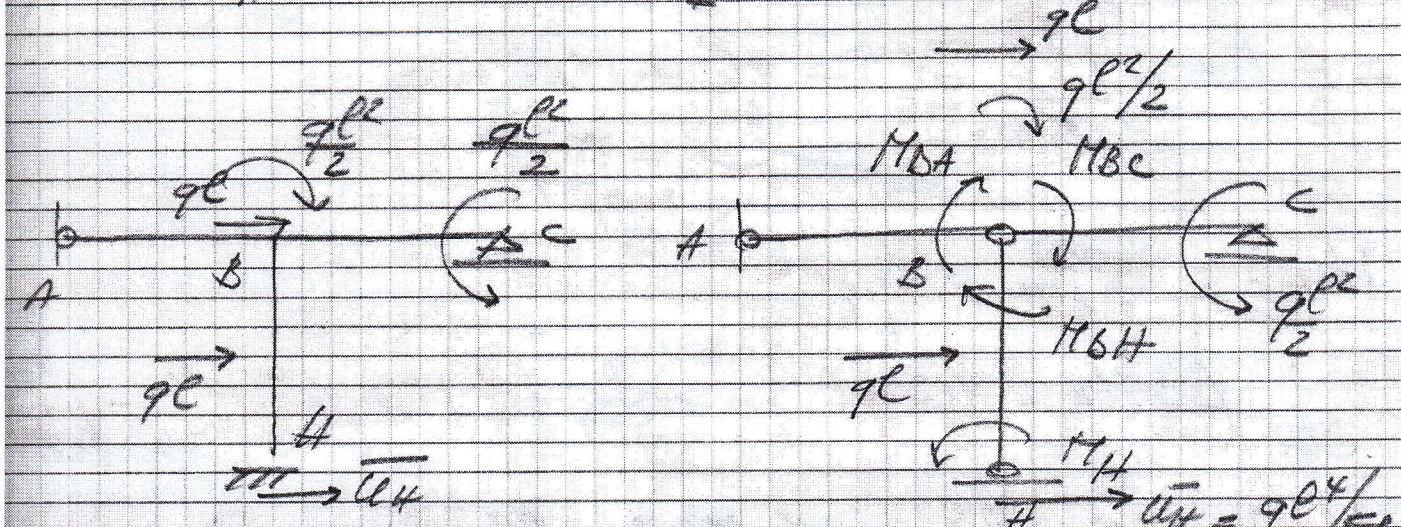
Sforzo



Taglio

(B) (1)





$$\begin{cases} \varphi_{BA} = \varphi_{BC} & \varphi_H = 0 \\ \varphi_{BA} = \varphi_{BH} & * M_{BA} + M_{BH} + M_{BC} = +q\ell^2/2 \end{cases}$$

$$* \varphi_{BA} = -\frac{M_{BA}\ell}{3EI} = \varphi_{BC} = -\frac{M_{BC}\ell}{3EI} - \frac{q\ell^3}{12EI}$$

$$* \varphi_{BA} = -\frac{M_{BA}\ell}{3EI} = \varphi_{BH} = -\frac{M_{BH}\ell}{3EI} - \frac{M_H\ell}{6EI} + \underbrace{\frac{q\ell^3}{EI} + \frac{7q\ell^3}{16EI}}_{\frac{17}{16}q\ell^3/EI}$$

$$* \varphi_H = \frac{M_H\ell}{3EI} + \frac{M_{BH}\ell}{6EI} + \underbrace{\frac{q\ell^3}{EI} - \frac{q\ell^3}{16EI}}_{\frac{15}{16}q\ell^2/EI} = 0$$

$$\boxed{M_{BH} = -\frac{2}{6EI}\frac{M_H\ell}{\ell} - \frac{3}{6EI}\frac{15}{16}\frac{q\ell^3}{8}\frac{2}{EI} = -2M_H - \frac{45}{8}q\ell^2}$$

$$-\frac{M_{BA}\ell}{3EI} = -\frac{\ell}{3EI}\left(-2M_H - \frac{45}{8}q\ell^2\right) - \frac{M_H\ell}{6EI} + \frac{17}{6}q\ell^3/EI =$$

$$= \frac{2 M_4 \ell}{3EI} + \frac{45}{24}\frac{q\ell^3}{EI} - \frac{M_4 \ell}{6EI} + \frac{17}{6}\frac{q\ell^3}{EI} =$$

$$= \frac{2 M_4 \ell}{2 \cdot 3EI} + \frac{113}{24}\frac{q\ell^3}{EI}$$

$$\underline{M_{BA} = -\frac{3EI}{\ell}\left(\frac{M_4 \ell}{2EI} + \frac{113}{24}\frac{q\ell^3}{EI}\right) = -\frac{3}{2}M_4 - \frac{113}{8}q\ell^2}$$

$$-\frac{3}{2}M_4 - \frac{113}{8}q\ell^2 - 2M_4 - \frac{45}{8}q\ell^2 + M_{BC} = +q\ell^2/\cdot 2$$

$$-\frac{7}{2}M_4 - \frac{158}{8}q\ell^2 - \ell^2 + M_{BC} = 0$$

$$-\frac{81}{4}q\ell^2$$

$$\underline{M_{BC} = \frac{7}{2}M_4 + \frac{81}{4}q\ell^2}$$

$$-\frac{\ell}{3EI}\left(-\frac{3}{2}M_4 - \frac{113}{8}q\ell^2\right) = -\frac{\ell}{3EI}\left(\frac{7}{2}M_4 + \frac{81}{4}q\ell^2\right) - \frac{q\ell^3}{12EI}$$

$$\frac{M_4 \ell}{2EI} + \frac{113}{24EI}q\ell^3 = -\frac{7}{6}\frac{M_4 \ell}{EI} - \frac{81}{12EI}q\ell^3 - \frac{q\ell^3}{12EI}$$

$$\frac{16}{3}\frac{M_4 \ell}{EI} = -\frac{113 - 162 - 2}{24EI}q\ell^3 = -\frac{277}{24EI}q\ell^3$$

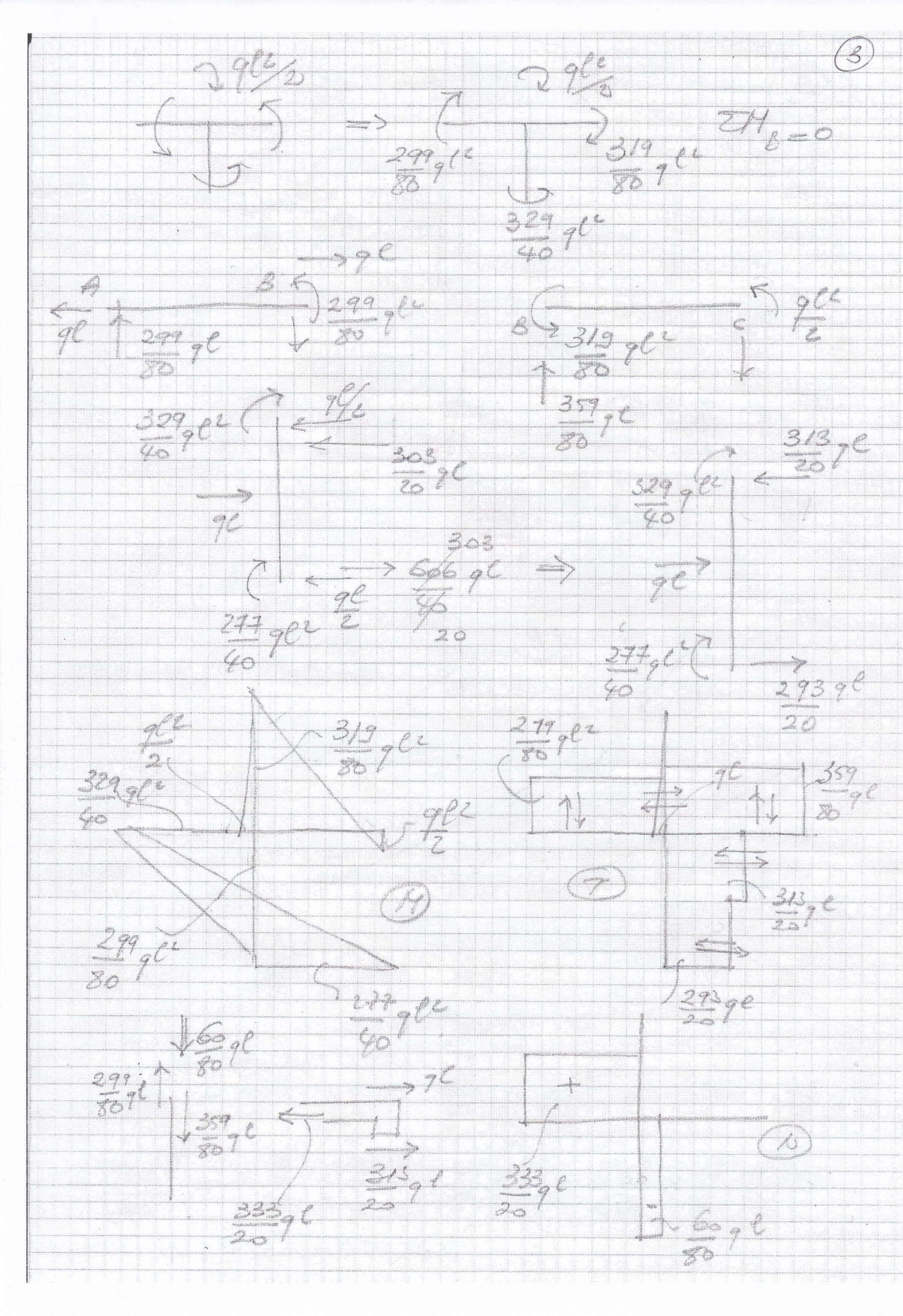
$$\underline{M_4 = -\frac{3EI}{5\ell}\cdot\frac{277}{24EI}q\ell^3 = -\frac{277}{40}q\ell^2}$$

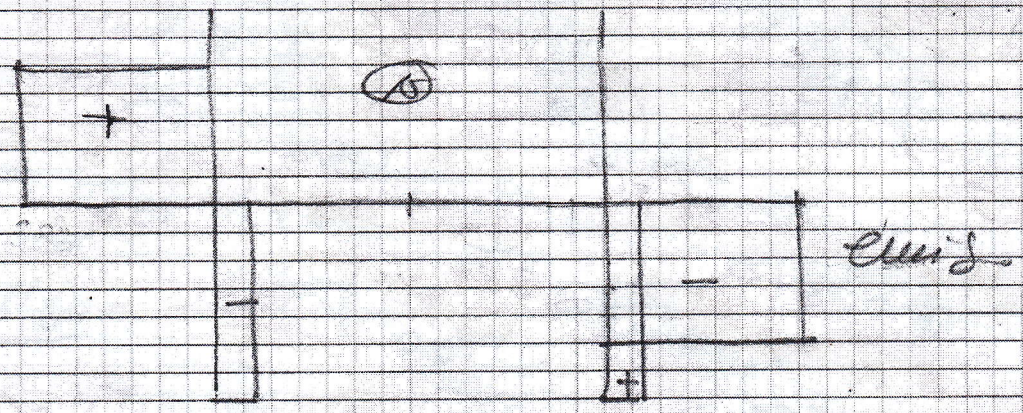
$$\underline{M_{BC} = \frac{81}{4}q\ell^2 - \frac{7}{2}\cdot\frac{277}{40}q\ell^2 = -\frac{319}{80}q\ell^2}$$

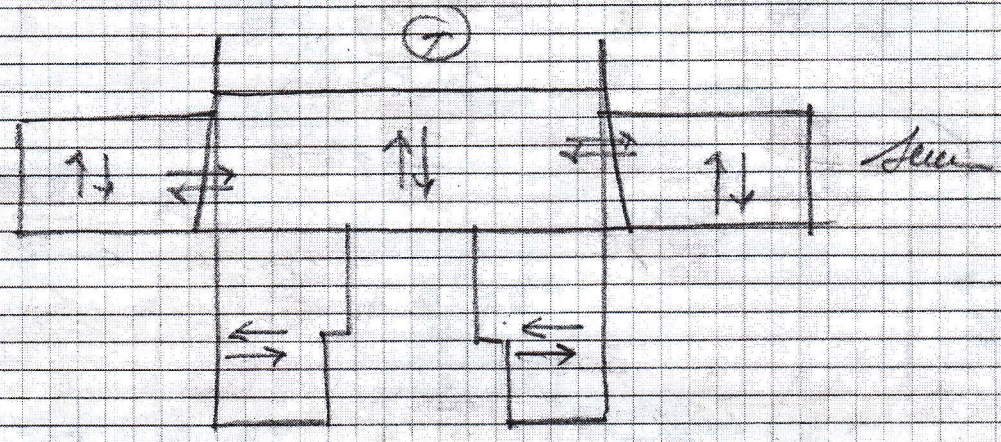
$$\underline{M_{BA} = +\frac{3}{2}\cdot\frac{277}{40}q\ell^2 - \frac{113}{8}q\ell^2 = \frac{831}{80}q\ell^2 - \frac{113}{80}q\ell^2}$$

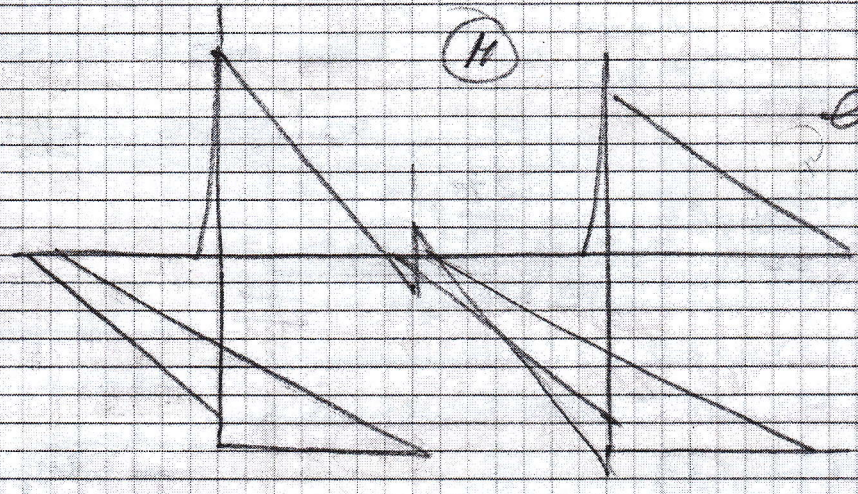
$$= -\frac{299}{80}q\ell^2$$

$$\underline{M_{DA} = \frac{277}{20}q\ell^2 - \frac{45}{8}q\ell^2 = \frac{329}{40}q\ell^2}$$

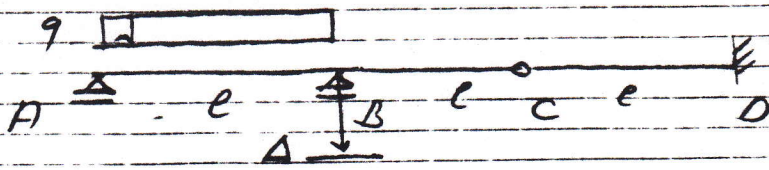

$\frac{299}{80}ql^2$
$\frac{319}{80}ql^2$
$\frac{329}{40}ql^2$
$\Sigma M_B = 0$
A
B
C
ql
$\frac{ql}{2}$
$\frac{313}{20}ql$
$\frac{303}{20}ql$
$\frac{359}{80}ql$
$\frac{277}{40}ql^2$
$\frac{293}{20}ql$
$\frac{279}{80}ql^2$
$\frac{60}{80}ql$
$\frac{215}{20}ql$
$\frac{335}{20}ql$
(9)
(7)
(10)

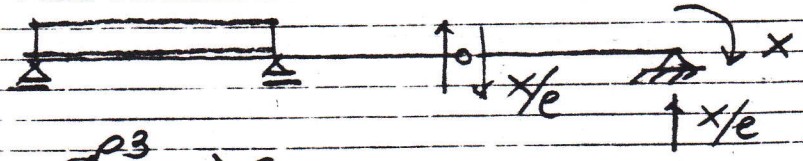

6
+
+
−
−
+
Эпюра

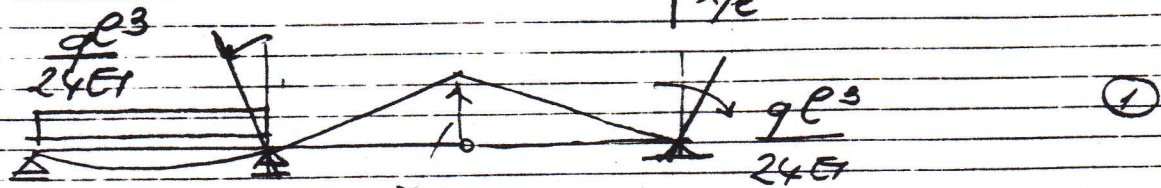

7
Эпюра


14
Эпюра

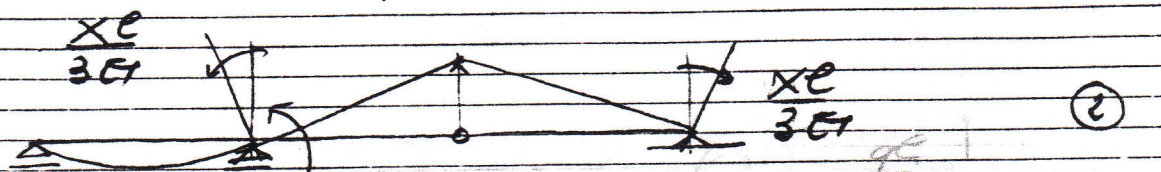
1B)

