Energia:

cinetica

$$E_{cin} = m \frac{w^2}{2} \quad [J]$$

(forma specifica)

$$e_{cin} = \frac{E_{cin}}{m} = \frac{w^2}{2} \quad [J/kg]$$

potenziale

(forma specifica)

$$E_{pot} = mgz \quad [J]$$

$$e_{pot} = \frac{E_{pot}}{m} = gz \quad [J/kg]$$

interna

(forma specifica)

$$U \quad [J]$$

$$u = \frac{U}{m} \quad [J/kg]$$

totale

(forma specifica)

$$E = U + E_{cin} + E_{pot} \quad [J]$$

$$e = \frac{E}{m} = u + e_{cin} + e_{pot} \quad [J/kg]$$

Portata massica: $\dot{m} = \rho \dot{V} = \rho A w_{avg} \quad [kg/s]$

Potenza associata a $\dot{m}$: $\dot{E} = \dot{m} e \quad [J/s] = [W]$

en. spec.	e	$\Delta e = e_2 - e_1$	[J/kg]
energia	$E = me$	$\Delta E = E_2 - E_1$	[J]
potenza	$\dot{E} = \dot{m}e$	$\Delta \dot{E} = \dot{E}_2 - \dot{E}_1$	[W]

energia meccanica : $e_{mecc} = \frac{P}{\rho} + \frac{w^2}{2} + gz$ $\begin{cases} \text{en. flusso} \\ \text{en. cinetica} \\ \text{en. potenziale} \end{cases}$

calore : Q

cal. rif. a unità di massa : $q = Q/m$ $\qquad Q = \int_{t_1}^{t_2} \dot{Q}\, dt = \int_1^2 \delta Q$ [J]

potenza termica : $\dot{Q}$

lavoro L

lav. rif. a unità di massa $l = L/m$ $\qquad L = \int_1^2 F\, ds = \int_1^2 \dot{E}\, dt$ [J]

potenza (e.g. meccanica) $\dot{L}$

I Principio

$$E_{in} - E_{out} = \Delta E_{sist} = E_2 - E_1$$

$$\Rightarrow \Delta E_{sist} = \Delta U + \Delta E_{cin} + \Delta E_{pot} = m\left(u_2 - u_1 + \frac{w_2^2 - w_1^2}{2} + g(z_2 - z_1)\right)$$

$$\stackrel{!}{=} Q - L$$

sist. stazionario $\Delta E = \Delta U$

ciclo $\Delta E_{sist} = 0 = Q_{net} - L_{net} \Rightarrow Q_{net} = L_{net}$

Proprietà delle Sostanze Pure

Entalpia $h = u + pv$ [J/kg]

$H = U + pV$ [J]

Titolo di una miscela satura liquido - vapore $x = \dfrac{m_{v,sat}}{m_{tot}}$

proprietà di una miscela omogenea

$$v_{avg} = (1-x)\, v_{l,sat} + x\, v_{v,sat} \qquad (u_{avg} = \dots\ ;\ s_{avg} = \dots\ ;\ h_{avg} = \dots)$$

proprietà di un liquido sottoraffreddato

$$y \sim y_{l,sat@T} \qquad h \sim h_{l,sat@T} + v_{l,sat@T}(p - p_{sat@T})$$

Gas Ideali:

Eq. di stato $\quad pv = RT \quad [J/kg] \quad ; \quad R = R_u / M \quad [J/kg\,K]$

$$pV = mRT = MNRT = NR_uT \qquad [J]$$

Gas Reali:

$$pv = ZRT \quad ; \quad Z = \frac{v_{eff}}{v_{id}}$$

se $\quad p_r = \dfrac{p}{p_{cr}} \ll 1 \quad \vee \quad T_r = \dfrac{T}{T_{cr}} > 2 \Rightarrow$ gas ideale

$$v_r = v_{eff} / (RT_{cr}/p_{cr})$$

Van der Waals

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT \quad ; \quad a = \frac{27 R^2 T_{cr}^2}{64\, p_{cr}}$$

$$b = \frac{RT_{cr}}{8\, p_{cr}}$$

Lavoro di Variazione del Volume

$$\delta L_v = F ds = p A ds = p dV$$

$$L_v = \int_1^2 p dV \quad [J] \quad ; \quad l_v = \int_1^2 p dv \quad [J/kg]$$

risolvibile se $p = f(V)$ noto e.g. $pV^n = \text{cost.} \Rightarrow$ (trasf. politropica)

$$L_v = \int_1^2 p dV = pV^n \int_1^2 \frac{dV}{V^n} = pV^n \left. \frac{V^{1-n}}{1-n} \right|_1^2 = \frac{p_2 V_2 - p_1 V_1}{1-n} \quad (n \neq 1)$$

$$L_v = \int_1^2 p dV = pV \int_1^2 \frac{dV}{V} = pV \ln \frac{V_2}{V_1} \quad (n = 1)$$

Bilancio Energetico

$$\Delta E_{sist} = E_{in} - E_{out} = (Q_{in} + L_{in}) - (Q_{out} + L_{out}) = Q - L$$

sist. staz. $\Delta U = Q - L$

forma spec. $\Delta e = q - l$

forma diff. $de = \delta q - \delta l$

ciclo $q = l$

Sistemi Chiusi

calore specifico

a volume cost. $\quad c_v \, dT = du \rightarrow c_v = \left(\frac{\partial u}{\partial T}\right)_v$ $\quad$ [J/kg K] [J/kg °C]

a pressione cost. $\quad c_p \, dT = dh \rightarrow c_p = \left(\frac{\partial h}{\partial T}\right)_p$ $\quad$ [J/kg K] [J/kg °C]

nei gas ideali

$$\left.\begin{aligned} u &= u(T) \\ h &= u + pv = u + RT = h(T) \end{aligned}\right\} \quad \partial \rightarrow d$$

$$\Delta u = \int_1^2 c_v(T)\, dT = c_{v,avg} \Delta T$$

$$\Delta h = \int_1^2 c_p(T)\, dT = c_{p,avg} \Delta T$$

$$c_p = c_v + R \quad ; \quad k = c_p / c_v$$

nei solidi e nei liquidi

$$v = 1/\rho = \text{cost.} \qquad c_p = c_v = c(T)$$

$$du = c\, dT \quad ; \quad dh = du + v\, dp$$

Sistemi Aperti (Volumi di Controllo)

portata massica $\delta \dot{m} = \rho w_n dA = \rho \vec{w} \cdot \vec{n} \, dA \Rightarrow \dot{m} = \int_A \rho w_n dA = \rho w_{avg} A$ [kg/s]

portata volumetrica $\dot{V} = \int_A w_n dA = w_{avg} A$ [m³/s] $\quad w_{avg} = \frac{1}{A} \int_A w_n dA$

$\dot{m} = \rho \dot{V} = \dot{V} / v$

conservazione della massa

$m_{in} - m_{out} = \Delta m_{vc}$ [kg]

$\dot{m}_{in} - \dot{m}_{out} = dm_{vc}/dt$ [kg/s]

$m_{vc} = \int_{vc} \rho \, dV \quad ; \quad dm_{vc} = \rho \, dV$

forma generale $\frac{dm_{vc}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$ [kg/s]

flusso stazionario $\sum \dot{m}_{in} = \sum \dot{m}_{out}$

flusso a una corrente $\dot{m}_{in} = \dot{m}_{out} \Rightarrow \rho_{in} w_{in} A_{in} = \rho_{out} w_{out} A_{out}$

flusso staz. incomprimibile $\sum \dot{V}_{in} = \sum \dot{V}_{out}$ [m³/s]

flusso staz. incomp. a una corrente $\dot{V}_{in} = \dot{V}_{out} \Rightarrow w_{in} A_{in} = w_{out} A_{out}$

Sistemi Aperti

Lavoro di pulsione $L_p = Fs = pAs = pV \quad [J]$

$$l_p = pv \quad [J/kg]$$

Energia Totale della corrente fluida $e = u + e_{cin} + e_{pot} + pv \quad [J/kg]$

$$= h + e_{cin} + e_{pot} = h + \frac{w^2}{2} + gz$$

$$\dot{E}_{in/out} = \frac{1}{A}\int_A \dot{m}_{in/out}\left(h_{in/out} + \frac{w_{in/out}^2}{2} + g z_{in/out}\right) dA$$

$$= \dot{m}_{i/o}\left(h_{i/o\,avg} + \frac{w_{i/o\,avg}^2}{2} + g z_{i/o\,avg}\right)$$

Sistemi a flusso stazionario

Bilancio di massa $\sum \dot{m}_{in} = \sum \dot{m}_{out}$

per una corrente $\dot{m}_{in} = \dot{m}_{out}$

Bilancio energetico $\dot{E}_{in} = \dot{E}_{out} \Rightarrow \dot{Q} - \dot{L} + \sum \dot{m}_{in}\left(h_{in} + \frac{w_{in}^2}{2} + g z_{in}\right) - \sum \dot{m}_{out}\left(h_{out} + \frac{w_{out}^2}{2} + g z_{out}\right) = 0$

