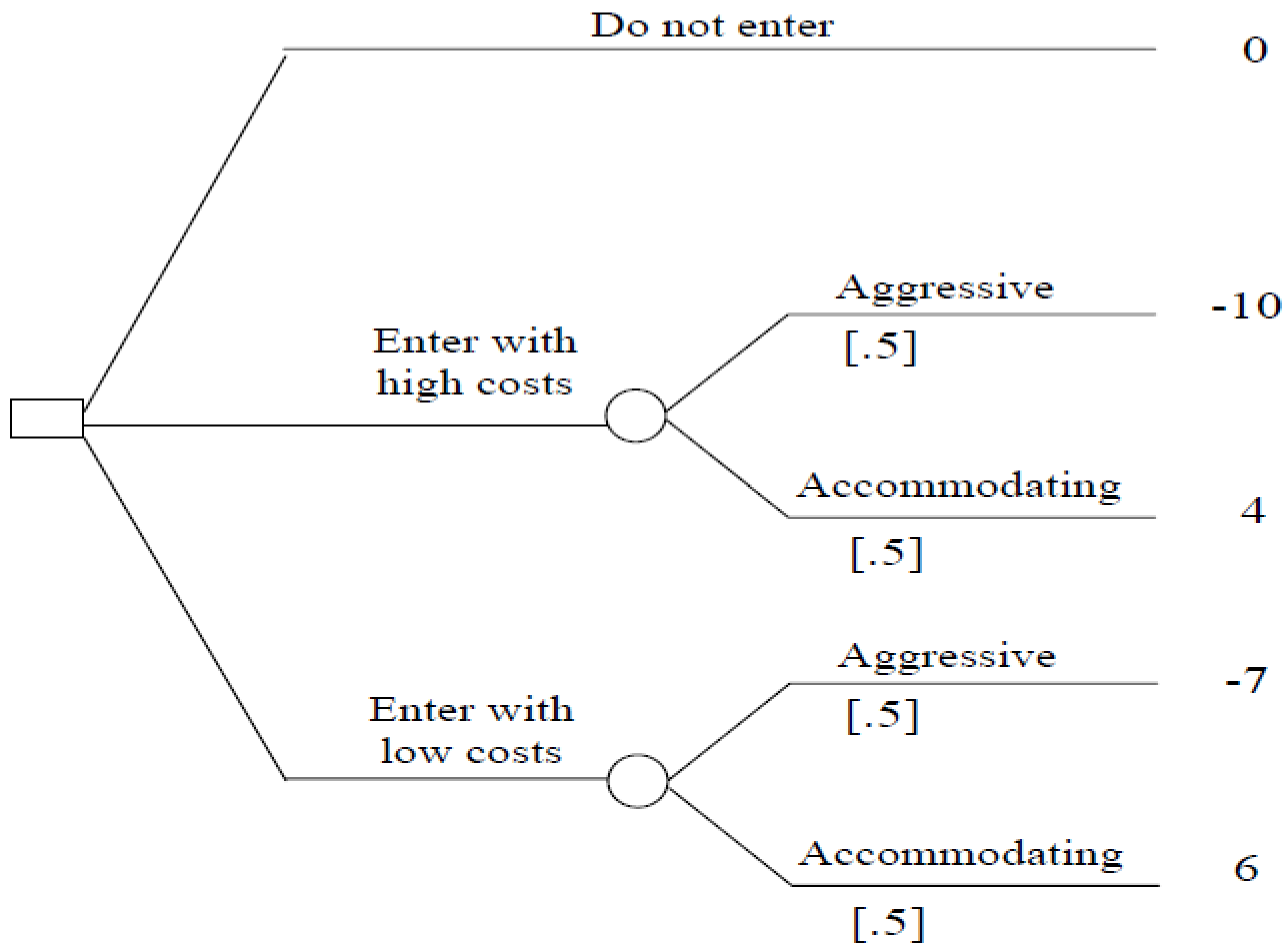
Game theory for business strategy

GT for Business strategy

- Given that each firm is part of a complex web of interactions, any business decision or action taken by a firm impacts multiple entities that interact with or within that firm, and vice versa.
- Ignoring these interactions could lead to unexpected and potentially very undesirable outcomes.