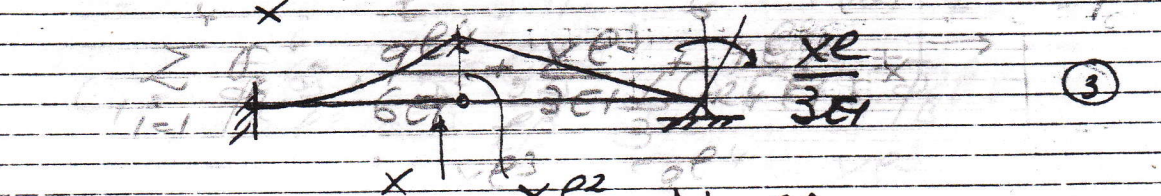


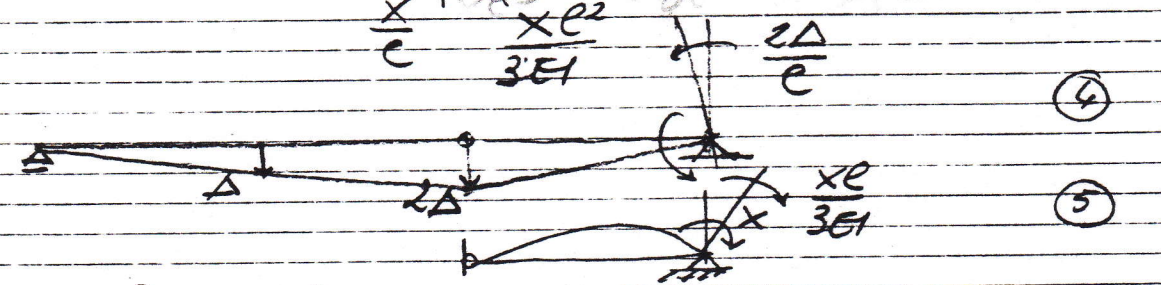




$\frac{qe^4}{24EI}$



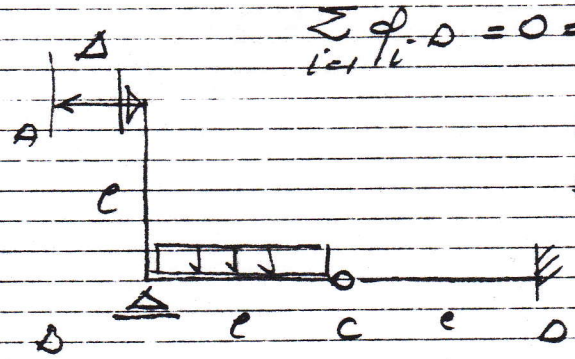


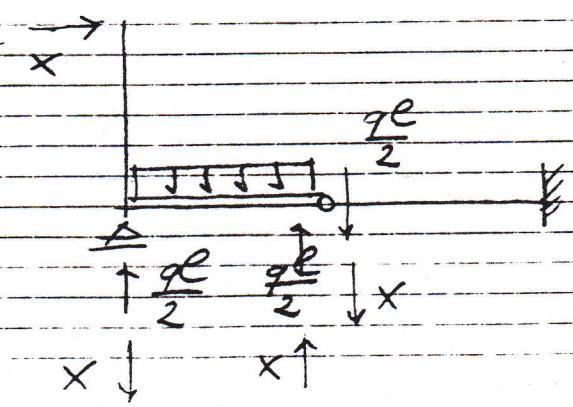


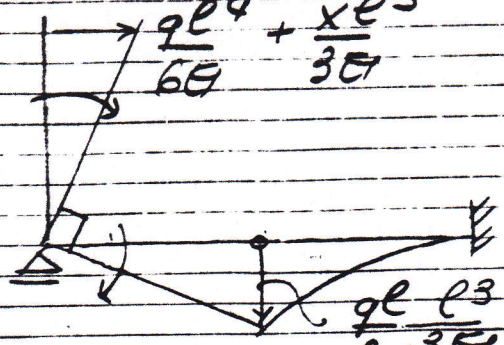
$$\sum_{i=1}^{5} q_i \cdot D = 0 = \frac{Xe}{EI} + \frac{qe^3}{24EI} - \frac{2\Delta}{e}$$

$$X = -\frac{qe^2}{24} + \frac{2\Delta EI}{e^2}$$

1C)



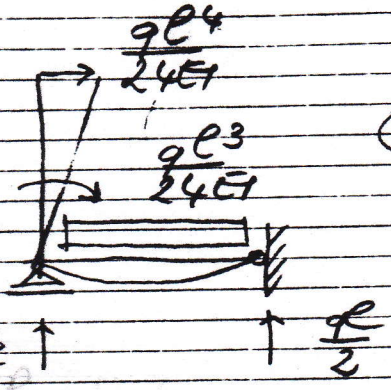




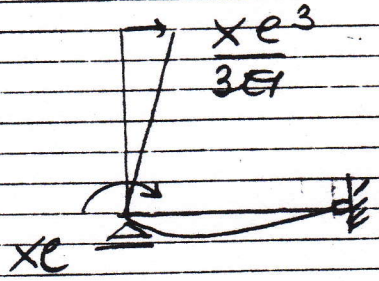
$\rightarrow$ +

③ ①

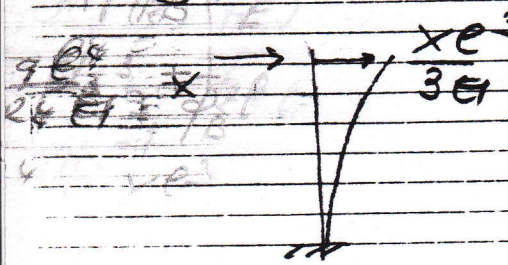
$$\frac{qe}{2}\cdot\frac{e^3}{3EI} + \frac{Xe^3}{3EI} = \frac{qe^4}{6EI} + \frac{Xe^3}{3EI}$$



②



③



④

$$\sum_{i=1}^{4} \delta_{iA} = -\frac{qe^4}{6EI} + \frac{Xe^3}{3EI} + \frac{qe^4}{24EI} + \frac{Xe^3}{3EI} + \frac{Xe^3}{3EI} = \frac{5qe^4}{24EI} + \frac{Xe^3}{EI} = -\Delta$$

$$\frac{5qe}{24} + X = -\frac{\Delta EI}{e^3} \qquad -\frac{5qe}{24} - \frac{\Delta EI}{e^3} = X$$

€ 9,40

Scienza delle Costruzioni - Applicazioni - I° Appello 3 settembre 2001							
Nome	Cognome					VOTAZIONE	INIZIALI
ANNO DI CORSO	III°	IV°	V°	FC			
Test da recuperare	I°	II°	III°	IV°	C.C.	Fila	Posto

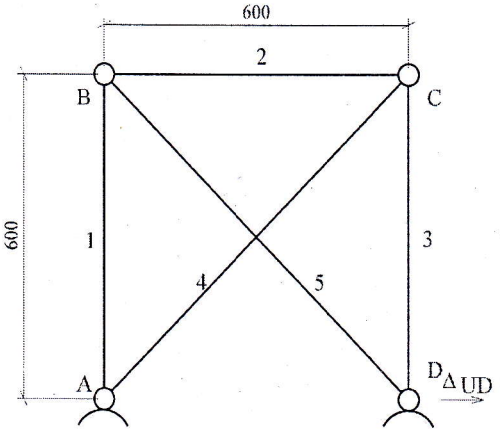
Tempi di consegna: ore 1.30' per 1 test. ore 2.30' per 2 test. ore 3.30' per 3 test. 4 ore compito completo.

I° TEST- Principio dei Lavori virtuali

Reticolare in tubi tondi FE 360:

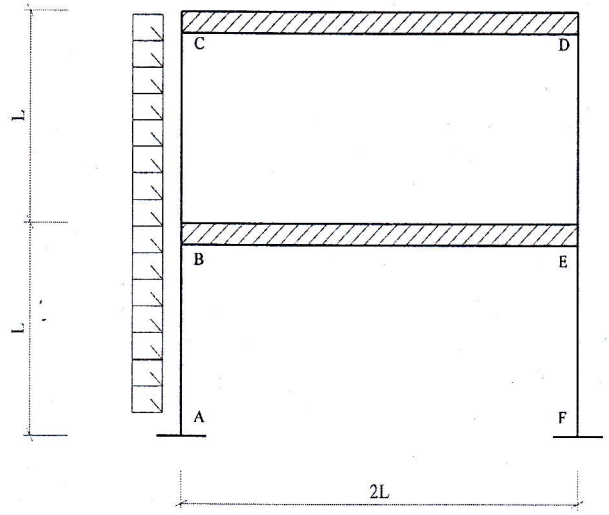
ø = 127mm
L = 600 cm
ΔU_D = 3 cm
A = 15.5 cm²
E = 2.100.000 Kg/cm²

Il vincolo esterno D è soggetto a un cedimento orizzontale di cm 3. Determinare gli sforzi nelle aste.



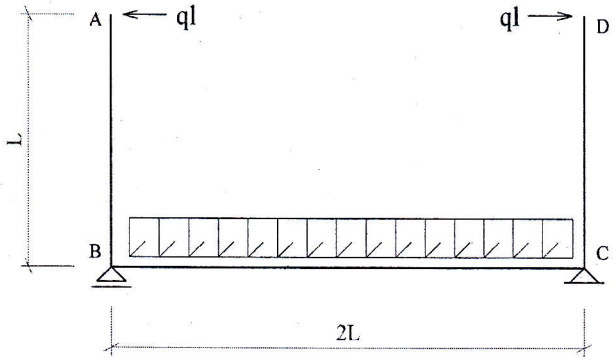
III° TEST- Metodo degli spostamenti.

Risolvere la seguente struttura attraverso il **Metodo degli spostamenti.** Eseguire l'analisi statica e tracciare i diagrammi quotati delle caratteristiche della sollecitazione (M T e N) del telaio in figura supponendo rigidi i traversi BE e CD.



II° TEST- Composizione cinematiche delle forz.

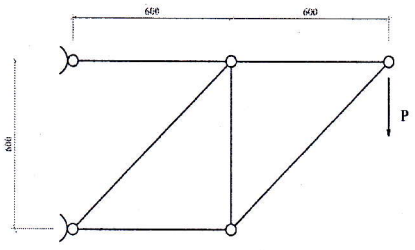
Determinare, via composizione cinematica degli spostamenti, lo spostamento orizzontale relativo tra i punti A e D.



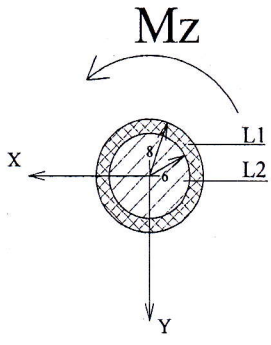
IV° TEST- Verifiche di resistenza.

a) La reticolare in figura, realizzata con tubolari tondi ø127 mm, Fe 360, ha i requisiti per poter incorrere in pericoli di instabilità. Determinare via metodo ω , il Pmax affinchè nessuna asta si instabilizzi.

L = 6m
A = 15.5 cm²
ρ = 4.35 cm
σ_{am} = 1600 Kg/ cm²



b) La sezione in figura, soggetta al Mz = 70 Kgm, è realizzata con due diverse essenze legnose 1 e 2 rese solidali tra loro, a cui corrispondono, τ_{am1} = 8Kg/cm²; G_1 = 40.000 Kg/cm² e τ_{am2} = 10Kg/cm² ; G_2 = 60.000 Kg/cm² .
Effettuare la verifica di resistenza.



Per simmetria:

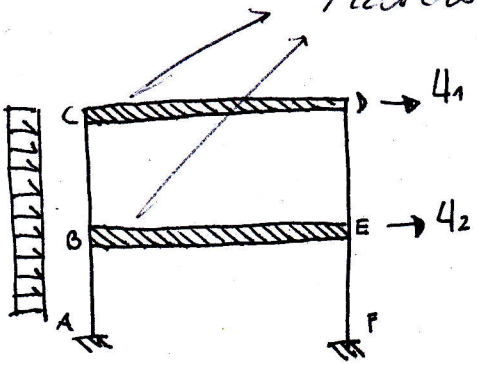
$$\Delta_D = \frac{7\,ql^4}{6EI} + \frac{ql^2}{EA}$$

$$\Delta\eta_{AD} = \Delta_D - \Delta_1 = \frac{14\,ql^4}{6EI} + \frac{2\,ql^2}{EA} = \frac{7\,ql^4}{3EI} + \frac{2ql^2}{EA}$$

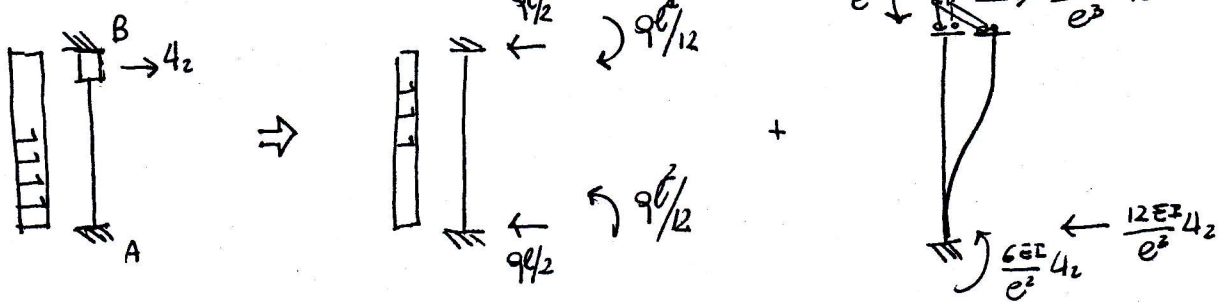
III)

Traversi supposti rigidi = solo traslazioni orizzontali. Pilastri due appoggi, indeformabili a.c. assialmente

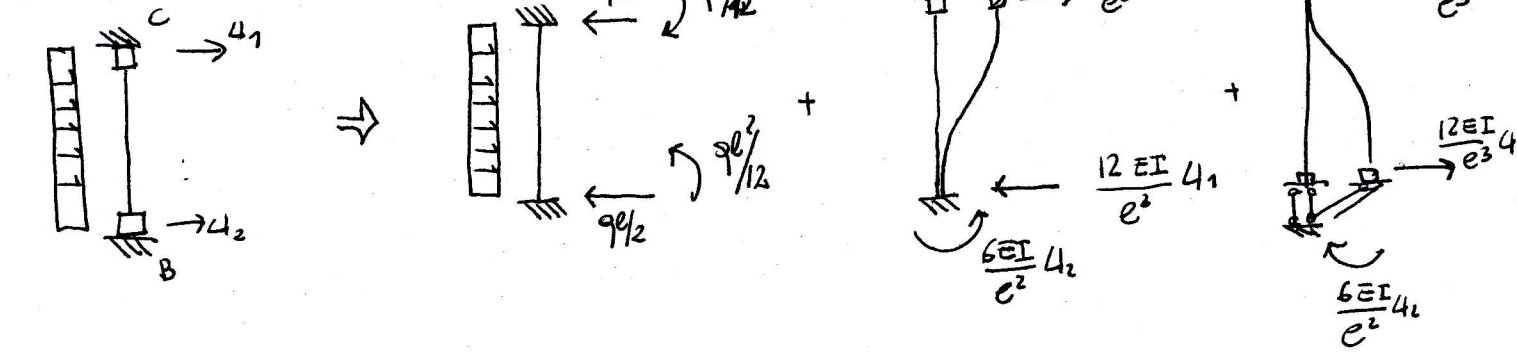
Unici spostamenti cinematicamente ammissibili.



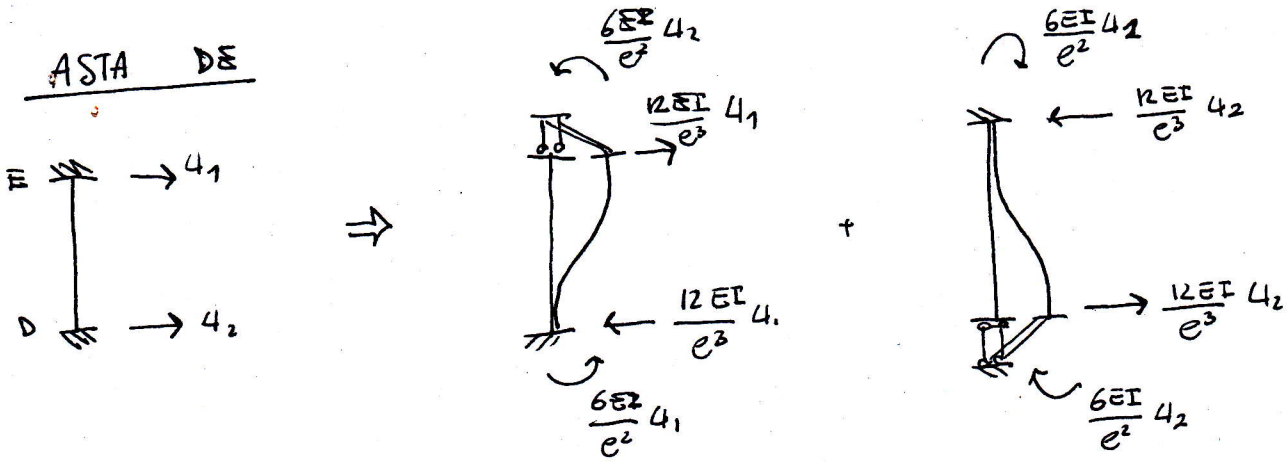
ASTA AB



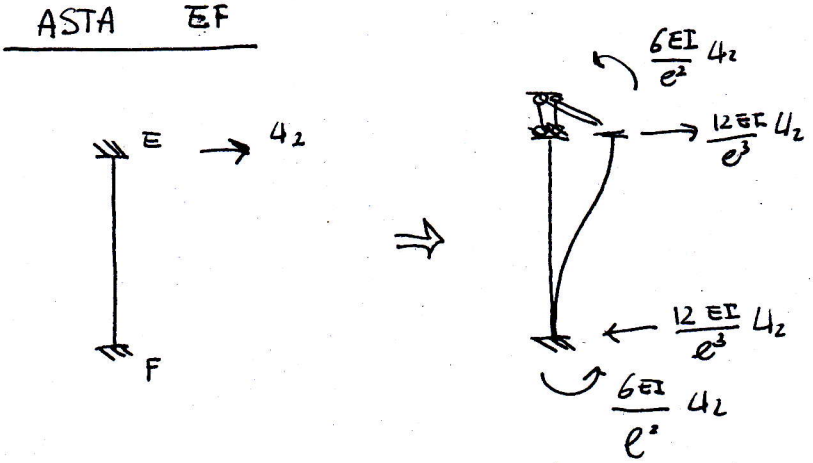
ASTA BC



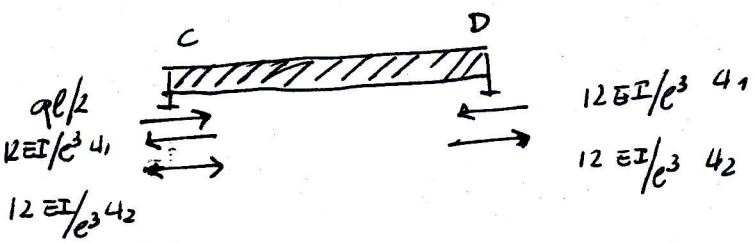
ASTA DE



ASTA EF

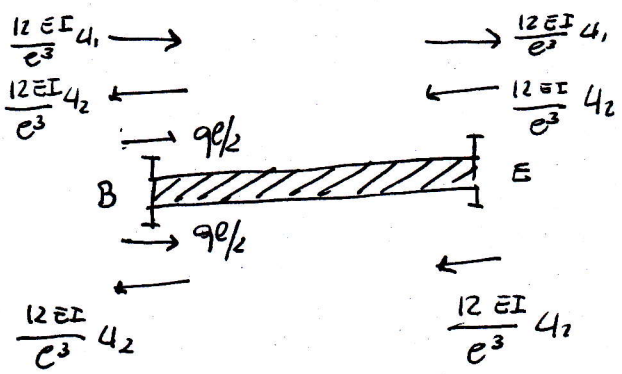


EQUAZIONI DI EQUILIBRIO



$$R_x = 0$$

$$\frac{ql}{2} + \frac{24EI}{e^3}\Delta_2 - \frac{24EI}{e^3}\Delta_1 = 0$$



$$R_x = 0$$

$$ql + \frac{24EI}{e^3}\Delta_1 - \frac{48EI}{e^3}\Delta_2 = 0$$

$$\begin{cases} \dfrac{q\ell}{2} + \dfrac{24\,EI}{\ell^3}u_2 - \dfrac{24\,EI}{\ell^3}u_1 = 0 \\[2ex] q\ell + \dfrac{24\,EI}{\ell^3}u_1 - \dfrac{48\,EI}{\ell^3}u_2 = 0 \end{cases}$$