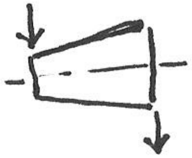
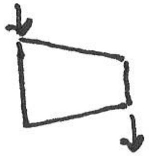
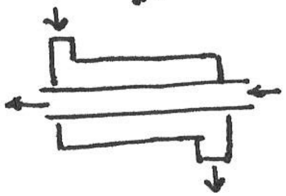
per una corrente $\dot{Q} - \dot{L} = \dot{m}\left[h_{out} - h_{in} + \frac{w_{out}^2 - w_{in}^2}{2} + g(z_{out} - z_{in})\right]$

$$q - l = h_{out} - h_{in} + \frac{w_{out}^2 - w_{in}^2}{2} + g(z_{out} - z_{in})$$

se en. cin. e pot. trascurabili $q - l = h_{out} - h_{in}$

Sistemi Aperti

Dispositivi a flusso stazionario

- ugelli e diffusori  $\Rightarrow q \sim 0 \quad l \sim 0 \quad \Delta e_{pot} \sim 0 \Rightarrow h_{in} + \frac{w_{in}^2}{2} = h_{out} + \frac{w_{out}^2}{2}$
- turbine  e compressori  $\Rightarrow$ $q \sim 0$, $\Delta e_{pot} \sim 0$, $l \gg$ $\Rightarrow -l = h_{out} - h_{in}$
- valvole di laminazione  $\Rightarrow q \sim l \sim \Delta e_{pot} \sim \Delta e_{cin} \sim 0 \Rightarrow h_{in} = h_{out}$
- camera di miscelazione (a 2 o + portate)  $\Rightarrow q \sim l \sim \Delta e_{pot} \sim \Delta e_{cin} \sim 0 \Rightarrow \Sigma \dot{m}_{in} = \Sigma \dot{m}_{out}$
  $\Sigma \dot{m}_{in} h_{in} = \Sigma \dot{m}_{out} h_{out}$
- Scambiatori di calore  $\Rightarrow q \sim l \sim \Delta e_{pot} \sim \Delta e_{cin} \sim 0 \Rightarrow \dot{m}_{in} = \dot{m}_{out} \ \forall$ corrente
  $\dot{m}(h_{out} - h_{in})_{fl1} = -\dot{m}(h_{out} - h_{in})_{fl2}$

Processi a flusso non stazionario

Bilancio di massa
$$m_{in} - m_{out} = \Delta m_{sist} = m_2 - m_1 \quad [kg]$$
$$\dot{m}_{in} - \dot{m}_{out} = dm_{sist}/dt \quad [kg/s]$$

Bilancio energetico
$$\begin{cases} E_{in} - E_{out} = \Delta E_{sist} = E_2 - E_1 & [J] \\ \dot{E}_{in} - \dot{E}_{out} = dE_{sist}/dt & [W] \end{cases}$$

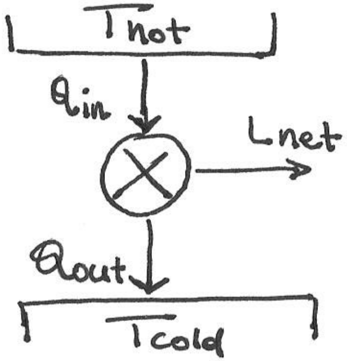
$$\Rightarrow Q - L + \Sigma m_{in}\left(h_{in} + \frac{w_{in}^2}{2} + g z_{in}\right) - \Sigma m_{out}\left(h_{out} + \frac{w_{out}^2}{2} + g z_{out}\right) = m_2 \overbrace{\left(u_2 + \frac{w_2^2}{2} + g z_2\right)}^{e_2} - m_1 \overbrace{\left(u_1 + \frac{w_1^2}{2} + g z_1\right)}^{e_1}$$

$$\dot{Q} - \dot{L} + \Sigma \dot{m}_{in}\left(h_{in} + \frac{w_{in}^2}{2} + g z_{in}\right) - \Sigma \dot{m}_{out}\left(h_{out} + \frac{w_{out}^2}{2} + g z_{out}\right) = \frac{d(e_{sist}\, m_{sist})}{dt}$$

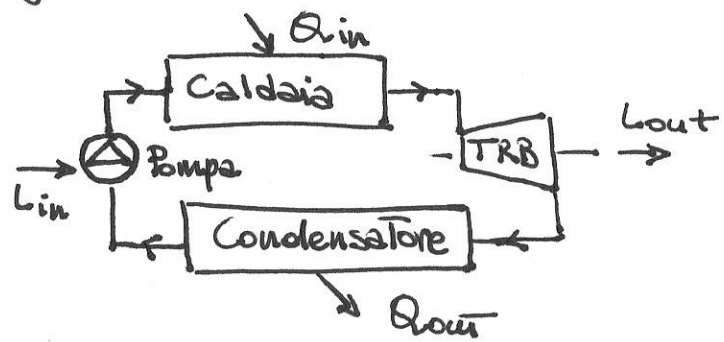
se en. cin. e pot. trascurabili
$$\dot{Q} - \dot{L} + \Sigma \dot{m}_{in} h_{in} - \Sigma \dot{m}_{out} h_{out} = \frac{d(m_{sist}\, u_{sist})}{dt} \sim \frac{m_2 u_2 - m_1 u_1}{\Delta t}$$

Secondo Principio della Termodinamica

Motori Termici



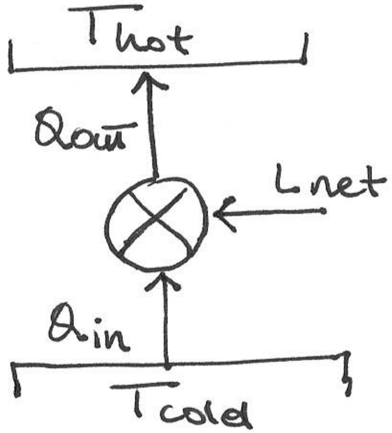
e.g. Impianto a Vapore



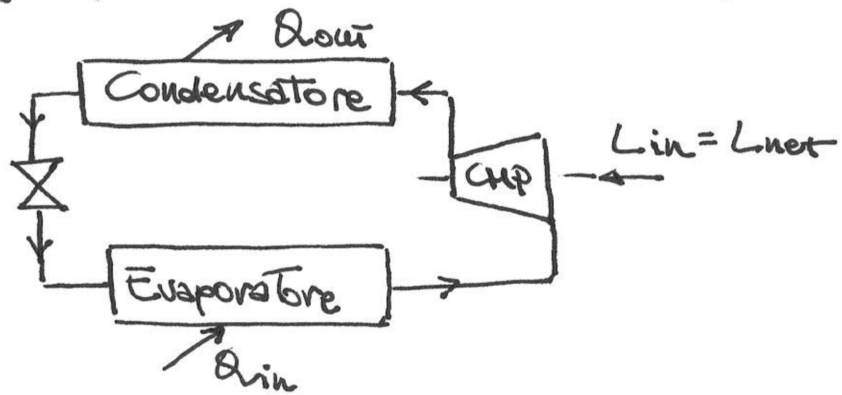
$$\Delta U = 0 \longrightarrow L_{net} = L_{out} - L_{in} = Q_{in} - Q_{out} < Q_{in}$$

$$\eta = \frac{L_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} < 1$$

Macchine Frigorifere e Pompe di Calore



e.g. Frigorifero a Compressione di Vapore



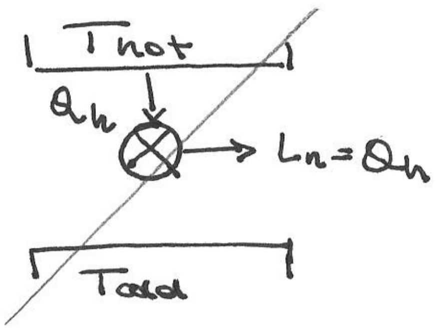
$$\Delta U = 0 \longrightarrow L_{in} + Q_{in} = Q_{out}$$

$$COP_{fr} = \frac{Q_{in}}{L_{in}} = \frac{Q_{in}}{Q_{out} - Q_{in}}$$

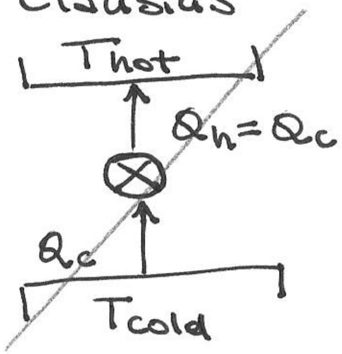
$$COP_{pdc} = \frac{Q_{out}}{L_{in}} = \frac{Q_{out}}{Q_{out} - Q_{in}} = \frac{L_{in} + Q_{in}}{L_{in}} = COP_{fr} + 1$$