Decision theory

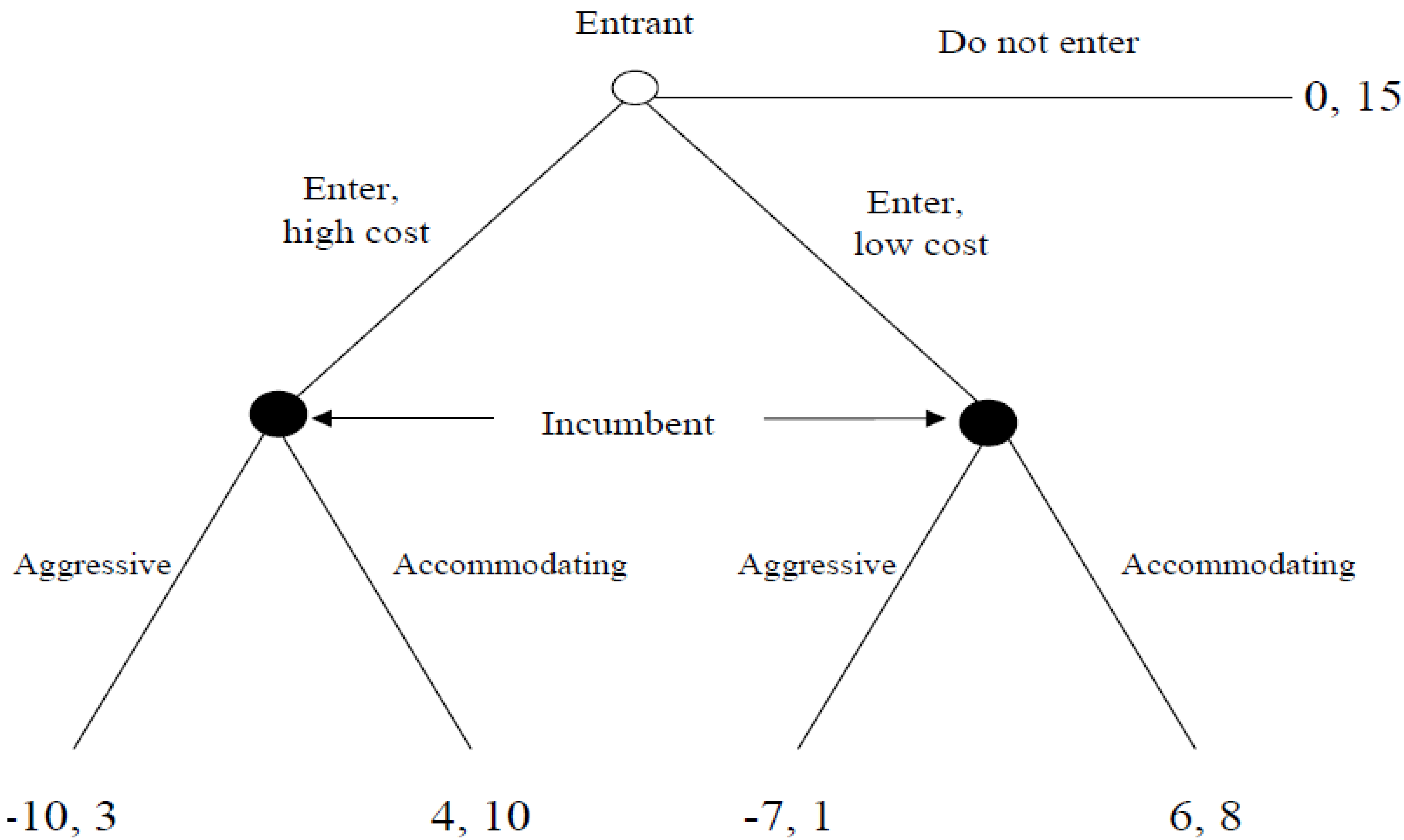


-10*0,5+4*0,5=-3 < 0

-7*0,5+6*0,5=-0,5 <0

Better not enter

Game Theory



Entrant
Do not enter
0, 15
Enter,
high cost
Enter,
low cost
Incumbent
Aggressive
Accommodating
Aggressive
Accommodating
-10, 3
4, 10
-7, 1
6, 8

Game theory

- ... a collection of tools for predicting outcomes of a group of interacting agents where an action of a single agent directly affects the payoff of other participating agents.
- ... the study of multiperson decision problems. (Gibbons)
- ... a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact. (Osborne and Rubinstein)
- ... the study of mathematical models of conflict and cooperation between intelligent rational (self interested) decision-makers. (Myerson)

The Game

1. The players who are involved.
2. The rules of the game that specify the sequence of moves as well as the possible actions and information available to each player whenever they move. (strategies)
3. The outcome of the game for each possible set of actions.
4. The (expected) payoffs based on the outcome.