$$\begin{cases} -\dfrac{24\,EI}{\ell^3}u_1 + \dfrac{24\,EI}{\ell^3}u_2 = -\dfrac{q\ell}{2} \\[2ex] \dfrac{24\,EI}{\ell^3}u_1 - \dfrac{48\,EI}{\ell^3}u_2 = -q\ell \end{cases}$$

$$0 \quad - \frac{24\,EI}{\ell^3}u_2 = -\frac{3\,q\ell}{2}$$

$$u_2 = \frac{3\,q\ell^4}{48\,EI} = \frac{q\ell^4}{16\,EI}$$

$$u_1 = \frac{\ell^3}{24\,EI}\cdot\left(-q\ell + \frac{48\,EI}{\ell^3}\cdot\frac{q\ell^4}{16\,EI}\right) = \frac{q\ell^4}{12\,EI}$$

$$u_1 = \frac{q\ell^4}{12\,EI}$$

$$u_2 = \frac{q\ell^4}{16\,EI}$$

Asta AB

$$M_{AB} = q\ell^2/12 + \frac{3}{8}q\ell^2 = \frac{11}{24}q\ell^2$$

$$T_{AB} = q\ell/2 + \frac{3}{4}q\ell = \frac{5}{4}q\ell \leftarrow$$

$$M_{BA} = q\ell^2/12 - \frac{3}{8}q\ell^2 = -\frac{7}{24}q\ell^2$$

$$T_{BA} = q\ell/2 - \frac{3}{4}q\ell = -\frac{1}{4}q\ell \rightarrow$$

ASTA BC

$$M_{BC} = \frac{ql^2}{12} + \frac{ql^2}{2} - \frac{3}{8}ql^2 = \frac{5}{24}ql^2 \quad \curvearrowleft$$

$$T_{BC} = ql/2 + ql - \frac{3}{4}ql = \frac{3}{4}ql \quad \leftarrow$$

$$M_{CB} = ql^2/12 - ql^2/2 + \frac{3}{8}ql^2 = -\frac{1}{24}ql^2 \quad \curvearrowleft$$

$$T_{CB} = ql/2 - ql + \frac{3}{4}ql = \frac{ql}{4} \quad \leftarrow$$

ASTA DE

$$M_{DE} = ql^2/2 - \frac{3}{8}ql^2 = ql^2/8 \quad \curvearrowleft$$

$$T_{DE} = ql - \frac{3}{4}ql = ql/4 \quad \rightarrow$$

$$M_{ED} = ql^2/2 - \frac{3}{8}ql^2 = ql^2/8 \quad \curvearrowleft$$

$$T_{ED} = ql - \frac{3}{4}ql = ql/4 \quad \leftarrow$$

ASTA EF

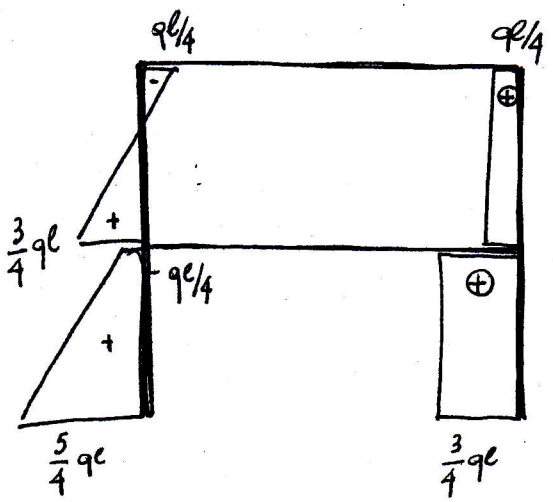
$$M_{EF} = \frac{3}{8}ql^2$$

$$T_{EF} = \frac{3}{4}ql$$

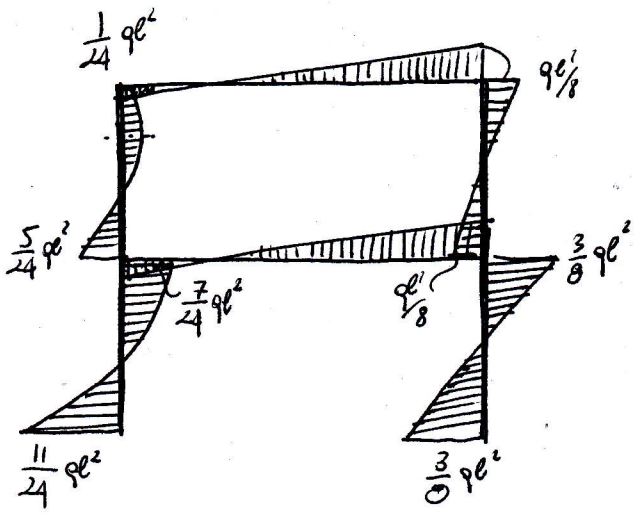
$$M_{FE} = \frac{3}{8}ql^2$$

$$T_{FE} = \frac{3}{4}ql$$

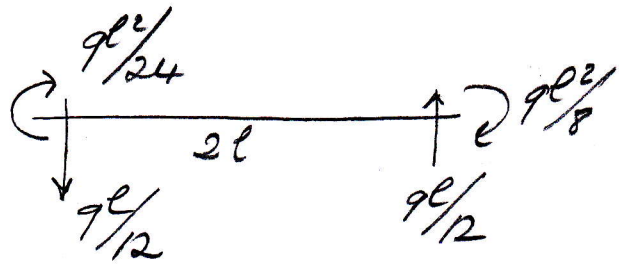
T



M

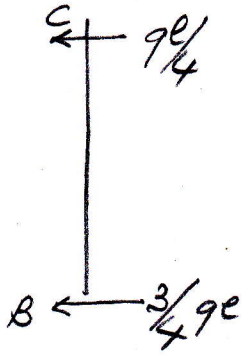


trave $\overline{CD}$ =

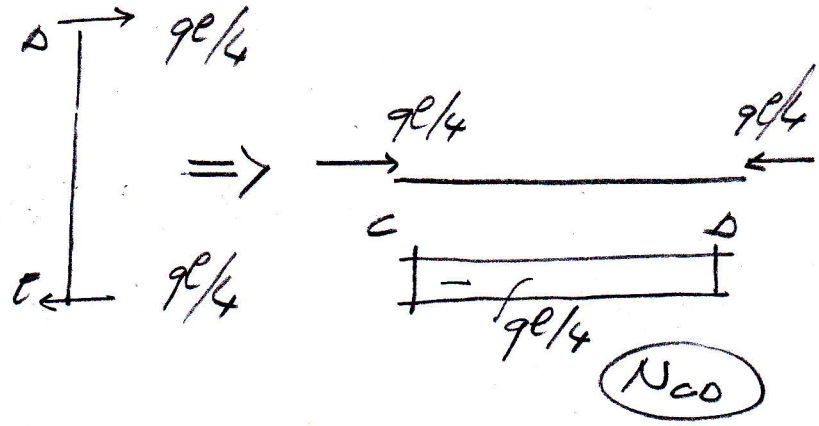


T su $\overline{CD}$

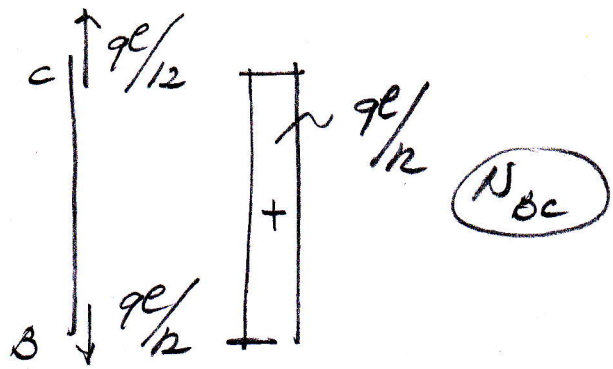
pilastro $\overline{BC}$:



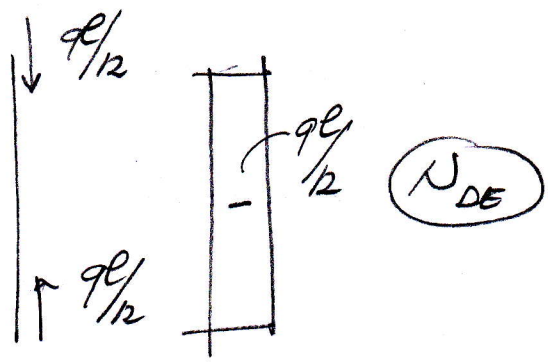
pilastro $\overline{DE}$.



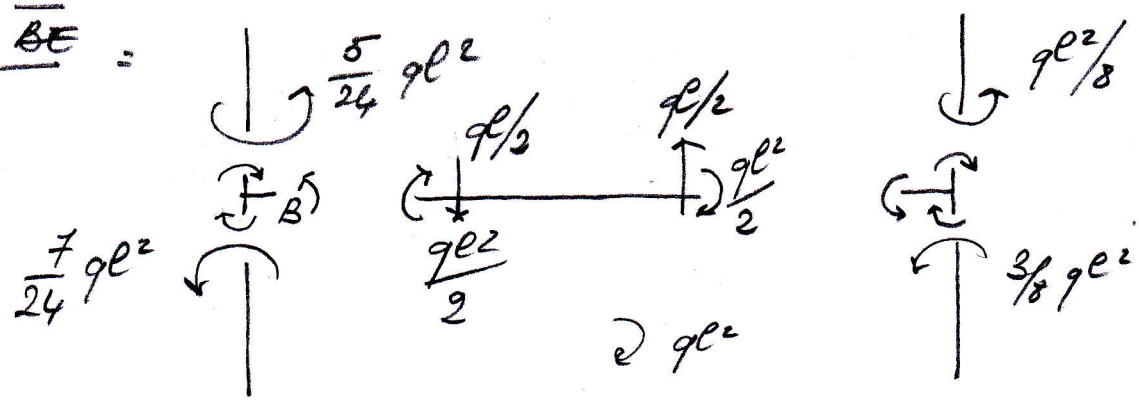
pilastro $\overline{BE}$:

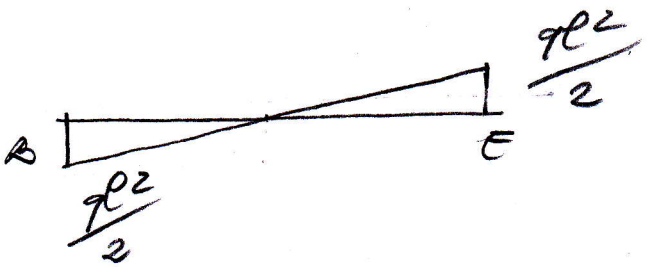


pilastro $\overline{DE}$ =



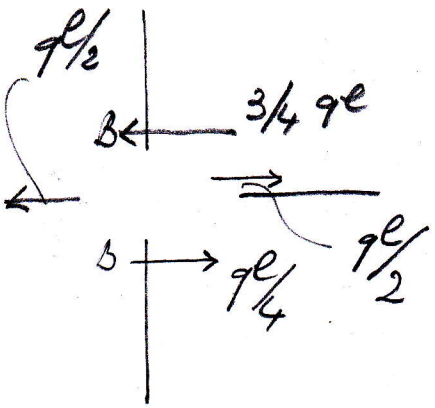
trave $\overline{BE}$:

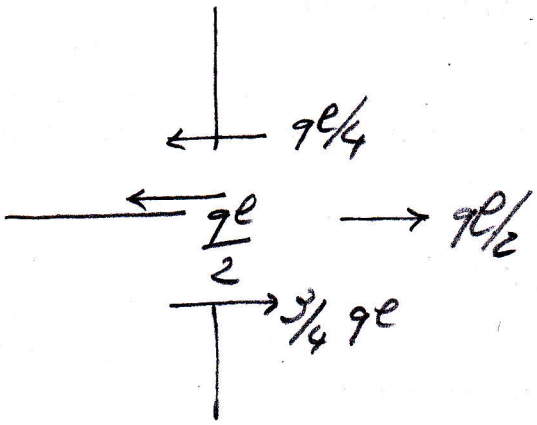


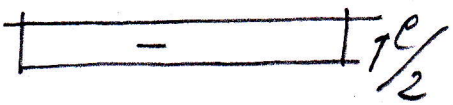


M_{BE}

T_{BE}

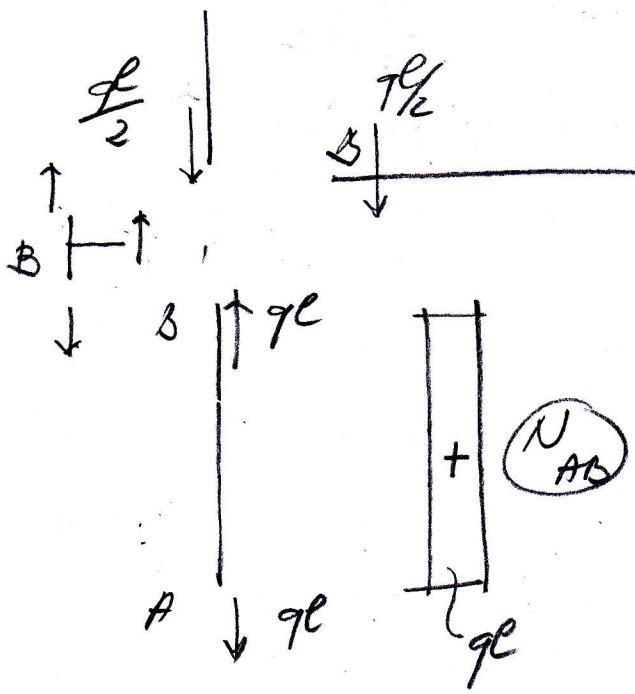




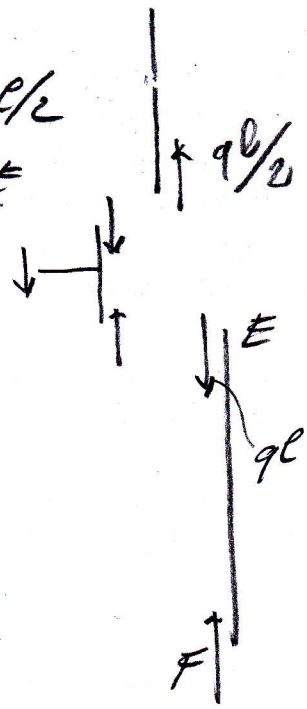


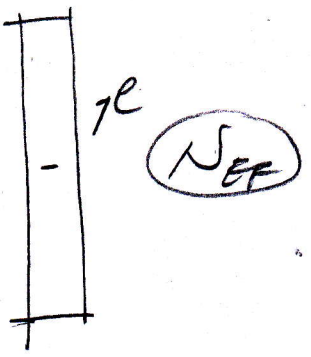
N_{BE}

Arkstro $\overline{AB}$:



Arkstro $\overline{EF}$:

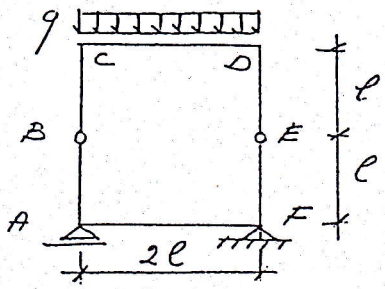




SCIENZA delle COSTRUZIONI PROVA SCRITTA 08/06/1999

Cognome.....Nome.....
Anno di Corso.....Tests da recuperare: 1 2 3

A.1 Risolvere via PLV la seguente struttura iperstatica determinando, in particolare, il diagramma del Momento flettente.

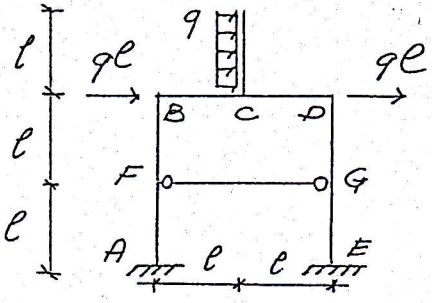


A.2 Dato il seguente tensore di sforzo

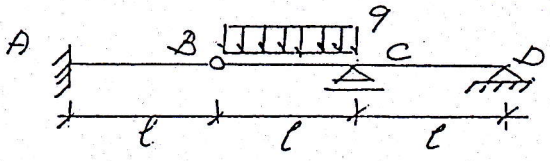
$$[\sigma_P]=\begin{bmatrix} -1500 & 0 & -800 \\ 0 & 500 & 0 \\ -800 & 0 & -2000 \end{bmatrix} \; kg/cmq$$

nell'intorno infinitesimo di P, definire tensioni principali e direzioni principali della tensione ossia autovalori e autovettori del tensore assegnato.

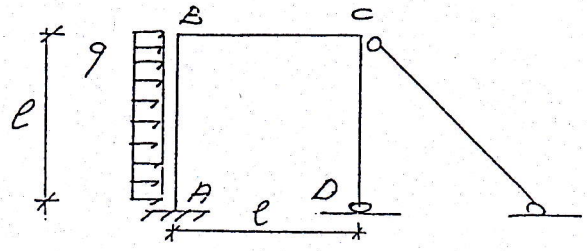
B.1 Determinare i parametri di spostamento nodali del seguente telaio mediante il Metodo degli Spostamenti.



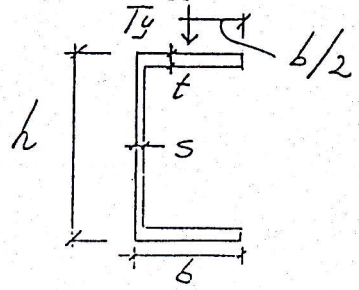
B.2 Determinare la componente di reazione verticale Y_D in D mediante Metodo delle Forze e composizione cinematica degli spostamenti.