Secondo Principio

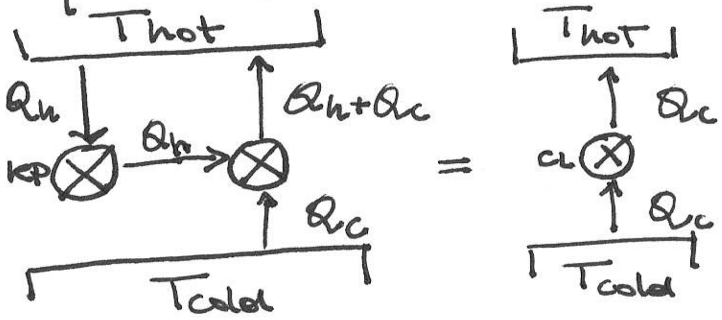
Kelvin - Planck



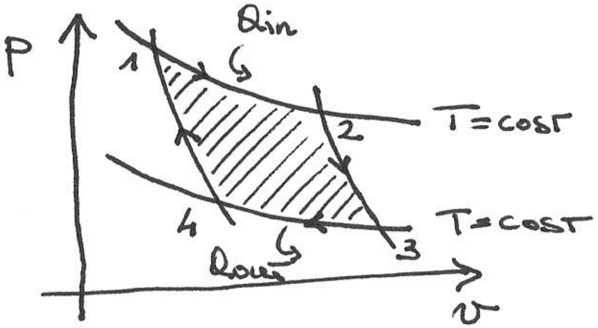
Clausius



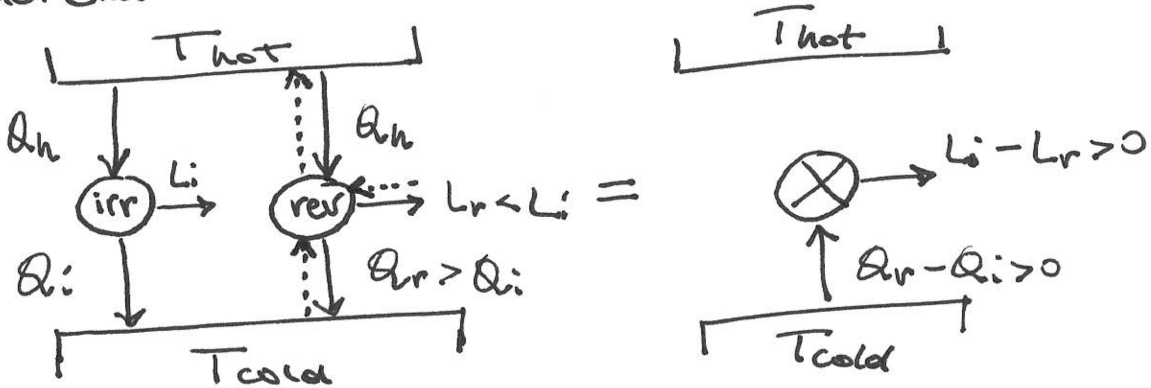
Equivalenza



Ciclo di Carnot (diretto)



Teoremi di Carnot



$$\eta_{car} = \eta_{rev} = \eta_f(T_{hot}, T_{cold}) = 1 - \left(\frac{Q_{cold}}{Q_{hot}}\right)_{rev} = 1 - \frac{T_{cold}}{T_{hot}} = \eta_{max} \left\{ \begin{array}{l} \text{cfr. kelvin} \\ \left(\frac{Q_{cold}}{Q_{hot}}\right)_{rev} = \frac{T_{cold}}{T_{hot}} \end{array} \right.$$

Ciclo di Carnot (inverso)

$$COP_{fr} = \frac{Q_{cold}}{Q_{hot} - Q_{cold}} = \frac{T_{cold}}{T_{hot} - T_{cold}} = \frac{1}{\frac{T_{hot}}{T_{cold}} - 1} = COP_{fr\,max}$$

$$COP_{pdc} = \frac{Q_{hot}}{Q_{hot} - Q_{cold}} = \frac{T_{hot}}{T_{hot} - T_{cold}} = COP_{pdc\,max}$$

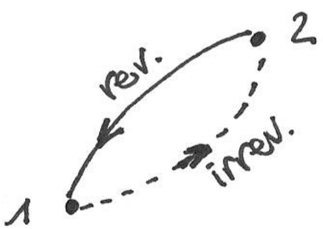
Entropia

Disuguaglianza di Clausius

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\oint \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev.}} = 0 \implies dS = \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}} \qquad \Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}} \qquad [J/K]$$

Principio di Aumento dell'Entropia



$$\oint \frac{\delta Q}{T} = \int_1^2 \frac{\delta Q}{T} + \int_2^1 \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}} = \int_1^2 \frac{\delta Q}{T} - \int_1^2 \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev.}} \leq 0 \implies$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}} \geq \int_1^2 \frac{\delta Q}{T} \implies \Delta S = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

$$S_{gen} \geq 0$$

$$S_{gen} = \Delta S_{tot} = \Delta S_{sist} + \Delta S_{amb} \geq 0$$

Relazioni del TdS

$$\delta Q_{\substack{int\\rev}} - \delta L_{\substack{int\\rev}} = dU = TdS - pdV \implies TdS = dU + pdV$$

$$dH = dU + pdV + Vdp \implies TdS = dH - Vdp$$

Entropia

Sistemi Chiusi $\quad l_v = \int_1^2 p\,dv$

Sistemi Aperti (flusso stazionario)

$$\delta q_{rev} - \delta l_{rev} = T ds - \delta l_{rev} = dh - v dp - \delta l_{rev} = dh + de_{cin} + de_{pot}$$

$$\Longrightarrow \delta l_{rev} = - v dp - \cancel{de_{cin}} - \cancel{de_{pot}} \Longrightarrow l_{rev} = - \int_1^2 v dp$$

fluido Incomprimibile

se $l = 0$

$$l_{rev} = - v (p_2 - p_1) - \Delta e_{cin} - \Delta e_{pot} \Longrightarrow v(p_2 - p_1) + \frac{w_2^2 - w_1^2}{2} + g(z_2 - z_1) = 0$$

$$\Longrightarrow p + \rho \frac{w^2}{2} + \rho g z = \text{cost.} \quad \text{(Eq. Bernoulli)}$$

Rendimento isoentropico di dispositivi a flusso stazionario $(-\delta l = dh)$

turbina $\quad \eta = \dfrac{l_{reale}}{l_{iso-s}} = \dfrac{h_1 - h_{2r}}{h_1 - h_{2s}}$

compressore $\quad \eta = \dfrac{l_{iso-s}}{l_{reale}} = \dfrac{h_1 - h_{2s}}{h_1 - h_{2r}}$

Exergia

entropia $\quad (s_2 - s_1) T_{min} = q + s_{gen} T_{min}$

sist. aperto $\quad q - l = h_2 - h_1$

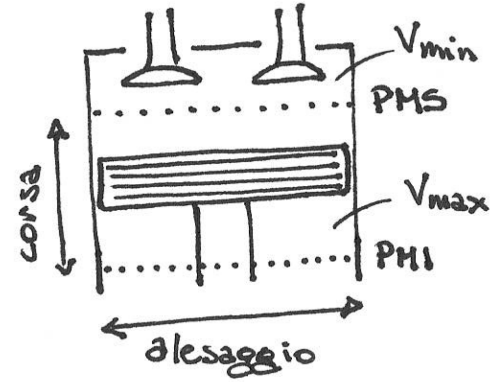
sottraggo $\quad l + s_{gen} T_{min} = (h_1 - T_{min} s_1) - (h_2 - T_{min} s_2) = \chi_1 - \chi_2$

$\eta_\chi = l / \chi_1 - \chi_2$

Energia = Exergia + Anergia

Cicli Termodinamici

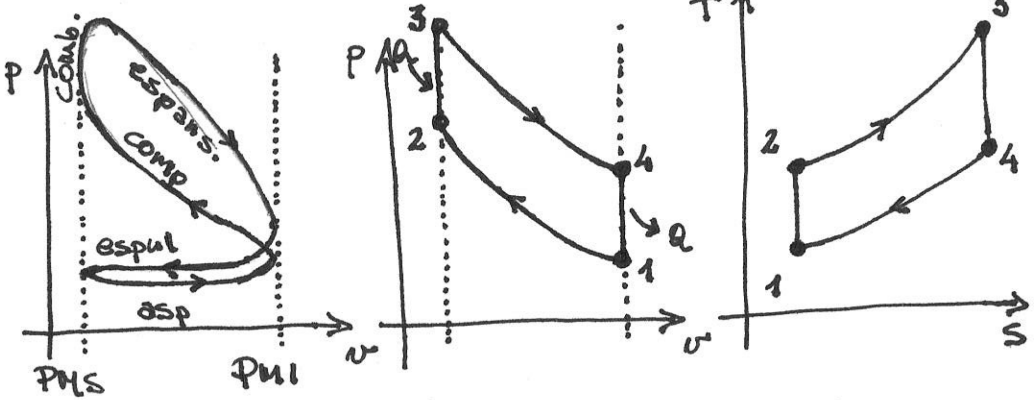
Motori alternativi



$$r = V_{max} / V_{min}$$

$$L_{net} = p_{me} (V_{max} - V_{min})$$

Ciclo Otto



ciclo chiuso $\rightarrow$ $q - \ell = \Delta u$ $\forall$ trasf.