Different games

- Non cooperative
- Cooperative
- Game with complete information
- Game with incomplete information (auction/ sealed bid – you don't know how valuable is a good for other bidders)
- Game with perfect information (chess - bargaining)
- Game with imperfect information
- Zero (costant) sum game (divide a pie)
- Non zero sum game
- Static game
- Dynamic game

Nash equlibrium

Definition 6 *A* **Nash Equilibrium (NE)** *is a profile of strategies* (s_i^*, s_{-i}^*) *such that each player's strategy is an optimal response to the other players' strategies:*

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*)$$

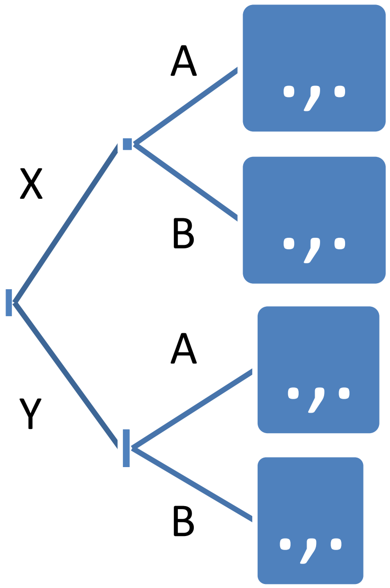
Static game – complete information (prisoner's dilemma)

		Prisoner 2	
		Cooperate (C)	Defect (D)
Prisoner 1	C	-1, -1	-9, 0
	D	0, -9	-6, -6

Normal form

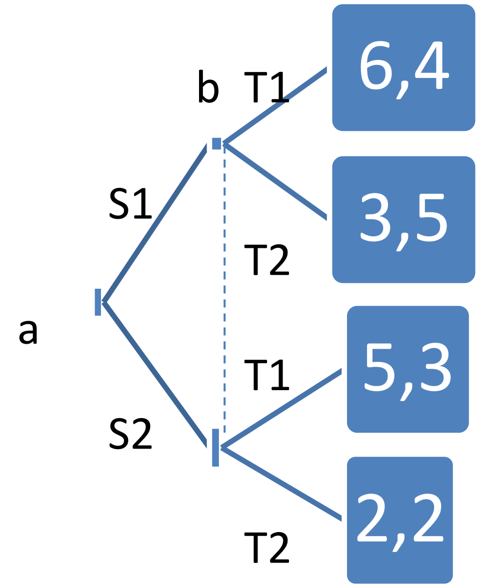
	A	B
X	.,.	.,.
Y	.,.	.,.

Game tree



Static game – complete and imperfect information

a\b	T1	T2
S1	6,4	3,5
S2	5,3	2,2



Equilibrium?

S1T1 NO given a S1 b – T2

S1T2 NE given b T2 a – S1 , given a S1 b – T2

S2T1 NO given b T1 a – S1

S2T2 NO given a S2 b – T1

Dominated stategies

a\b	T1	T2	T3
S1	3,4	0,4	4,-2
S2	4,2	1,1	-1,1

Step 1 a doesn't have dominated strategies
Step 2 B – T3 is dominated (T1 always better)
Step 3 Without T3 for a S1 is a dominated strategy
NE S2T1

Battle of sexs: going to the Opera or to a soccer game?