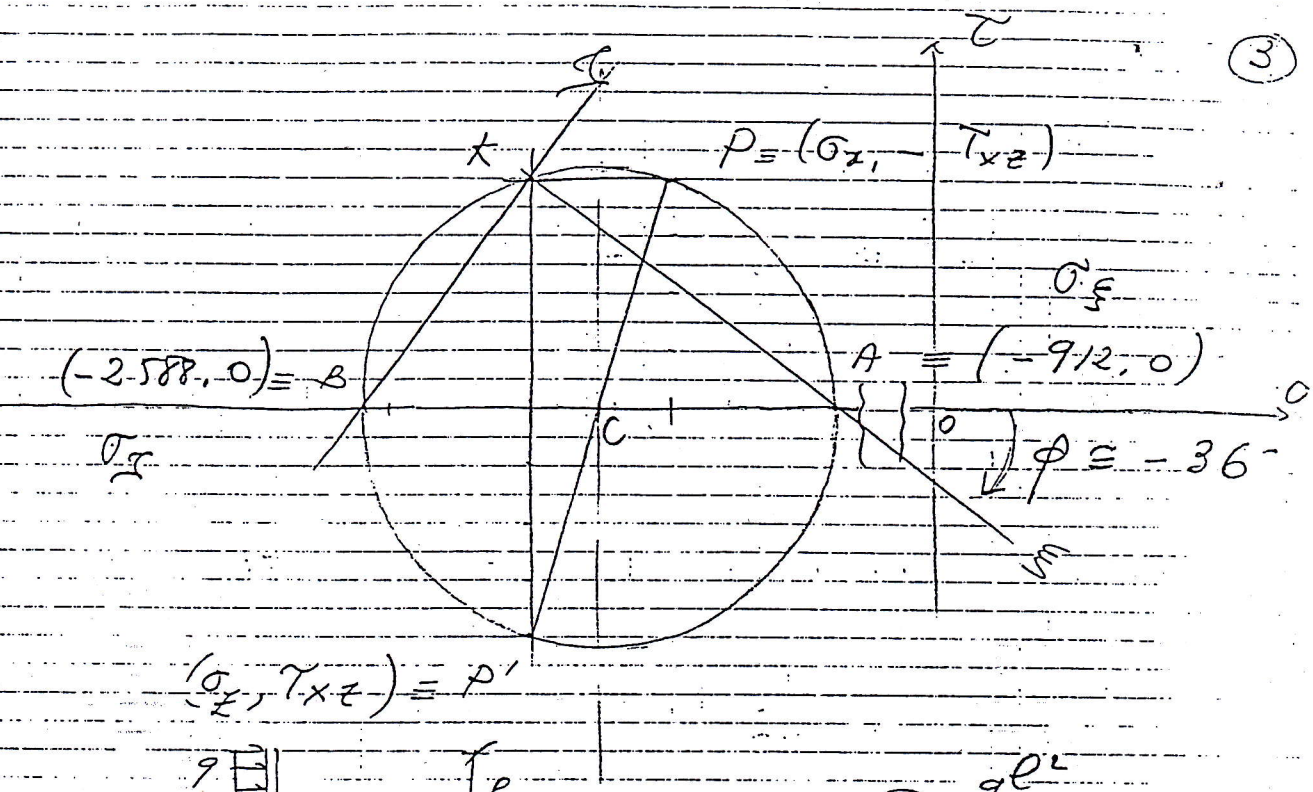
C.1 Determinare con il Metodo di Cross i Momenti alle estremità di ciascuna asta e le rotazioni dei nodi B e C.



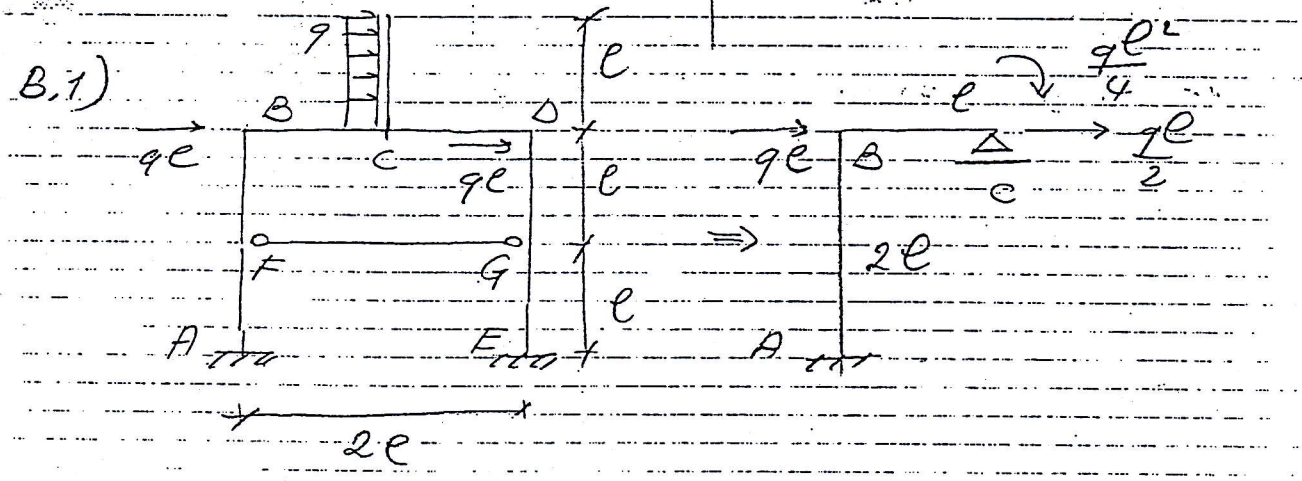
C.2 Verificare la seguente sezione in acciaio Fe 360, di dimensioni b = 10 cm, h = 30 cm, s = 1 cm, t = 1,2 cm, soggetta al taglio Ty = 1 t applicato alla metà dell'ala superiore.



$I_x = 8028\ cm^4$



B,1)



$\frac{3EI}{e}\varphi_B$ $\frac{qe^2}{8}$

$\frac{4EI}{2e}\varphi_B$ $\Sigma H_b = 0$

$\frac{6EI}{4e^2}w_B$ $\frac{qe^2}{8} - \frac{10EI}{2e}\varphi_B - \frac{6EI}{4e^2}w_B = 0$

$*$ $\frac{qe^2}{8} - \frac{5EI}{e}\varphi_B - \frac{3}{2}\frac{EI}{e^2}\psi_B = 0$

$\rightarrow \frac{3}{2}ql \qquad \sum X = 0$

$\leftarrow \frac{12EI}{(2l)^3} w_B \qquad \frac{3}{2}ql - \frac{12EI}{8l^3} w_B - \frac{6EI}{4l^2}\varphi_B = 0$

$\leftarrow \frac{6EI}{(2l)^2}\varphi_B \quad * \quad \frac{3}{2}ql - \frac{3EI}{2l^3} w_B - \frac{3}{2}\frac{EI}{l^2}\varphi_B = 0$

$$\varphi_B = \left(\frac{3}{2}ql - \frac{3EI}{2l^3} w_B\right)\frac{2l^2}{3EI} = \frac{ql^3}{EI} - \frac{w_B}{l}$$

$$\frac{ql^2}{8} - \frac{5EI}{l}\left(\frac{ql^3}{EI} - \frac{w_B}{l}\right) - \frac{3EI}{2l^2} w_B =$$

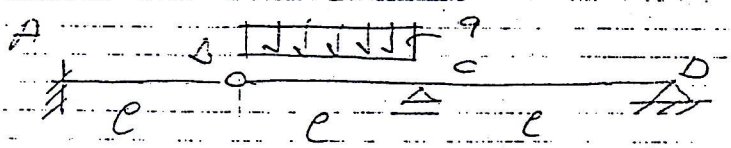
$$= \frac{ql^2}{8} - 5ql^2 + \frac{5EI}{l^2} w_B - \frac{3EI}{2l^2} w_B =$$

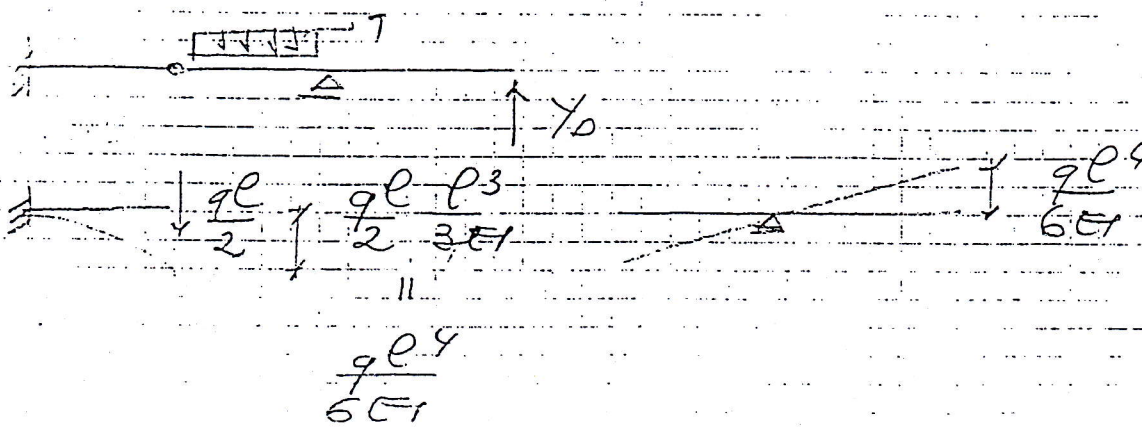
$$= -\frac{39}{8}ql^2 + \frac{7EI}{2l^2} w_B = 0$$

$$w_B = \frac{39}{4} ql^2 \frac{l^2}{7EI} = \frac{39}{28}\frac{ql^4}{EI}$$

$$\varphi_B = \frac{ql^3}{EI} - \frac{39}{28}\frac{ql^3}{EI} = -\frac{11}{28}\frac{ql^3}{EI}$$

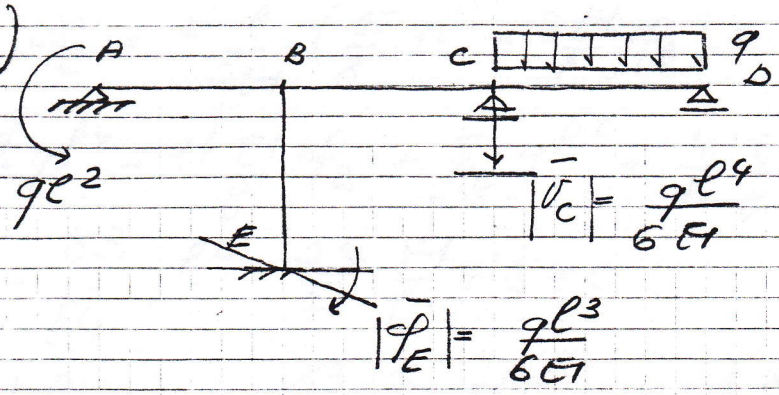
B.2)





1)

1)



$$M^0_{BA} = \frac{q\ell^2}{2} \qquad M^0_{BE} = -\frac{2EI}{\ell}\bar{\varphi}_E = -\frac{2EI}{\ell}\frac{q\ell^3}{6EI} = -\frac{q\ell^2}{3}$$

$$M^0_{BC} = \frac{6EI}{\ell^2}\bar{v}_C = \frac{6EI}{\ell^2}\frac{q\ell^4}{6EI} = q\ell^2$$

$$M^0_{CD} = -\frac{3EI}{\ell^2}\bar{v}_C + \frac{q\ell^2}{8} = -\frac{3EI}{\ell^2}\frac{q\ell^4}{6EI} + \frac{q\ell^2}{8} = -\frac{q\ell^2}{2} + \frac{q\ell^2}{8} =$$

$$= -\frac{3}{8}q\ell^2 \qquad M^0_{EB} = -\frac{4EI}{\ell}\cdot\frac{q\ell^3}{6EI} = -\frac{2}{3}q\ell^2$$

$$M^0_{CB} = \frac{6EI}{\ell^2}\bar{r}_C = q\ell^2$$

$$\bar{M}_{BA} = \frac{q\ell^2}{2} + \frac{3EI}{\ell}\varphi_B \qquad \bar{M}_{CB} = q\ell^2 + \frac{4EI}{\ell}\varphi_C + \frac{2EI}{\ell}\varphi_B$$

$$\bar{M}_{BE} = -\frac{q\ell^2}{3} + \frac{4EI}{\ell}\varphi_B \qquad \bar{M}_{CD} = -\frac{3}{8}q\ell^2 + \frac{3EI}{\ell}\varphi_C$$

$$\bar{M}_{BC} = q\ell^2 + \frac{4EI}{\ell}\varphi_B + \frac{2EI}{\ell}\varphi_C$$

<u>nodo B</u>

$$\Sigma M_{Bi} = -\frac{q\ell^2}{2} - \frac{3EI}{\ell}\varphi_B + \frac{q\ell^2}{3} - \frac{4EI}{\ell}\varphi_B - q\ell^2 -$$

$$-\frac{4EI}{\ell}\varphi_B - \frac{2EI}{\ell}\varphi_C =$$

$$= -\frac{7}{6}q\ell^2 - \frac{11EI}{\ell}\varphi_B - \frac{2EI}{\ell}\varphi_C = 0$$

$$\Sigma M_{Ci} = -q\ell^2 - \frac{4EI}{\ell}\varphi_C - \frac{2EI}{\ell}\varphi_B + \frac{3}{8}q\ell^2 - \frac{3EI}{\ell}\varphi_C =$$

$$= -\frac{5}{8}q\ell^2 - \frac{2EI}{\ell}\varphi_B - \frac{7EI}{\ell}\varphi_C = 0$$

$$\varphi_C = -\frac{l}{7EI}\left(\frac{5}{8}ql^2 + \frac{2EI}{l}\varphi_B\right) = -\frac{5}{56}\frac{ql^3}{EI} - \frac{2}{7}\varphi_B$$

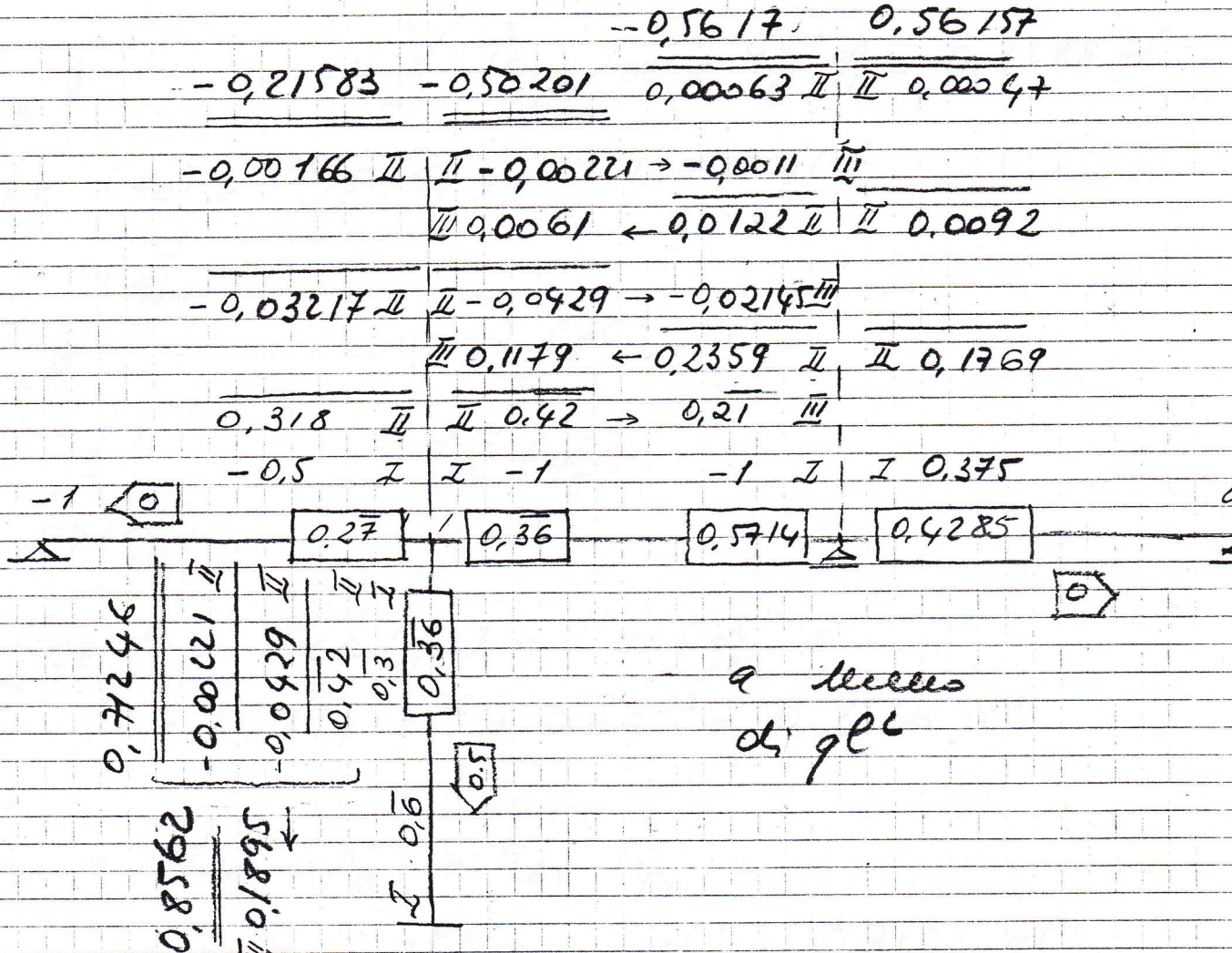
$$-\frac{7}{6}ql^2 - 11\frac{EI}{l}\varphi_B + \frac{5}{28}ql^2 + \frac{4}{7}\frac{EI}{l}\varphi_B =$$

$$= -\frac{83}{84}ql^2 - \frac{73}{7}\frac{EI}{l}\varphi_B = 0$$

$$\varphi_B = -\frac{83}{84}\cdot\frac{7}{73}\frac{ql^3}{EI} = -\frac{581}{6132}\frac{ql^3}{EI} = -0.0947\frac{ql^3}{EI}$$

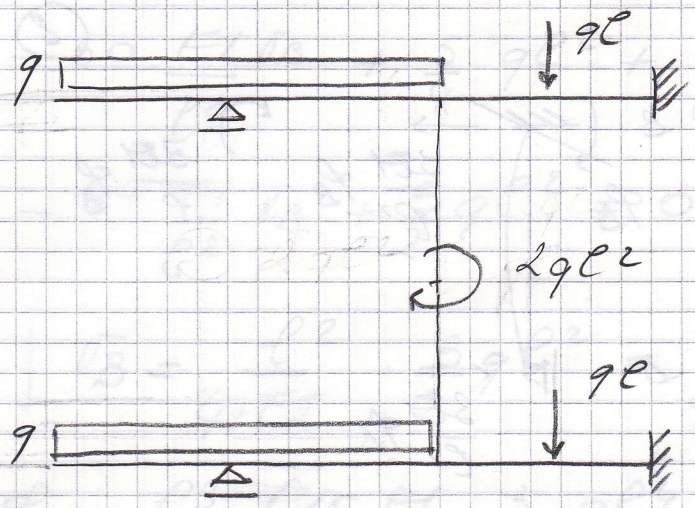
$$\varphi_C = -\frac{5}{56}\frac{ql^3}{EI} + \frac{2}{7}\cdot\frac{581}{6132}\frac{ql^3}{EI} = -0.0622\frac{ql^3}{EI}$$