1-2) compr. iso-s $\rightarrow$ $\ell_{12} = -c_v(T_2 - T_1) < 0$

2-3) comb. iso-v $\rightarrow$ $q_{23} = c_v(T_3 - T_2) > 0$

3-4) espans. iso-s $\rightarrow$ $\ell_{34} = -c_v(T_4 - T_3) > 0$

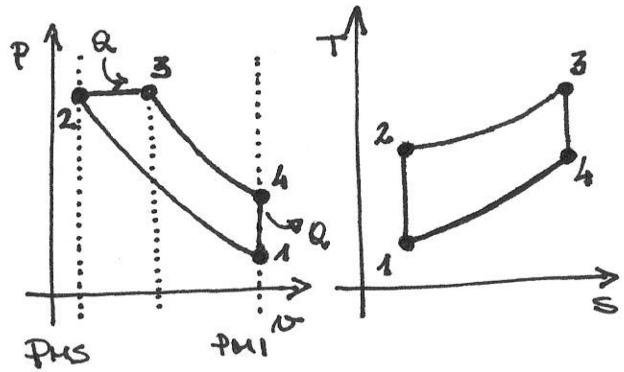
4-1) raffr. iso-v $\rightarrow$ $q_{41} = c_v(T_1 - T_4) < 0$

trasf. iso-s : $T v^{k-1} = \text{cost.}$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \frac{1}{r^{k-1}} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3}$$

$$\eta_t = \frac{\ell_{net}}{q_{in}} = 1 - \frac{|q_{out}|}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1 (T_4/T_1 - 1)}{T_2 (T_3/T_2 - 1)} = 1 - \frac{1}{r^{k-1}}$$

Ciclo Diesel



ciclo chiuso $\rightarrow$ $q - \ell = \Delta u$ $\forall$ trasf.

1-2) compr. iso-s $\rightarrow$ $\ell_{12} = -c_v(T_2 - T_1) < 0$

2-3) comb. iso-p $\rightarrow$ $\begin{cases} \ell_{23} = p(v_3 - v_2) = R(T_3 - T_2) > 0 \\ q_{23} = (c_v + R)(T_3 - T_2) = c_p(T_3 - T_2) > 0 \end{cases}$

3-4) espans. iso-s $\rightarrow$ $\ell_{34} = -c_v(T_4 - T_3) > 0$

4-1) raffr. iso-v $\rightarrow$ $q_{41} = c_v(T_1 - T_4) < 0$

$\tau = v_3 / v_2$ rapp. vol. intr.

trasf. iso-s : $T v^{k-1} = \text{cost.}$

trasf. iso-p : $T / v = \text{cost.}$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = r^{k-1} \quad ; \quad \frac{T_3}{T_2} = \frac{v_3}{v_2} = \tau$$

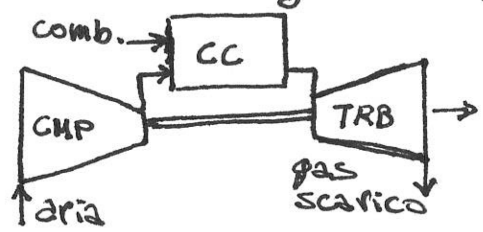
$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{k-1} = \left(\frac{v_3}{v_2}\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{\tau}{r}\right)^{k-1}$$

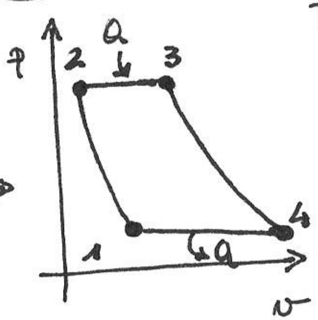
$$\frac{T_4}{T_1} = \frac{T_4}{T_3}\frac{T_3}{T_2}\frac{T_2}{T_1} = \tau^k$$

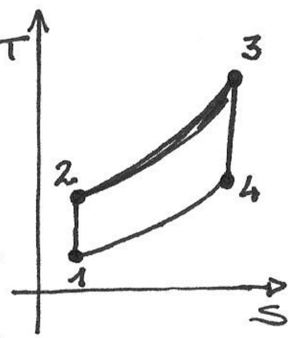
$$\eta_t = \frac{\ell_{net}}{q_{in}} = 1 - \frac{|q_{out}|}{q_{in}} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{1}{k}\frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)} = 1 - \frac{1}{r^{k-1}}\frac{\tau^k - 1}{k(\tau - 1)}$$

Cicli Termodinamici

Ciclo Brayton







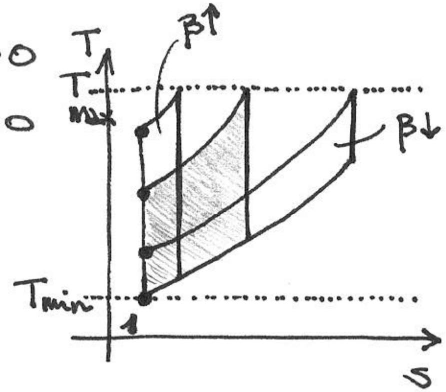
dispositivi come sistemi aperti → $q - l = \Delta h \ \forall$ trasf.

1-2) compr. iso-S: $l_{12} = -c_p(T_2 - T_1) < 0$

2-3) comb. iso-p: $q_{23} = c_p(T_3 - T_2) > 0$

3-4) espans. iso-S: $l_{34} = -c_p(T_4 - T_3) > 0$

4-1) raffr. iso-p: $q_{41} = c_p(T_1 - T_4) < 0$



$\beta = p_2/p_1$ rapp. manom. compr.

trasf. iso-S: $T\, p^{\frac{1-k}{k}} = \text{cost.}$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \beta^{\frac{k-1}{k}} = \left(\frac{p_3}{p_4}\right)^{\frac{k-1}{k}} = \frac{T_3}{T_4}$$

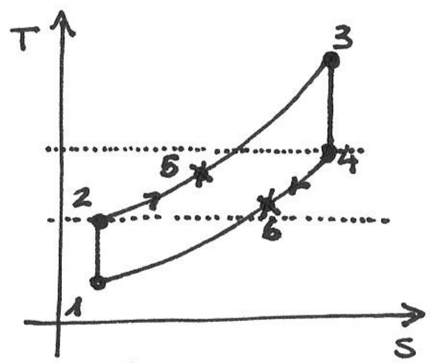
$$\eta_t = \frac{l_{net}}{q_{in}} = 1 - \frac{|q_{out}|}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)} = 1 - \frac{1}{\beta^{\frac{k-1}{k}}}$$

$$l_{net} = l_{trb} - l_{cmp} = q_{in}\,\eta_t = c_p\left(T_3 - T_1\beta^{\frac{k-1}{k}}\right)\left(1 - \frac{1}{\beta^{\frac{k-1}{k}}}\right) = c_p(T_3 - T_1 x)\left(1 - \frac{1}{x}\right)$$

date T_{min} e T_{max}, derivando, l_{max} si ha per: $-T_1 + T_3/x^2 = 0 \rightarrow x^2 = T_3/T_1 \rightarrow \beta = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$

η_{max} invece si ha per (ma a lavoro nullo) $\eta_t = \eta_{carnot} = 1 - \frac{1}{\beta^{\frac{k-1}{k}}} = 1 - \frac{T_1}{T_3} \rightarrow x = T_3/T_1 \rightarrow \beta = \left(\frac{T_3}{T_1}\right)^{\frac{k}{k-1}}$

Ciclo Brayton con Rigenerazione

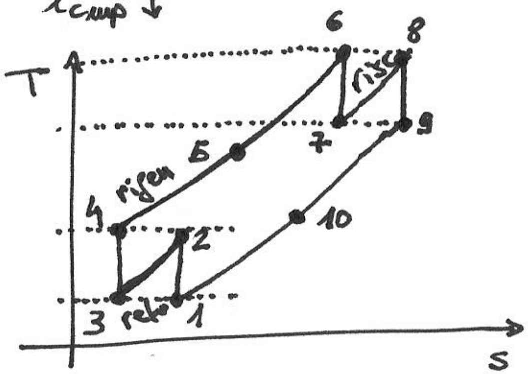


$$\varepsilon = \frac{q_{rig}}{q_{rig,max}} = \frac{h_5 - h_2}{h_4 - h_2} \cong \frac{T_5 - T_2}{T_4 - T_2}$$

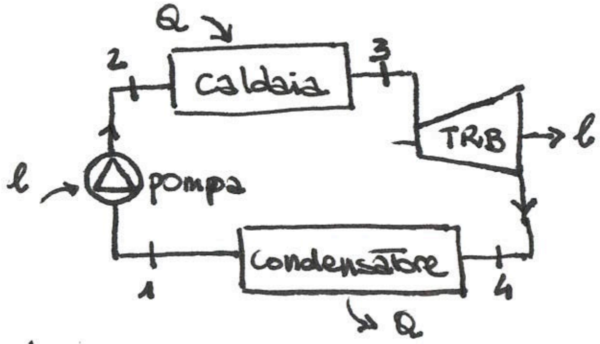
$$\eta_{trig} = 1 - \frac{T_6 - T_1}{T_3 - T_5}$$

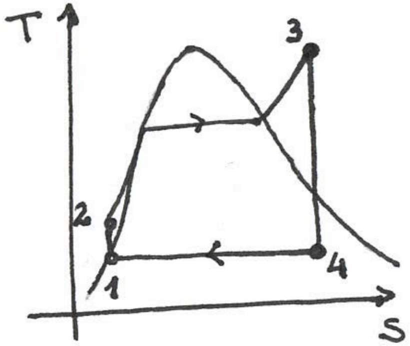
Ciclo Brayton Interrefrigerato / Interriscaldato

$l_{trb} \uparrow \quad l_{cmp} \downarrow$



Ciclo Rankine





dispositivi come sistemi aperti → $q - l = \Delta h \ \forall$ trasf.