M\F	S	O
S	5,4	1,1
O	0,0	4,5

We have 2 NE – we need another criterium to decide

Mix strategies

a\b	L	R
A	0,0	0,-1
B	1,0	-1,3

No NE in pure strategies

a\b	(β) L	(1-β) R
(α) A	0,0	0,-1
(1-α)B	1,0	-1,3

NE in Mixed strategies

$E\pi a(\alpha^*,\beta^*) \geq E\pi a(\alpha,\beta^*)$

$E\pi b(\alpha^*,\beta^*) \geq E\pi a(\alpha^*,\beta)$

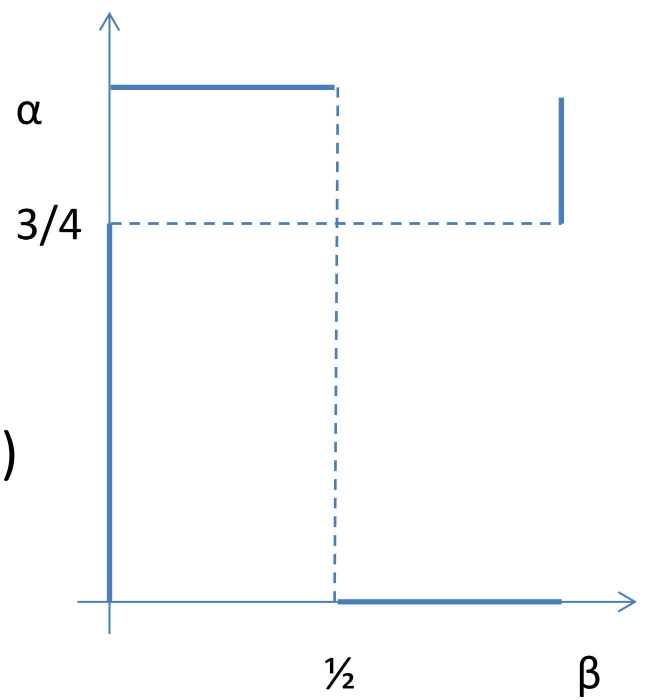
solution

a – supposes b plays L prob. β and R prob (1- β)

Eπa(A) = 0 x β + 0 (1- β) = 0
Eπa(B) = 1 x β + (-1) (1- β) = 2β -1
When is a indifferent?
Eπa(A)= Eπa(B) => 0=2β -1 => β=1/2
If β>1/2 a plays B if β<1/2 a plays A

The same for B
b – supposes a plays A prob. A and B prob. (1- α)

Eπb(L) = 0 x α + 0 (1- α) = 0
Eπb(R) = (-1) x α + 3(1- α) = 3-4 α
When is a indifferent?
Eπb(L)= Eπb(R) => 0= 3-4 α => α=3/4
If α >3/4 b plays L if α <3/4 a plays R



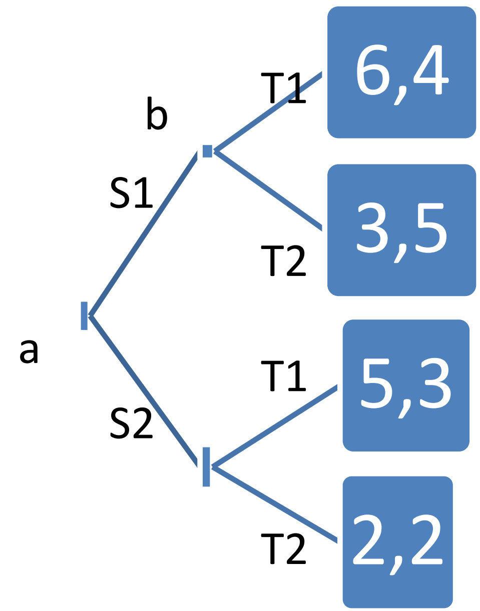
NE in mix strategies
a => (A prob ¾, B prob ¼)
B=> (L prob ½, R prob ½)

Dynamic game with complete and perfect information

a first move

b has 4 strategies

a\b	T1T1	T1T2	T2T1	T2T2
S1	6,4	6,4	3,5	3,5
S2	5,3	2,2	5,3	2,2



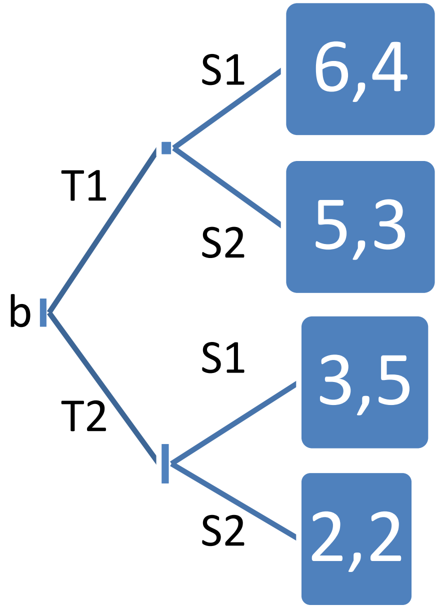
(S1, T1T2) NE no sub game perfect

(S2, T2T1) NE SGP

If B first

a\b	T1	T2
S1,S1	6,4	3,5
S1,S2	6,4	2,2
S2,S1	5,3	3,5
S2,S2	5,3	2,2

NE
(T2, S1S1) SGP
(T1, S1S2) no SGP
(T2, S2S1) no SGP



Game with incomplete information (static)