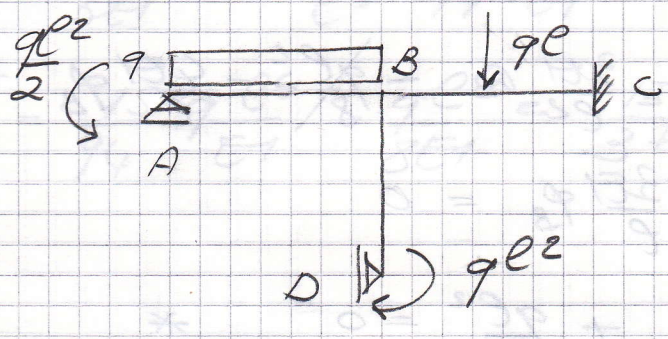
3.1)



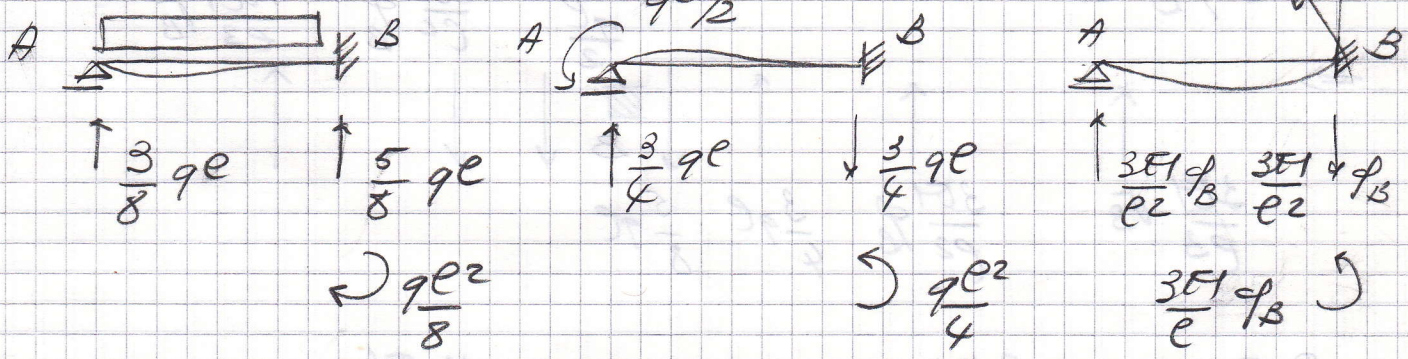
węzeł i	pręt ij	W_{ij}	μ_{ij}	c_{ij}
B	BA	3EI/l	0,27	0
	BC	4EI/l	0,36	0,5
	BE	4EI/l	0,36	0,5
C	CB	4EI/l	0,5714	0,5
	CD	3EI/l	0,4285	0

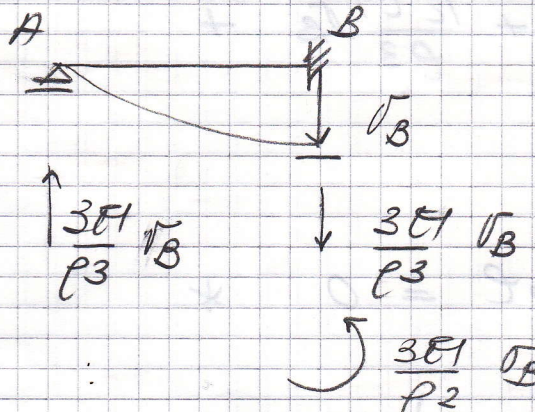
Metodo degli spostamenti



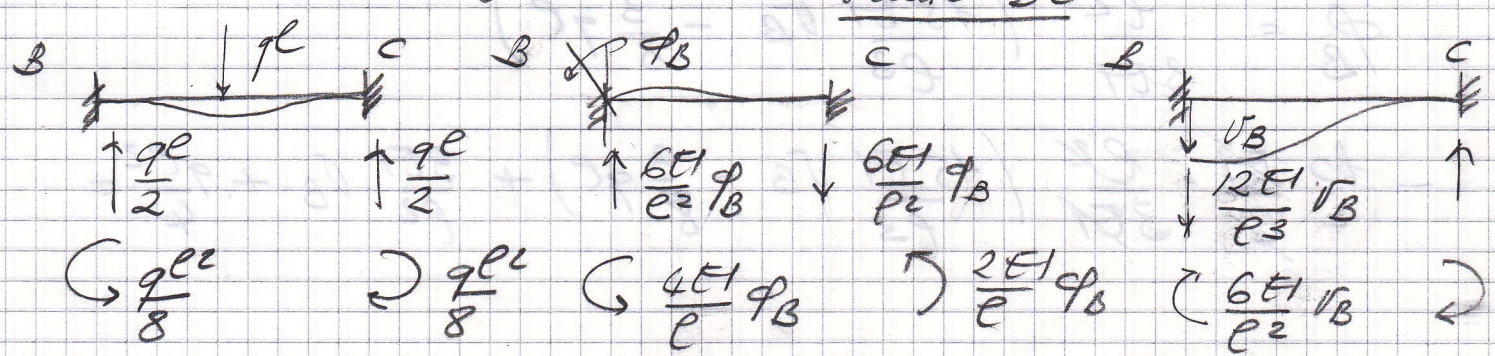


tratto $\overline{AB}$

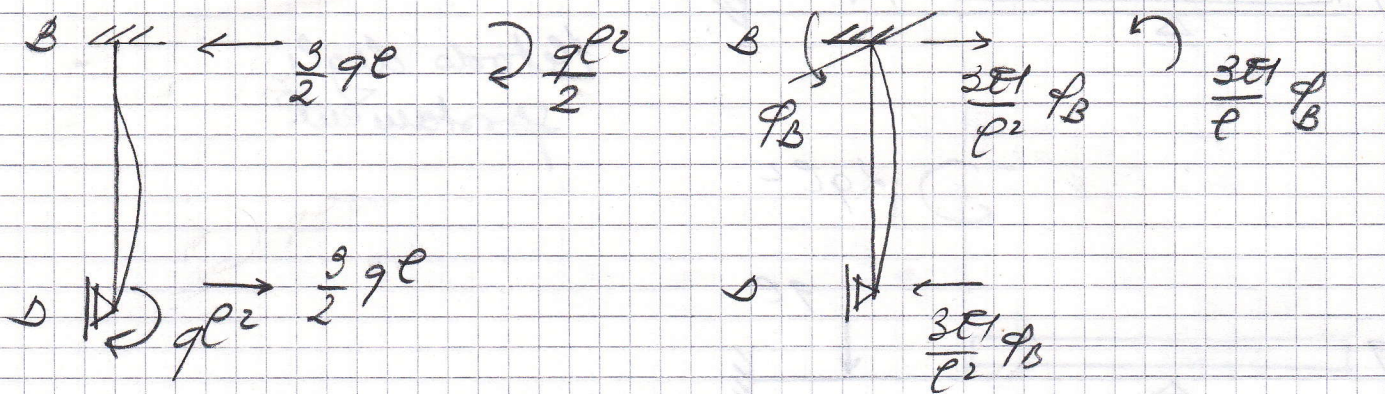




tratto $\overline{BC}$



Стан 3D



$$\Sigma M_B = 0$$

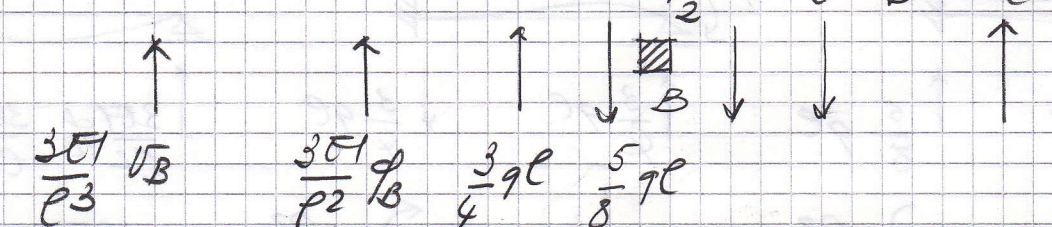
$$\Rightarrow \cancel{\frac{ql^2}{8}} - \frac{ql^2}{4} - \frac{3EI}{l}\varphi_B - \frac{3EI}{l^2}v_B - \cancel{\frac{ql^2}{8}} - \frac{4EI}{l}\varphi_B +$$

$$+ \frac{6EI}{l^2}v_B + \frac{ql^2}{2} - \frac{3EI}{l}\varphi_B = 0$$

$$-10\frac{EI}{l}\varphi_B + \frac{3EI}{l^2}v_B + \frac{ql^2}{4} = 0 \quad *$$

$$\Sigma Y_B = 0$$

$$\Rightarrow$$



$$\frac{3EI}{l^2}\varphi_B - \frac{6EI}{l^2}\varphi_B + \frac{3EI}{l^3}v_B + \frac{12EI}{l^3}v_B +$$

$$+ \frac{3}{4}ql - \frac{5}{8}ql - \frac{ql}{2} = 0$$

$$-\frac{3EI}{l^2}\varphi_B + 15\frac{EI}{l^3}v_B - \frac{3}{8}ql = 0 \quad *$$

$$\varphi_B = \frac{l^2}{3EI}\left(15\frac{EI}{l^3}v_B - \frac{3}{8}ql\right)$$

$$-10\cancel{\frac{EI}{l}}\cdot\frac{l^2}{3\cancel{EI}}\left(15\frac{EI}{l^3}v_B - \frac{3}{8}ql\right) + \frac{3EI}{l^2}v_B + q\frac{l^2}{4} =$$

$$-50\frac{EI}{\ell^2}\delta_B+\frac{5}{4}q\ell^2+\frac{3EI}{\ell^2}r_B+\frac{q\ell^2}{4}=$$

$$-47\frac{EI}{\ell^2}r_B+\frac{3}{2}q\ell^2=0$$

$$\underline{r_B}=\frac{\ell^2}{47EI}\cdot\frac{3}{2}q\ell^2=\frac{3}{94}\frac{q\ell^4}{EI}=0{,}0319\,\frac{q\ell^4}{EI}$$

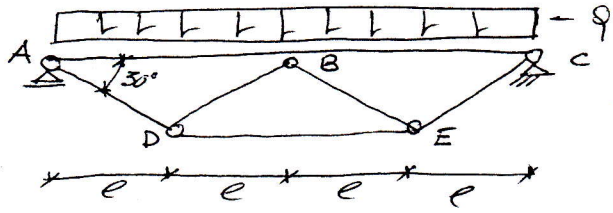
$$\underline{\varphi_B}=\frac{\ell^3}{3EI}\left(15\frac{EI}{\ell^3}\cdot\frac{3}{94}\frac{q\ell^4}{EI}-\frac{3}{8}q\ell\right)=$$

$$=\frac{15}{94}q\frac{\ell^4}{EI}-\frac{q\ell^4}{8EI}=\frac{13}{376}q\frac{\ell^4}{EI}=0{,}0345\,q\frac{\ell^4}{EI}$$

SCIENZA delle COSTRUZIONI **PROVA SCRITTA** **25 / 06 / 99**

Cognome.....**Nome**.....
Anno di Corso.....**Tests da recuperare:** 1 2 3

A.1 Risolvere via PLV la seguente struttura iperstatica determinando, in particolare, lo sforzo nell'asta DE, ed i diagammi quotati di N t e M sulla trave AC. Sia l =2,5 m ; q = 2000 Kg/m , A= 15,5 cm^2 e I_x = 2609 cm^4(Tubo Diametro esterno 127 mm e cassone 200x140 mm).



A.2 Dato il seguente campo di spostamenti:

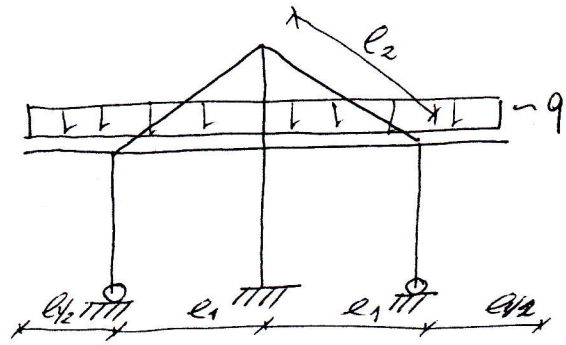
$$u = 0,1x^2y + 0,2yz^2$$

$$v = 0,4xyz + 0,3y^2$$

$$w = 0,1xz^2$$

definito nell'intorno infinitesimo di P=(1; -2; 3), determinare il coefficiente di dilatazione cubica.

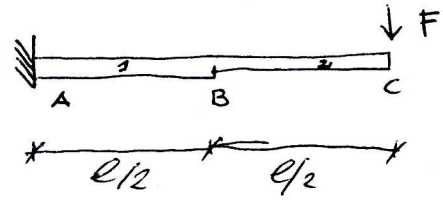
B.1 Risolvere il seguente telaio mediante il Metodo degli Spostamenti, determinando i diagrammi di T e M sulla travata AI. (l_1=4 m ; l_2 = 5 m)



$l_1 = 4\,m$

$l_2 = 5\,m$

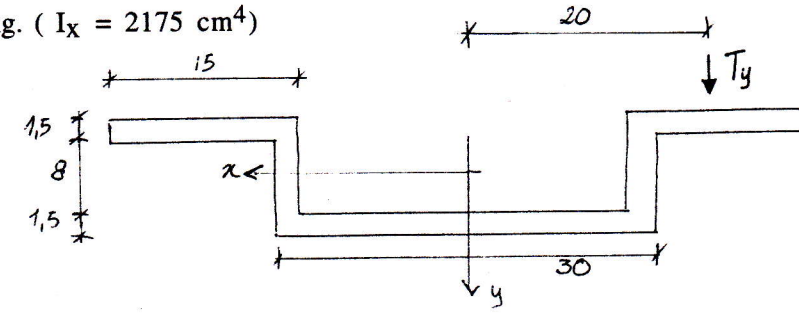
B.2 Determinare la componente di spostamento verticale V_C e la rotazione Φ_C dell'estremo C mediante l'equazione della linea elastica, eventualmente verificare i risultati ottenuti con la composizione cinematica degli spostamenti (sia $I_1 = 2\,I_2$)



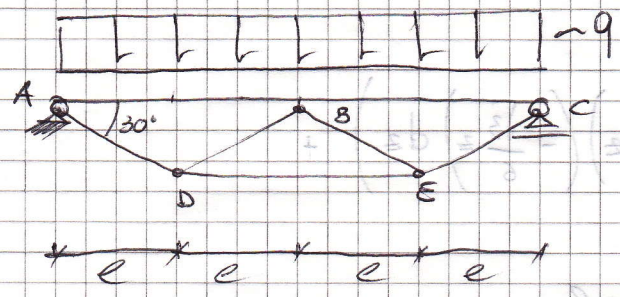
$I_1 = 2I_2$

C.1 Risolvere il telaio di **B. 1** con il metodo di Cross, determinando i diagrammi di T e M per la travata AI.

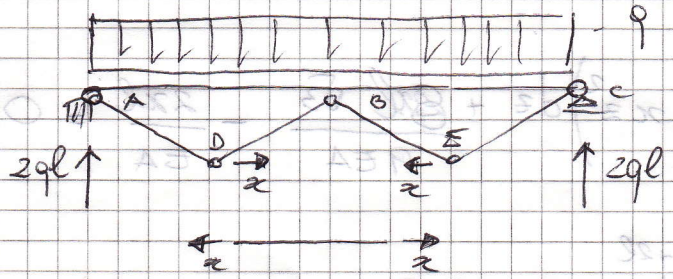
C.2 Verificare la seguente sezione in multistrato di legno di betulla (τ_{am}=10 Kg/cm^2), soggetta al taglio T_y = 25 kg. (I_x = 2175 cm^4)

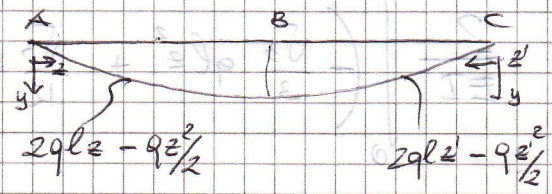


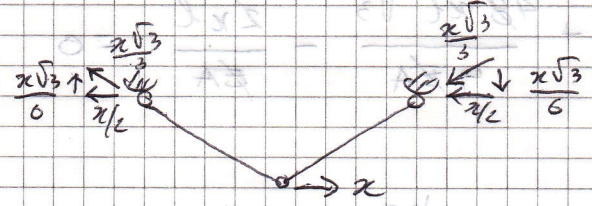
1.1



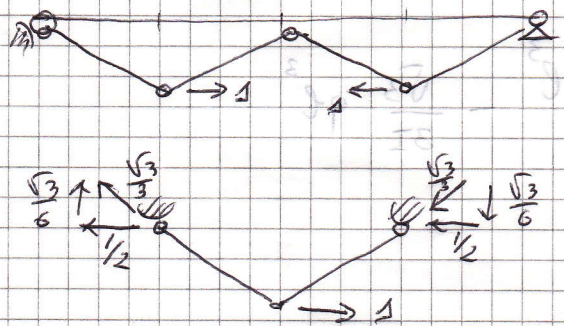
SD

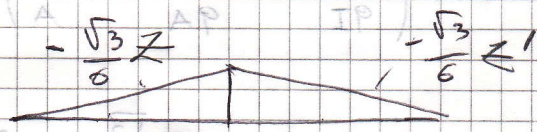






FT





ASTA	ℓ	N^z	N^A	M^z	M^r
AB	2ℓ	$-x/2$	$-1/2$	$2q\ell z - qz^2/2 - \frac{x\sqrt{3}}{6}z$	$-\frac{\sqrt{3}}{6}z$
CB	2ℓ	$-x/2$	$-1/2$	$2q\ell z' - qz^2/2 - \frac{x\sqrt{3}}{6}z'$	$-\frac{\sqrt{3}}{6}z'$
AD	$\frac{2\ell\sqrt{3}}{3}$	$\frac{x\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$	0	0
DB	$\frac{2\ell\sqrt{3}}{3}$	$-\frac{x\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$	0	0
BE	$\frac{2\ell\sqrt{3}}{3}$	$-\frac{x\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$	0	0
EC	$\frac{2\ell\sqrt{3}}{3}$	$-\frac{x\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$	0	0

$$L_{Ve} = 1 \cdot \frac{x\ell}{EA} = L_{Vi}$$

$$L_{Vi} = 2\left(\frac{1}{EI}\int_0^{2\ell}\left(2q\ell z - \frac{qz^2}{2} - \frac{x\sqrt{3}}{6}z\right)\left(-\frac{\sqrt{3}}{6}z\right)dz\right) +$$

$$+ 4\left(\frac{\frac{2\ell\sqrt{3}}{3}\cdot\frac{x\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{3}}{EA}\right) = \frac{2x\ell}{EA}$$