1-2) compr. iso-s (pmp) $\quad l_{12} = -(h_2 - h_1) = -v_1(p_2 - p_1) < 0$

2-3) comb. iso-p (cld) $\quad q_{23} = h_3 - h_2 > 0$

3-4) espans. iso-s (trb) $\quad l_{34} = -(h_4 - h_3) > 0$

4-1) condens. iso-p (cnd) $\quad q_{41} = h_1 - h_4 < 0$

$$\eta_t = \frac{l_{net}}{q_{cld}} = 1 - \frac{|q_{cnd}|}{q_{cld}} = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

$$l_{net} = l_{trb} - |l_{cmp}|$$

e.g.

$$h_1 = h_{l,sat@p_1}$$

$$h_2 = h_1 + v_{l\,sat@T_1}(p_2 - p_1)$$

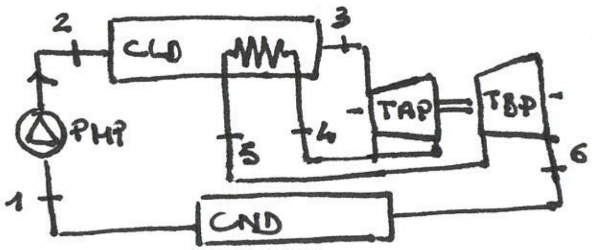
$$h_3 = h_{p_3,T_3}$$

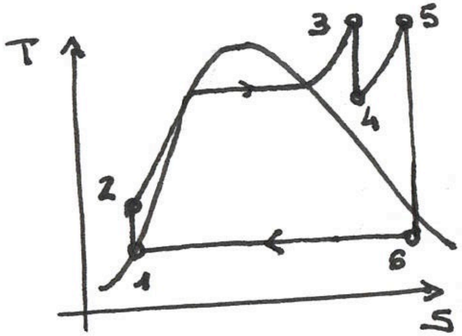
$$s_3 = s_{p_3,T_3}$$

$$s_4 = s_3 \Rightarrow x = \frac{s_4 - s_{l\,sat@p_1}}{s_{v\,sat@p_1} - s_{l\,sat@p_1}}$$

$$h_4 = h_{l\,sat@p_1} + x\,(h_{v\,sat@p_1} - h_{l\,sat@p_1})$$

Ciclo Rankine con Surriscaldamento

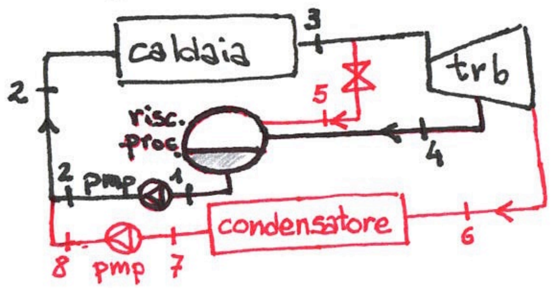




$$q_{cld} = q_{23} + q_{45} = (h_3 - h_2) + (h_5 - h_4)$$

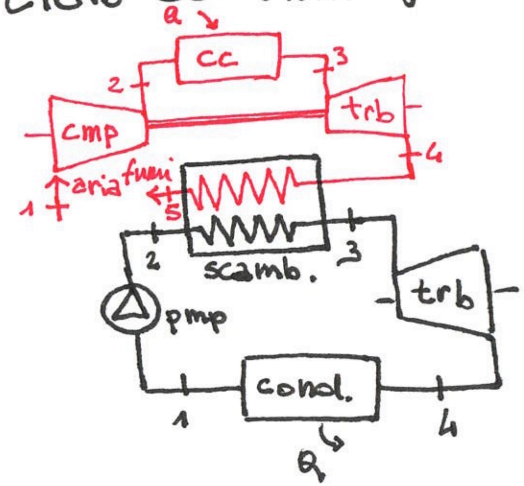
$$l_{trb} = l_{34} + l_{56} = (h_3 - h_4) + (h_5 - h_6)$$

Cogenerazione

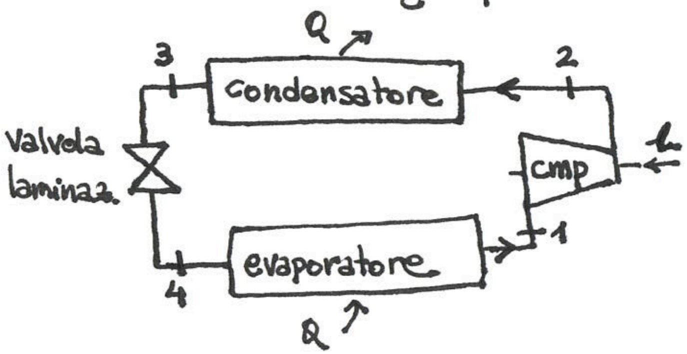


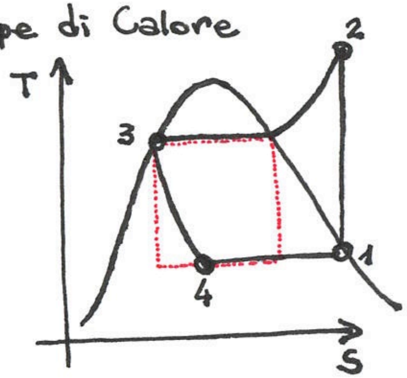
$$\varepsilon_u = \frac{L_{net} + Q_{prc}}{Q_{in}}$$

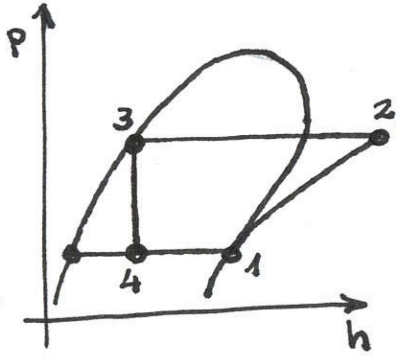
Ciclo Combinato Gas-Vapore



Macchine Frigorifere e Pompe di Calore







dispositivi come sistemi aperti $\rightarrow q - \ell = \Delta h \quad \forall$ trasf.

12 - cmp) compr. iso-s $\qquad \ell_{12} = -(h_2 - h_1) < 0$

23 - cond) condens. iso-p $\qquad q_{23} = h_3 - h_2 < 0$

34 - lam.) laminaz. adiab. $\qquad h_3 = h_4$

41 - evap.) evaporaz. iso-p $\qquad q_{41} = h_1 - h_4 > 0$

$$COP_{fr} = \frac{q_{in}}{|\ell_{in}|} = \frac{h_1 - h_4}{h_2 - h_1} \qquad COP_{pdc} = \frac{|q_{out}|}{|\ell_{in}|} = \frac{h_2 - h_3}{h_2 - h_1} = COP_{fr} + 1$$

e.g. $h_1 = h_{v\,sat\,@p_1}$

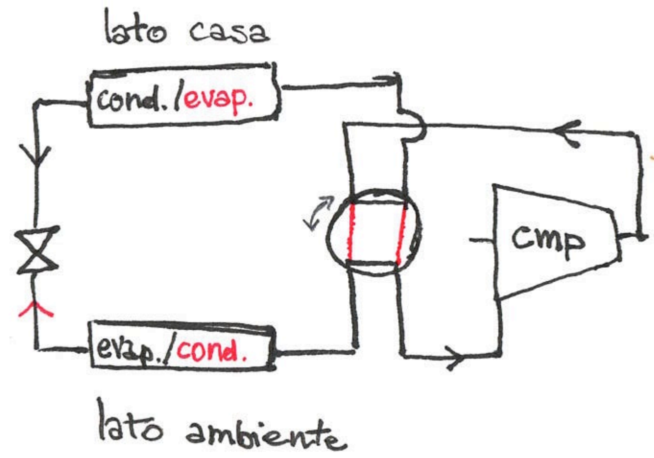
$s_1 = s_{v\,sat\,@p_1} = s_2$

$h_2 = h_{@p_2, s_2}$

$h_3 = h_{l\,sat\,@p_2} = h_4$

$$x_4 = \frac{h_4 - h_{l\,sat\,@p_1}}{h_{v\,sat\,@p_1} - h_{l\,sat\,@p_1}}$$

Pompa di calore con valvola di inversione



— modalità riscaldamento

— modalità raffrescamento

Miscele di gas (ideali)

massa $m_m = \Sigma_i m_i$; fraz. ponderale $mf_i = \dfrac{m_i}{m_m}$

moli $N_m = \Sigma_i N_i$; fraz. molare $y_i = \dfrac{N_i}{N_m}$

$m = N M$ [g]

massa molare app. $M_m = \dfrac{m_m}{N_m} = \Sigma_i y_i M_i$ [g/mol]

costante app. $R_m = \dfrac{R_u}{M_m}$ [J/kg K]

legge di Dalton $P_m = \Sigma_i p_i (T_m, V_m)$

legge di Amagat-Leduc $V_m = \Sigma_i V_i (T_m, p_m)$

$\Longrightarrow \dfrac{p_i}{p_m} = y_i$; $\dfrac{V_i}{V_m} = y_i$

Proprietà estensive : e.g. $\Delta U_m = \Sigma_i \Delta U_i = \Sigma_i m_i \Delta u_i = \Sigma_i N_i \Delta \bar{u}_i$
(somma contributi)