A new CEO has been hired. He can be good or bad

There is CFO close to retirement and is tired, he prefers not to work hard

But if CEO detect him he doesn't get the annual bonus

CEO good meand higher profits and lower cost effort to control CFO

CEO = A CFO = B

I have 2 games

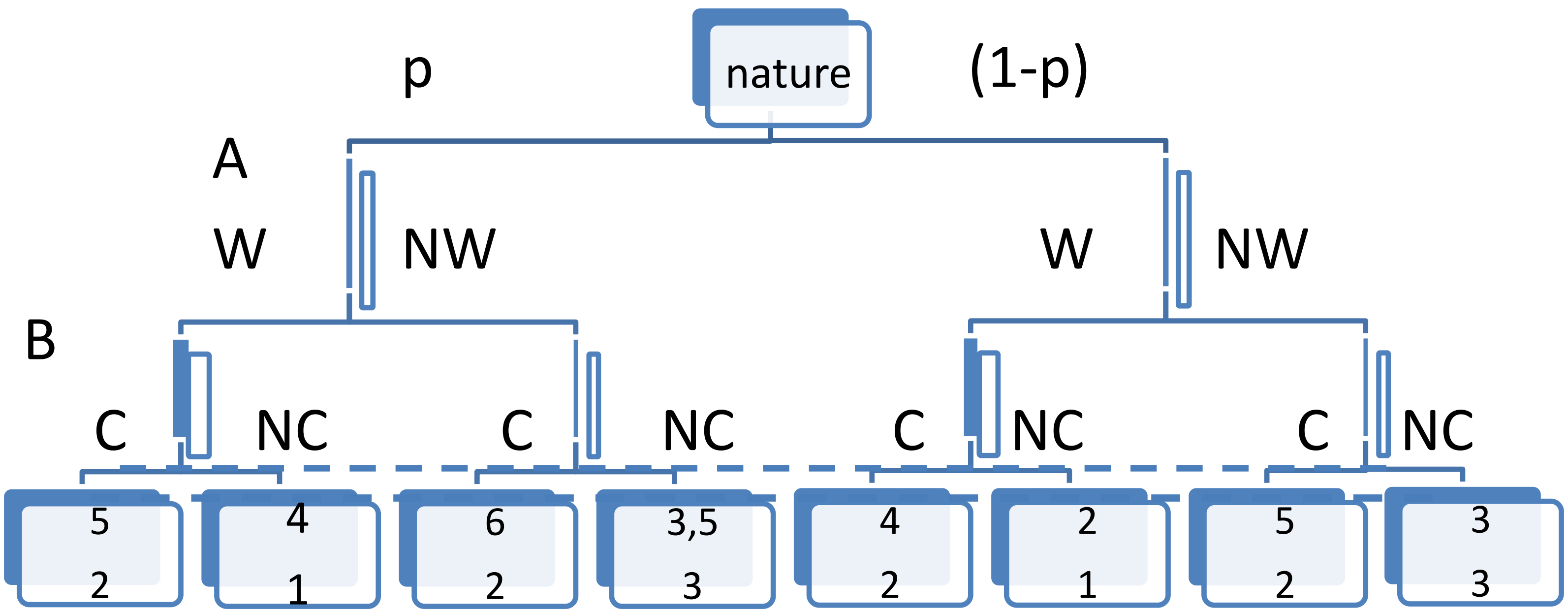
Good

	W	NW
C	5;2	4;1
NC	6;2	3,5;3

Bad

	W	NW
C	4,2	2,1
NC	5,2	3,3

- How can I find an Equlibrium?
- I change the Game in one with complete but imperfect information



E payoff B (W,p) = p(2) + (1-p)2=2

E payoff B(NW,p) =(1-p)3 + p(1)= 3-2p

2=3-2p => p=1\2 if p<1/2 low probability to be detected =>[NW,(C,NC)] B doesn’t work and A control if is Good

if P>1/2 [W,(NC,NC)] B work and A doesn’t control (Bayesian NE)

Repeated Prisoner's dilemma

a\b	Lb	Hb
La	10;10	1;11
Ha	11;1	3;3

NE (Ha;Hb)
Max profit (La,Lb)
How to increase profit?
Change game

a\b	Lb	Hb	R
La	10;10	1;11	0,0
Ha	11;1	3;3	1,0
R	0,0	0,1	0,0

T is a dominated strategy
Play 2 times
First NE (M both games)
Other: first period play P and second M if you in the first P otherwise T)
Pay off if no deviations (10+3) (10+3)
If deviation (12+1) 10(1+1) non convinient
But not SGP, not credible

Other solutions?

- To play the game N times? NO. In the last period someone will deviate
- To play infinitely? Not T is credible

a 10,10,10,10,10,….1,0,0,0,0,….

b 10,10,10,10,10,….11,1,1,1,1,…..

We need to compare profit today and tomorrow.

Caveat: it is not renegotiation proof.