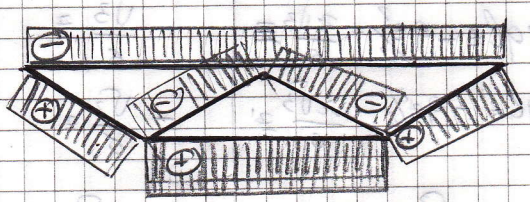
$$\frac{2}{EI}\int_0^{2\ell}\left(-\frac{\sqrt{3}}{3}q\ell z^2 + \frac{\sqrt{3}}{12}qz^3 + \frac{1}{12}xz^2\right)dz + \frac{8x\ell\sqrt{3}}{9EA} - \frac{2x\ell}{EA} = 0$$

$$\frac{2}{EI}\left[-\frac{\sqrt{3}}{9}q\ell z^3 + \frac{\sqrt{3}}{48}qz^4 + \frac{xz^3}{36}\right]_0^{2\ell} + \frac{48x\ell\sqrt{3}}{9EA} - \frac{2x\ell}{EA} = 0$$

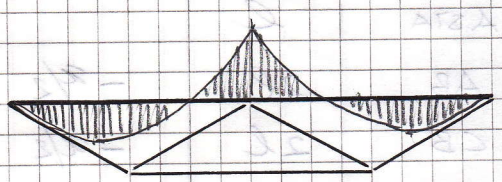
$$-\frac{8\sqrt{3}q\ell^4}{9I} + \frac{\sqrt{3}}{3I}q\ell^4 + \frac{2x\ell^3}{9I} + \frac{4x\ell\sqrt{3}}{9A} - \frac{x\ell}{A} = 0$$

$$x\left(\frac{2\ell^2}{9I} + \frac{4\sqrt{3}}{9A} - \frac{1}{A}\right) = \frac{8\sqrt{3}}{9I}q\ell^3 - \frac{\sqrt{3}}{3I}q\ell^3$$

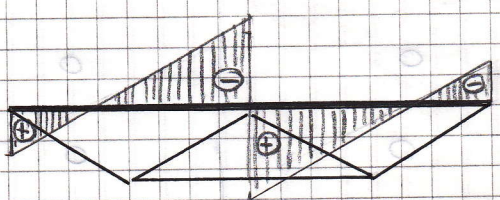
$$x = \frac{\frac{5\sqrt{3}}{9I}q\ell^3}{\frac{2\ell^2}{9I} + \frac{4\sqrt{3}\cdot 9}{9A}}$$



N



M

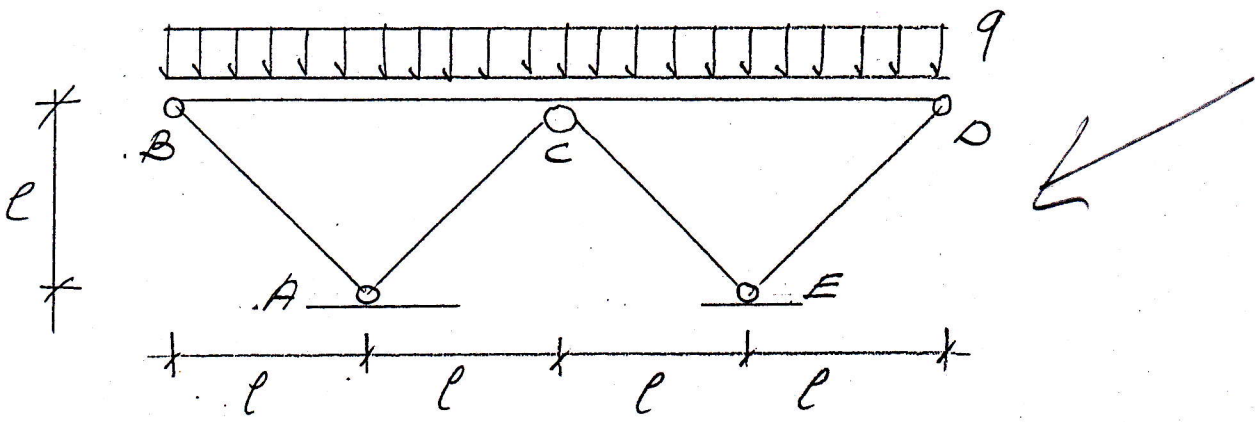


T

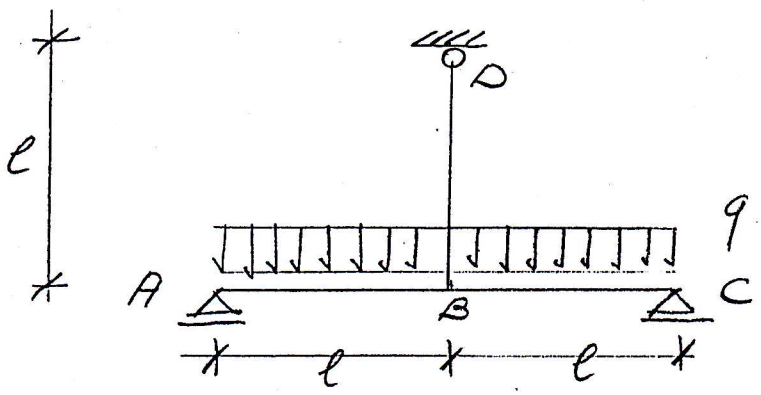
SCIENZA DELLE COSTRUZIONI

APPELLO 12 Luglio 1996

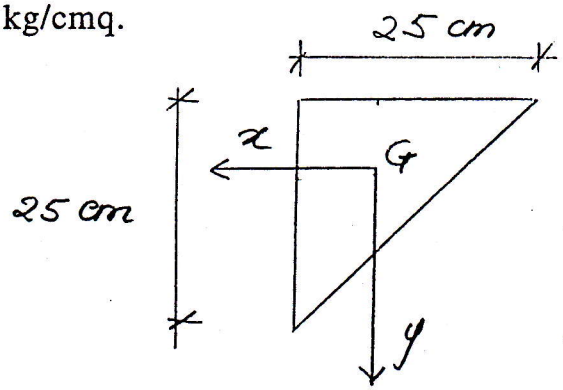
1 Determinare via P.L.V. il Momento flettente in C nella seguente struttura:.



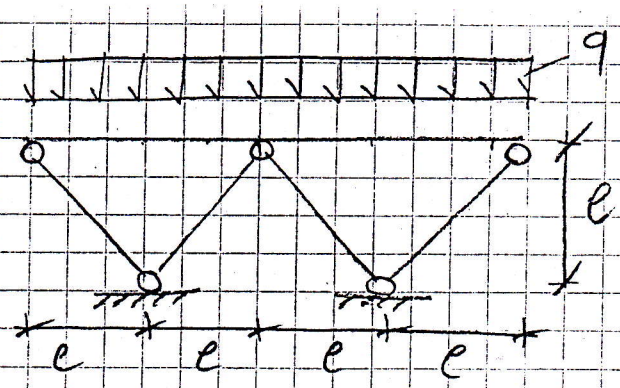
2. Determinare con il Metodo delle Forze la reazione verticale in D nella seguente struttura:

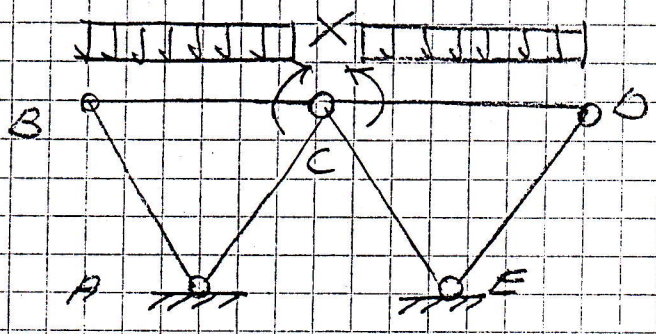


3. Effettuare la verifica di resistenza nella seguente sezione di un gradino in legno soggetta al Momento flettente M_x = - 400 Kgm. Sia σ_{amm} = 100 kg/cmq.

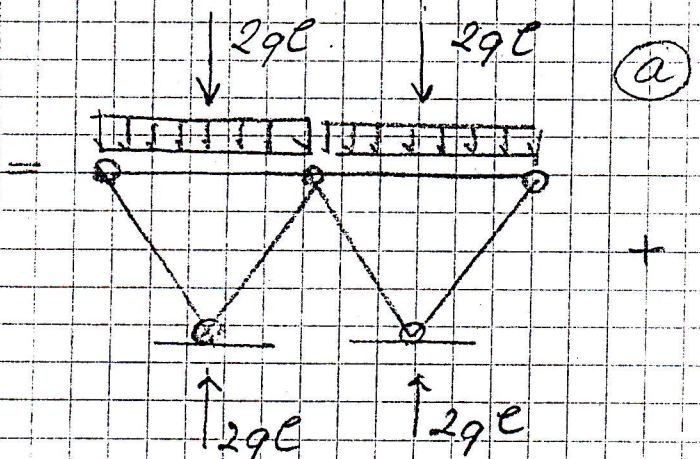


1)

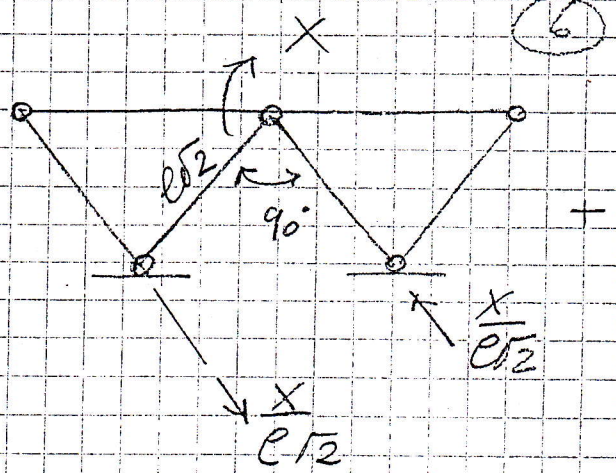

q
e
e
e
e
e


X
B
C
D
A
E

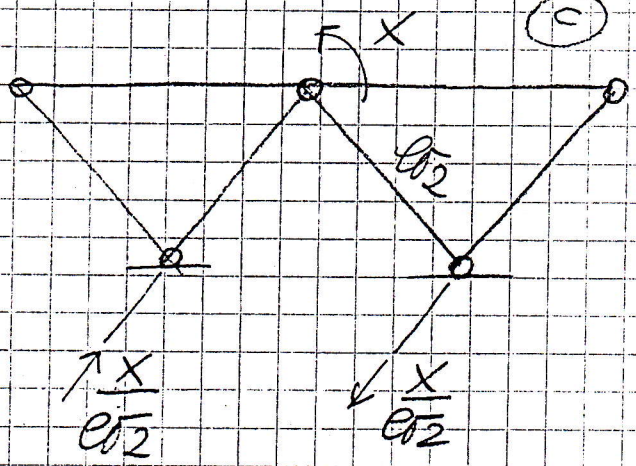
=


2qe
2qe
a
2qe
2qe

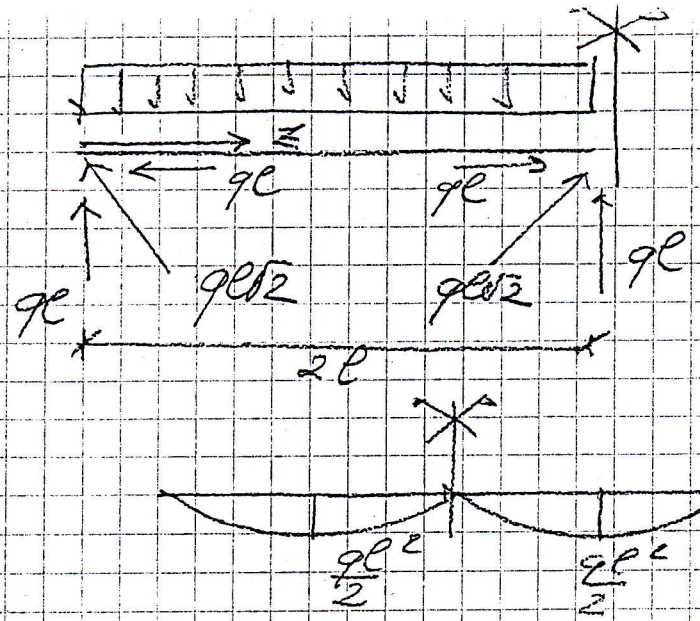
+


X
b
eb2
90°
X/eb2
X/eb2

+

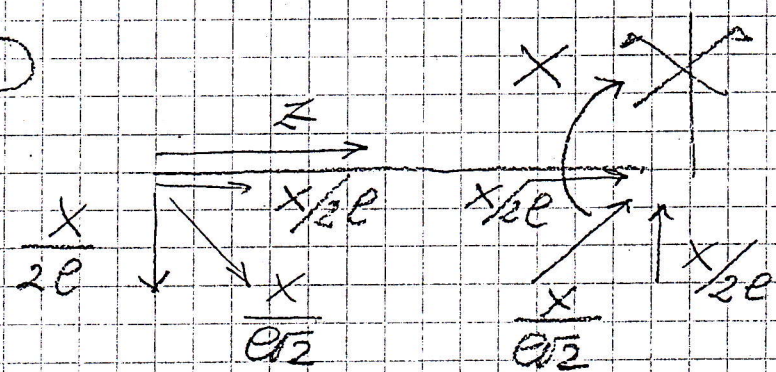

X
c
eb2
X/eb2
X/eb2

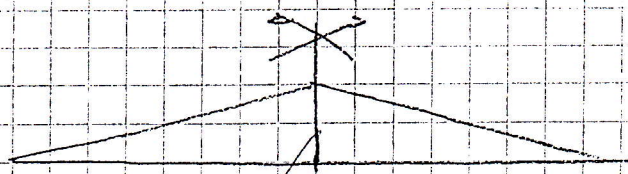
(a)



$$M_{max} = q\ell \cdot \ell - q\ell \cdot \frac{\ell}{2} = \frac{q\ell^2}{2}$$

(b) + (c)





$$X = M_{max}$$

$$N^{\Sigma} = N(q) + N^{f} \cdot X$$

$$M^{\Sigma} = M(q) + M^{f} X$$

	AB	AC	BC
ℓ_i	$\ell\sqrt{2}$	$\ell\sqrt{2}$	2ℓ
N^f	$1/\ell\sqrt{2}$	$-1/\ell\sqrt{2}$	$-1/2\ell$
M^f	0	0	$-\frac{1}{2\ell} Z$
$M(q)$	0	0	$q\ell Z - q\frac{Z^2}{2}$
$N(q)$	$-q\ell\sqrt{2}$	$-q\ell\sqrt{2}$	$q\ell$

$$\alpha \delta_{ik} = 1 \cdot \Delta\varphi = 0 =$$

$$= \alpha\delta_{11} = 2\left[e\sqrt{2}\left(\frac{1}{e\sqrt{2}}\right)\left(-q e\sqrt{2} + \frac{X}{e\sqrt{2}}\right)\frac{1}{EA} + \right.$$

$$\left. + e\sqrt{2}\left(-\frac{1}{e\sqrt{2}}\right)\left(-q e\sqrt{2} - \frac{X}{e\sqrt{2}}\right)\frac{1}{EA}\right] +$$

$$+ \frac{1}{EI}\int_0^{2e}\left(-\frac{1}{2e}z\right)\left(q e z - \frac{q z^2}{2} - \frac{X}{2e}z\right)dz\Bigg] =$$

$$= 2\left(-q e\sqrt{2} + \frac{X}{e\sqrt{2}} + q e\sqrt{2} + \frac{X}{e\sqrt{2}}\right)\cdot\frac{1}{A} +$$

$$+ \frac{1}{I}\int_0^{2e}\left(-\frac{q z^2}{2} + \frac{q z^3}{4e} + \frac{X z^2}{4e^2}\right)dz =$$

$$= \frac{X\,2\sqrt{2}}{eA} + \frac{1}{I}\left(-\frac{q\,8e^3}{6} + \frac{q\,16e^4}{16e} + \frac{X\,8e^3}{12e^2}\right) =$$

$$= \frac{X 2\sqrt{2}}{eA} + \frac{X 2e}{3I} - \frac{q e^3}{3I} = X\left(\frac{2\sqrt{2}}{eA} + \frac{2e}{3I}\right) - \frac{q e^3}{3I}$$

$$\boxed{X = \frac{q e^3}{3I} \Big/ \left(\frac{2\sqrt{2}}{eA} + \frac{2e}{3I}\right)}$$

€ 0,64

SOLUZIONI

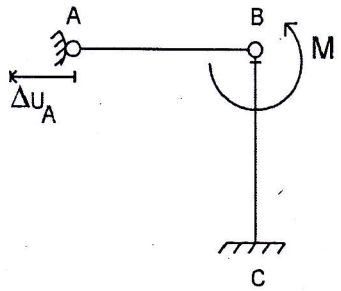
Facoltà di Architettura di Ferrara a.a. 2001-2002

Scienza delle Costruzioni								TEST A	
STUDENTE								VOTAZIONE	INIZIALI
ANNO DI CORSO	III°	IV°	V°	FC	Aula	A2	D3		
EX TEMPORE I 13 Ottobre 2001								Fila	Posto

N.B. - I risultati positivi dei test, sono validi fino a tutto settembre 2002.
Il presente foglio deve essere consegnato unitamente allo svolgimento del compito.
Tempo Massimo 4 ore.

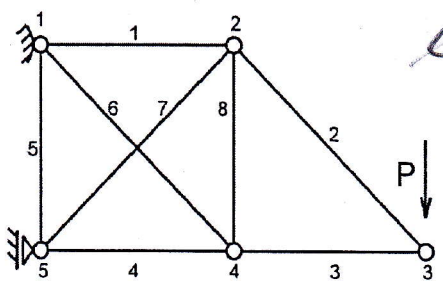
A1) Nella seguente struttura inflessa determinare l'incognita iperstatica via PLV.
E' obbligatorio predisporre la tabella delle caratteristiche reali e fittizie della sollecitazione e disegnare i corrispondenti diagrammi.

HEA 240
L = 400 cm
E = 2100000 Kg/cm²
I = 7763 cm⁴
A = 76.8 cm²
ΔU_A = 3 cm
M = 4000 Kgm



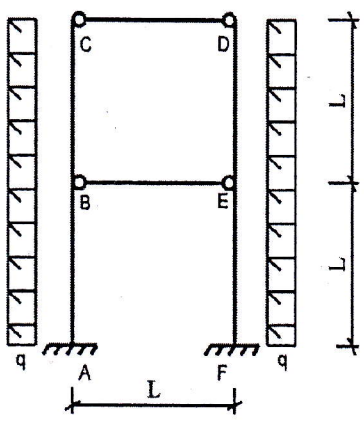
A2) Nella seguente struttura reticolare iperstatica determinare l'incognita iperstatica via PLV.
E' obbligatorio predisporre la tabella degli sforzi assiali reali e fittizi.