Proprietà intensive : e.g. $\Delta u_m = \dfrac{\Delta U_m}{m_m} = \Sigma_i mf_i \Delta u_i$; $\Delta \bar{u}_m = \dfrac{\Delta U}{N_m} = \Sigma_i y_i \Delta \bar{u}_i$
(media pesata)

$c_{v,m} = \Sigma_i mf_i c_{v,i}$ $\qquad \bar{c}_{v,m} = \Sigma_i y_i \bar{c}_{v,i}$

Psicrometria

miscela $P = P_a + P_{vap}$

aria secca $h_a = c_p T = 1{,}005 \cdot T \quad [kJ/kg]$

vapor d'acqua $h_{vap} \cong h_{v,sat}(T) = h_{v,sat}(0°C) + c_p T = 2501{,}3 + 1{,}82\,T \quad [kJ/kg]$

umidita' assoluta $$x = \frac{m_{vap}}{m_a} = \frac{P_{vap} V / R_{vap} T}{P_a V / R_a T} = \frac{R_a}{R_{vap}} \frac{P_{vap}}{P_a} = \frac{0{,}622\, P_{vap}}{P - P_{vap}} \quad \left[\frac{kg\ vap}{kg\ a}\right]$$

umidita' relativa $$\varphi = \frac{m_{vap}}{m_{v,sat}} = \frac{P_{vap} V / R_{vap} T}{P_{vsat} V / R_{vap} T} = \frac{P_{vap}}{P_{v,sat}}$$

$$\Longrightarrow \quad \varphi = \frac{x P}{(0{,}622 + x) P_{v,sat}} \quad ; \quad x = \frac{0{,}622\, \varphi\, P_{v,sat}}{P - \varphi P_{v,sat}}$$

entalpia $$H = H_a + H_{vap} = m_a h_a + m_{vap} h_{vap} \quad [kJ]$$

$$h = \frac{H}{m_a} = h_a + x\, h_{vap} = 1.005\,T + x\,(2501.3 + 1.82\,T) \quad [kJ/kg_a]$$

saturazione adiabatica

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$$

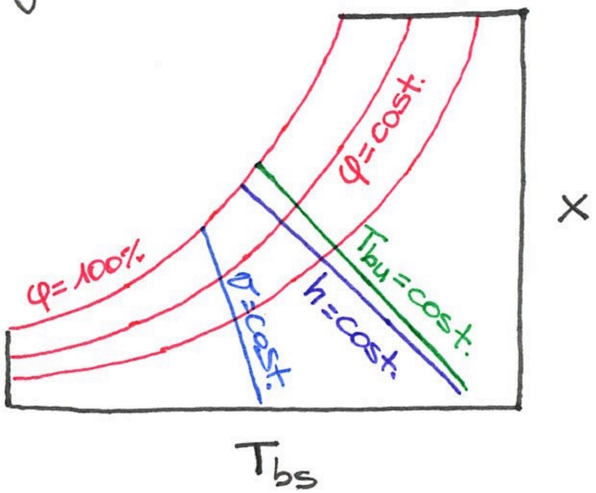
$$\dot{m}_{vap1} + \dot{m}_{liq} = \dot{m}_{vap2} \Longrightarrow \dot{m}_{liq} = \dot{m}_a (x_2 - x_1)$$

$$\dot{m}_a h_1 + \dot{m}_{liq} h_{liq} = \dot{m}_a h_2 \Longrightarrow h_1 + (x_2 - x_1) h_{liq} = h_2$$

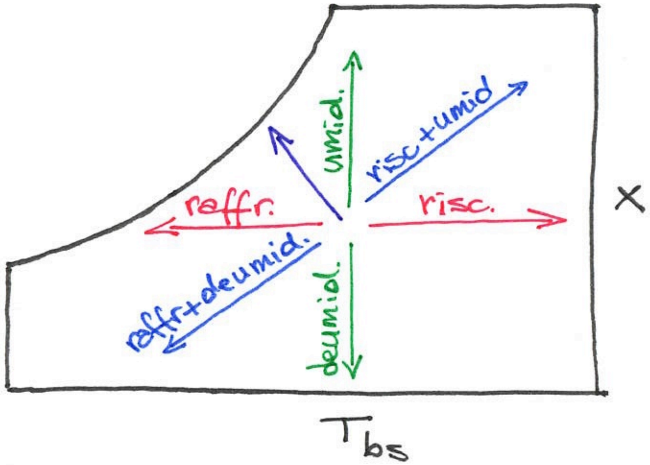
$$\Longrightarrow \quad x_1 = \frac{h_{a2} - h_{a1} + x_2 (h_{vap2} - h_{liq})}{h_{vap1} - h_{liq}}$$

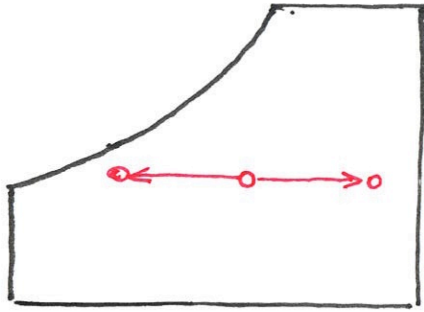
Psicrometria

Diagramma Psicrometrico

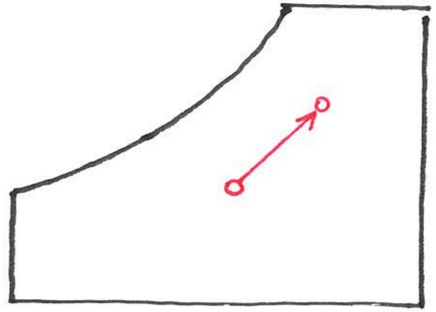


Trasformazioni

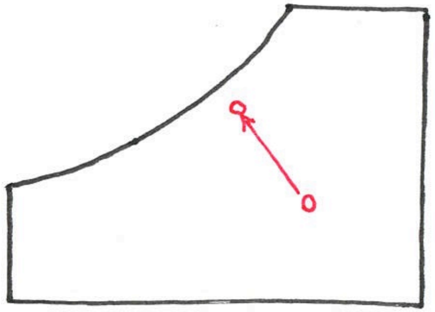




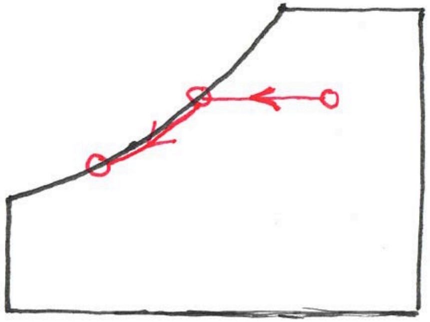
riscaldamento
raffreddamento
semplice



riscaldamento
+umidificazione
(vapore)



raffreddamento
per evaporazione



raffreddamento
con
deumidificazione

Bilanci nelle Trasformazioni

aria $\sum \dot{m}_{a,in} = \sum m_{a,out}$

vapore $\sum \dot{m}_{a,in} x_{in} = \sum \dot{m}_{a,out} x_{out}$

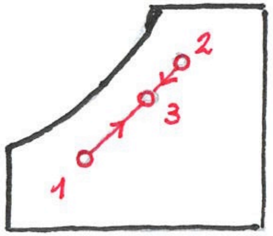
energia $\dot{Q} - \cancel{\dot{L}} + \sum \dot{m}_{a,in} h_{in} - \sum \dot{m}_{a,out} h_{out} = 0$

Miscelazione adiabatica di 2 correnti:

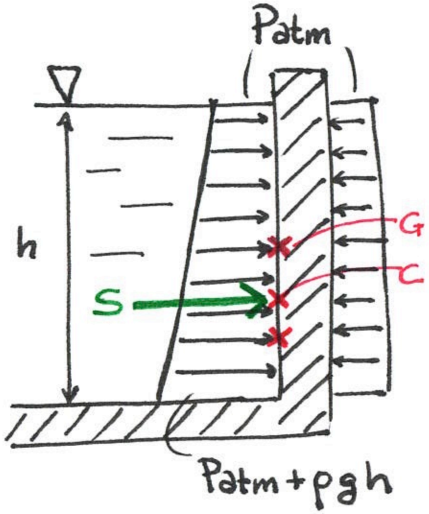
$$\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$$

$$\dot{m}_{a1} x_1 + \dot{m}_{a2} x_2 = \dot{m}_{a3} x_3 \Rightarrow \frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{x_2 - x_3}{x_3 - x_1} = \frac{h_2 - h_3}{x_3 - x_1}$$

$$\dot{m}_{a1} h_1 + \dot{m}_{a2} h_2 = \dot{m}_{a3} h_3$$



Statica dei fluidi



spinta sulla parete

$$S = p_G A = \rho g z_G A$$

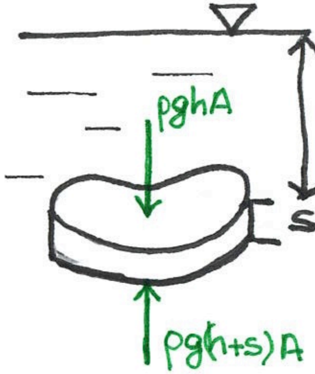
centro di spinta

$$z_c = z_G + \frac{I_G}{M}$$

Principio di Archimede

Spinta di galleggiamento

$$S_g = \rho g s A = \rho g V$$



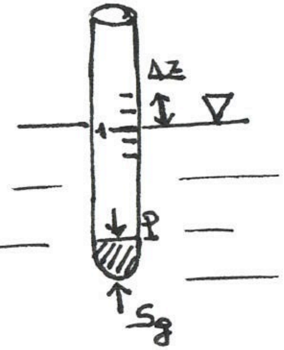
e.g. galleggiante

$$\rho g V_i = \rho_c g V_c = \frac{V_i}{V_c} = \frac{\rho_c}{\rho}$$

e.g. densimetro

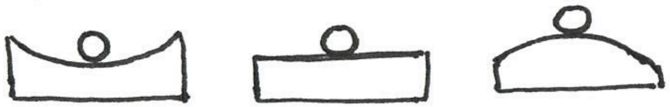
$$P = \rho g A z_0 = \rho_{liq} g A (z_0 + \Delta z)$$

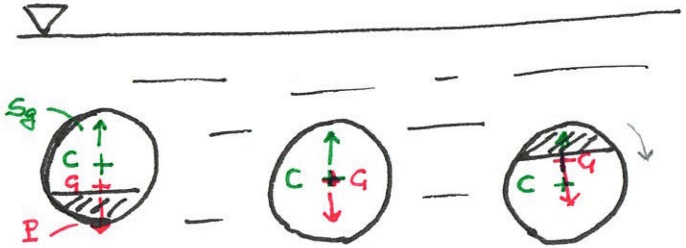
$$\frac{\rho_{liq}}{\rho} = \frac{z_0}{z_0 + \Delta z}$$

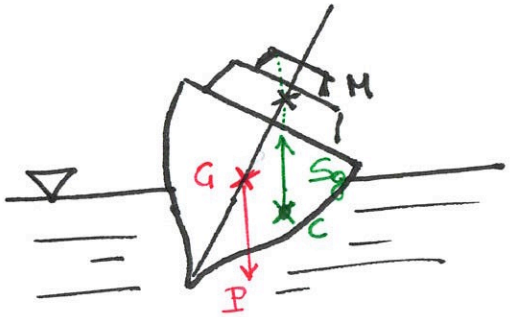


Equilibrio corpi immersi o galleggianti

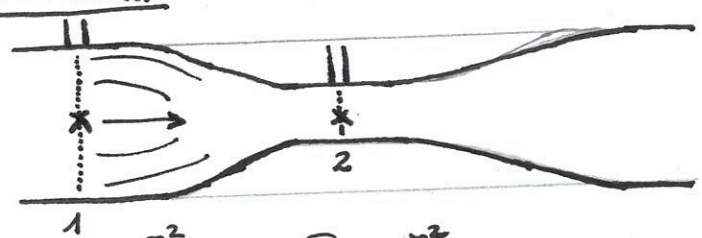
eq. stabile indifferente instabile







Bernoulli Tubo di Venturi



$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad ; \; Q_v = v A = v \frac{\pi D^2}{4}$$

$$\left(z_1 + \frac{p_1}{\rho g}\right) - \left(z_2 + \frac{p_2}{\rho g}\right) = \delta \; ;$$

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \delta = \frac{Q_v^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) \Rightarrow Q_v = \frac{A_1 \sqrt{2g\delta}}{\sqrt{D_1^4/D_2^4 - 1}}$$

Bernoulli

portata in massa $Q_m = \int_A \rho v_n \, dA$

portata in volume $Q_v = \int_A v_n \, dA$

eq. continuità $Q_{m,in} - Q_{m,out} = dm/dt$

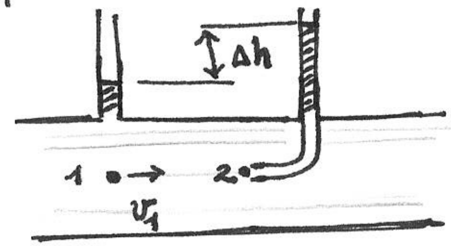
moto permanente $Q_{m,in} = Q_{m,out}$

energia meccanica $e_m = gz + \frac{p}{\rho} + \frac{v^2}{2}$ $\quad [J/kg] = [m^2/s^2]$

carico $H = z + \frac{p}{\rho g} + \frac{v^2}{2g}$ $\quad [m]$

teor. Bernoulli $H = z + \frac{p}{\rho g} + \frac{v^2}{2g} = \text{cost.}$

pressione statica e dinamica

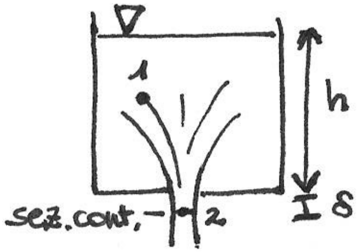


$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\rho g}$$

$$h_1 + \frac{v_1^2}{2g} = h_2 \Rightarrow \Delta h = \frac{v_1^2}{2g}$$

$$v_1 = \sqrt{2g\Delta h}$$

Processi di efflusso



$$z_1 + \frac{p_1}{\rho g} = z_2 + \frac{v_2^2}{2g}$$

$$z_1 + \frac{p_1}{\rho g} - z_2 = h + \delta$$

$$v_2 = \sqrt{2g(h+\delta)} \cong \sqrt{2gh}$$

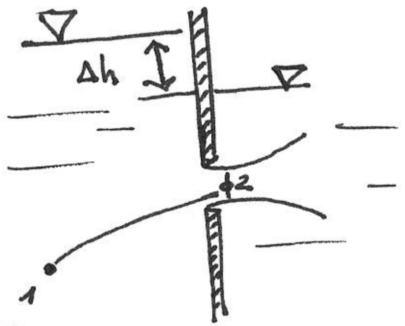
$$Q_v = CA\sqrt{2gh}$$

$$\frac{v_c}{v_t} = C_v$$

$$\frac{A_c}{A} = C_c$$

$$C = C_v C_c$$

coeff. efflusso

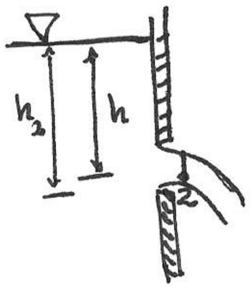


$$z_1 + \frac{p_1}{\rho g} = z_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\left(z_1 + \frac{p_1}{\rho g}\right) - \left(z_2 + \frac{p_2}{\rho g}\right) = \Delta h$$