Tubolare Ø127 mm
L = 300 cm
E = 2100000 Kg/cm²
I = 293 cm⁴
A = 15.5 cm²
P = 2000 Kg

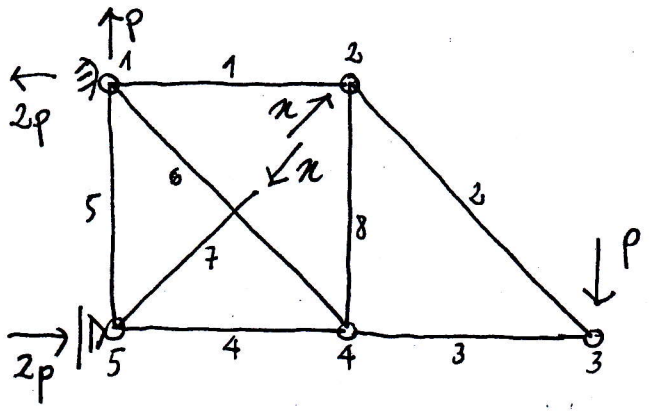


A3) OPZIONALE - Determinare via PLV lo spostamento orizzontale in sommità dei due piedritti.

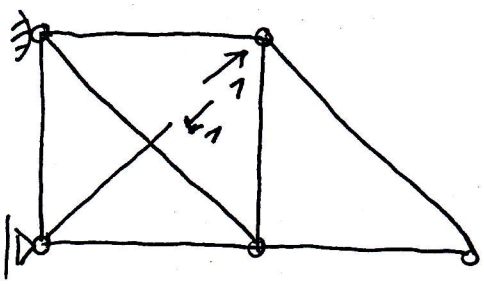
HEA 240
L = 600 cm
E = 2100000 Kg/cm²
I = 7763 cm⁴
A = 76.8 cm²
q = 2000 Kg/m



A2_



SD



SISTEMA AUTOEQUILIBRATO

ASTA	ℓ	N^F	N^ℓ x	P
1	ℓ	$\sqrt{2}/2$	$\sqrt{2}/2\, x$	+ P
2	$\ell\sqrt{2}$	0	0	$+P\sqrt{2}$
3	ℓ	0	0	− P
4	ℓ	$\sqrt{2}/2$	$\sqrt{2}/2\, x$	−2P
5	ℓ	$\sqrt{2}/2$	$\sqrt{2}/2\, x$	0
6	$\ell\sqrt{2}$	− 1	− x	$P\sqrt{2}$
7	$\ell\sqrt{2}$	− 1	− x	0
8	ℓ	$\sqrt{2}/2$	$\sqrt{2}/2\, x$	− P

$$L_{VE} = 1 \cdot \Delta\eta_2 = 0$$

$$L_{V1} = \frac{1}{EA}\left[\frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}x+P\right)\ell + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}x-2P\right)\ell + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}x\right)\ell - 1\left(-x+P\sqrt{2}\right)\ell\sqrt{2} + \right.$$

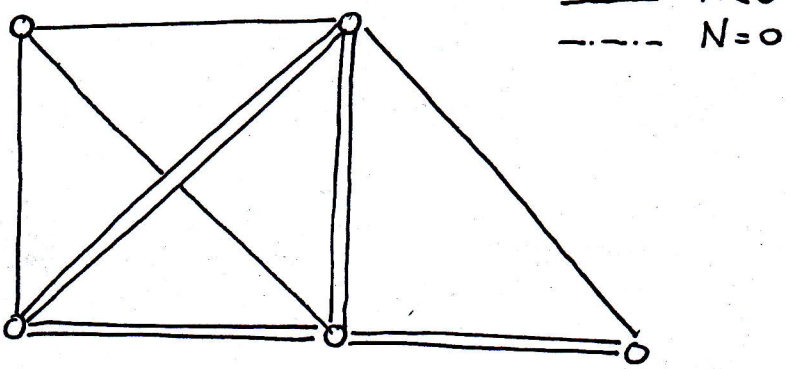
$$\left. -1\left(-x\right)\ell\sqrt{2} + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}x-P\right)\ell\right] = 0$$

$$\frac{x}{2} + \frac{P\sqrt{2}}{2} + \frac{x}{2} - P\sqrt{2} + \frac{x}{2} + x\sqrt{2} - 2P + x\sqrt{2} + \frac{x}{2} - \frac{P\sqrt{2}}{2} = 0$$

$$2x + 2x\sqrt{2} = P\sqrt{2} + 2P \qquad 2x = P\,\frac{(2+\sqrt{2})}{1+\sqrt{2}}$$

$$\kappa = \frac{P\left(2+\sqrt{2}\right)}{2\left(1+\sqrt{2}\right)} = 1414$$

ASTA	N
1	2999
2	2828
3	-2000
4	-3000
5	1000
6	1414
7	-1414
8	-1000



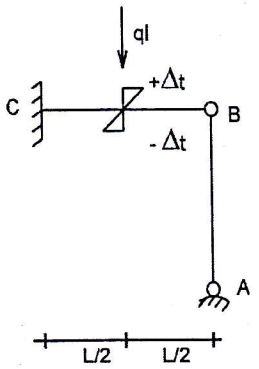
Facoltà di Architettura di Ferrara a.a. 2001-2002

Scienza delle Costruzioni								TEST C	
STUDENTE								VOTAZIONE	INIZIALI
ANNO DI CORSO	III°	IV°	V°	FC	Aula	A2	D3		
EX TEMPORE I 13 Ottobre 2001								Fila	Posto

N.B. - I risultati positivi dei test, sono validi fino a tutto settembre 2002.
Il presente foglio deve essere consegnato unitamente allo svolgimento del compito.
Tempo Massimo 4 ore.

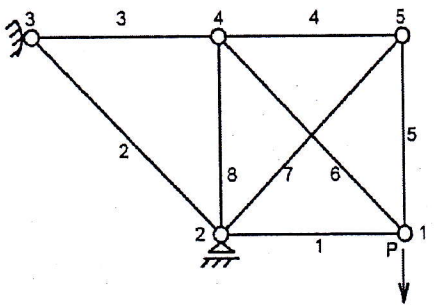
C1) Nella seguente struttura inflessa determinare l'incognita iperstatica via PLV.
E' obbligatorio predisporre la tabella delle caratteristiche reali e fittizie della sollecitazione e disegnare i corrispondenti diagrammi.

HEA 240
L = 400 cm
E = 2100000 Kg/cm^2
I = 7763 cm^4
A = 76.8 cm^2
$\Delta t = 50°$
q = 2000 Kg/m
h = 22 cm



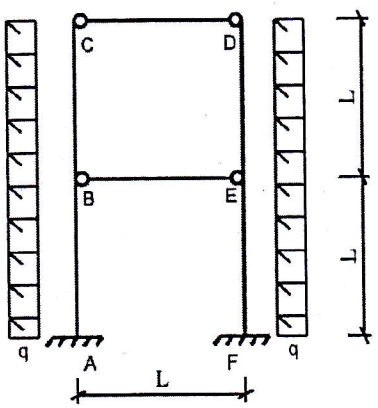
C2) Nella seguente struttura reticolare iperstatica determinare l'incognita iperstatica via PLV.
E' obbligatorio predisporre la tabella degli sforzi assiali reali e fittizi.

Tubolare Ø127 mm
L = 300 cm
E = 2100000 Kg/cm^2
I = 293 cm^4
A = 15.5 cm^2
P = 2000 Kg

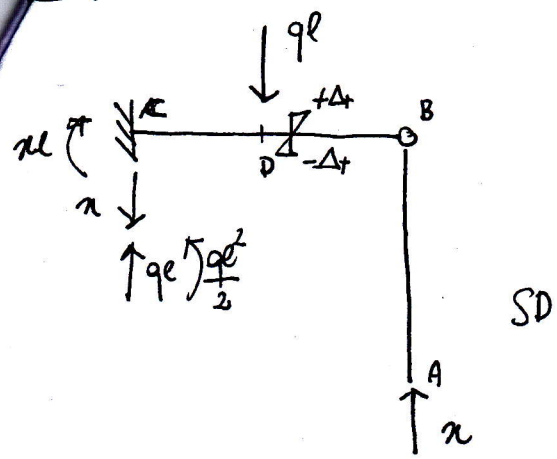


C3) OPZIONALE - Determinare via PLV lo spostamento orizzontale in sommità dei due piedritti.

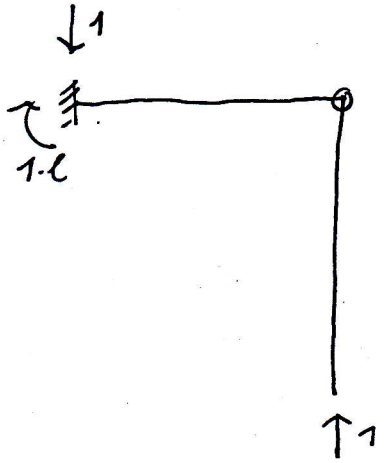
HEA 240
L = 600 cm
E = 2100000 Kg/cm^2
I = 7763 cm^4
A = 76.8 cm^2
q = 2000 Kg/m



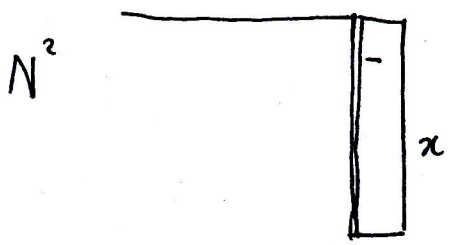
1

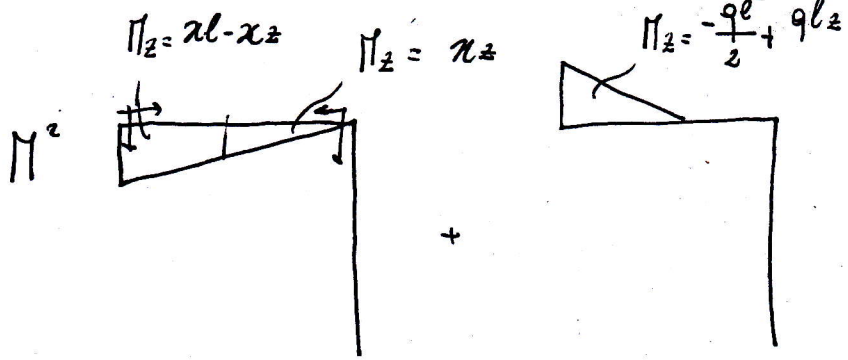


SD

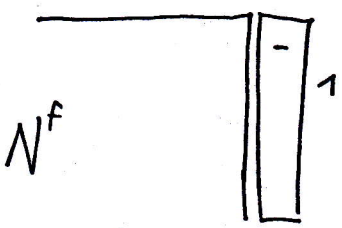


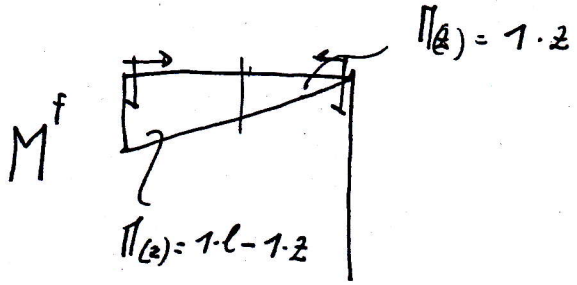
- DIAGRAMMI REALI -





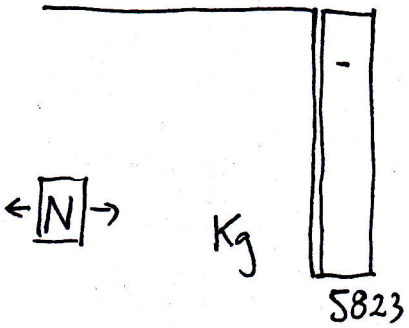
- DIAGRAMMI FITTIZI -

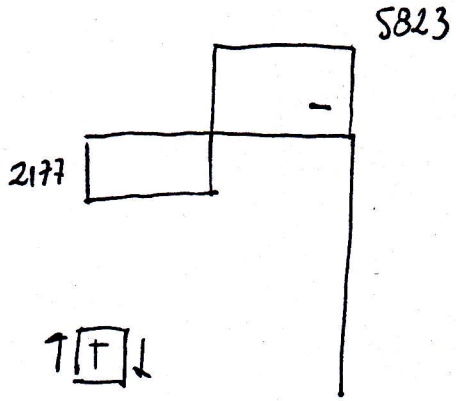


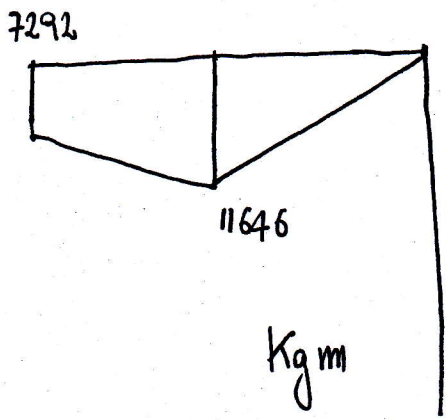


ASTA	ℓ	N^f	N^2	M^f	M^2 x	M^2 q	$\lambda_{\Delta t}$
CD	$\ell/2$	0	0	$1\cdot l - 1\cdot z$	$xl - x_2$	$-\frac{ql^2}{2} + ql_2$	$-\frac{2\alpha\Delta t}{h}$
BD	$\ell/2$	0	0	$1\cdot z$	x_2	0	$-\frac{2\alpha\Delta t}{h}$
AB	ℓ	-1	$-x$	0	0	0	0

- DIAGRAMMI -







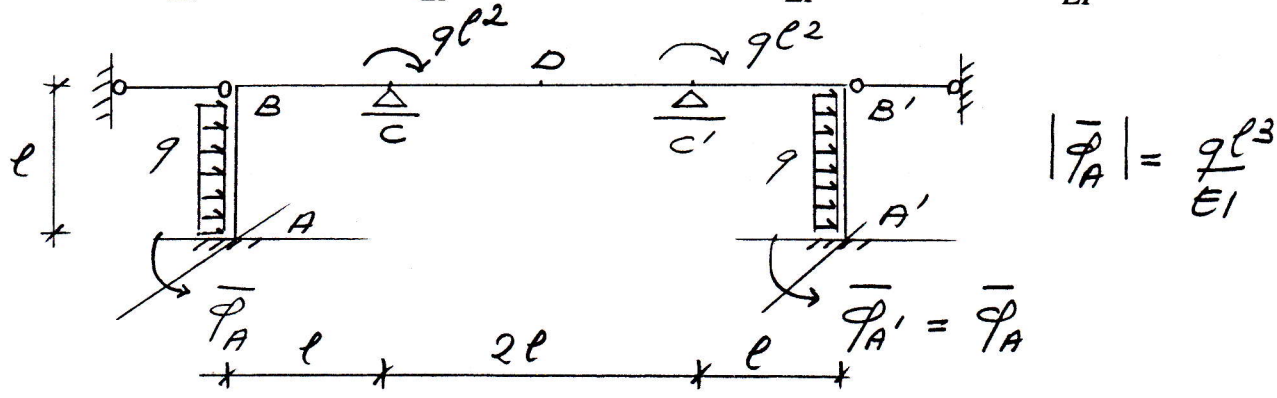
SCIENZA delle COSTRUZIONI **TEST 2** **14/11/1997**

Cognome.....Nome.....
Anno di Corso.....

1 Determinare con il Metodo degli Spostamenti le rotazioni dei nodi B e C:

1: $\Phi_B = -0.2195\frac{ql^3}{EI}$ $\Phi_C = -0.0801\frac{ql^3}{EI}$; 2: $\Phi_B = 0.9023\frac{ql^3}{EI}$ $\Phi_C = -0.2784\frac{ql^3}{EI}$;

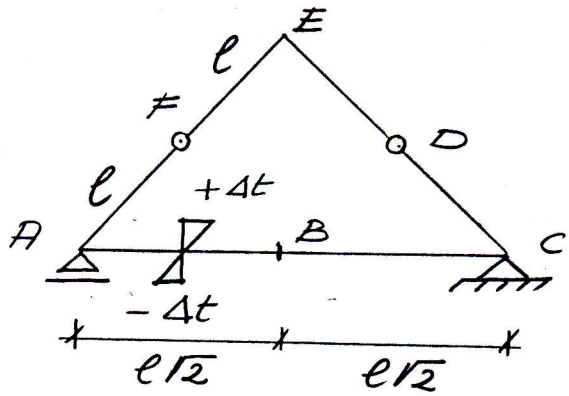
3: $\Phi_B = -1.1824\frac{ql^3}{EI}$ $\Phi_C = 0.3248\frac{ql^3}{EI}$; 4: $\Phi_B = -1.7624\frac{ql^3}{EI}$ $\Phi_C = -0.8532\frac{ql^3}{EI}$



$|\bar{\varphi}_A| = \frac{q\ell^3}{EI}$

2 Risolvere via P.L.V. la seguente struttura inserendo una cerniera in B:

1: $X = \pm\frac{6EI\alpha\Delta t}{h(3+\sqrt{2})}$; 2: $X = \pm\frac{6EI\alpha\Delta t}{h(2+\sqrt{3})}$; 3: $X = \pm\frac{3EI\alpha\Delta t}{h(3+\sqrt{2})}$; 4: $X = \pm\frac{3EI\alpha\Delta t}{h(2+\sqrt{3})}$



3 Dato il seguente campo di spostamenti determinare l'angolo Φ_ξ che la direzione principale ξ della deformazione forma con l'asse di riferimento x nell'intorno del punto P=(2, -1, 0):

$u = -3xy + 2y^2 \quad v = -4y^2x + x^2 \quad w = 5z^3$

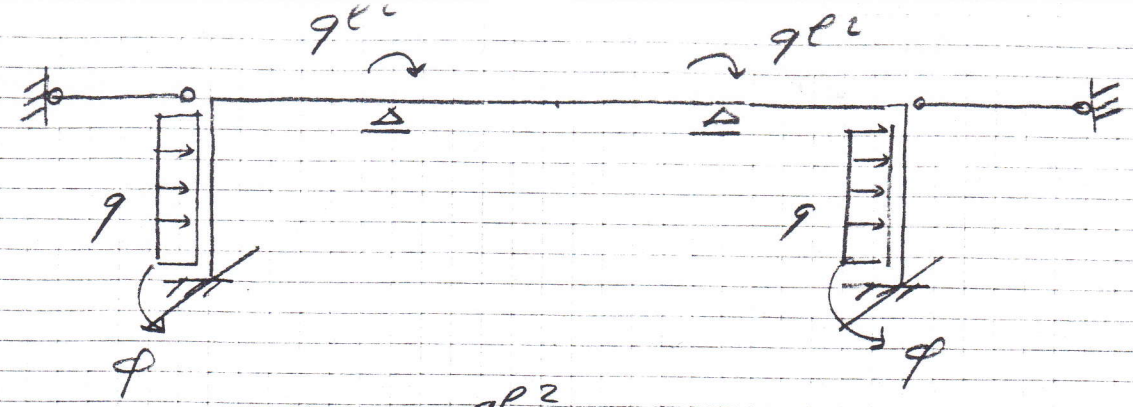
1: −27.3145; 2: −8.5687; 3: 18.7852; 4: 34.8965; 5: −42.2315

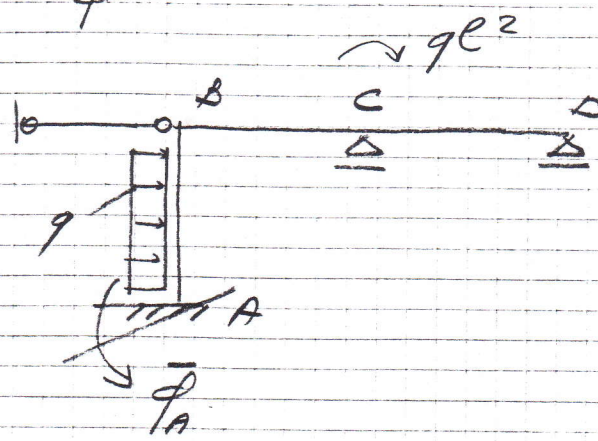
Verificare il risultato ottenuto con il procedimento grafico di Mohr.

4 Enunciazione e dimostrazione del Teorema di reciprocità fra componenti di spostamento relative a punti appartenenti ad uno stesso intorno.

5 Significato meccanico delle componenti diagonali del tensore della deformazione.

1.A)





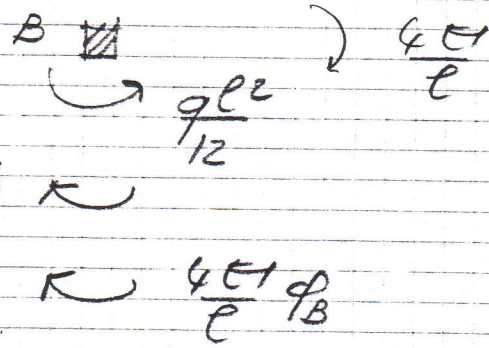
$$|\bar{\varphi}_A| = \frac{q\ell^3}{EI}$$

$$M_{BA}^{0} = -\frac{q\ell^2}{12} + \frac{2EI}{\ell}\bar{\varphi}_A \qquad M_{CB}^{0} = 0$$

$$M_{BC}^{0} = 0 \qquad M_{CD}^{0} = 0$$

$$M_{BA} = \frac{4EI}{\ell}\varphi_B \qquad M_{CB} = \frac{4EI}{\ell}\varphi_C + \frac{2EI}{\ell}\varphi_B$$

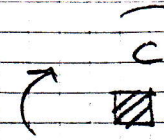
$$M_{BC} = \frac{4EI}{\ell}\varphi_B + \frac{2EI}{\ell}\varphi_C \qquad M_{CD} = \frac{3EI}{\ell}\varphi_C$$



$$\Sigma M_B = 0$$

$$\frac{q\ell^2}{12} - \frac{8EI}{\ell}\varphi_B - \frac{2EI}{\ell}\cdot\frac{q\ell^3}{EI}\cdot\frac{1}{2} - \frac{2EI}{\ell}\varphi_C =$$

$$= -\frac{23}{12}q\ell^2 + \frac{8EI}{\ell}\varphi_B + \frac{2EI}{\ell}\varphi_C = 0 \qquad *$$



$$\frac{4EI}{\ell}\varphi_C + \frac{2EI}{\ell}\varphi_B \quad \frac{3EI}{\ell}\varphi_C$$

$$\Sigma M_C = 0 \qquad ql^2 + \frac{2EI}{\ell}\varphi_B + \frac{7EI}{\ell}\varphi_C = 0 \quad *$$

$$\varphi_B = -\frac{\ell}{2EI}\left(ql^2 + \frac{7EI}{\ell}\varphi_C\right)$$

$$\frac{23}{12}ql^2 - \frac{8EI}{\ell}\cdot\frac{\ell}{2EI}\left(ql^2 + \frac{7EI}{\ell}\varphi_C\right) + \frac{2EI}{\ell}\varphi_C = 0$$

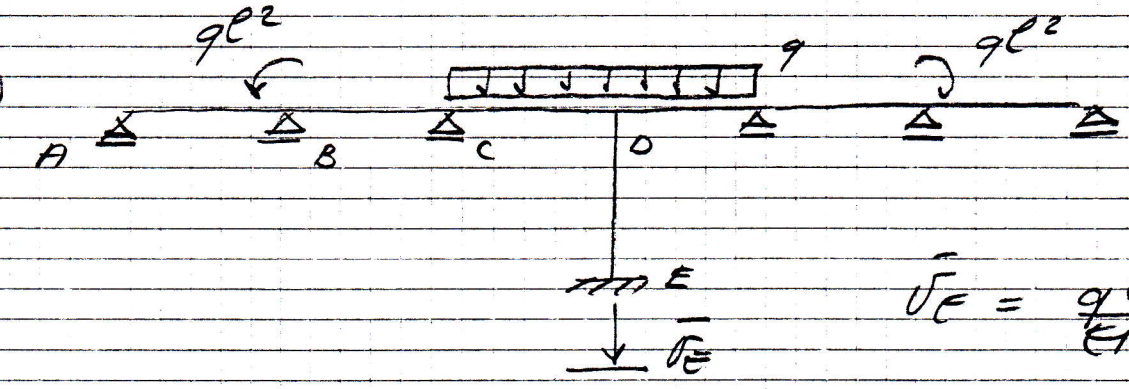
$$\frac{23}{12}ql^2 - 4ql^2 - 28\frac{EI}{\ell}\varphi_C + \frac{2EI}{\ell}\varphi_C =$$

$$= -\frac{25}{12}ql^2 - 26\frac{EI}{\ell}\varphi_C = 0$$

$$\boxed{\varphi_C = -\frac{\ell}{26EI}\cdot\frac{25}{12}ql^2 = -\frac{25}{312}\frac{ql^3}{EI} = -0{,}0801\frac{ql^3}{EI}}$$

$$\boxed{\varphi_B = -\frac{\ell}{2EI}\left(ql^2 - \frac{175}{312}ql^2\right) = -\frac{137}{624}\frac{ql^3}{EI} = -0{,}2195\frac{ql^3}{EI}}$$

1. B)



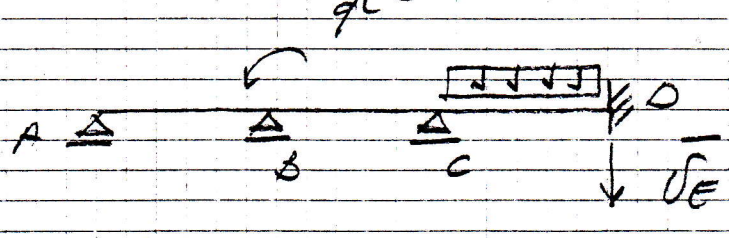
$$\bar{\delta}_E = \frac{q\ell^4}{EI}$$

$M_{BA}^{\circ} = 0$

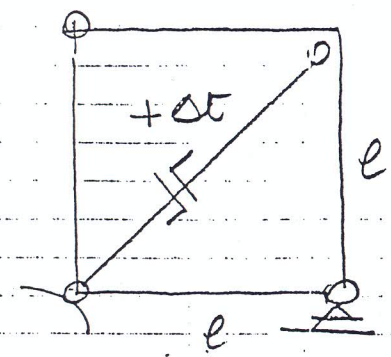
$M_{BC}^{\circ} = 0$

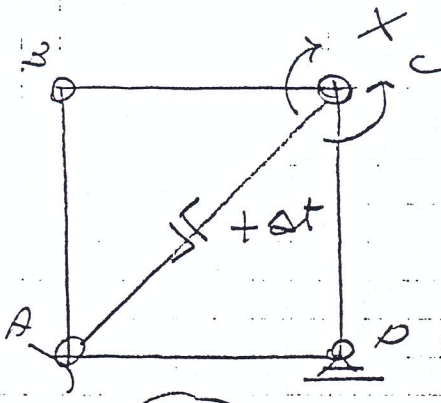
$M_{CB}^{\circ} = 0$

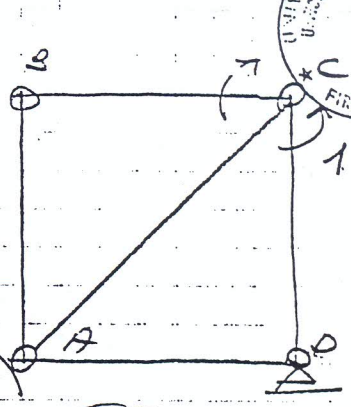
$$M_{CD}^{\circ} = \frac{ql^2}{12} + \frac{6EI}{\ell^2}\bar{\delta}_E = \frac{ql^2}{12} + 6ql^2$$



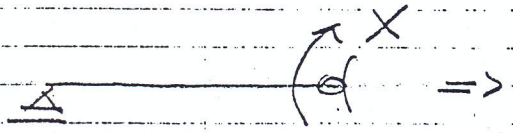
1)







SD FT



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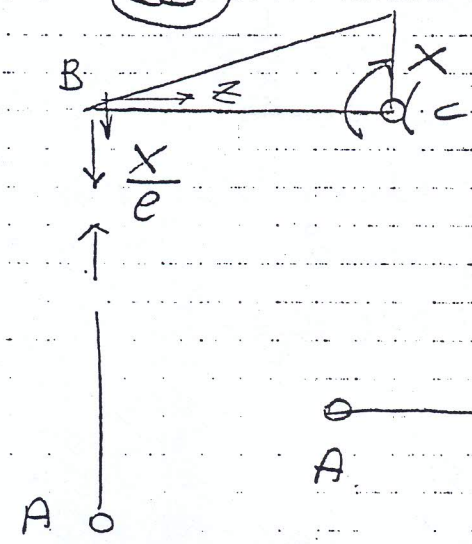
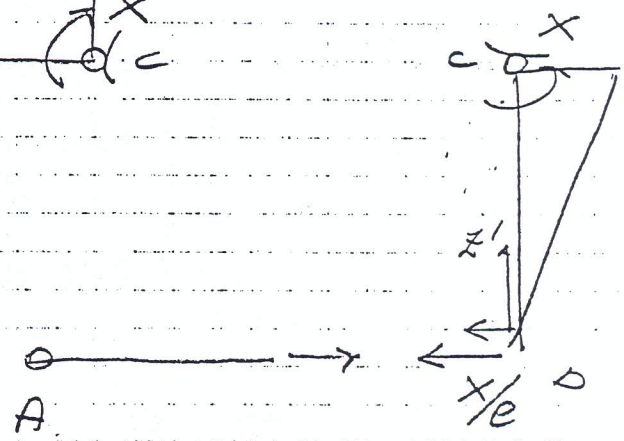
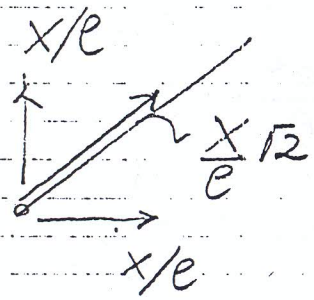






	AB	BC	CD	AD	AC
N^t	$1/e$	0	0	$1/e$	$\sqrt{2}/e$
N^z	X/e	0	0	X/e	$X\sqrt{2}/e$
M^t	0	$-\frac{1}{e}z$	$-\frac{1}{e}z'$	0	0
M^z	0	$-\frac{X}{e}z$	$-\frac{X}{e}z'$	0	0
Δe^z	0	0	0	0	$\alpha e\sqrt{2}\Delta t$
e	e	e	e	e	$e\sqrt{2}$

RO

$$\alpha_{1c} = 1\varphi^D - 1\varphi^S = 1\Delta\varphi = \frac{1}{EI}\int_0^{\ell}\left(\frac{z}{\ell}\right)\left(\frac{X z}{\ell}\right)dz + \quad (2)$$

$$\overline{BC}$$

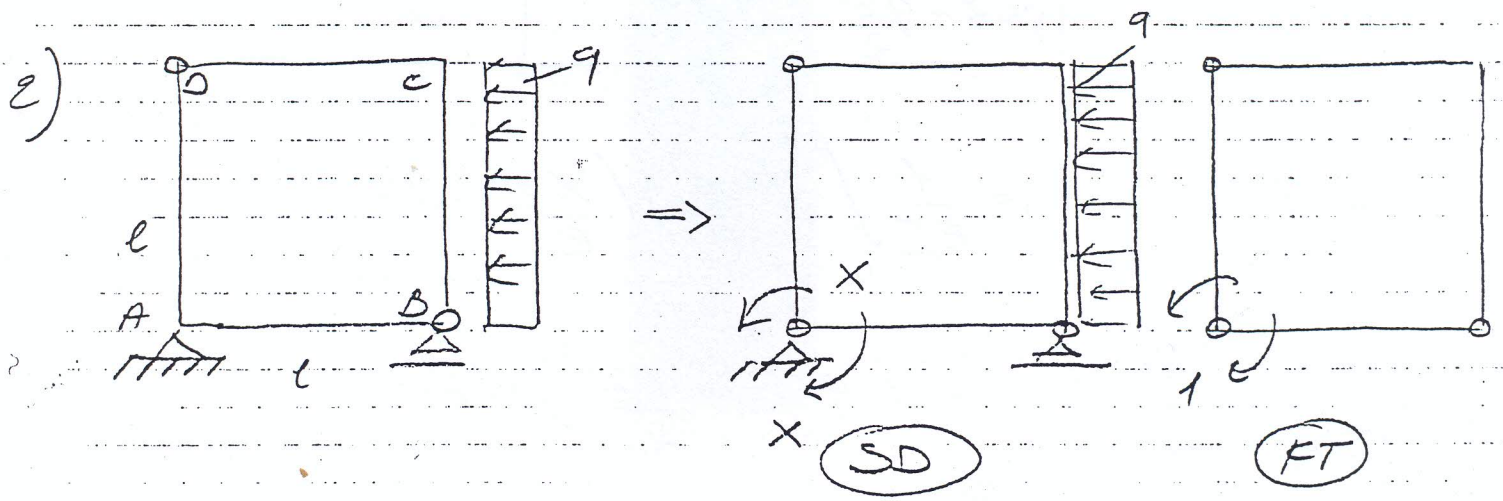
$$+\frac{1}{EI}\int_0^{\ell}\left(\frac{z'}{\ell}\right)\left(\frac{X z'}{\ell}\right)dz' + \frac{1}{\ell}\cdot\frac{X\ell}{\ell EA} + \frac{1}{\ell}\frac{X\ell}{\ell EA} +$$

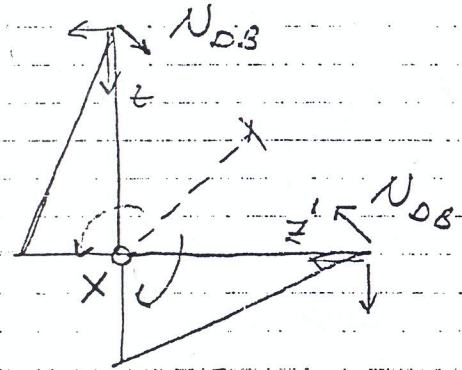
$$\overline{CD} \qquad \overline{AB} \qquad \overline{AD}$$

$$+\frac{\sqrt{2}}{\ell}\left(\frac{X\sqrt{2}\ell}{\ell EA} + \alpha\ell\sqrt{2}\Delta t\right) = 0$$

$$\frac{1}{EI}\frac{X\ell^3}{3\ell^2} + \frac{1}{EI}\frac{X\ell^3}{3\ell^2} + \frac{2X}{\ell EA} + \frac{2X}{\ell EA} + 2\alpha\Delta t =$$

$$= \frac{2X\ell}{3EI} + \frac{4X}{\ell EA} + 2\alpha\Delta t = 0 \quad \Rightarrow X$$

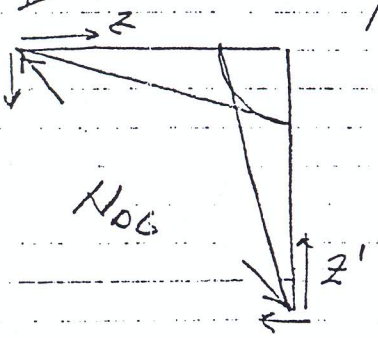


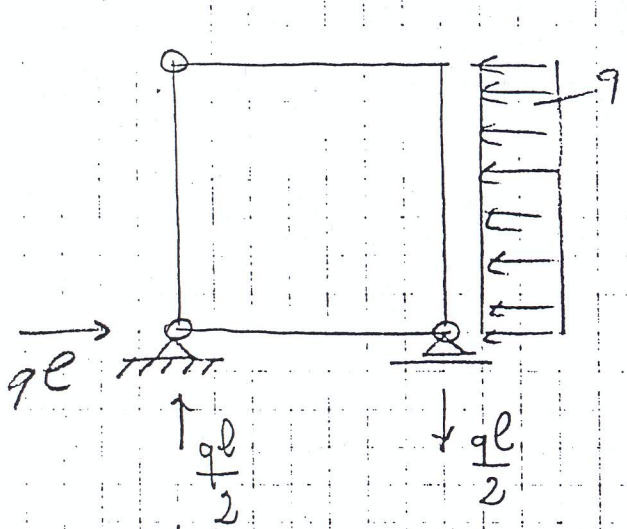


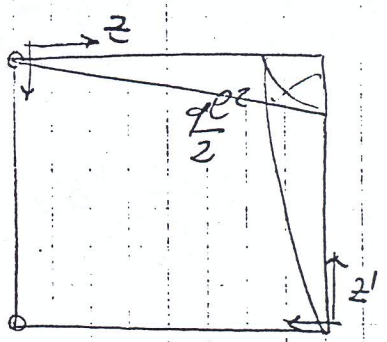
$$N_{DB}\cdot\frac{\ell\sqrt{2}}{2} - X = 0$$

$$N_{DB} = \frac{2X}{\ell\sqrt{2}} = \frac{2\sqrt{2}X}{2\ell} = \frac{\sqrt{2}}{\ell}X$$

$N^c(x)$







$M^z(q)$

	AB	BC	CD	DA
M^t	$\frac{1}{\ell} z'$	$\frac{1}{\ell} z'$	$\frac{1}{\ell} z$	$\frac{1}{\ell} z$
$M^z(x)$	$\frac{x}{\ell} z'$	$\frac{x}{\ell} z'$	$\frac{x}{\ell} z$	$\frac{x}{\ell} z$
$M^z(q)$	0	$q\ell z' - q\frac{z'^2}{2}$	$\frac{q\ell}{2} z$	0

$$\mathcal{L}_{ve} = 1 \cdot \Delta\varphi = 0 = \frac{1}{EI}\int_0^\ell \underbrace{\left(\frac{z'}{\ell}\right)\left(\frac{x}{\ell} z'\right)}_{\overline{AB}} dz' +$$

$$+ \frac{1}{EI}\int_0^\ell \underbrace{\left(\frac{z'}{\ell}\right)\left(\frac{x}{\ell} z' + q\ell z' - q\frac{z'^2}{2}\right)}_{\overline{BC}} dz' +$$

$$+ \frac{1}{EI}\int_0^\ell \underbrace{\left(\frac{z}{\ell}\right)\left(\frac{x}{\ell} z + \frac{q\ell}{2} z\right)}_{\overline{CD}} dz +$$

$$+ \frac{1}{EI}\int_0^\ell \left(\frac{z}{\ell}\right)\left(\frac{x}{\ell} z\right) dz \quad \Rightarrow x$$