$$v_2 = \sqrt{2g\Delta h}$$

$$Q_v = CA\sqrt{2g\Delta h}$$

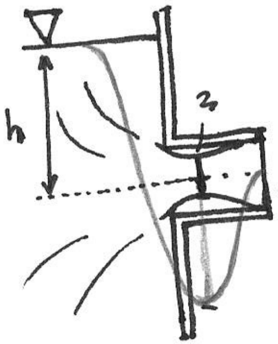


$$z_1 + \frac{p_1}{\rho g} = z_2 + \frac{v_2^2}{2g}$$

$$z_1 + \frac{p_1}{\rho g} - z_2 = h_2$$

$$v_2 = \sqrt{2gh_2}$$

$$Q_v = CA\sqrt{2gh}$$



$$z_1 + \frac{p_1}{\rho g} = z_2 - \frac{3}{4}h + \frac{v_2^2}{2g}$$

$$z_1 + \frac{p_1}{\rho g} - z_2 = h$$

$$v_2 = \sqrt{2g\frac{7}{4}h}$$

$$Q_v = CA\sqrt{2gh}\sqrt{\frac{7}{4}} \cong 0{,}8A\sqrt{2gh}$$

Viscosità

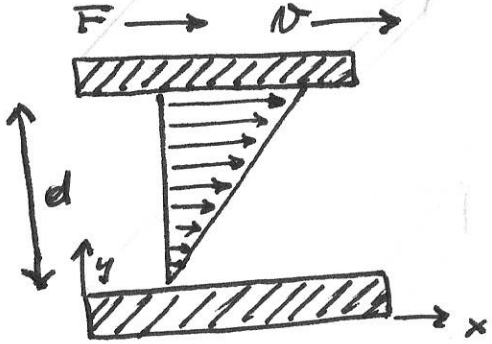
$$\tau = F/A$$

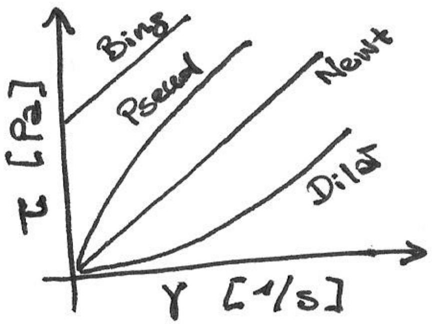
$$v_x(y) = \frac{y\,v}{d}$$

$$\frac{dv_x}{dy} = \frac{v}{d} = \gamma \propto \tau$$

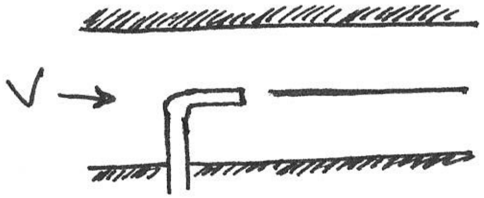
$$\tau = \mu \frac{dv_x}{dy}$$

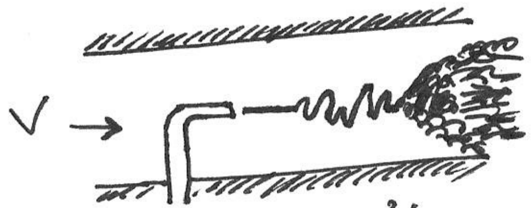
$$\nu = \mu/\rho$$





Numero di Reynolds

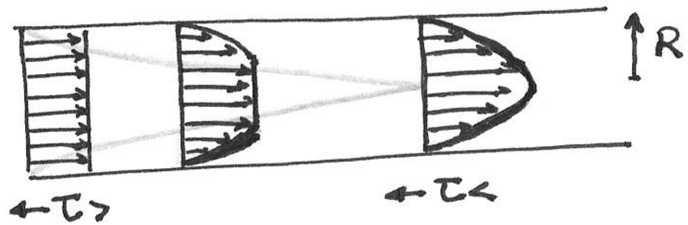




$$Re = \frac{f.\,inerz}{f.\,visc.} = \frac{\rho V^2/L}{\mu V/L^2} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

$$D_h = 4A/P \qquad Re_{cr} = 2300$$

Regione di Ingresso (Strato limite)



Moto Laminare

$$v(r) = \frac{R^2}{4\mu}\frac{\Delta p}{L}\left(1-\frac{r^2}{R^2}\right) = 2V\left(1-\frac{r^2}{R^2}\right)$$

$$V = \frac{R^2}{8\mu}\frac{\Delta p}{L} \Longrightarrow \Delta p = \frac{32\mu L V}{D^2}$$

$$Q_v = VA = \frac{\pi \Delta p D^4}{128 \mu L}$$

Darcy-Weisbach : $\frac{\Delta p}{L} \propto \frac{en.cin.}{D}$

$$\Delta p = \lambda \frac{L}{D}\frac{\rho V^2}{2} \Longrightarrow \lambda = \frac{2D}{\rho V^2}\frac{\Delta p}{L} = \dots = \frac{64}{Re}$$

$$\Delta H = \lambda \frac{L}{D}\frac{V^2}{2g} = R_{dist}$$

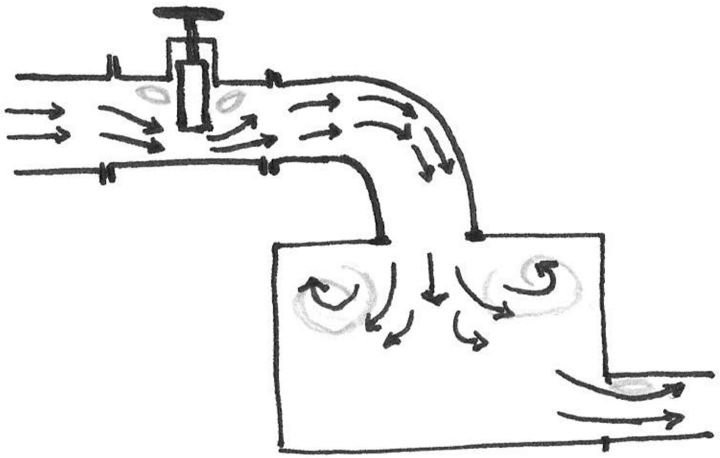
$$P_p = Q_v \Delta p / \eta_p = \frac{\rho g Q_v \Delta H}{\eta_p}$$

Moto Turbolento

Colebrook $\frac{1}{\sqrt{\lambda}} = -2 \log_{10}\left(\frac{2{,}51}{Re\sqrt{\lambda}} + \frac{\varepsilon/D}{3{,}71}\right)$

Blasius $\lambda = 0{,}316 / Re^{0{,}25}$

Perdite di carico



distribuite

$$R_{dist} = \lambda \frac{L}{D}\frac{V^2}{2g} = \Delta H \quad [m] \quad ; \quad \Delta p = \lambda \frac{L}{D}\frac{\rho V^2}{2} \quad [Pa]$$

concentrate

$$R_{conc} = \beta \frac{V^2}{2g} = \Delta H \quad [m] \quad ; \quad \Delta p = \beta \frac{\rho V^2}{2} \quad [Pa]$$

e.g.

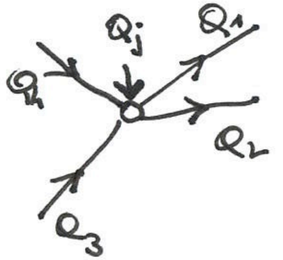
brusco allargamento sez.	$\left(1 - \frac{d^2}{D^2}\right)^2$
brusco restringimento sez.	$\frac{1}{2}\left(1 - \frac{d^2}{D^2}\right)^2$
da condotto a serbatoio	1
da serbatoio a condotto	1/2
... con sifone	1,16
con raccordi	$\beta << $
curva 90°	0.3
curve 180°	0.4

Bernoulli

$$H_1 = z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + R = H_2 + R$$

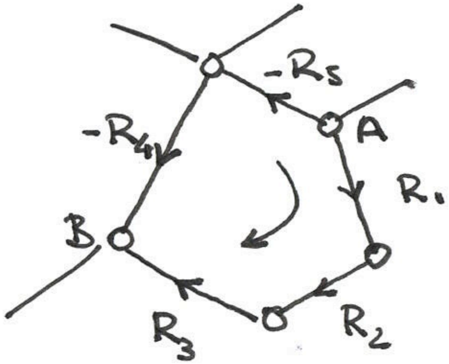
Reti Idrauliche

$\forall$ nodo



$$\Sigma Q_i + Q_j = 0$$

$\forall$ maglia

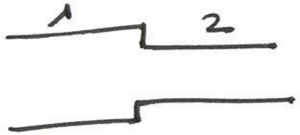


$$\Sigma R_i = 0$$

Serie

$$Q_1 = \dots = Q_n$$

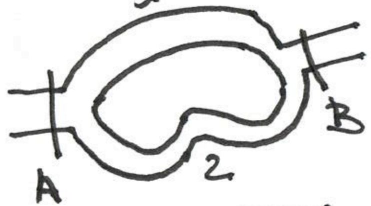
$$\Delta H = \Sigma R_i$$



parallelo

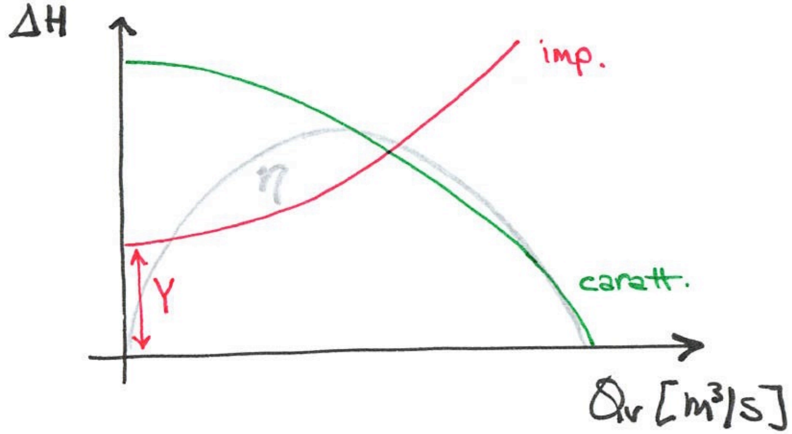
$$Q = \Sigma Q_i$$

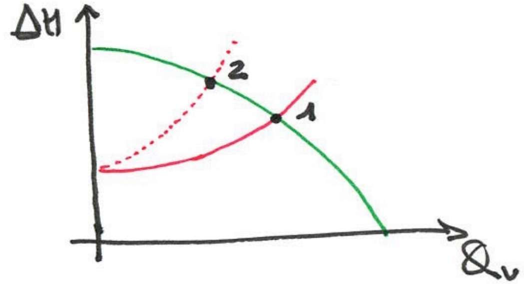
$$R_1 = \dots = R_n$$

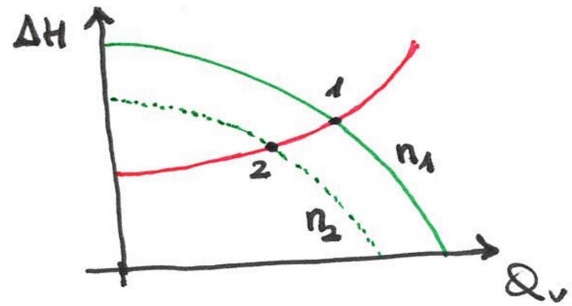


$$\frac{Q_{vi}}{Q_{vj}} = \sqrt{\frac{\lambda_j L_j D_i^5}{\lambda_i L_i D_j^5}}$$

Condizioni di Funzionamento







Eq. forma generale

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \Delta H_p = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \Delta H_t + R$$